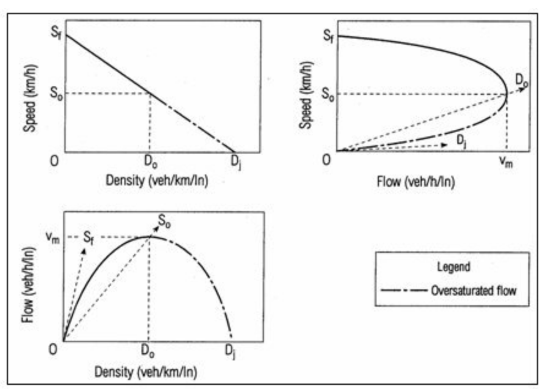
3.2 Flow Density Relationships

Flow-Density Relationships

- Objective: Provide fundamental relationships among the traffic stream characteristics for uninterrupted flow conditions:
  - **speed-density**
  - **flow-density**
  - **speed-flow**



Flow-Density Relationships

1. Flow = density × space mean speed

$$q = k\bar{u}_s$$

- Each of the variables depends on several other factors:
  - Characteristics of the roadway,
  - Characteristics of the vehicle,
  - Characteristics of the driver,
  - Environmental factors such as the weather.

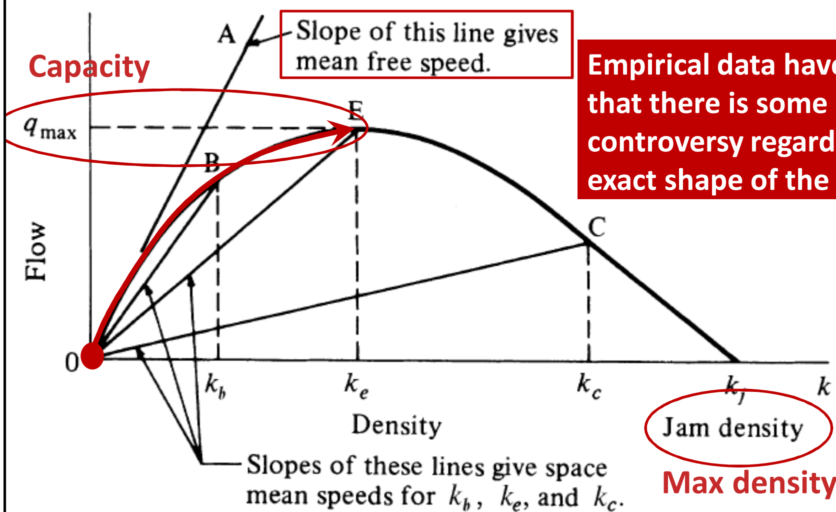
Flow-Density Relationships

2. $\bar{u}_s = q\bar{d}$
3. $\bar{d} = (1/k)$
4. $k = q\bar{t}$
5. $\bar{d} = \bar{u}_s\bar{h}$
6. $\bar{h} = \bar{t}\bar{d}$

k is the density
$\bar{u}$ is the space mean speed
q is the flow rate
$\bar{t}$ is the travel time for unit distance
h is the average time headway
$\bar{d}$ is the average space headway

Fundamental Diagrams of Traffic Flow

Fundamental Diagram of Traffic Flow

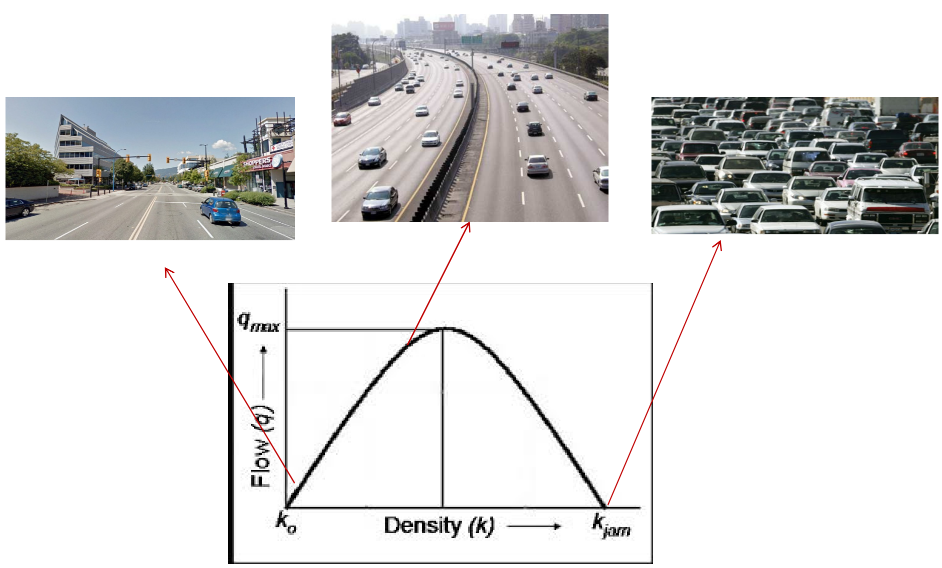


Empirical data have shown that there is some controversy regarding the exact shape of the curve

Figure 6.4 (a) Flow versus density

5

Fundamental Diagrams of Traffic Flow



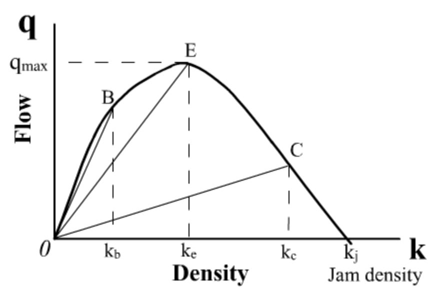
Flow versus density

Fundamental Diagrams of Traffic Flow

- **The following theory has been postulated with respect to the shape of the curve Figure 6.4(a):**

1. When the density on the highway is 0, the flow is also 0 because there are no vehicles on the highway.
2. As the density increases, the flow also increases.
3. However, when the density reaches its maximum, generally referred to as the *jam density (**k_j**), the flow must be 0 because vehicles will tend to line up end* to end.
4. It follows that as density increases from 0, the flow will also initially increase from 0 to a maximum value. Further continuous increase in density will then result in continuous reduction of the flow, which will eventually be 0 when the density is equal to the jam density.

Fundamental Diagrams of Traffic Flow



(a) Flow versus density

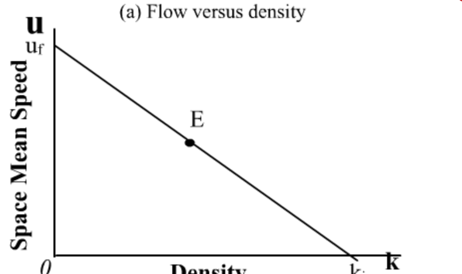
Select a Diagram

Flow and Density

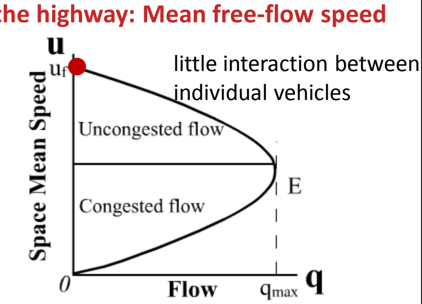
Space Mean Speed and Density

Space Mean Speed and Flow

Maximum speed that can be attained on the highway: Mean free-flow speed



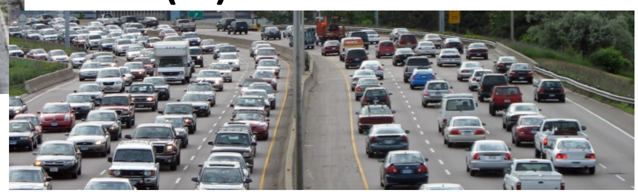
(b) Space mean speed versus density Jam density



(c) Space mean speed versus volume

Flow-Density Relationships

- **It is desirable for highways to operate at densities not greater than that corresponding to maximum flow (K_e)**



Mathematical Relationships Describing Traffic Flow

- Mathematical relationships are classified into:
  a. **Macroscopic** : considers traffic stream and develops algorithms that relate the flow to the density and space mean speeds.

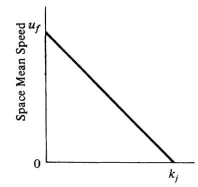
  b. **Microscopic:** considers spacings between vehicles and speeds of individual vehicles.

Mathematical Relationships Describing Traffic Flow

- **Macroscopic Approach**

① **Greenshields Model**: It is hypothesized that a linear relationship existed between speed and density

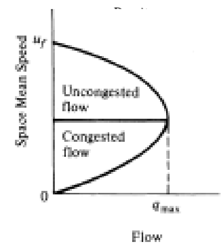
$$\overline{u}_s = u_f - \frac{u_f}{k_j} k$$



parabolic relationships

$$\overline{u}_s^2 = u_f \overline{u}_s - \frac{u_f}{k_j} q$$

$$q = u_f k - \frac{u_f}{k_j} k^2$$



Mathematical Relationships Describing Traffic Flow

- **Macroscopic Approach**

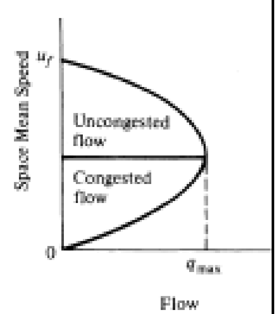
① **Greenshields Model**: the space speed and density at the maximum flow (Capacity)

At maximum flow

$$u_o = \frac{u_f}{2}$$

$$k_o = \frac{k_j}{2}$$

$$q_{\max} = \frac{k_j u_f}{4}$$



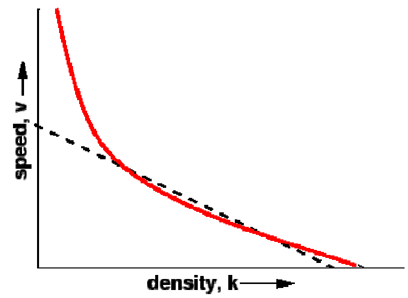
Mathematical Relationships Describing Traffic Flow

- **Macroscopic Approach**

② **Greenberg Model**: A major contributions using the fluid-flow analogy was developed by Greenberg in the form of:

$$\bar{u}_s = c \ln \frac{k_j}{k}$$

$$q = ck \ln \frac{k_j}{k}$$



At the maximum flow (Capacity)

$$\ln \frac{k_j}{k_o} = 1 \qquad u_o = c$$

u_o: speed at maximum flow
K_o: density at maximum flow

Mathematical Relationships Describing Traffic Flow

- **Macroscopic Approach:** Model application

Use of these macroscopic models depends on whether they satisfy the boundary criteria of the fundamental traffic flow diagrams

① **Greenshields Model**: satisfies the boundary conditions when the density is zero and jam density

- **It can be used for light or dense traffic**

② **Greenberg Model**: satisfies the boundary conditions when the density is approaching the jam density only

- **It can be used only for dense traffic conditions**

Calibration of Macroscopic Traffic Flow Models

- Traffic models discussed before can be used to estimate speed and density at which maximum flow occurs and the jam density of a facility.

This requires appropriate data which have to be fitted using suitable model

Regression analysis

By minimizing the squares of the differences between the observed and expected values of a dependent variable

Calibration of Macroscopic Traffic Flow Models

- Assuming that the dependent variable is linearly related to the independent variable

Linear regression analysis

If the relationship is with two or more independent variables
Multiple Linear regression analysis

$$y = a + bx$$

$$a = \frac{1}{n}\sum_{i=1}^{n} y_i - \frac{b}{n}\sum_{i=1}^{n} x_i = \overline{y} - b\overline{x}$$

$$b = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2}$$

n = number of sets of observations
x_i = ith observation for x
y_i = ith observation for y

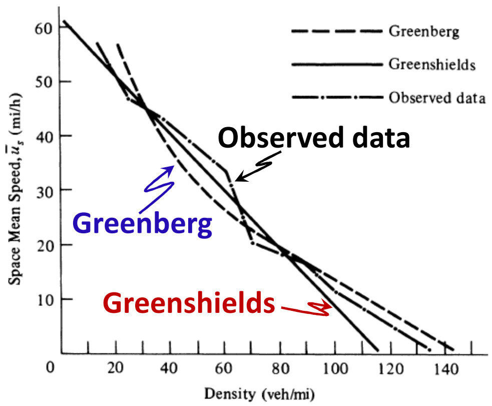
Calibration of Macroscopic Traffic Flow Models

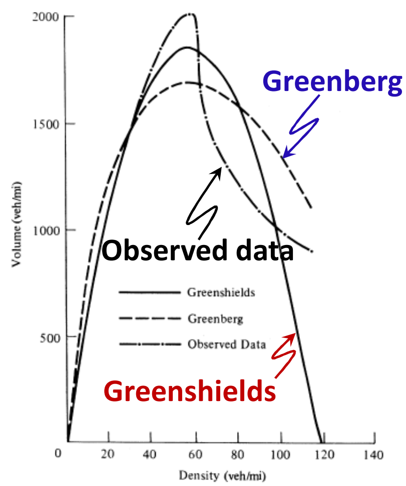
- The suitability of an estimated regression function is usually determined using the coefficient of determination ***R*²**

$$R^2 = \frac{\sum_{i=1}^{n}(Y_i - \overline{y})^2}{\sum_{i=1}^{n}(y_i - \overline{y})^2}$$

- Y_i is the value of the dependent variable as computed from the regression equations
- The closer ***R*²** is to 1, the better the regression fits

Calibration of Macroscopic Traffic Flow Models





Example 3.2

A section of highway is known to have a free-flow speed of 90 km/h and a capacity of 3300 veh/h. In a given hour, 2100 vehicles were counted at a specified point along this highway section. If the linear speed-density relationship shown in Eq. 5.15 applies, what would you estimate the space-mean speed of these 2100 vehicles to be?

Solution:

The jam density is first determined from Eq. 5.20 as

$$k_j = \frac{4q_{max}}{u_f} = \frac{4 \times 3300}{90} = 146.7 \text{ veh/km}$$

Rearranging Eq. 5.22 to solve for u,

$$\frac{k_j}{u_f}u^2 - k_j u + q = 0$$

Substituting,

$$\frac{146.7}{90}u^2 - 146.7u + 2100 = 0$$

which gives u = 72.14 km/h or 17.86 km/h. Both of these speeds are feasible, as shown in Fig. 5.3.