7 COORDINATE GEOMETRY, AND TRAVERSE SURVEYING

7.1 INTRODUCTION

The rapid advances in surveying technology and the increasing use of computers in all aspects of engineering planning and design made the use of coordinates to define geographic positions of survey points a necessity, rather than just being convenient. There are now computer programs available for performing many of the basic surveying calculations by the use of coordinates. Examples of these program packages are the PC SURVEY for Microsoft Windows written by Soft-Art, Inc., Softdesk 8 and the COCOLA package written by the author.

The fundamental principles of coordinate geometry and traverse surveying will be discussed in this chapter, and emphasis will be given to horizontal coordinates only. It should be noted here that this book uses the coordinate system utilized by the Palestinian Survey Department where the x-axis is taken to coincide with the north direction, while the y-axis coincides with the east direction (see Figure 7.1).

7.2 COORDINATE GEOMETRY

Several methods are used to locate and calculate the coordinates of a point with respect to a known line (a known line is one for which the coordinates of the end points are known or the coordinates of the beginning point and the azimuth of the line are known). These methods include: location by angle and distance, distance and offset, intersection by angles, intersection by distances and resection.

7.2.1 THE INVERSE PROBLEM

If the y and x coordinates of two points are known, the horizontal distance and azimuth of the line joining them can be computed. Figure 7.1 illustrates the problem. The horizontal distance d_{ij}:

$$d_{ij} = \sqrt{(y_j - y_i)^2 + (x_j - x_i)^2} \quad \ldots\ldots\ldots\ldots (7.1)$$

The azimuth α_{ij} of the line going from *i* to *j* is:

$$\alpha_{ij} = \tan^{-1}\frac{(y_j - y_i)}{(x_j - x_i)} + c \quad \ldots\ldots\ldots\ldots (7.2)$$

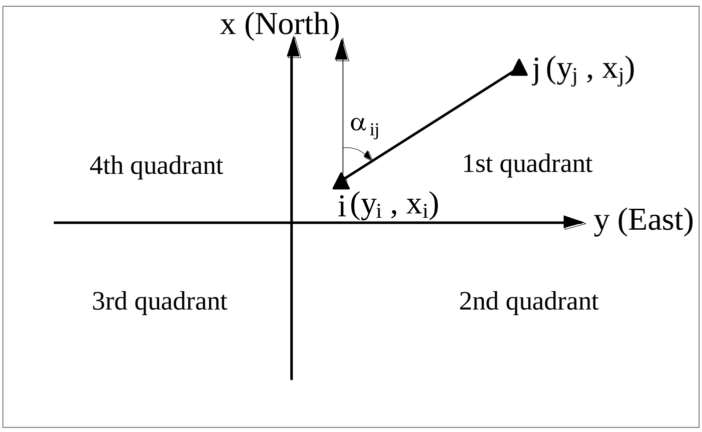

FIGURE 7.1: The inverse problem.

Where
c = 0° if Δy is positive and Δx is positive (1st quadrant)
c = 180° if Δy is positive and Δx is negative (2nd quadrant)
c = 180° if Δy is negative and Δx is negative (3rd quadrant)
c = 360° if Δy is negative and Δx is positive (4th quadrant)

Traditionally, $\left(\Delta y = y_j - y_i\right)$ is called the departure of line *ij*, and $\left(\Delta x = x_j - x_i\right)$ is called the latitude of line *ij*.

EXAMPLE 7.1:

Given the following horizontal coordinates for two points *i* and *j* in Nablus area:

y_i = 174410.56 m x_i = 181680.76 m
y_j = 174205.31 m x_j = 181810.22 m

Compute the horizontal distance (d_{ij}) and azimuth (α_{ij}) of line *ij*.

SOLUTION:

$\Delta y = y_j - y_i$ = 174205.31 – 174410.56 = -205.25 m

$\Delta x = x_j - x_i$ = 181810.22 - 181680.76 = 129.46 m

$\Rightarrow d_{ij} = \sqrt{(-205.25)^2 + (129.46)^2} = 242.67 \text{ m}$

Since Δy is negative and Δx is positive $\Rightarrow$ the line is in the 4th quadrant (c=360°),

$$\Rightarrow \alpha_{ij} = \tan^{-1}\frac{\Delta y}{\Delta x} + c = \tan^{-1}\frac{-205.25}{129.46} + 360 = 302°\ 14'\ 29''$$

7.2.2 LOCATION BY ANGLE AND DISTANCE

Referring to Figure 7.2, let *i* and *j* be two points of known coordinates. The horizontal coordinates of a new point, such as *k*, can be determined by measuring the horizontal angle β and the distance d_{ik}. The azimuth α_{ij} is calculated from Equation (7.2).

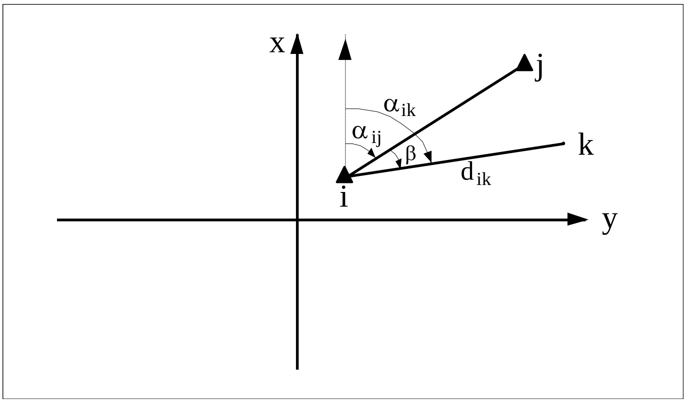

FIGURE 7.2: Location by angle and distance.

Then,

$$\alpha_{ik} = \alpha_{ij} + \beta \qquad (7.3)$$

Note: If α_{ik} is found to be more than 360°, then subtract 360° from it.

From Figure 7.2, the coordinates (y_k, x_k) of point *k* are computed as follows:

$$y_k = y_i + d_{ik} \sin \alpha_{ik} \qquad (7.4)$$

$$x_k = x_i + d_{ik} \cos \alpha_{ik} \qquad (7.5)$$

EXAMPLE 7.2:

For the two points *i* & *j* in example 7.1, a total station was set up at point *i* and directed towards point *j*. The following horizontal angle and distance were then measured to point *k*:

β = 111° 27' 45", d_{ik} = 318.10 m

Compute the horizontal coordinates of point *k*.

SOLUTION:

From example 7.1, α_{ij} = 302° 14' 29"

$$\alpha_{ik} = \alpha_{ij} + \beta = 302° 14' 29'' + 111° 27' 45'' = 413° 42' 14'' - 360°$$
$$= 53° 42' 14''$$

From equations (7.4) and (7.5):

$$y_k = 174410.56 + 318.10 \sin(53° 42' 14'') = 174666.94 \text{ m}$$
$$x_k = 181680.76 + 318.10 \cos(53° 42' 14'') = 181869.06 \text{ m}$$

7.2.3 LOCATING THE NORTH DIRECTION AT A POINT

Suppose that you are standing with your theodolite or total station at point *i* whose coordinates are known, and facing another point *j* whose coordinates are also known. Now, in order to locate the direction of the north at point *i*, perform the following steps:

1. Calculate the azimuth of line *ij* (α_{ij}).
2. Let the horizontal circle reading of your instrument read the value of α_{ij} while sighting point *j*.
3. Rotate the instrument in a counterclockwise direction. The horizontal circle reading will start decreasing, and when it becomes 0°, the telescope of the instrument will point in the north direction.

Now, let us assume that the coordinates of the point at which the north direction is to be located are not known. For this case, measurements are needed to calculate the coordinates of this point using the procedure of location by angle and distance in the previous section, or any of the procedures that will be explained in the following sections. Now that the coordinates of this point are known, set up the instrument over the point and direct it towards another point of known coordinates, and then repeat the previous three steps.

7.2.4 LOCATION BY DISTANCE AND OFFSET

Referring to Figure 7.3, let *i* and *j* be two points of known coordinates. The horizontal coordinates of a new point, such as *p*, can be determined by measuring the horizontal distance *im* along the line *ij* and the offset o_1 to point *p*. The azimuth α_{ij} is calculated from Equation (7.2).

The coordinates of point *p*, which lies to the left of line *ij* are calculated from the following equations:

$$y_p = y_i + d_{im}.\sin\alpha_{ij} + o_1.\sin(\alpha_{ij} - 90°)$$

$$x_p = x_i + d_{im}.\cos\alpha_{ij} + o_1.\cos(\alpha_{ij} - 90°)$$

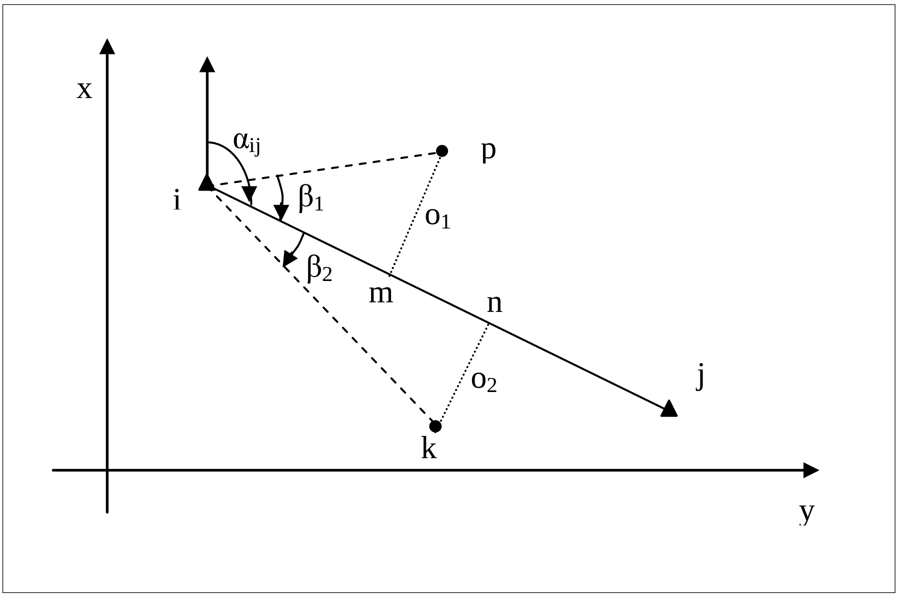

FIGURE 7.3: Location by distance and offset.

But, $\sin(\alpha_{ij} - 90°) = -\cos\alpha_{ij}$ and $\cos(\alpha_{ij} - 90°) = \sin\alpha_{ij}$, therefore:

$$y_p = y_i + d_{im}.\sin\alpha_{ij} - o_1.\cos\alpha_{ij}$$
$$x_p = x_i + d_{im}.\cos\alpha_{ij} + o_1.\sin\alpha_{ij} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.6)$$

However, if the point lies to the right of line *ij*, such as point *k* (Figure 7.3), the coordinates are calculated from the following equations:

$$y_k = y_i + d_{in}.\sin\alpha_{ij} + o_2.\sin(\alpha_{ij} + 90°)$$
$$x_k = x_i + d_{in}.\cos\alpha_{ij} + o_2.\cos(\alpha_{ij} + 90°)$$

But, $\sin(\alpha_{ij} + 90°) = \cos\alpha_{ij}$ and $\cos(\alpha_{ij} + 90°) = -\sin\alpha_{ij}$, therefore:

$$y_k = y_i + d_{in}.\sin\alpha_{ij} + o_2.\cos\alpha_{ij}$$
$$x_k = x_i + d_{in}.\cos\alpha_{ij} - o_2.\sin\alpha_{ij} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.7)$$

Alternatively, the coordinates of points *p* and *k* can be calculated as follows:

A) For points which lie on the left side of the line ij (such as point *p* in Figure 7.3):

1. Calculate the horizontal distance from *i* to *p* ($d_{ip} = \sqrt{d_{im}^2 + o_1^2}$)
2. Calculate angle $\beta_1 = \tan^{-1}\dfrac{o_1}{d_{im}}$
3. Calculate the azimuth of line ip ($\alpha_{ip} = \alpha_{ij} - \beta_1$)
4. Calculate the coordinates of point p as follows:

$$y_p = y_i + d_{ip}.\sin\alpha_{ip}$$
$$x_p = x_i + d_{ip}.\cos\alpha_{ip}$$

B) For points which lie on the right side of the line ij (such as point *k* in Figure 7.3):

1. Calculate the horizontal distance from i to k ($d_{ik} = \sqrt{d_{in}^2 + o_2^2}$)
2. Calculate angle $\beta_2 = \tan^{-1}\dfrac{o_2}{d_{in}}$
3. Calculate the azimuth of line ik ($\alpha_{ik} = \alpha_{ij} + \beta_2$)
4. Calculate the coordinates of point k as follows:

$$y_k = y_i + d_{ik}.\sin\alpha_{ik}$$
$$x_k = x_i + d_{ik}.\cos\alpha_{ik}$$

Now, let us assume that you are given the coordinates of the end points of a line such as *i* and *j* as well as the coordinates of a third point *p*, and you want to know if point *p* is located to the left or right of line *ij* and at what distance and offset. The following procedure is followed (refer to Figure 7.3):

1. Calculate both azimuths of lines *ip* and *ij* (i.e. α_{ip} & α_{ij}).
2. Calculate the angle β between the lines *ip* and *ij* ($\beta = \alpha_{ip} - \alpha_{ij}$). If angle β is negative, this means that point *p* is to the left of the line *ij*, else it is to the right.
3. Calculate the distance between points *i* and *p* ($d_{ip} = \sqrt{(y_p - y_i)^2 + (x_p - x_i)^2}$).
4. The distance *im* and offset o (Figure 7.3) are calculated as follows:
$$d_{im} = d_{ip}.\cos\beta$$
$$o = d_{ip}.\sin\beta$$

Again, a negative offset means that point *p* lies on the left side of line *ij*, while a positive offset means that point *p* is on the right side of line *ij*.

EXAMPLE 7.3:

The coordinates of the end points of a chain line *ij* are as follows:

y_i = 1000.00 m x_i = 1000.00 m
y_j = 1050.00 m x_j = 975.00 m

An edge of a building *k* is located at a distance of 30.00 m and offset of 10.00 m to the right of line *ij*. Calculate the coordinates of this edge point *k*.

SOLUTION:

$$\alpha_{ij} = \tan^{-1}\frac{1050.00 - 1000.00}{975.00 - 1000.00} + 180^\circ = 116°\ 33'\ 54'' \quad \text{(second quadrant)}$$

Using equations (7.7):

y_k = 1000.00 + 30.00 x sin (116˚ 33′ 54″) + 10.00 . cos (116˚ 33′ 54″)
= 1022.36 m

x_k = 1000.00 + 30.00 x cos (116˚ 33′ 54″) - 10.00 . sin (116˚ 33′ 54″)
= 977.64 m

7.2.5 INTERSECTION BY ANGLES

The horizontal coordinates of a new point can be determined by measuring angles from two points of known coordinates. In Figure 7.4, the coordinates of points *i* and *j* are known and the coordinates of point *k* are to be calculated. This procedure is mostly used when the surveyor has a theodolite and does have access to an EDM, or when point *k* is difficult to be reached such as the top of a mosque minaret or a church.

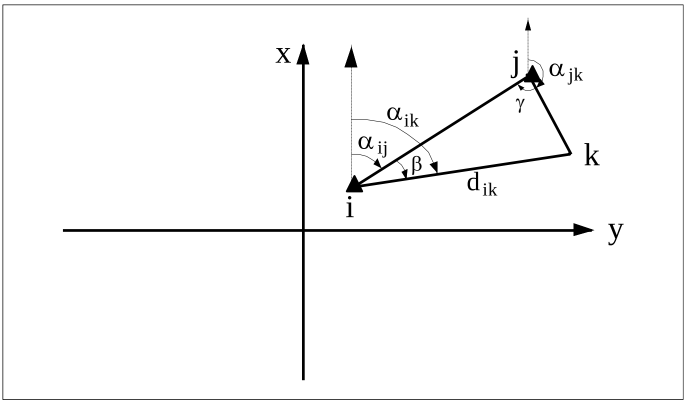

FIGURE 7.4: Intersection by angles.

The azimuth α_{ij} and distance d_{ij} can be first be computed by equations (7.1) and (7.2). The azimuths α_{ik} and α_{jk} can then be computed from the azimuth α_{ij} and the measured angles β and γ. Let d_{ik} and d_{jk} represent the lengths of lines *ik* and *jk* respectively. By the Sine law:

$$\frac{d_{ik}}{\sin\gamma} = \frac{d_{ij}}{\sin(180 - \gamma - \beta)}$$

$$\Rightarrow \quad d_{ik} = \frac{d_{ij}\sin\gamma}{\sin(180 - \gamma - \beta)} \qquad \ldots (7.8)$$

Then,

$$y_k = y_i + d_{ik}\sin\alpha_{ik} \qquad \ldots (7.9)$$

$$x_k = x_i + d_{ik}\cos\alpha_{ik} \qquad \ldots (7.10)$$

Similarly,

$$d_{jk} = \frac{d_{ij}\sin\beta}{\sin(180 - \gamma - \beta)} \qquad \ldots (7.11)$$

$$y_k = y_j + d_{jk}\sin\alpha_{jk} \qquad \ldots (7.12)$$

$$x_k = x_j + d_{jk}\cos\alpha_{jk} \qquad \ldots (7.13)$$

EXAMPLE 7.4:

Referring to Figure 7.4, let *i* & *j* be two points of known coordinates, and point *k* be the cross of a church whose coordinates are to be calculated. Given:

y_i = 175329.41 m x_i = 184672.66 m
y_j = 176321.75 m x_j = 185188.24 m
β = 31° 26' 30" γ = 42° 33' 41"
Compute the horizontal coordinates y_k and x_k.

SOLUTION:

$y_j - y_i = 176321.75 - 175329.41 = 992.34$ m
$x_j - x_i = 185188.24 - 184672.66 = 515.58$ m

$$\Rightarrow d_{ij} = \sqrt{(992.34)^2 + (515.58)^2} = 1118.29 \text{ m}$$

$$\alpha_{ij} = \tan^{-1}\frac{992.34}{515.58} + 0 = 62° 32' 44'' \text{ (1}^{st}\text{ quadrant)}$$

$\alpha_{ik} = \alpha_{ij} + \beta = 62° 32' 44'' + 31° 26' 30'' = 93° 59' 14''$
$180° - \beta - \gamma = 180° - 31° 26' 30'' - 42° 33' 41'' = 105° 59' 49''$

$$d_{ik} = \frac{1118.29 \sin(42° 33' 41'')}{\sin(105° 59' 49'')} = 786.86 \text{ m}$$

Then,
$y_k = 175329.41 + 786.86 \sin(93° 59' 14'') = 176114.37$ m
$x_k = 184672.66 + 786.86 \cos(93° 59' 14'') = 184617.95$ m

7.2.6 INTERSECTION BY DISTANCES

The coordinates of a new point can also be determined by measuring distances from (or to) two points of known coordinates. In Figure 7.4, the coordinates (y_k, x_k) of new point k can be determined by measuring the distances d_{ik} and d_{jk} instead of β and γ. This procedure is preferable when point *k* is accessible; especially when measuring distances is easier and faster. This method is also widely used when calculating the coordinates of unknown boundary points when there is a need to fix boundaries of land parcels on the ground.

The solution here is similar to that used for the method of intersection by angles. The angle β is computed using the Cosine law.

$$d_{jk}^{2} = d_{ij}^{2} + d_{ik}^{2} - 2d_{ij}d_{ik}\cos\beta$$

$$\Rightarrow \quad \beta = \cos^{-1}\left[\frac{d_{ij}^{2} + d_{ik}^{2} - d_{jk}^{2}}{2d_{ij}\cdot d_{ik}}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.14)$$

The coordinates of point *k* are then calculated by using equations (7.3), (7.4) and (7.5).

EXAMPLE 7.5:

In example 7.4 and referring to Figure 7.4, the following distance measurements were made from points *i & j* to point *k:*

$d_{ik} = 888.86\,\text{m}$, $\quad d_{jk} = 950.55\,\text{m}$

Compute the horizontal coordinates y_k and x_k.

SOLUTION:

From equation (7.14)

$$\beta = \cos^{-1}\left[\frac{d_{ij}^{2} + d_{ik}^{2} - d_{jk}^{2}}{2d_{ij}\cdot d_{ik}}\right]$$

$$= \cos^{-1}\left[\frac{1118.29^{2} + 888.86^{2} - 950.55^{2}}{2x1118.29x888.86}\right] = 55^{\circ}\ 06'\ 42''$$

From example 7.4, $\alpha_{ij} = 62°\ 32'\ 44''$

$\alpha_{ik} = \alpha_{ij} + \beta$ = 62° 32' 44" + 55° 06' 42" = 117° 39' 26"

Then,

$y_k = y_i + d_{ik}\sin\alpha_{ik}$ = 175329.41 + 888.86 sin(117° 39' 26")
= 176116.71 m

$x_k = x_i + d_{ik}\cos\alpha_{ik}$ = 184672.66 + 888.86 cos(117° 39' 26")
= 184260.07 m

7.2.7 RESECTION

The horizontal position of a new point can also be determined by measuring angles from a point to three points of known coordinates. This method is called *resection*. It is mainly used when the surveyor is standing at a point of unknown coordinates and is facing three points of known coordinates that may be far away or inaccessible. In Figure 7.5, A, B and C are points of known coordinates, and hence distances b and c can be calculated. The coordinates of point P are to be found by measuring angles M and N.

Let,

$$J = \beta + \gamma \qquad (7.15)$$

Then,

$$J = 360° - (M + N + R) \qquad (7.16)$$

By the Sine law,

$$\frac{AP}{\sin\beta} = \frac{c}{\sin M} \quad \Rightarrow \quad AP = \frac{c \cdot \sin\beta}{\sin M} \qquad (7.17)$$

Similarly,

$$AP = \frac{b \cdot \sin\gamma}{\sin N}$$

Therefore,

$$\frac{c \cdot \sin\beta}{\sin M} = \frac{b \cdot \sin\gamma}{\sin N} \qquad (7.18)$$

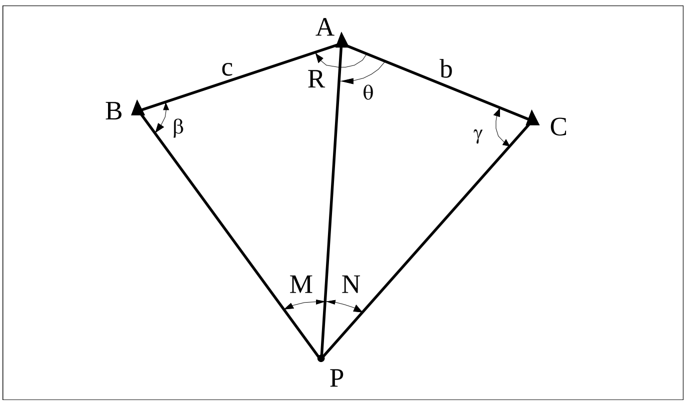

FIGURE 7.5: Resection.

Let,

$$H = \frac{\sin \beta}{\sin \gamma} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.19)$$

Then, from Equation (7.18)

$$\Rightarrow \quad H = \frac{b \sin M}{c \sin N} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.20)$$

Also from Equation (7.19)

$$H \sin \gamma = \sin \beta = \sin(J - \gamma)$$

Or $\quad H \sin \gamma = \sin J \cos \gamma - \cos J \sin \gamma$

Dividing both sides by cos γ and rearranging the terms:

$$\Rightarrow \quad \tan\gamma = \frac{\sin J}{H + \cos J} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.21)$$

Also, $\quad \theta = 180° - N - \gamma \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.22)$

The following general procedure is used to compute the coordinates of point P:

1. Compute b, c, azimuths α_{AB} and α_{AC}, and R from the known coordinates of points A, B and C.
2. Compute J using Equation (7.16).
3. Compute H using Equation (7.20)
4. Compute the angle γ using Equation (7.21).
5. Compute the angle θ using Equation (7.22).
6. Compute the azimuth α_{AP} of line AP, $\alpha_{AP} = \alpha_{AC} + \theta$
7. Compute AP, from Equation (7.17).
8. Compute y_P and x_P as follows:

$$y_P = y_A + AP \sin \alpha_{AP}$$
$$x_P = x_A + AP \cos \alpha_{AP}$$

EXAMPLE 7.6:

Given the following coordinates for points A, B and C in Figure 7.5:

Point	y-coordinate	x-coordinate
A	146732.41 m	138111.26 m
B	142139.65 m	136781.33 m
C	149822.47 m	137266.32 m

The measured angles are: M = 37° 21' 33" N = 41° 03' 56"

Compute the coordinates (y_P, x_P) of point P.

SOLUTION:

$$c = \sqrt{(142139.65 - 146732.41)^2 + (136781.33 - 138111.26)^2}$$
$$= 4781.44 \text{ m}$$

$$b = \sqrt{(149822.47 - 146732.41)^2 + (137266.32 - 138111.26)^2}$$
$$= 3203.50 \text{ m}$$

$$\alpha_{AB} = \tan^{-1}\left(\frac{142139.65 - 146732.41}{136781.33 - 138111.26}\right) + 180° = 253°\,51'\,02''$$

$$\alpha_{AC} = \tan^{-1}\left(\frac{149822.47 - 146732.41}{137266.32 - 138111.26}\right) + 180° = 105°\,17'\,35''$$

$$R = \alpha_{AB} - \alpha_{AC} = 148°\ 33'\ 27''$$

$$J = 360° - (M + N + R) = 133°\ 01'\ 04''$$

$$H = \frac{b \sin M}{c \sin N} = \frac{3203.50 \sin(37°\ 21'\ 33'')}{4781.44 \sin(41°\ 03'\ 56'')} = 0.61887727$$

$$\tan\gamma = \frac{\sin J}{H + \cos J} = \frac{\sin(133°\,01'\,04'')}{0.61887727 + \cos(133°\,01'\,04'')} = -11.5416786$$

$\Rightarrow \gamma = 94°\ 57'\ 07''$ (because the tangent is negative, then γ has to be between 90° & 180°)

$$\theta = 180° - N - \gamma = 43°\ 58'\ 57''$$

$$AP = \frac{b\sin\gamma}{\sin N} = \frac{3203.50\sin(94^\circ 57' 07'')}{\sin(41^\circ 03' 56'')} = 4858.33\ \text{m}$$

$$\alpha_{AP} = \alpha_{AC} + \theta = 105^\circ\ 17'\ 35'' + 43^\circ\ 58'\ 57''$$
$$= 149^\circ\ 16'\ 32''$$

$$y_P = y_A + AP\sin\alpha_{AP}$$
$$= 146732.41 + 4858.33\sin(149^\circ\ 16'\ 32'') = 149214.58\ \text{m}$$

$$x_P = x_A + AP\cos\alpha_{AP}$$
$$= 138111.26 + 4858.33\cos(149^\circ\ 16'\ 32'') = 133934.87\ \text{m}$$

7.2.8 MAPPING DETAILS USING EDM

To illustrate how a theodolite-EDM combination or a total station can be used to measure and map field details, let us refer to Figure 7.6 which shows a sketch for a land parcel which is to be surveyed and mapped. To do so, the instrument is set up at a known station, such as P which can be inside or outside the area to be surveyed, and from which all points can be seen. The telescope is then directed towards another known point so that the azimuth between station P and this known point can be calculated from the known coordinates. The horizontal circle of the theodolite or total station is then set to read zero. If no points of known coordinates were available in the area, or if national grid coordinates for the area were not required, a station P with assumed coordinates and an arbitrary azimuth can be chosen. The resulting coordinates with respect to the assumed datum can be transformed to national grid coordinates using the equations in the next section if the national grid coordinates of two points were known, in addition to their local coordinates.

The instrument is then directed to points 1, 2, 3, etc., and for each point the following data are recorded: point number, slope distance, horizontal circle reading, vertical circle reading, reflector height and any notes about the point such as being an iron angle, pole, etc. Sometimes, the horizontal distance and elevation difference ($\Delta H'$) between the instrument and the reflector are recorded instead of the slope distance and the vertical circle reading. Table 7.1 shows a typical page from a field book used to record the EDM data.

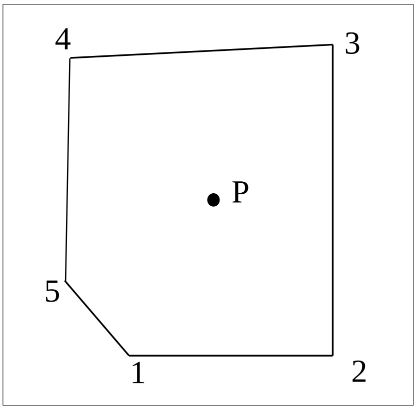

FIGURE 7.6: A sketch for a land parcel.

EXAMPLE 7.7

For the land parcel shown in Figure 7.6, the instrument was stationed at point P and the following measurements were taken:

Station No. (محطة الجهاز) P **H.I.** (ارتفاع الجهاز) 1.50 (m)

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رقم النقطة Point #	المسافة المائلة S (m)	الزاوية الأفقية (H.A)			الزاوية السمتية (Z.A.)			ارتفاع العاكس HT (m)	ملاحظات Notes
		°	'	"	°	'	"		
1	16.85	0	00	00	83	20	55	1.60	+ (صليب)
2	22.29	294	51	07	88	33	25	1.60	سور
3	32.75	188	05	03	92	11	42	1.60	سور
4	31.22	133	58	47	89	05	11	1.60	سور
5	15.07	48	26	24	82	23	24	1.60	A.I.

Given that the coordinates and elevation of point P are (Y = 100.00, X = 100.00, H = 300.00) and that the azimuth of the line from P to 1 is 195° 00' 00", calculate the elevations and coordinates of all the points.

SOLUTION:

1) Elevations: the elevation (H) of any point is calculated from the following equation:

$$H = H_p + S.\cos z + i - t$$

The calculations are shown in the next table:

Point #	Slope Dist. S	Zenith Angle (z) ° ' "	Horizontal Dist. D = S.sin z	t	S.cos z ($\Delta H'$)	Elev. Diff. $\Delta H = \Delta H' + i - t$	Elevation $H = H_p + \Delta H$
1	16.85	83 20 55	16.74	1.60	1.95	1.85	301.85
2	22.29	88 33 25	22.28	1.60	0.56	0.46	300.46
3	32.75	92 11 42	32.73	1.60	-1.25	-1.35	298.65
4	31.22	89 05 11	31.22	1.60	0.50	0.40	300.40
5	15.07	82 23 24	14.94	1.60	2.00	1.90	301.90

2) Coordinates: the coordinates (y and x) of any point *i* are calculated from the following equations:

$$y_i = y_p + d_{pi} \sin \alpha_{pi} \text{ , and}$$

$$x_i = x_p + d_{pi} \cos \alpha_{pi} \text{ ,}$$

where α_{pi} is the azimuth of line *pi* and is equal to the azimuth of line *p1* (given as 195° 00' 00") plus the horizontal angle between lines *p1* and *pi*. The calculations are shown in the next table:

Point #	Horiz. Dist. (D)	Horiz. Angle ° ' "	Azimuth (α_{pi}) ° ' "	$\Delta y = d_{pi}.\sin\alpha_{pi}$	$\Delta x = d_{pi}.\cos\alpha_{pi}$	$y = y_p + \Delta y$	$x = x_p + \Delta x$
1	16.74	00 00 00	195 00 00	-4.33	-16.17	95.67	83.83
2	22.28	294 51 07	129 51 07	17.11	-14.28	117.11	85.72
3	32.73	188 05 03	23 05 03	12.83	30.11	112.83	130.11
4	31.22	133 58 47	328 58 47	-16.09	26.75	83.91	126.75
5	14.94	48 26 24	243 26 24	-13.36	-6.68	86.64	93.32

TABLE 7.1: A typical page from an EDM field book.

Station No. ________________ H.I. (ارتفاع الجهاز) __________ (m)

/ / : التاريخ ____ : اسم المساح

رقم النقطة Point #	المسافة المائلة S.D. (m)	المسافة الأفقية H.D. (m)	ΔH' (m)	(H.A.) الزاوية الأفقية			(Z.A.) الزاوية السمتية			ارتفاع العاكس HT (m)	ملاحظات Notes
				°	'	"	°	'	"		

Now, assume that some points (features) cannot be seen from the selected instrument location (such as P). To illustrate this problem, let us refer again to Figure 7.6 and assume that an obstacle between station P and point 3 (such as a building) prevents seeing the reflector at this point (Figure 7.7). The following procedure solves this problem:

1. Choose another station (such as Q) from which the remaining points (which could not be seen from station P) can be seen.

2. With the instrument still located at P, take measurements to this new station so that its coordinates can be calculated.

3. Move the instrument and set it up at the new station Q (the coordinates of Q are known at this stage).

4. Direct the instrument towards the old station P or any other point with known coordinates and set the horizontal angle to read zero. Now rotate the instrument in a clockwise direction and start taking measurements at the points that have not been observed from the previous station (point 3 in Figure 7.7).

5. Calculate the coordinates of these points in the same manner as explained in example 7.7.

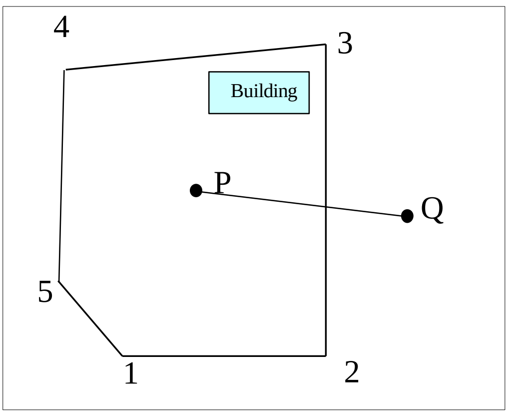

FIGURE 7.7:A land parcel with an obstacle.

6. It is a good practice to repeat the readings at a point that has been read from the previous station. The coordinates computed for this point from both stations should be identical. If they differ by a big amount, this means that an error has occurred when moving from the first station to the second.

7. If there are still some points, which could not be observed from the first and second stations, a third station can be chosen, and the previous process repeated until all points are observed.

7.2.9 TRANSFORMATION OF COORDINATES

When horizontal control points of known coordinates are not available in the area to be surveyed, the surveyor can set up his instrument (total station or theodolite - EDM combination) at any suitable station (national grid coordinates are unknown) and direct it to an assumed azimuth. In this case, the calculated coordinates of all the surveyed points will be referenced to the assumed local datum. If later, the national grid coordinates of at least two of the surveyed points are known, the local coordinates of all the other surveyed points can be transformed to the national grid coordinate system using what is called similarity transformation.

To illustrate this transformation from one coordinate system to another, let us refer to Figure 7.8. Assume that the local coordinate system is represented by the (y' - x') axes and the national grid coordinate system is represented by the (y - x) axes. Assume also that the angle between the two coordinate systems is β (β is positive counterclockwise and negative clockwise). In order to bring the local (y' - x') coordinate axes to coincide with the grid y - x coordinate system, the following steps are to be taken:

1 - Rotate the (y' - x') coordinate system by an angle β (see Figure 7.8) so that the resulting (y'' - x'') axes will become parallel to the y - x axes.

2 - If there is a scale difference between the objects in the (y' - x') system and their counterparts in the (y – x) system, the rotated coordinates in step 1 should be multiplied by a scale factor (s).

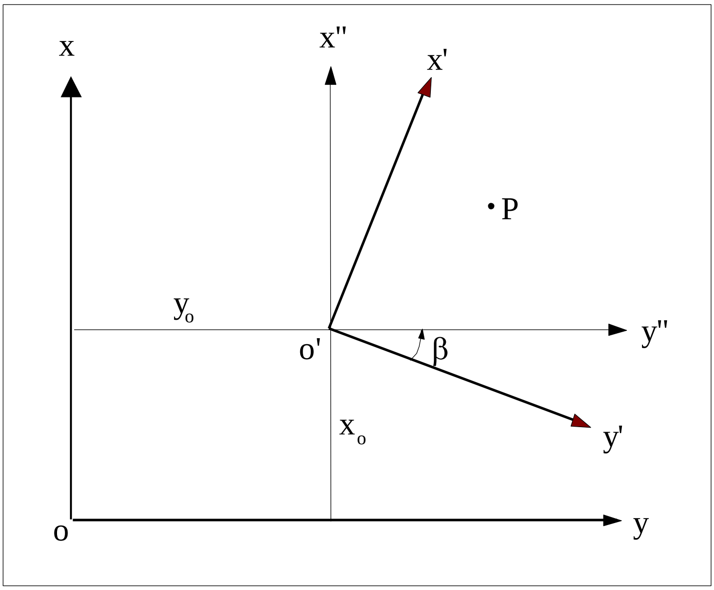

FIGURE 7.8: Relation between two-dimensional coordinate systems.

3 - Translate the o' to o by moving it a distance x_0 parallel to the x - axis and a distance y_0 parallel to the y - axis.

Mathematically, assume that the local coordinates of point P are (y'_p , x'_p), then the grid coordinates (y_p , x_p) are given by:

$$y_p = y_0 + s\left(y'_p \cdot \cos\beta + x'_p \cdot \sin\beta\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (7.23)$$

$$x_p = x_0 + s\left(-y'_p \cdot \sin\beta + x'_p \cdot \cos\beta\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (7.24)$$

Equations (7.23) and (7.24) have four unknowns which are: y_0, x_0, s and β. In order to solve for these unknowns, four equations are needed. This means that two points of known coordinates in both coordinate systems are needed. Each point will provide two equations. In most field surveying problems, the scale between the local and the national grid coordinate systems is the same (i.e., s = 1). The angle β between the two coordinate systems can be calculated as follows:

$$\beta = AZ_g - AZ_\ell \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.25)$$

Where AZ_g is the grid azimuth of the line joining the two known points, and
AZ_ℓ is the local azimuth of the line joining the two known points.

The remaining two unknowns (y_0 and x_0) are calculated from equations (7.23) and (7.24) as follows:

$$y_0 = y_p - \left(y'_p \cdot \cos\beta + x'_p \cdot \sin\beta\right) \quad \ldots\ldots\ldots\ldots (7.26)$$

$$x_0 = x_p - \left(-y'_p \cdot \sin\beta + x'_p \cdot \cos\beta\right) \quad \ldots\ldots\ldots\ldots (7.27)$$

> **Note:** If the national grid coordinates are known for more than 2 points, then the 4 unknowns of the similarity transformation (s, β, x_0, y_0) are calculated using the principle of least squares adjustment (see Chapter 10).

EXAMPLE 7.8:

For the land parcel in Example 7.7, assume that the grid coordinates of points 1 & 2 are as follows: 1 (182790.00 , 174519.32),
2 (182811.51 , 174519.32).

Calculate the transformed coordinates of points 3, 4 and 5.

SOLUTION

From Equation (7.25), $\beta = AZ_g - AZ_\ell$

The grid azimuth

$$AZ_g = \tan^{-1}\left(\frac{182811.51 - 182790.00}{174519.32 - 174519.32}\right) = 90° \ 00' \ 00''$$

The local azimuth $AZ_\ell = \tan^{-1}\left(\frac{117.10 - 95.67}{85.72 - 83.83}\right) = 84° \ 57' \ 36''$

$\Rightarrow \beta = 90° \ 00' \ 00'' - 84° \ 57' \ 36'' = 5° \ 02' \ 24''$

From Equations (7.26) and (7.27),

$$y_0 = 182790.00 - \left(95.67\cos(5°\ 02'\ 24'') + 83.83\sin(5°\ 02'\ 24'')\right)$$
$$= 182687.34$$

$$x_0 = 174519.32 - \left(-95.67\sin(5°\ 02'\ 24'') + 83.83\cos(5°\ 02'\ 24'')\right)$$
$$= 174444.22$$

Now applying Equations (7.23) and (7.24) on the assumed coordinates of points 3, 4 and 5 (with s=1) gives:

$$y_3 = 182687.34 + \left(112.83\cos(5°\ 02'\ 24'') + 130.10\sin(5°\ 02'\ 24'')\right)$$
$$= 182811.16$$

$$x_3 = 174444.22 - \left(-112.83\sin(5°\ 02'\ 24'') + 130.10\cos(5°\ 02'\ 24'')\right)$$
$$= 174563.90$$

In the same manner, the coordinates of points 4 & 5 are:
4 (182782.07 , 174563.11) , 5 (182781.85 , 174529.57)

7.3 TRAVERSE SURVEYING

Traverse surveying is a measurement procedure used for determining the horizontal relative positions (y & x coordinates) of a number of survey points. While leveling is used to establish the elevations of points, traverse surveying is used to determine the horizontal coordinates of these points. Basically, it consists of repeated application of the method of locating by angle and distance (Section 7.2.2). By starting from a point of known horizontal coordinates and a line of known direction, the location of a new point is determined by measuring the distance and angle from the known point. Then, the location of another new point is determined by angle and distance measurement from the newly located point. This procedure is repeated from point to point. The resulting geometric figure is called a ***traverse***.

7.3.1 PURPOSE OF THE TRAVERSE

The traverse serves several purposes among which are:

1 - Property surveys to establish boundaries.

2 - Location and construction layout surveys for highways, railways and other works.

3 - Providing Ground control points for photogrammetric mapping.

7.3.2 TYPES OF TRAVERSE

There are two main types of traverse:

a) ***Open traverse*** (Figure 7.9). This originates at a point that could be of known or unknown position and terminates at a different point of unknown position. To minimize errors, distances can be measured twice, angles repeated, and magnetic azimuth observed on all lines. This type is used in certain works such as locating the centerline of a tunnel during construction. For projects requiring high accuracy, it is preferable not be used.

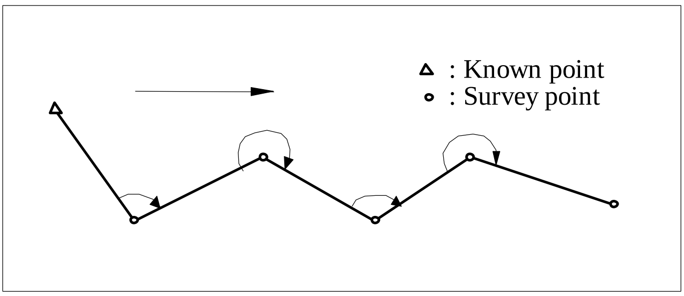

FIGURE 7.9: Open traverse.

b) ***Closed traverse*** (Figure 7.10). This type originates at a point (of known or assumed position) and terminates at the same point yielding a closed loop traverse (Figure 7.10a), or originates at a line of known coordinates and ends at another line of known coordinates (or a coordinate and direction) yielding a closed connecting traverse (Figure 7.10b). This is a requirement by the Survey Department in Palestine. The closed traverse is preferred to the open traverse because it provides a check on errors.

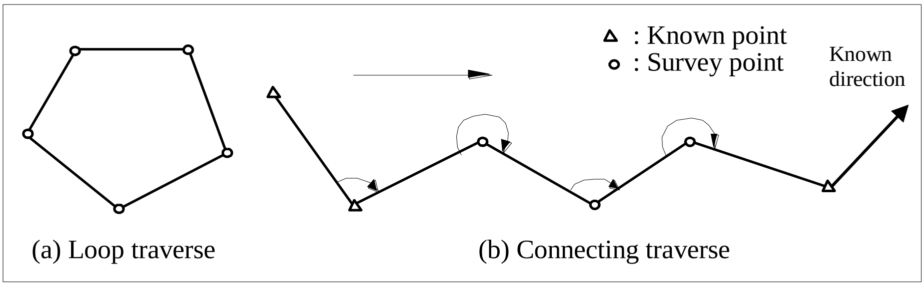

FIGURE 7.10: Closed traverse

7.3.3 CHOICE OF TRAVERSE STATIONS

Traverse stations should be located so that:

1 - Traverse lines should be as close as possible to the details to be surveyed.

2 - Distances between traverse stations should be approximately equal and the shortest line should be greater than one third of the longest line.

3 - Stations should be chosen on firm ground, or monumented in a way to make sure that they are not easily lost or damaged.

4 - When standing on one station, it should be easy to see the backsight and foresight stations.

7.3.4 TRAVERSE COMPUTATIONS AND CORRECTION OF ERRORS

The following computations and error correction are usually associated with traverse surveying.

A) Azimuth of a line:

The requirement here is to calculate the azimuth (α_2) of line BC from the known azimuth (α_1) of line AB and the clockwise measured angle (ϕ) between the two lines (see Figure 7.11). Two cases can be distinguished here:

1. When ($\alpha_1 + \phi$) > 180°. From Figure 7.11:

$$\alpha_2 = \phi - (180° - \alpha_1)$$

$$\Rightarrow \alpha_2 = \phi + \alpha_1 - 180° \quad \ldots\ldots\ldots\ldots (7.28)$$

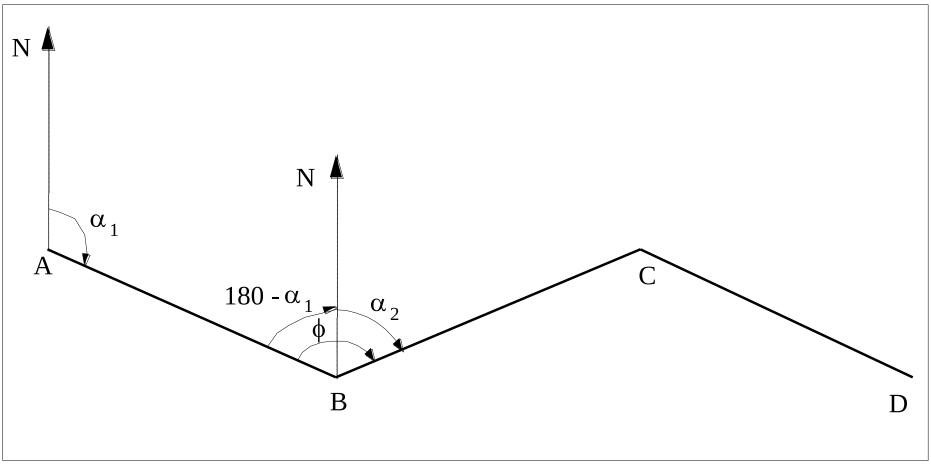

FIGURE 7.11: Case (1): $\alpha_1 + \phi > 180°$

2. When ($\alpha_1 + \phi$) < 180°. From Figure 7.12):

$$\alpha_2 = \phi + 180° + \alpha_1$$

$$\Rightarrow \alpha_2 = \phi + \alpha_1 + 180° \quad \ldots\ldots\ldots\ldots (7.29)$$

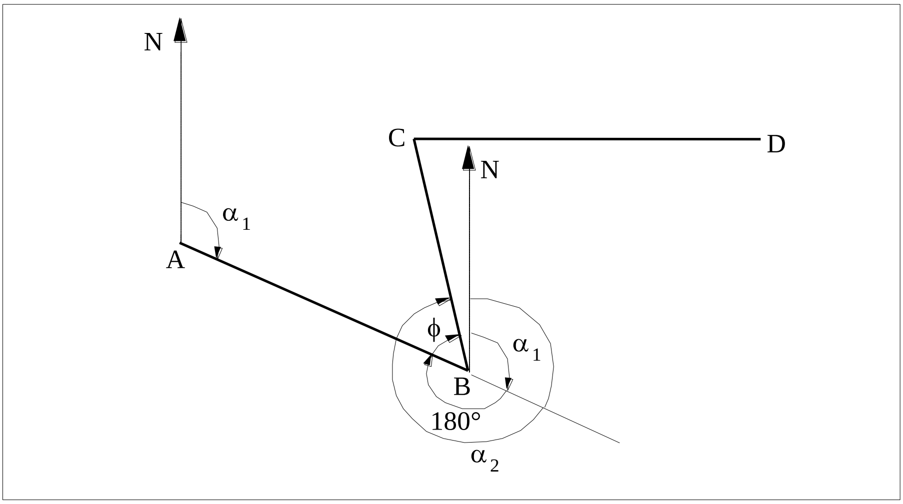

FIGURE 7.12: Case (2): $\alpha_1 + \phi < 180°$

In general,

$$\alpha_2 = \alpha_1 + \phi \pm 180° \qquad \text{(7.30)}$$

Where α_2 = the azimuth of the following line,
α_1 = the azimuth of the previous line, and
ϕ = the clockwise angle between the previous line and the following line.

If $\alpha_1 + \phi < 180°$ ⇒ add 180° to get α_2
If $\alpha_1 + \phi > 180°$ ⇒ subtract 180° to get α_2

Now, if the coordinates of point B are known, then the coordinates of point C are calculated as follows:

$$y_C = y_B + d_{BC} \cdot \sin \alpha_2$$

$$x_C = x_B + d_{BC} \cdot \cos \alpha_2$$

B) Checks and Correction of Errors:

When performing the traverse calculations, the following conditions must be satisfied for a closed traverse (Figure 7.13):

$$y_{\text{last point}} - y_{\text{first point}} = \Sigma\Delta y_{\text{all lines}} \quad \text{.......................} (7.31)$$
$$x_{\text{last point}} - x_{\text{first point}} = \Sigma\Delta x_{\text{all lines}} \quad \text{.......................} (7.32)$$

When the last point is the same as the first point, then: $\Sigma\Delta y = 0$ and $\Sigma\Delta x = 0$.

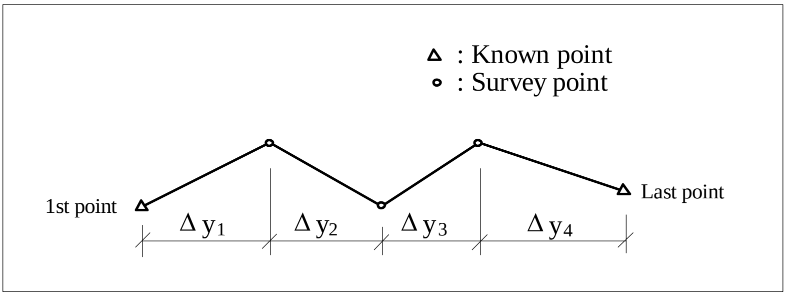

FIGURE 7.13: A closed connecting traverse.

In order to meet the previous two conditions (equations 7.31 and 7.32), the following checks and corrections are performed:

(1) ***Angle Correction***. This can be done in two ways:

a) Closed loop traverse. For a closed loop traverse of *n* sides (Figure 7.14):

Sum of internal angles = (n - 2) x 180° .... (7.33)

For the traverse in Figure 7.14:
Σ Internal angles = (5 - 2) x 180° = 540°
If the sum is found to be 540° 00' 15", ⇒ Error = +15"

No. of internal angles = 5
⇒ Correction for each angle = -15"/5 = -3"
⇒ Subtract 3" from each angle

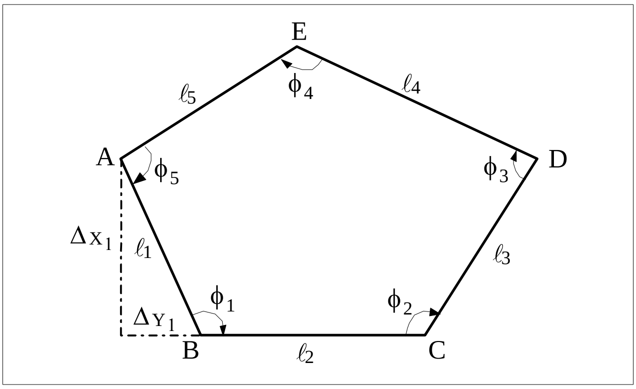

FIGURE 7.14: A closed loop traverse.

b) *For both loop and connecting closed traverses*. If the azimuth of the last line in the traverse is known, this azimuth is compared with the calculated azimuth, and the error is distributed between the angles. Assume that the known azimuth of last line is α_n , and the calculated one is α_c, then the error ε_α is:

$$\varepsilon_\alpha = \alpha_c - \alpha_n \qquad \ldots\ldots\ldots\ldots (7.34)$$

For n measured angles:

$$\Rightarrow \text{Correction/angle} = -\frac{\varepsilon_\alpha}{n} \qquad \ldots\ldots\ldots\ldots (7.35)$$

This correction is added to each angle in the traverse and the azimuths of all lines are recalculated.

To save time and effort, the azimuth correction can be applied to the preliminary computed azimuths directly in order to get to the corrected ones. Let α'_i be the initially computed azimuth of the *i-th* line in the traverse, then the corrected azimuth α_i of this line is:

$$\alpha_i = \alpha'_i - i \cdot \left(\frac{\varepsilon_\alpha}{n}\right) \qquad \ldots\ldots\ldots\ldots (7.36)$$

(2) ***Position Correction***. It always happens to have small errors in the values of $\Sigma\Delta y$ and $\Sigma\Delta x$ in such a way that they will not meet the condition of equations (7.31) and (7.32). This happens even after correcting the angles. The reason is that the errors in the measured distances have not yet been accounted for.

Assume that the calculated and known coordinates of the last point are (y_c, x_c) and (y_n, x_n) respectively. Then:

Closure error in the y-direction $(\varepsilon_y) = y_c - y_n$ . . . . (7.37)

Closure error in the x-direction $(\varepsilon_x) = x_c - x_n$ . . . . (7.38)

Closure error in the position of the last point:

$$\varepsilon = \sqrt{\varepsilon_y^2 + \varepsilon_x^2} \quad \text{. . . . . (7.39)}$$

This is called the linear error of closure and it is equal to the length of the error vector connecting the calculated point with the known point.

<u>How to correct and distribute this error?</u>

Different methods can be used for the correction of the position closure error. These include the compass rule and the transit rule. Only the compass rule will be discussed here.

<u>Compass Rule (Bowditch Rule):</u>

This rule distributes the correction on the departures (Δy) and latitudes (Δx) of the measured traverse lines as follows:

$$\text{Correction to } \Delta y \text{ of line ij} = -\frac{\text{length of line ij}}{\text{total traverse length}} \cdot \varepsilon_y \quad \text{. . . . . (7.40)}$$

$$\text{Correction to } \Delta x \text{ of line ij} = -\frac{\text{length of line ij}}{\text{total traverse length}} \cdot \varepsilon_x \quad \text{. . . . .(7.41)}$$

The computation of the coordinates is now repeated using the corrected Δy's and Δx's. Alternatively, the preliminary computed coordinates of the traverse points can be corrected directly in the same manner like the azimuths. This is performed as follows:

$$cy_i = -\frac{L_i}{D} \cdot \varepsilon_y \qquad (7.42)$$

$$cx_i = -\frac{L_i}{D} \cdot \varepsilon_x \qquad (7.43)$$

Where cy_i and cx_i are the corrections to be applied to the computed coordinates y_i and x_i of station *i*,

L_i = cumulative traverse distance up to station *i*

D = total length of the traverse

The corrected coordinates of station *i* $(y_i^{'} , x_i^{'})$ are:

$$y_i^{'} = y_i + cy_i \qquad (7.44)$$

$$x_i^{'} = x_i + cx_i \qquad (7.45)$$

7.3.5 ALLOWABLE ERRORS IN TRAVERSE SURVEYING

The Department of Surveying in the West Bank allows the following errors in traverse surveying:

	Allowable error	
	Important areas (example: urban areas)	Less important areas (example: rural areas)
Measured distances	$\Delta\ell = 0.0005\ell + 0.03$ m	$\Delta\ell = 0.0007\ell + 0.03$ m
Measured angles	$\Delta = 60''\sqrt{n}$	$\Delta = 90''\sqrt{n}$
Closure error	$\varepsilon = 0.0006\ \Sigma\ell + 0.20$ m	$\varepsilon = 0.0009\ \Sigma\ell + 0.20$ m

Where ℓ = measured length, Δ = angle closure error in seconds

n = number of measured angles, $\varepsilon = \sqrt{\varepsilon_y^{2} + \varepsilon_x^{2}}$ (Equation 7.39)

$\Delta\ell$ = allowable error in the measured distance

EXAMPLE 7.9:

Perform the following calculations for the traverse shown in Figure 7.15.

(a) Distribute the angular error of closure and compute the corrected azimuths of all lines.

(b) Calculate the linear and relative errors of closure.

(c) Balance the traverse by the Compass rule and compute the coordinates of the traverse stations.

(d) Compute the distance, azimuth and reduced bearing of each line using the final coordinates.

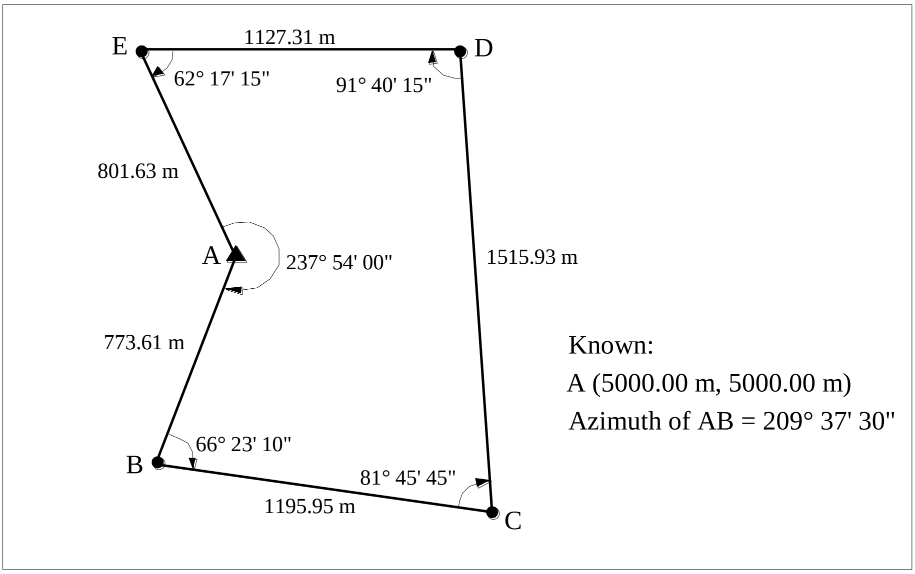

FIGURE 7.15: Traverse of Example 7.9.

SOLUTION:

Line	Preliminary Azimuth	Correction	Corrected Azimuth
AB	209° 37' 30"		
$+\hat{B}$	66 23 10		
	276 00 40		
	- 180		
BC	96 00 40	- 5"	96° 00' 35"
$+\hat{C}$	81 45 45		
	177 46 25		
	+180		
CD	357 46 25	- 10"	357° 46' 15"
$+\hat{D}$	91 40 15		
	449 26 40		
	-180		
DE	269 26 40	- 15"	269° 26' 25"
$+\hat{E}$	62 17 15		
	331 43 55		
	-180		
EA	151 43 55	- 20"	151° 43' 35"
$+\hat{A}$	237 54 00		
	389 37 55		
	-180		
AB	209 37 55	- 25"	209° 37' 30"
	209 37 30		

Closure error = + 25"
Correction per angle = -25"/5 = -5"

Note: It is also possible to correct the angles by comparing their sum to (n-2).180 = 540°, correcting the angles individually, and then recalculating the azimuths again.

TABLE 7.2: Preliminary coordinates.

Station	Corrected Azimuth (α_{ij})	Distance d_{ij} (m)	Departure $\Delta y_{ij} = d_{ij} \sin\alpha_{ij}$	Latitude $\Delta x_{ij} = d_{ij} \cos\alpha_{ij}$	Preliminary coordinates y	x
A					5000.00	5000.00
	209° 37′ 30″	773.61	-382.41	-672.48		
B					4617.59	4327.52
	96° 00′ 35″	1195.95	1189.38	-125.21		
C					5806.97	4202.31
	357° 46′ 15″	1515.93	-58.96	1514.78		
D					5748.01	5717.09
	269° 26′ 25″	1127.31	-1127.26	-11.01		
E					4620.75	5706.08
	151° 43′ 35″	801.63	379.72	-705.99		
A					5000.47	5000.09

$$\sum d_{ij} = 5414.43\,\text{m}$$

Closure error: $\varepsilon_y = +0.47$ m

$\varepsilon_x = +0.09$ m

$$\text{Linear error of closure} = \sqrt{(0.47)^2 + (0.09)^2} = 0.48\,\text{m}$$

$$\text{Relative error of closure} = \frac{1}{5414.43/0.48} \cong \frac{1}{11{,}000}$$

TABLE 7.3: Corrected coordinates.

Station	Cumulative Distance	y-coordinate Preliminary	y-coordinate Correction	y-coordinate Final	x-coordinate Preliminary	x-coordinate Correction	x-coordinate Final
A	0	5000.00	0	5000.00	5000.00	0	5000.00
B	773.61	4617.59	-0.07	4617.52	4327.52	-0.01	4327.51
C	1969.56	5806.97	-0.17	5806.80	4202.31	-0.03	4202.28
D	3485.49	5748.01	-0.30	5747.71	5717.09	-0.06	5717.03
E	4612.80	4620.75	-0.40	4620.35	5706.08	-0.08	5706.00
A	5414.43	5000.47	-0.47	5000.00	5000.09	-0.09	5000.00

TABLE 7.4: Final Results.

Line	Final Distance	Final Azimuth	Final Reduced Bearing
A – B	773.65	209° 37′ 45″	S 29° 37′ 45″ W
B – C	1195.86	96° 00′ 39″	S 83° 59′ 21″ E
C – D	1515.90	357° 45′ 57″	N 2° 14′ 03″ W
D – E	1127.41	269° 26′ 22″	S 89° 26′ 22″ W
E – A	801.60	151° 43′ 52″	S 28° 16′ 08″ E

EXAMPLE 7.10:

A traverse starts from the line AB and goes through points C, D and E. Given that:

- The coordinates of points A and B are:

	East or Y (m)	North or X (m)
A	8300.40	6310.30
B	8358.30	6031.73

- The angles measured in a clockwise direction are:
  $A\hat{B}C = 92° \ 30' \ 10''$
  $B\hat{C}D = 164° \ 18' \ 20''$
  $C\hat{D}E = 94° \ 55' \ 48''$
- The length of BC = 60.00 m, and
- The reduced bearing of line DE = N 20° W

Calculate:

a) The corrected azimuth for all lines, and
b) The coordinates of point C.

SOLUTION:

a) The corrected azimuth of all lines:
The azimuth of line AB (α_{AB}):

$$\alpha_{AB} = \tan^{-1}\left(\frac{y_B - y_A}{x_B - x_A}\right) = \tan^{-1}\left(\frac{8358.30 - 8300.40}{6031.73 - 6310.30}\right)$$

$$= \tan^{-1}\left(\frac{57.90}{-278.57}\right) = -11° \ 44' \ 30'' + 180° = 168° \ 15' \ 30''$$

The 180° was added because line AB lies in the second quadrant.

The known azimuth of line DE = 360° - 20° = 340°
The calculation of the corrected azimuths is done as follows:

Line	Preliminary Azimuth	Correction	Corrected Azimuth
AB	168° 15' 30"		
$+\hat{B}$	92 30 10		
	260 45 40		
	- 180		
BC	80 45 40	+ 4"	80° 45' 44"
$+\hat{C}$	164 18 20		
	245 04 00		
	-180		
CD	65 04 00	+ 8"	65° 04' 08"
$+\hat{D}$	94 55 48		
	159 59 48		
	+180		
DE	339 59 48	+ 12"	340° 00' 00"

Closure error = - 12"

Correction per angle = +12"/3 = +4"

b) The coordinates of point C are:

$$y_C = y_B + BC \sin AZ_{BC} = 8358.30 + 60.00 \text{ x } \sin(80^\circ\, 45'\, 44'')$$
$$= 8417.52 \text{ m}$$
$$x_C = x_B + BC \cos AZ_{BC} = 6031.73 + 60.00 \text{ x } \cos(80^\circ\, 45'\, 44'')$$
$$= 6041.36 \text{ m}$$

PROBLEMS

7.1 Figure 7.16 shows the locations of the beginning point 1 and end point 2 of a new tunnel with respect to four survey points: A, B, C and D . The coordinates of two control points A & D and the design coordinates of points 1 & 2 are given below :

POINT	Y-COORDINATE	X-COORDINATE
A	112598.38 m	211121.62 m
D	114985.01 m	209880.34 m
1	113525.91 m	210475.76 m
2	114624.23 m	210837.84 m

The following measurements were made to points B & C:
- Horizontal Distance AB = 1746.76 m,
- Horizontal Distance DB = 1572.87 m
- Horizontal Angle CDA = 29° 59' 35",
- Horizontal Angle DAC = 43° 41' 9.8"

Calculate:

a. The coordinates of point B.
b. The coordinates of point C.
c. The azimuth and distance of the line from point D to point 2.
d. The coordinates of point M located on the centerline of the tunnel mid-way between points 1 & 2.
e. The coordinates of points E and F which are at distances 10.00 m and 15.00 m from point 1 along line 1-2 with offsets +3.00 m and -4.00 m respectively.
f. Calculate the distance and offset of point B from line 1-2.

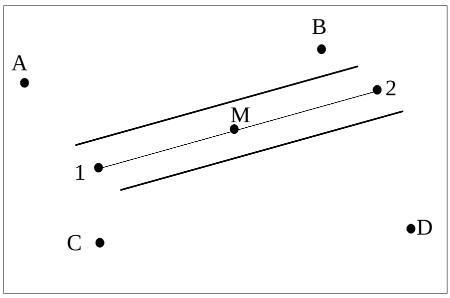

FIGURE 7.16

7.2 Two circles intersect at points 1 & 2. The coordinates of the centers of these circles are (100.00,100.00) & (200.00,250.00) m, while the radii are 100.00 m and 120.00 m respectively. Calculate the coordinates of the points of intersection 1 & 2.

7.3 In a bridge construction project, the position of the pile-driving barge is determined by the method of resection. A theodolite is set up on the barge, and angles are measured to three control points: A, B and C (Figure 7.17). The coordinates of the control points are:

Control Point	Y-Coordinate	X-Coordinate
A	146298.36 m	92175.27 m
B	143721.45 m	89332.15 m
C	150036.79 m	91628.21 m

The following angles are measured at the barge position P:

BPA = 37° 23' 08''
APC = 33° 47' 36''

Compute the coordinates of the barge position P.

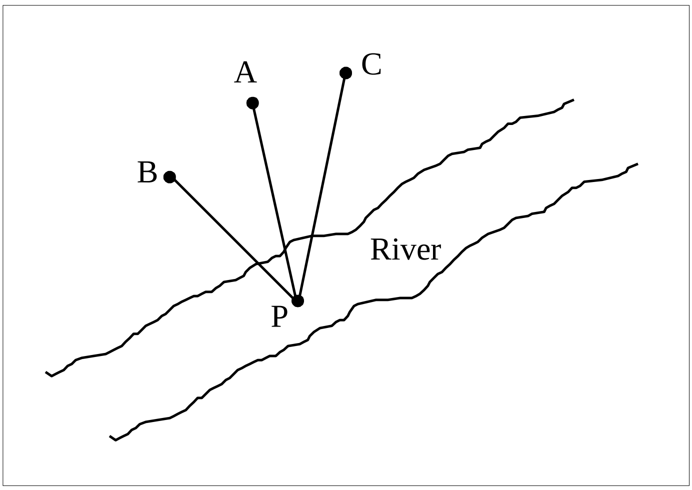

FIGURE 7.17

7.4 For a certain land parcel, the total station was stationed at point 100 and the following measurements were taken:

Station No. (محطة الجهاز): 100 H.I. (ارتفاع الجهاز): 1.60 (m)

التاريخ: 23/ 7 / 2003 اسم المساح : نضال صادق تميم

رقم النقطة Point #	المسافة المائلة S (m)	الزاوية الأفقية (H.A.)			الزاوية السمتيـة (Z.A.)			ارتفاع العاكس HT (m)	ملاحظات Notes
		°	'	"	°	'	"		
1	116.85	0	00	00	93	20	55	1.55	+ (صليب)
2	112.29	12	51	07	90	33	25	2.75	سلسال
3	122.75	18	05	03	88	11	42	3.60	A.I.
4	145.22	10	58	47	89	55	11	1.55	سلسال
5	150.07	356	26	24	84	23	24	1.55	A.I.
6	130.19	15	00	00	95	17	00	3.00	ع.ك

Given that the elevation and coordinates of point 100 are (Y = 500.00 m, X = 500.00 m, H = 200.00) and that the azimuth of the line from 100 to 1 is 0° 00' 00", calculate the elevations and coordinates of all the points.

7.5 In problem 7.4, given that the grid coordinates of points 100 & 1 are as follows:

100 (170320.00 , 185814.50), 1 (170340.26 , 185929.38).

Calculate the transformed coordinates of points 2, 3, 4, 5 and 6.

7.6 The coordinates of points A & B are:

Point	Y-coordinate	X-coordinate
A	368120.30 m	172315.28 m
B	367813.20 m	172603.13 m

A traverse starting at B continues over the points C & D. Compute the coordinates of points C & D if:

- Clockwise angle at B = 95° 16' 30" ,
- Clockwise angle at C = 285° 32' 20"
- Distance BC = 212.50 m,
- Distance CD = 172.30 m

What should be available to check the calculated coordinates?

7.7 Compute the adjusted coordinates of the points of the following closed loop traverse (Figure 7.18):

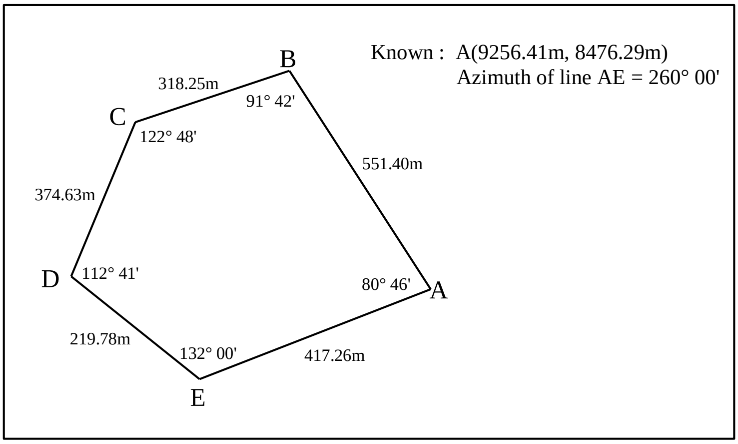

FIGURE 7.18

If one of the five angles in the above figure was wrong (i.e., has a blunder), explain how to find this erroneous angle.

7.8 A five-sided loop traverse has been computed giving the coordinate differences (departures and latitudes) shown in the next table:

Side	ΔY (m)	ΔX (m)
AB	-43.62	-61.39
BC	+70.45	-34.71
CD	+50.85	+48.10
DE	-23.01	+73.37
EA	-53.73	-25.86

(a) Determine the error in both Y and X directions as well as the linear misclosure of the traverse.

(b) The linear misclosure indicates a blunder of +1 m in the length of one of the sides of the traverse. Find which side contains the blunder and, after eliminating its effect, re-compute the requirements of part (a).

7.9 A closed connecting traverse starts at point 221B(y=166238.10m, x=180067.29m) with the total-station directed towards point 203M(y=176000.14m, x=178658.08m), and goes over points 100 and 200, and closes at point 693W(y=164095.24m, x=177510.91m) with the total-station directed towards point 679W (y=168816.43m, x=173371.62m) (Figure 7.19). Does this traverse comply with the specifications of the Department of Surveying in the West Bank? If yes, calculate the corrected coordinates of all points.

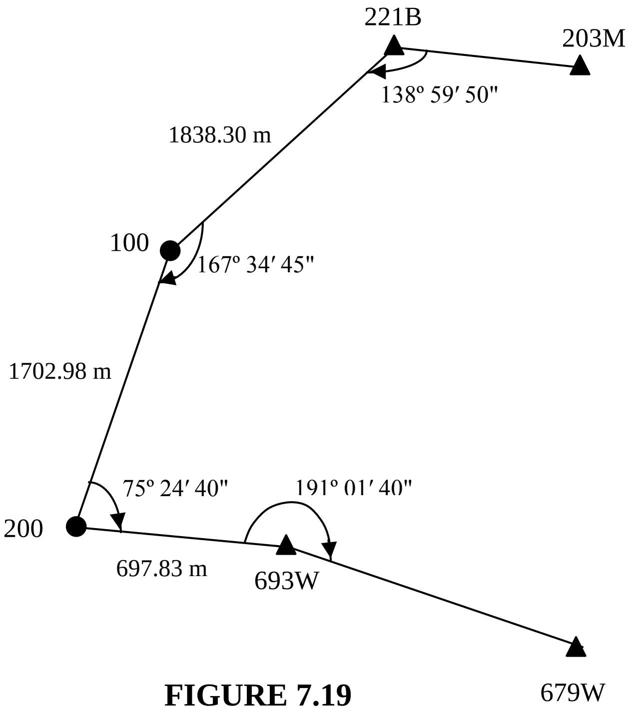

FIGURE 7.19