5 ANGLES, DIRECTIONS, AND ANGLE MEASURING EQUIPMENT

5.1 INTRODUCTION

Basically, surveying aims to determine the relative location of points on or near the surface of the earth. To locate a point, both distance and angular measurements are usually required. Such angular measurements are either horizontal or vertical, and they are most commonly accomplished with instruments called transits or theodolites.

5.2 HORIZONTAL, VERTICAL AND ZENITH ANGLES

In surveying, angles are measured either in a horizontal plane, yielding horizontal angles, or in a vertical plane, yielding vertical angles. In Figure 5.1, points A, B and C are three points located on the earth's surface. Points A', B' and C' are the projections of points A, B and C onto a horizontal plane. Angles $A'\hat{B}'C'$, $B'\hat{C}'A'$ and $C'\hat{A}'B'$ are the horizontal angles.

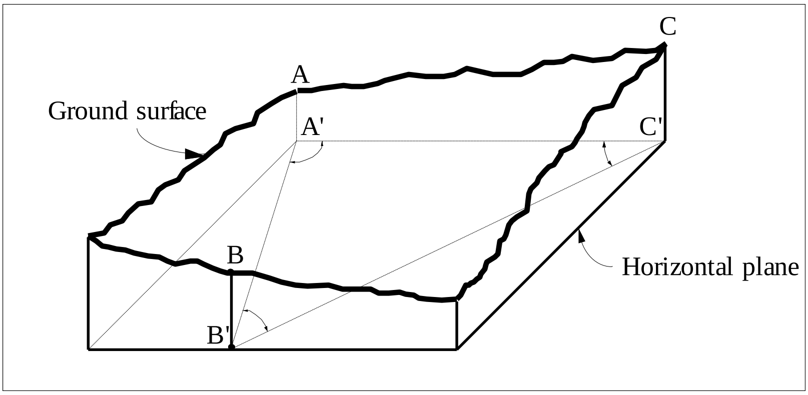


FIGURE 5.1: Horizontal angles.

A vertical angle, on the other hand, is measured in a vertical plane using the horizontal plane as a reference plane. When the point being sighted on is above the horizontal plane, the vertical angle is called an angle of elevation and is considered to be a positive angle. When the point being sighted on is below the horizontal plane, the angle is termed an angle of depression and is considered a negative angle. The value of a vertical angle can range from -90° to +90° (Figure 5.2).

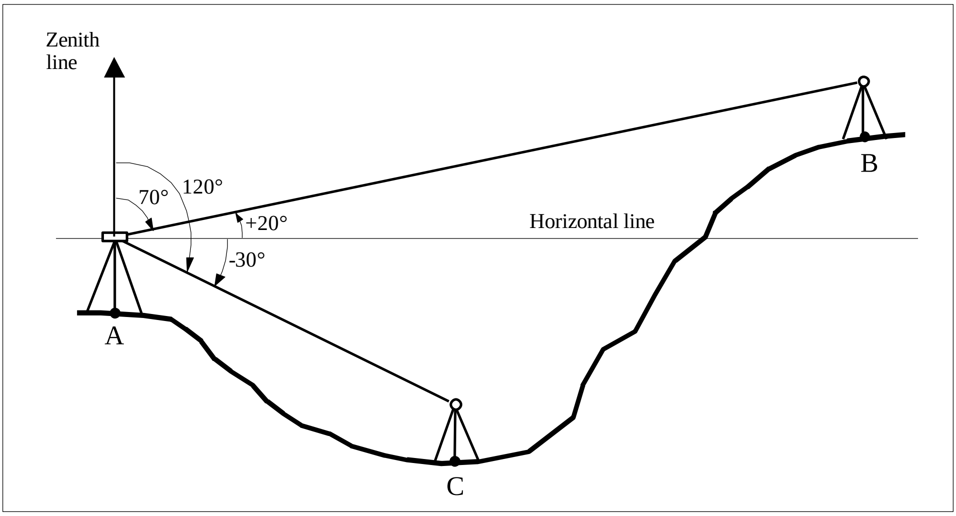


FIGURE 5.2: Vertical and zenith angles.

A zenith angle is also measured in a vertical plane but uses the overhead extension of the plumb line as a reference line (also known as zenith direction). Its value ranges from 0° to 180°. In Figure 5.2, the vertical angles measured at station A to target points B and C are 20° and -30°, respectively. The corresponding zenith angles are 70° and 120°. As can be seen, the summation of the vertical and zenith angles of any line is equal to 90°.

5.3 REFERENCE DIRECTION

It is convenient to choose a reference line to which directions of all the lines of a survey are referenced. Showing this reference direction on the map is also very basic to the map-reader to know how the different elements of the map are oriented in reality. Three different types of direction lines are traditionally used in surveying. These are the true or geographic north, the magnetic north and the assumed north.

5.3.1 TRUE OR GEOGRAPHIC NORTH

The true or geographic north at a point is defined as the direction of the line that connects this point to the North Pole of the earth. This line usually coincides with the great circle (also termed meridian) that passes through the point and the true (geographic) north and south poles of the earth (Figure 5.3). The north direction is usually used in the northern hemisphere, while the south direction is used in the southern hemisphere instead. Since this direction is steady in a given place and does not change with time, it is usually preferred to be used for referencing maps.

The geographic north at a point can be determined either approximately or precisely through astronomical and global positioning system measurements. It can also be determined using the coordinate geometry principles as will be explained later in chapter 7. However, two approximate methods will be described here.

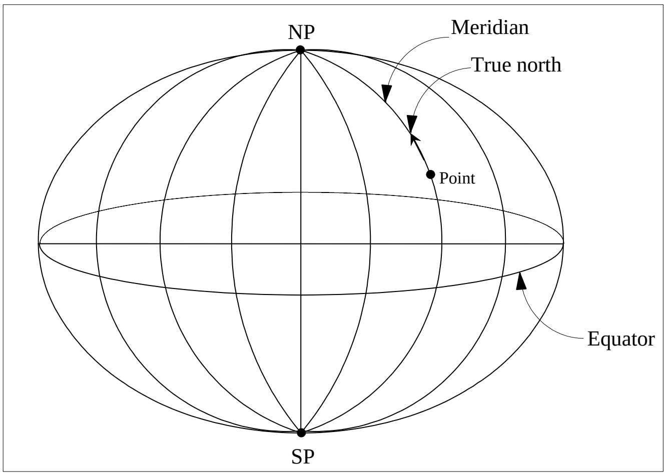


FIGURE 5.3: The true (geographic) north.

a) The watch method:

At noon (around 12:00), the direction of the line joining the observer's eye with the sun points toward the south. At other times, the south direction is determined using a watch with handles (non-digital). To do this, hold the watch in such a way that the hours handle points towards the sun. Bisect the angle between the hour handle and the line joining the center of the watch with the number 12 on the watch (number 1 for summer saving time). This bisecting line will point towards the south, and its opposite should, of course, be towards the north direction (Figure 5.4).

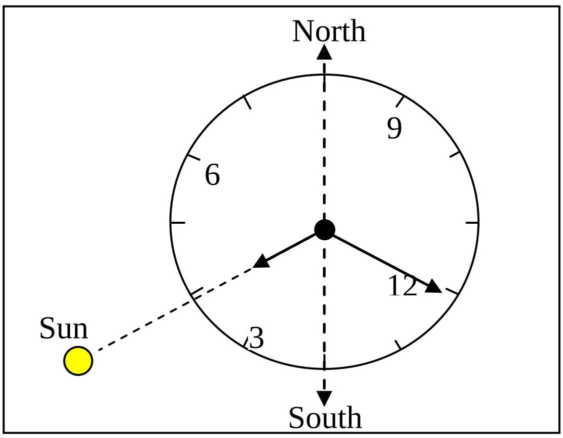


FIGURE 5.4: The watch method for determining the north direction.

b) The shadow method:

In this method, a straight stick 2-3 m long is supported at one of its ends on the ground, and around two-thirds of its length it is supported on an X-like support as shown in figure 5.5. The stick is made to point approximately towards north, and a plumb bob is hung from the free end of the stick to project it down on the ground at point A. Next, and two hours before noon (around 10:00 am), we start watching the shadow of the stick. The end point of the shadow (point B in figure 5.5) is allocated, and a circular arc whose center is at A and radius equal to AB is made using a pin. After this point, the shadow of the stick will start getting shorter and shorter until noon when it will start getting longer again, but towards the other side of the arc. Watch the shadow until it touches the circular arc at point C (should be around 2:00 pm). Bisect line BC at D. The north direction will then coincide with the line AD.

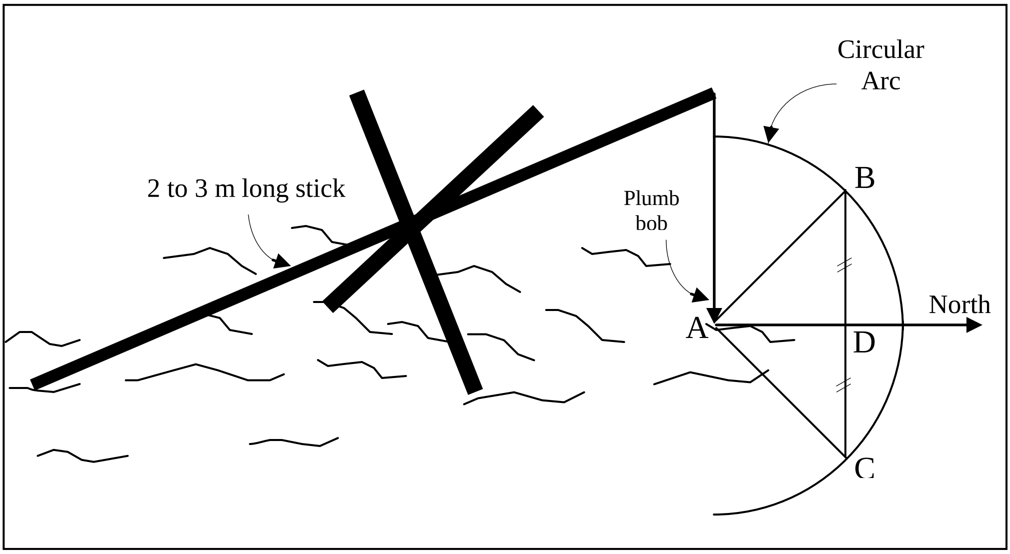


FIGURE 5.5: The shadow method for determining the north direction.

5.3.2 MAGNETIC NORTH

A magnetic needle, when let to come to rest in the earth's magnetic field (away from the effect of other magnetic fields), will point in the direction of the magnetic north pole of the earth. The direction in which the magnetic needle rests is called the magnetic meridian, and this meridian usually does not coincide with the true meridian. They make a small angle with each other called the magnetic declination, which could be to the east or west of the true meridian. Thus a declination of $\gamma°$W means that the magnetic meridian is $\gamma°$ west of the true meridian (Figure 5.6). As shown in this figure, line AB deflects from the geographic and magnetic norths by the angles α and β respectively.

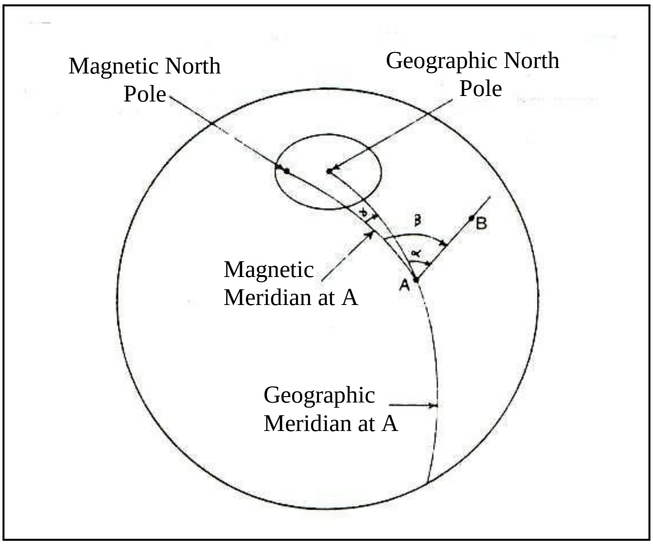
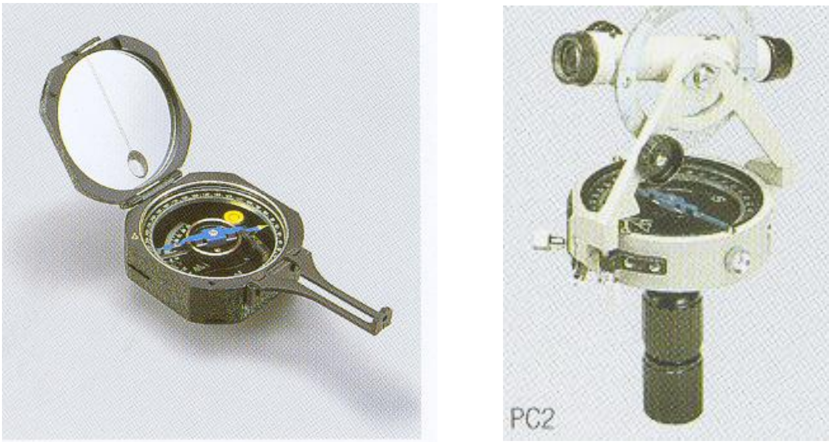


FIGURE 5.6: Relationship between true (geographic) and magnetic norths.

The value of the magnetic declination varies from one location to another, and at a given location, also changes with time. For this reason, it is preferable to reference maps according to the fixed true or geographic north. Usually, maps are prepared which give the value of the magnetic declination at a given location in a given year. If the angle that a line makes with the magnetic north is measured using a magnetic compass (such that shown in Figure 5.7), and if the magnetic declination at this point is known, then the direction of the true north at this location can be found.



(a) Pocket compass (b) Surveyor's compass

FIGURE 5.7: Magnetic compass.

5.3.2 ASSUMED NORTH

If neither the geographic north, nor the magnetic north is known in a certain local area for which the surveyor is making measurements, a line can be arbitrarily chosen and assumed to be in the direction the north. This line is called the assumed north. If later, the angle between the assumed and geographic or magnetic norths is known, then all the obtained survey measurements including any prepared maps can be corrected by simply rotating them to point in the right direction.

5.4 REDUCED BEARING OF A LINE

The reduced bearing of a line is the acute angle that the line makes with the reference meridian (whether being geographic or magnetic). It is expressed as "North (or South) so many degrees to the East (or West)". Thus the true reduced bearing of line OA in Figure 5.8 is N 70° E and for line OC is S 85° 36' 20" W. The true reduced bearing of a line never exceeds 90° and is never referenced to the east or west line.

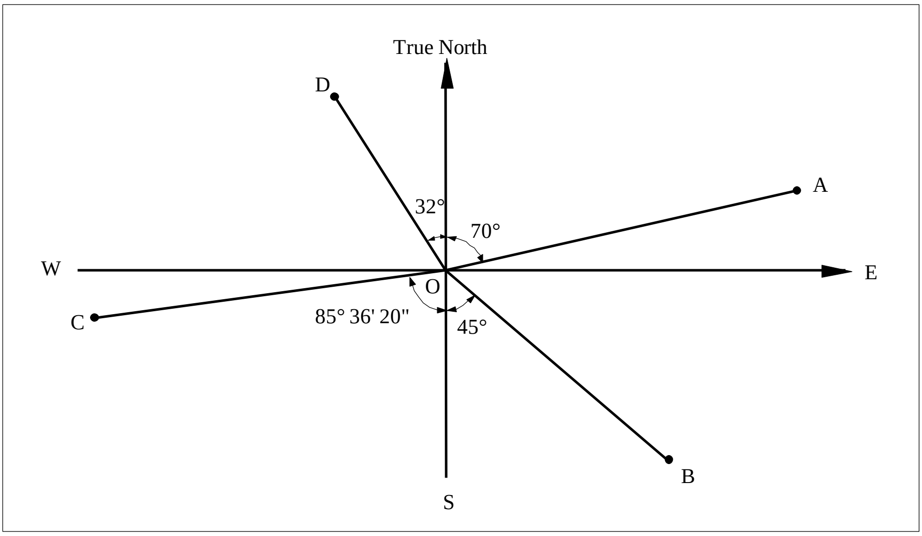


FIGURE 5.8: Reduced bearings.

The magnetic reduced bearing of a line is the acute angle that the line makes with the magnetic meridian. In Figure 5.9, the magnetic reduced bearings of lines OE, OF, OG and OH are: N 40° E, S 63° E, S 81° W and N 47° W, respectively. Since the magnetic meridian is 3º east of the true meridian, the corresponding true reduced bearings for lines OE, OF, OG and OH are: N 43° E, S 60° E, S 84° W and N 44° W, respectively.

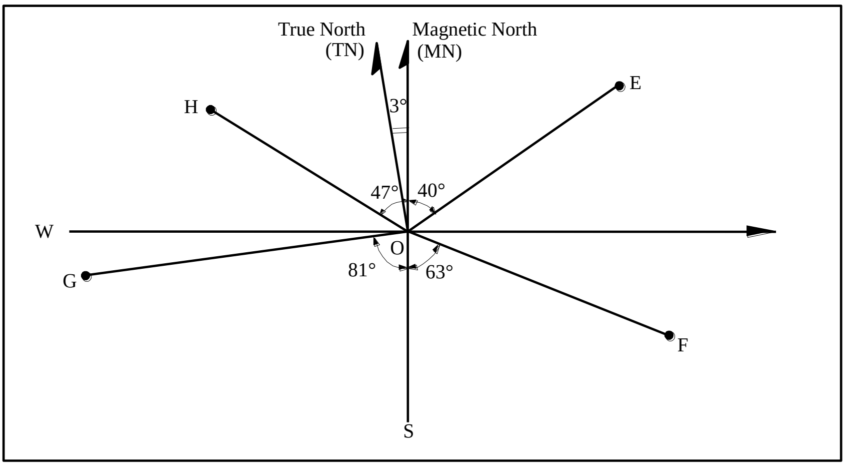


FIGURE 5.9: Magnetic bearings.

5.5 AZIMUTH OR WHOLE CIRCLE BEARING

The direction of a line may also be expressed by its azimuth. The azimuth of a line is the clockwise horizontal angle that the line makes with the north end of the reference meridian. For example, the azimuths of lines OA, OB, OC and OD in Figure 5.10 are: 70°, 135°, 265° 36' 20" and 328° respectively relative to the true north.

It is obvious from the previous values that the magnitude of an azimuth ranges from 0° to 360°. Sometimes azimuths are referred to the south (especially in the southern hemisphere) but all azimuths in a surveying project should refer to the same end of the meridian. In this book, the term azimuth will refer to the north end of the meridian.

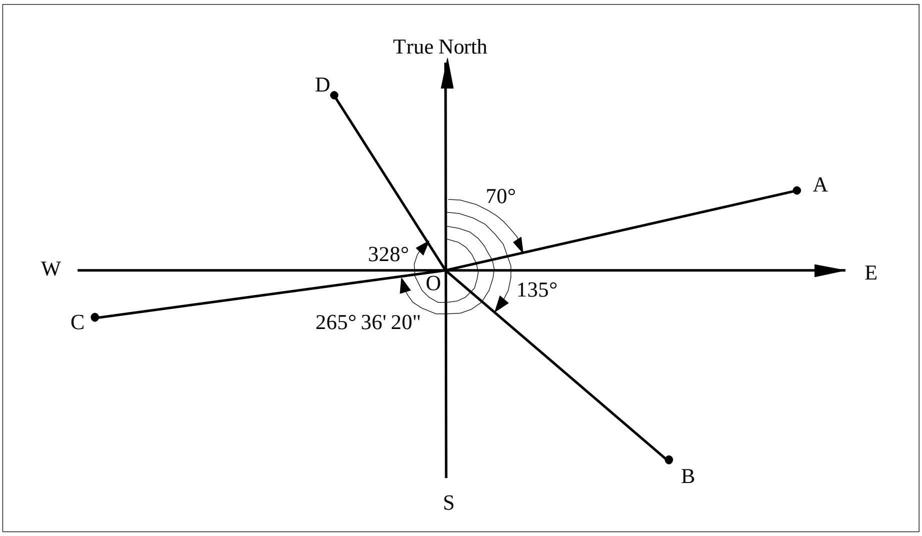


FIGURE 5.10: Azimuth of a line.

5.6 BACK REDUCED BEARING AND BACK AZIMUTH

The back reduced bearing of a line OA is the reduced bearing of the same line going from A to O. In Figure 5.11, the true reduced bearing of line OA going from O to A is N 60° E. The back reduced bearing of the same line

OA (but going from A to O) is S (60° +θ) W, where θ is an angle due to the convergence of the meridians. The magnitude of θ depends on the east-west distance between the two end points (A and O) and the average latitude along the line. For short lines, θ will be small and can be safely ignored. Similarly, the back azimuth of line OA in Figure 5.11 is 60° + 180° + θ = 240° + θ .

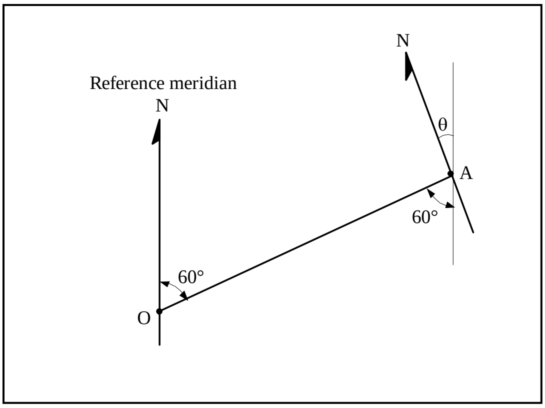


FIGURE 5.11: Back bearing.

5.7 PRINCIPAL ELEMENTS OF AN ANGLE-MEASURING INSTRUMENT

All angle-measuring instruments have the following basic elements:

1 - A line of sight,
2 - A horizontal axis about which the line of sight revolves,
3 - A vertical axis about which the line of sight can be rotated,
4 - A graduated horizontal circle for measuring horizontal angles, and
5 - A graduated vertical circle for measuring vertical angles.

These five basic elements are illustrated in Figure 5.12. When the instrument is in perfect adjustment and ready for use, the following conditions exist:

1. The line of sight is perpendicular to the horizontal axis.
2. The horizontal axis is perpendicular to the vertical axis.
3. The horizontal axis is perpendicular to the vertical circle.
4. The vertical axis is perpendicular to the horizontal circle.

Figure 5.13 shows an example of a scale-reading (manual) theodolite, while Figure 5.14 shows an example of a digital theodolite.

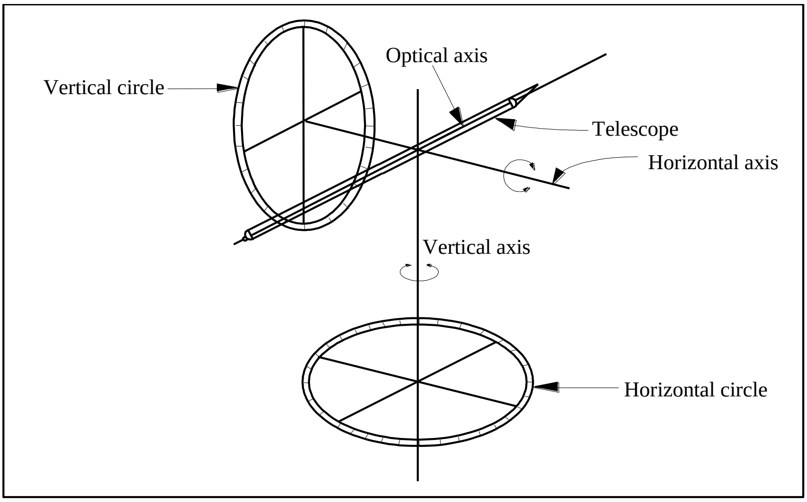


FIGURE 5.12: Principal elements of an angle measuring instrument.

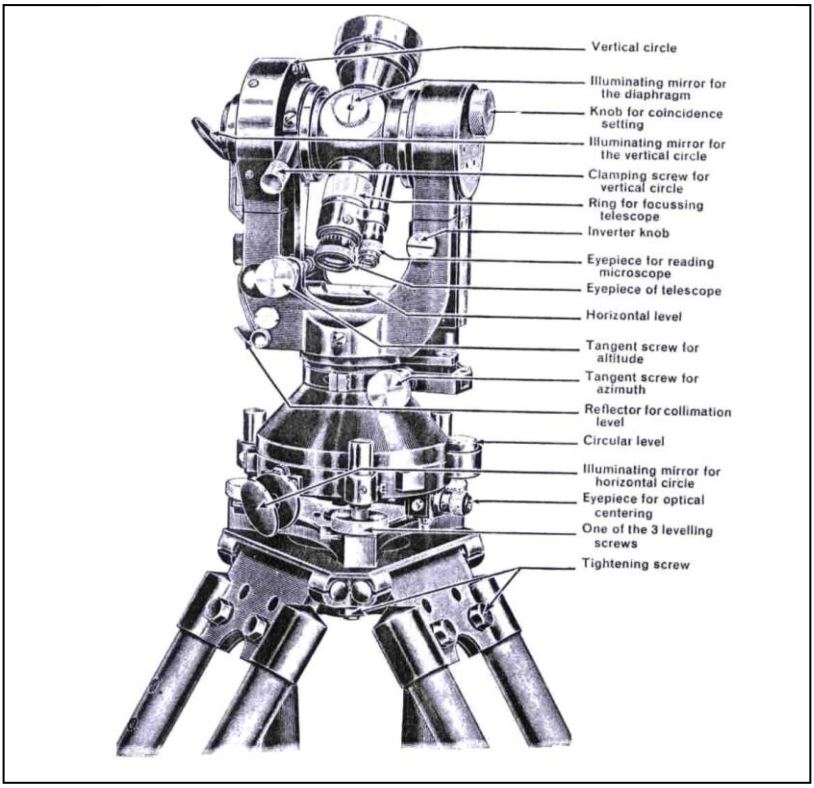


FIGURE 5.13: An example of a scale-reading (manual) Wild-T2 theodolite.

FIGURE 5.14: An example of a digital theodolite.

5.8 SETTING UP A THEODOLITE

A theodolite is usually equipped with an optical plummet for centering over a survey point, a bull's-eye bubble level for preliminary leveling, and one plate bubble for precise leveling of the instrument. The following procedure is used to set up a theodolite equipped with these features and an extension leg tripod:

1. Set the extension legs of the tripod to the proper length for the instrument operator.

2. Attach a plumb bob to the tripod head. Although an optical plummet is available with the instrument, it is usually more convenient to perform the preliminary centering over the survey point using a plumb bob.

3. Keeping the tripod head level, set the tripod approximately over the survey point. Step on the spur of each tripod leg to drive it firmly into the ground.

4. Center the plumb bob over the survey point by adjusting the length of the tripod legs. If the tripod head looks excessively off level, lift the tripod from the ground and repeat steps 3 and 4.

5. Mount the theodolite on the tripod, and level it with the bull's-eye bubble (see section 4.6).

6. If steps 3 and 4 have been performed properly, the survey point should be within the view of the optical plummet. The theodolite is then exactly centered over the survey point by loosening the fastening screw on the tripod head and moving the theodolite around slowly. During this operation the theodolite should always be held firmly by its standard with one hand. Once it is centered properly, the fastening screw is tightened firmly.

7. Check to see if the bull's-eye bubble is still centered. If it has moved off center, repeat steps 6 and 7.

8. Rotate the theodolite so that its plat bubble is parallel to a line joining any two of the foot screws, say screws A and B as shown in Figure 5.15a. The level bubble is centered using the same two foot screws.

9. The theodolite is rotated 180° so that the plate bubble assumes the position shown in Figure 5.15b. If the plate level is in perfect adjustment, the bubble should remain centered. If the bubble has

moved off center, it is brought halfway back towards the center using the same two foot screws (A and B).

10 Rotate the theodolite so that the plate bubble is now perpendicular to the line joining the two previous foot screws (A and B) (see Figure 5.15c). Center the bubble with the third foot screw (C) only.

11. Rotate the theodolite 180° so that the bubble assumes the position shown in Figure 5.15d. If the bubble has moved off center, bring it back halfway using the third foot screw (C).

12. Check the optical plummet to make sure that the instrument is still properly centered over the survey point. If not, repeat steps 6 to 12.

13. The theodolite is now ready for making angle measurements.

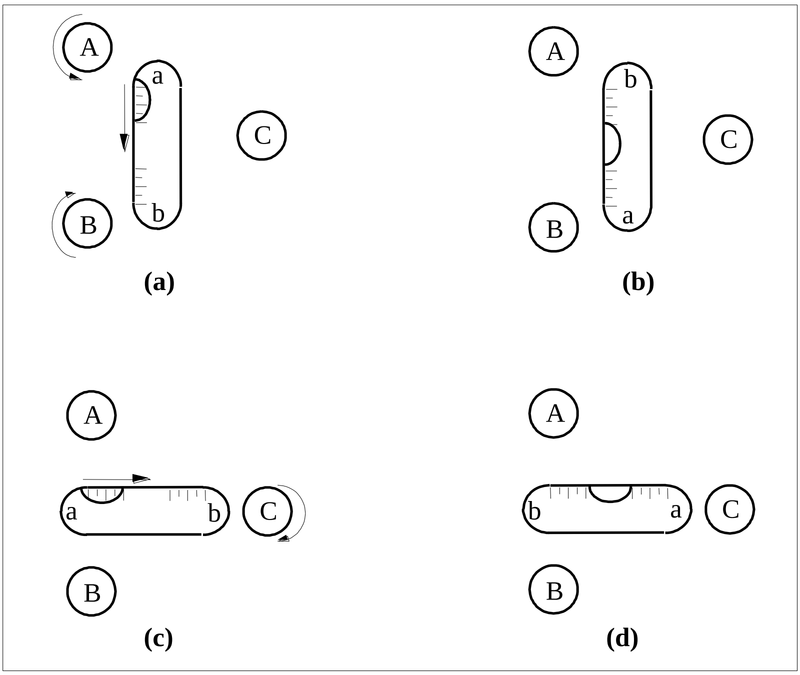


FIGURE 5.15: Leveling a three foot screw instrument.

5.9 MEASUREMENT OF A HORIZONTAL ANGLE

A horizontal angle $A\hat{B}C$ (Figure 5.16) can be simply measured in the field as follows:

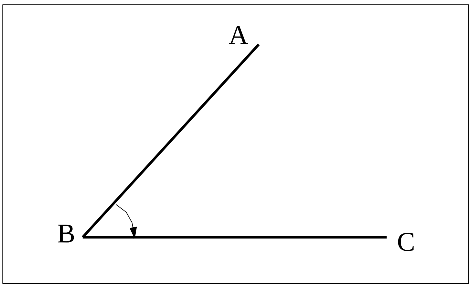


FIGURE 5.16: A horizontal angle.

1) Set up the theodolite over station B according to the steps given in section 5.8.

2) Direct the telescope of the theodolite to sight station A, and set the horizontal circle to read 0° 00' 00", or simply record the initial reading of the horizontal circle (e.g. 50° 20' 15").

3) Direct the telescope in a clockwise direction to sight station C and record the horizontal circle final reading (e.g. 120° 30' 47").

4) The value of the angle will be the difference between the final and the initial readings of the horizontal circle. For the above readings, the angle will be: 120° 30' 47" - 50° 20' 15" = 70° 10' 32".

However, it is possible that small errors will result due to imperfect leveling of the instrument or due to the maladjustment of the instrument when the three principal axes are not mutually perpendicular to each other. These errors can be minimized by using the following procedure for measuring the angle $A\hat{B}C$:

1) Set up the theodolite over station B.

2) With the theodolite in the direct (D) position, backsight to station A and set the horizontal circle to read 0° 00' 00".

3) Rotate the telescope in a clockwise direction and foresight to station C. Read and record the horizontal circle (e.g. 112° 50' 18").

4) Rotate the telescope through 180° in the vertical plane (i.e. about the horizontal axis), and then through 180° in the horizontal plane to sight on station C again. The horizontal circle is read and recorded as reverse (R) reading (e.g. 292° 50' 30"). This reading should differ from the reading obtained from step 3 by 180° plus or minus a few seconds.

5) Rotate the telescope in a counterclockwise direction and backsight to station A. Read and record the horizontal circle reading (e.g., 180° 00' 06").

6) Calculate the angle in both direct (D) and reverse (R) positions as follows:
Direct position: Angle = 112° 50' 18" - 0° 00' 00" = 112° 50' 18"
Reverse position: Angle = 292° 50' 30" - 180° 00' 06" = 112° 50' 24"

7) The value of the angle will be the average of the two values obtained in step 6, i.e.

$$\text{Angle} = \frac{112°\ 50'\ 18'' + 112°\ 50'\ 24''}{2} = 112°\ 50'\ 21''$$

Note 1: In some books, the terms face-right and face-left are used instead of direct and reverse positions

Note 2: When measuring a horizontal angle such as the angle $A\hat{B}C$, it is necessary that the theodolite be stationed exactly over point B. If, however, the theodolite were off station B by a small distance (say positioned at B' in Figure 5.17), the horizontal angle $A\hat{B}'C$ would be mistakably measured instead of angle $A\hat{B}C$.

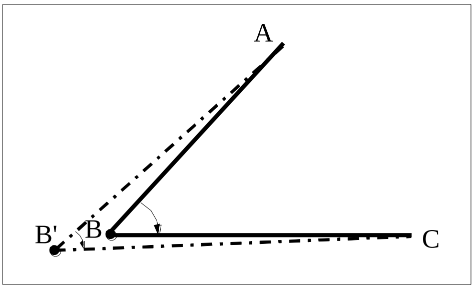


FIGURE 5.17: A wrong setup of the theodolite over station B.

5.10 MAIN APPLICATIONS OF THE THEODOLITE

As explained in the previous sections, the main use of the theodolite is to measure horizontal and vertical angles. These angles, in turn, are used for the calculation of object heights, distances, as well as, point coordinates. The following sections will deal with the main applications of the theodolite. Other applications will be dealt with in the next two chapters.

5.10.1 MEASUREMENT OF OBJECT HEIGHTS

The vertical or zenith angles measured with the theodolite can be used in conjunction with a distance measuring equipment for the measurement of object heights. Two cases can be distinguished here:

1) The determination of the elevation of a point whose horizontal distance from the theodolite can be directly measured.

2) The *determination* of the elevation of a point whose horizontal distance from the theodolite is difficult to be directly measured.

<u>CASE (1):</u>* *Points whose horizontal distance from the theodolite is directly measured

Assume that in Figure 5.18, the elevation of point C is to be determined by measuring the vertical angle α and the horizontal distance D. Given that the elevation of point A is known (H_A), and the height of the theodolite above A is (i), then the elevation of point C (H_C) is:

$$H_C = H_A + i + D \cdot \tan\alpha \qquad \text{(5.1)}$$

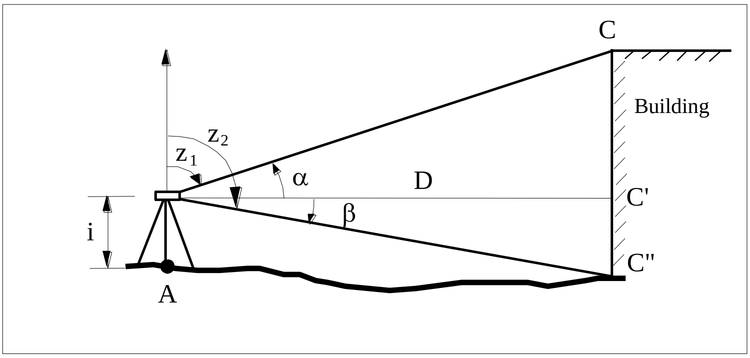


FIGURE 5.18: An object whose horizontal distance (D) from the theodolite can be measured directly.

If point C and C" lie on a vertical line, such as being a vertical edge of a building, then the height difference (ΔH) between these two points can be calculated as follows:

$$\Delta H = D \cdot \tan\alpha - D \cdot \tan\beta = D(\tan\alpha - \tan\beta)$$
$$= D\left(\frac{1}{\tan z_1} - \frac{1}{\tan z_2}\right) \qquad \text{(5.2)}$$

Where α is the vertical angle to point C

β is the vertical angle to point C". (*Notice that β is negative in Figure 5.18*),

z_1 and z_2 are the zenith angles to points C and C" respectively.

CASE (2): ***Points whose horizontal distance from the theodolite is difficult to measure***

Assume that the object whose elevation is to be determined is the top of a minaret (C). In this case, it is difficult to measure the horizontal distance D between the theodolite and the top of the minaret. To overcome this problem, the following procedure is used (Figure 5.19):

1) Choose two nearby points A and B such that the horizontal triangle A'B'C' is approximately equilateral.

2) Using a distance measuring equipment such as a tape, measure the horizontal distance AB.

3) Using the theodolite, measure the two horizontal angles a and b. The angle c is then calculated as follows: c = 180° - a - b

4) From the sine law, calculate the horizontal distances A'C' and B'C':

$$\frac{A'C'}{\sin b} = \frac{B'C'}{\sin a} = \frac{A'B'}{\sin c}$$

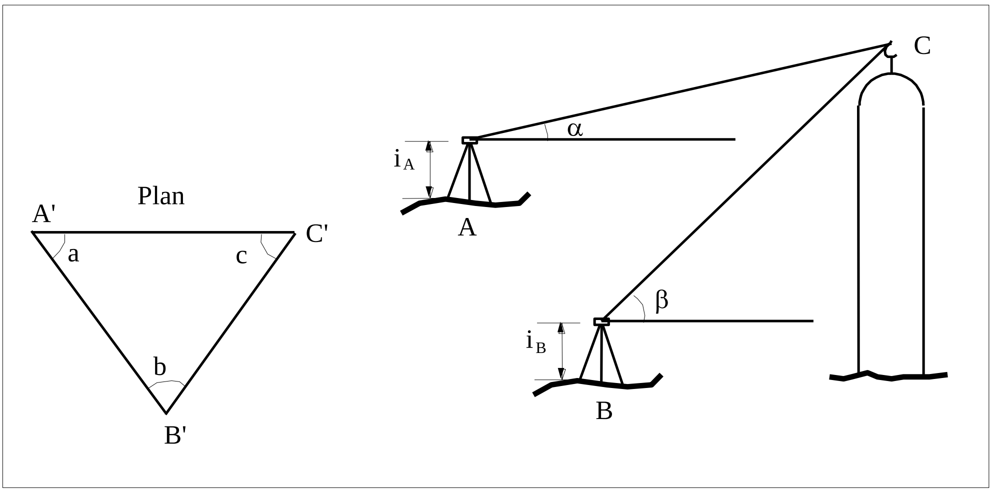


FIGURE 5.19: An object whose horizontal distance from the theodolite is difficult to be measured directly.

$\Rightarrow$

$$A'C' = \frac{A'B'}{\sin c} \cdot \sin b$$
$$B'C' = \frac{A'B'}{\sin c} \cdot \sin a \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (5.3)$$

5) Measure the two vertical angles α and β as shown in Figure 5.19 (z_A and z_B if the theodolite measures zenith angles instead of vertical angles).

6) The elevation of point C (H_C) can be calculated as follows:

$$H_C = H_A + i_A + A'C' \cdot \tan\alpha = H_A + i_A + \frac{A'C'}{\tan z_A} \qquad \ldots\ldots\ (5.4)$$

Or

$$H_C = H_B + i_B + B'C' \cdot \tan\beta = H_B + i_B + \frac{B'C'}{\tan z_B} \qquad \ldots\ldots\ (5.5)$$

5.10.2 TACHEOMETRY

In this branch of surveying, distances and elevation differences are determined from instrumental readings alone, these usually being taken with a specially adapted theodolite. The chaining operation is eliminated, and tacheometry is therefore very useful in broken terrain, e.g. river valleys, standing crops, etc., where direct linear measurements would be difficult and inaccurate.

There are several methods of tacheometry that include: the tangential method, stadia method, subtense bar method and the optical wedge method. Only the tangential and the stadia methods will be discussed here.

5.10.2.1 TANGENTIAL METHOD

In this method, both horizontal distances, as well as, elevation differences can be measured (Figure 5.20). To do this, the theodolite is set up at point A and two vertical angles θ and ϕ are measured on a staff held vertically at B, at two points such M and N respectively. Now, from Figure 5.20:

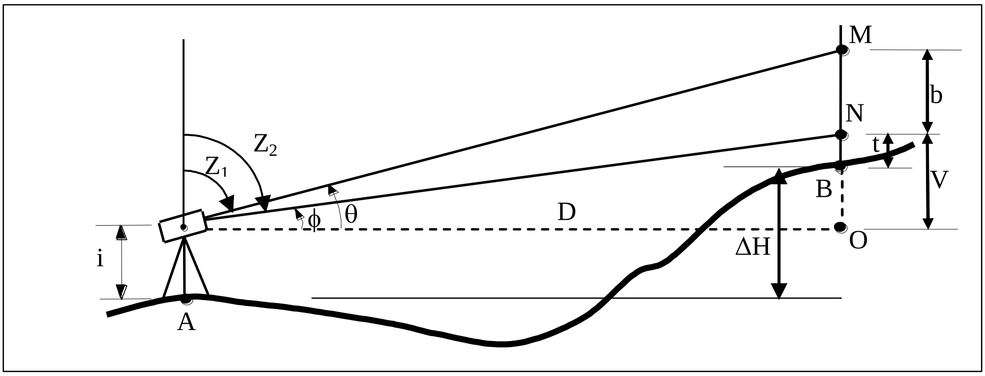


FIGURE 5.20: Tangential method.

$$OM = D \cdot \tan\ \theta$$
$$ON = D \cdot \tan\ \phi$$
$$OM - ON = b = D\left(\tan\theta\ -\ \tan\phi\right)$$

$$\Rightarrow \quad D = \frac{b}{\left(\tan\theta - \tan\phi\right)} = \frac{b}{\left(\frac{1}{\tan z_1} - \frac{1}{\tan z_2}\right)} \qquad \text{. . . . . (5.6)}$$

Where D is the horizontal distance between points A and B
b is the difference between the two staff readings

Also from Figure 5.20, the elevation difference (ΔH) between points A and B is calculated as follows:

$$\Delta H = i + V - BN = i + D \cdot \tan\phi - t \qquad \text{. . . . . (5.7)}$$

Where i is the theodolite height over A
t is the staff reading at N

EXAMPLE 5.1:

The following readings were taken on a staff held vertically at point B.

Vertical Angle	Staff Reading
6° 15' 20" ± 5"	3.50 ± 0.005 m
5° 10' 45" ± 5"	1.00 ± 0.005 m

If you know that the theodolite is 1.65 m above A,

(a) Calculate the horizontal distance and elevation difference between points A and B, as well as, their standard errors.
(b) Do you recommend the tangential method for precise surveying, and why?

SOLUTION:

(a) b = 3.50 - 1.00 = 2.50 m

$$D = \frac{b}{(\tan\theta - \tan\phi)} = \frac{2.50}{\tan(6° \ 15' \ 20'') - \tan(5° \ 10' \ 45'')} = 131.75 \text{ m}$$

$$\Delta H = i + D \cdot \tan\phi - t$$
$$= 1.65 + 131.75 \text{ x } \tan(5° \ 10' \ 45'') - 1.00 = 12.59 \text{ m}$$

From the Law of propagation of random errors:

$$\sigma_b^2 = (0.005)^2 + (0.005)^2 = 0.00005$$

For small angles θ and φ:

$$D = \frac{b}{(\tan\theta - \tan\phi)} \approx \frac{b}{\theta - \phi}$$, where θ and φ are in radian,

$$\sigma_D^2 = \left(\frac{\partial D}{\partial b}\right)^2 \cdot \sigma_b^2 + \left(\frac{\partial D}{\partial \theta}\right)^2 \cdot \sigma_\theta^2 + \left(\frac{\partial D}{\partial \phi}\right)^2 \cdot \sigma_\phi^2$$

$$= \left(\frac{1}{\theta - \phi}\right)^2 \cdot \sigma_b^2 + \left(\frac{-b}{(\theta - \phi)^2}\right)^2 \cdot \sigma_\theta^2 + \left(\frac{b}{(\theta - \phi)^2}\right)^2 \cdot \sigma_\phi^2$$

Substitute $\theta = 0.10918$ radian, $\phi = 0.09039$ radian, $b = 2.50$ m,
$\sigma_b^2 = 0.00005$ and $\sigma_\theta = \sigma_\phi = 2.424 \times 10^{-5}$ radian

$\Rightarrow \sigma_D^2 = 0.2005 \text{ m}^2$

$\Rightarrow \sigma_D = \pm\sqrt{0.2005} = \pm 0.45 \text{ m}$

$$\sigma_{\Delta h}^2 = \sigma_i^2 + (\tan\phi)^2 \sigma_D^2 + \left(D \cdot \sec^2\phi\right)^2 \sigma_\phi^2 + \sigma_t^2$$

Consider $\sigma_i = 0.0$,

$\Rightarrow$

$$\sigma_{\Delta h}^2 = \left(\tan\left(5° \ 10' \ 45''\right)\right)^2 \cdot (0.2005) + \left(131.75 \cdot \sec^2\left(5° \ 10' \ 45''\right)\right)^2 \cdot \left(2.424 \times 10^{-5}\right)^2 + (0.005)^2$$

$\Rightarrow \sigma_{\Delta h} = \pm 0.04 \text{ m}$

Final results:
Horizontal distance = D = 131.75 ± 0.45 m
Elevation difference = ΔH = 12.59 ± 0.04 m

(b) From part (a), we notice the high values of σ_D and $\sigma_{\Delta h}$ which makes the tangential method not suitable for precise surveying. In general, tacheometry gives rapid results and is easy to do, but does not give highly accurate results. It is generally used for topographic mapping.

5.10.2.2 STADIA METHOD

For this method, a theodolite equipped with a rectile that has one vertical and three horizontal cross-wires (Figure 5.21) should be available. The upper and lower horizontal cross-wires are called stadia wires.

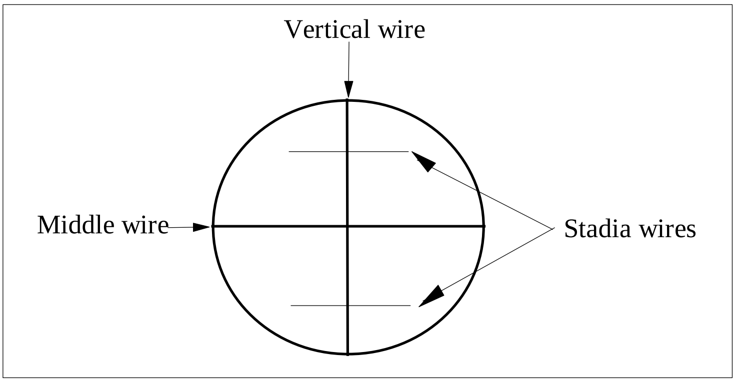


FIGURE 5.21: Stadia wires.

Figure 5.22 illustrates the method. The theodolite is positioned at A and sighted on the rod held at B. The vertical angle θ to the middle hair is recorded. The graduated rod (staff) is also read at all three horizontal cross hairs. The difference between the upper and lower cross-wire readings is called the interval or intercept (r). Its magnitude is a function of the distance between the rod and the theodolite. Thus, the horizontal distance (D) can be computed from this interval. In addition, from the middle cross-wire reading, the difference in elevation between points A and B can be computed by the equations developed for trigonometric leveling.

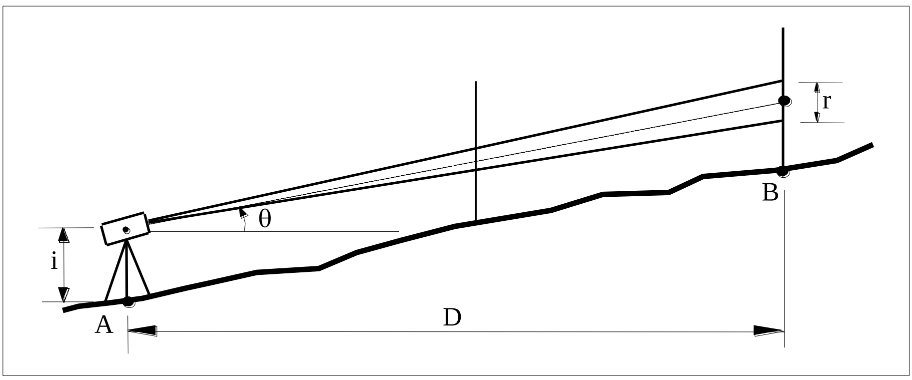


FIGURE 5.22: Stadia method.

Stadia Geometry for Horizontal Sight:

The theory of the stadia method will be first considered for the case of a horizontal sight with an external-focusing telescope. Figure 5.23 illustrates a telescope, the plumb line, and a rod intercept (r). Two rays of vision are shown, namely those emanating from the objective lens and pass through a focal point at a distance F in front of the lens, and intersect the rod as shown.

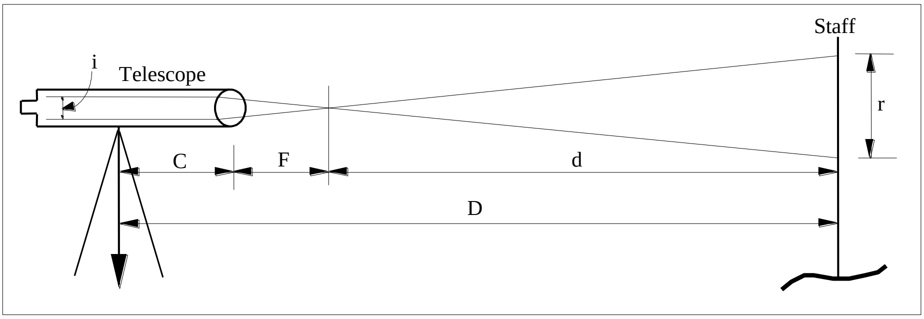


FIGURE 5.23: Stadia geometry for horizontal sight.

If the distance between the stadia cross-wires is represented by (i), then from the similar triangles:

$$\frac{d}{r} = \frac{F}{i} \quad \Rightarrow \quad d = \frac{F}{i} \cdot r$$

From Figure 5.23, the total distance from the staff to the plumb bob is:

$$D = \left(\frac{F}{i}\right) r + (F + C) \qquad \text{.............................. (5.8)}$$

The distance F is the focal length of the lens and is a constant. Also the quantity (F + C) is practically a constant, since the distance C varies only by a negligible amount when the telescope is focused on different objects. For most instruments the value of (F + C) is about 0.25 - 0.3 m and it can be safely neglected.

Theodolites of recent design have internal focusing telescopes where (F+C) is a constant which can be disregarded under all conditions.

Since F and i are constants for any telescope, (F/i) is also a constant, say k. Then neglecting (F+C), Equation (5.8) may be simplified to:

$$D = k\,r \qquad (5.9)$$

The constant k is called the stadia coefficient and is equal to 100 for most instruments.

Stadia Geometry for Inclined Sight:

Refer to Figure 5.24 and suppose that the theodolite is in position at station A and the staff is held vertically at station B with an intercept = mn = r. The inclination of the line of sight is θ from the horizontal, and m'n' = r' is the intercept that would be read on the staff if it were held perpendicular to the line of sight.

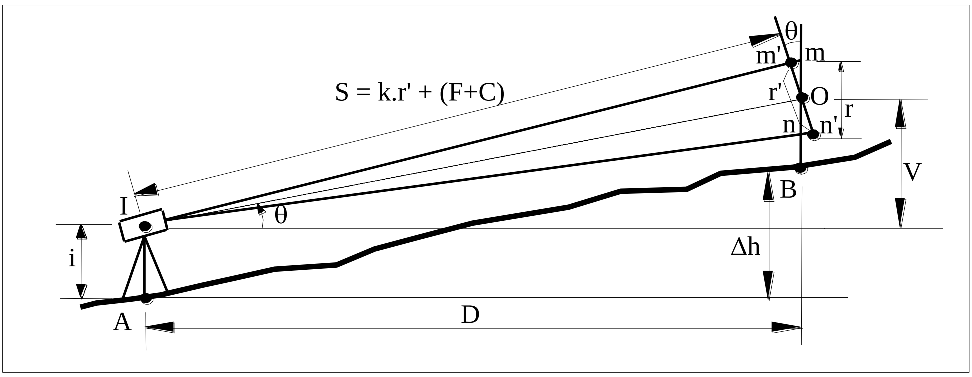


FIGURE 5.24: Stadia geometry for inclined sight.

Angle Om'm = Angle On'n ≈ 90°

$$m'n' = mn \cos\theta \text{ , or } r' = r\cos\theta \qquad (5.10)$$

$$S = kr' + (F + C) \qquad (5.11)$$

Substituting Equation (5.10) into (5.11)

$$\Rightarrow \quad S = kr\cos\theta + (F + C) \quad \ldots\ldots (5.12)$$

The difference in elevation between I and O is:

$$V = S\sin\theta = kr\sin\theta\cos\theta + (F + C)\sin\theta$$

From which:

$$V = \frac{1}{2}kr\sin 2\theta + (F + C)\sin\theta \quad \ldots\ldots (5.13)$$

The horizontal distance D between A and B is:

$$D = S\cos\theta = kr\cos^2\theta + (F + C)\cos\theta \quad \ldots\ldots (5.14)$$

From Figure 5.24, it is evident that the difference in elevation (Δh) between A and B is:

$\Delta h = V + IA - OB$

If distance OB is taken equal to IA, then, $\Delta h = V$. This is done by setting the middle cross-hair on the rod equal to the height of the instrument (HI).

For reasons stated in the previous section, (F + C) can be disregarded and equations (5.13) and (5.14) become:

$$V = \frac{1}{2}kr\sin 2\theta \quad \ldots\ldots (5.15)$$

$$D = kr\cos^2\theta \quad \ldots\ldots (5.16)$$

Since the zenith angle (z) = 90° - θ, Equations (5.15) and (5.16) may be written as:

$$V = \frac{1}{2}kr\sin 2z \quad \ldots\ldots (5.17)$$

$$D = kr\sin^2 z \quad \ldots\ldots (5.18)$$

When z = 90° (line of sight is horizontal)

$$\Rightarrow \quad V = \frac{1}{2}kr\sin(2 \times 90°) = 0$$

$$D = kr\sin^2(90°) = kr$$

which are the equations derived for the case of horizontal sight.

EXAMPLE 5.2:

The following readings were taken on a vertical staff with a theodolite having a constant k = 100 and F + C = 0.

Staff Station	Azimuth	Stadia Readings	Vertical Angle
A B	27° 30' 207° 30'	1.000 1.515 2.025 1.000 2.055 3.110	+ 8° 00' - 5° 00'

Calculate the mean slope between A and B.

SOLUTION:

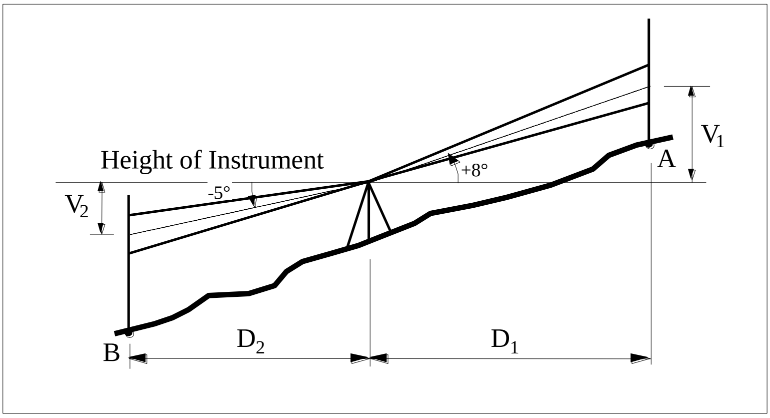


FIGURE 5.25

From Figure 5.25:

(1) Staff at Station A:
Staff intercept r = 2.025 - 1.000 = 1.025 m
Mid-reading = 1.515 m

$$D = kr\cos^2\theta$$
$$V = \frac{1}{2}kr\sin 2\theta$$
$$\Rightarrow D_1 = 100 \times 1.025 \times \cos^2(8°) = 100.515\,\text{m}$$

$$V_1 = \frac{1}{2} \text{ x } 100 \text{ x } 1.025 \text{ x } \sin(16°) = 14.126 \text{ m}$$

(2) Staff at Station B:
Staff intercept r = 3.110 - 1.000 = 2.110 m
Mid-reading = 2.055 m

$$\Rightarrow \quad D_2 = 100 \text{ x } 2.110 \text{ x } \cos^2(-5°) = 209.397 \text{ m}$$

$$V_2 = \frac{1}{2} \text{ x } 100 \text{ x } 2.110 \text{ x } \sin(-10°) = -18.320 \text{ m}$$

Let h = height of instrument above datum, then

Elevation of point A= h + 14.126 - 1.515 = h + 12.611
Elevation of point B= h - 18.320 - 2.055 = h - 20.375
Elevation difference between B and A (ΔH_{BA}):
(ΔH_{BA}) = (h + 12.611) - (h - 20.375) = 32.986 m

From a consideration of azimuths, it will be seen that A, B and the instrument lie on a straight line (207° 30' - 27° 30' = 180°), so that the mean slope =

$$\frac{\text{Elevation difference}}{D_1 + D_2} = \frac{32.986}{100.515 + 209.397}$$

$$= \frac{1}{9.4} = 1 \text{ in } 9.4$$

$$= 0.1064 = 10.64\%$$

PROBLEMS

5.1 Given that the azimuth of line AB with respect to the geographic north is 153°, and that the magnetic declination at this location is 2°E. Calculate:

- The true back azimuth of line AB.
- The true reduced bearing of line AB.
- The true back reduced bearing of line AB.
- The magnetic azimuth of line AB.
- The magnetic reduced bearing of line AB.

5.2 A horizontal angle was measured using a theodolite in both face right and face left positions. The readings were:

Face right: 0° 00′ 00″ , 45° 19′ 55″

Face left: 180° 00′ 02″ , 225° 20′ 03″

Calculate the accepted value of the angle.

5.3 You are standing on one side of a river (Figure 5.26) with a theodolite, tape and ranging rods. Using this equipment, show how to measure the distance between two points A and B located on the other side of the river without actually crossing to this side. Clarify with a sketch and show all the involved mathematics.

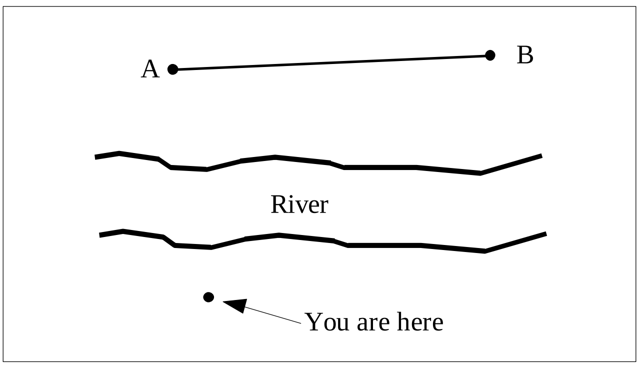


FIGURE 5.26

5.4 Using a theodolite and a tape measure, show how to make a parallel line such as CD to a given line AB and with a separation of 10 m between the two lines.

5.5 Using a theodolite, explain how to allocate on the ground the point of intersection of two intersecting lines AB and CD.

5.6 To measure the elevation of the top of a tower C, two points A and B were chosen near the tower, and the following measurements were made with a theodolite whose stadia coefficient is equal to 100 and (F+C = 0):

- With the theodolite at A, stadia readings at the staff held vertically at B are: 1.00, 1.52, 2.04 m. Vertical angle = +8°
- Height of instrument (i) at A = 1.65 m
- Horizontal angle CAB = 58° 16' 24"
- Horizontal angle ABC = 61° 22' 37"
- Vertical angle measured at A when the theodolite was directed towards C was +30° 18' 00"
- Elevation of point A = 970.34 m AMSL.

Calculate the elevation of the top of the tower C.

5.7 The following data were obtained during a tacheometric survey work using a theodolite whose constants are 100 & 0.

Station	Point	Vertical Circle Reading	Stadia Readings (m)	Instrument Height (m)
A	BM	- 5° 30'	2.15 ,1.95 ,1.75	1.40
A	B	+1° 30'	1.80 ,1.65 ,1.50	
B	C	+12° 00'	2.21 ,2.05 ,1.89	1.30

Knowing that the reduced level of the BM. = 500.00 m, Calculate:
a. The elevations of B & C.
b. Distances AB & BC.

5.8 The following readings were taken on a vertical staff with a theodolite having a constant k=100 and (F+C=0):

Staff Station	Azimuth	Stadia Readings	Zenith Angle
A	30° 20' 40"	1.000 1.515 2.025	82° 00'
B	140° 40' 20"	1.000 2.055 3.110	95° 00'

Calculate the mean slope between points A and B.