4 LEVELING

4.1 INTRODUCTION

The elevation of a point is defined as the vertical distance between the point and a reference level surface called *datum*. The most commonly used datum is the mean sea level (MSL). The elevation of a point can be considered as its vertical coordinate, and is considered to be positive if the point is above the datum (e.g. Jerusalem), and negative if the point is below the datum (e.g. Jerico).

Leveling may be simply defined as the process by which the elevation of a point above a reference elevation datum, or the elevation difference between two or more points on the earth's surface is determined. Its purpose may be to provide spot heights or contour lines on a plan, to provide data for making longitudinal and cross-sections, or to provide a level or inclined surface in the setting out of construction works. Leveling can be done in several ways, which include:

1) *Chain Surveying*. By measuring the slope distance and angle of inclination for a uniformly sloping ground, the elevation difference between two points can be calculated (Section 3.3.1).

2) *Barometric Leveling*. This is the process of determining elevation by measurement of atmospheric pressure, and is based on the principle that atmospheric pressure decreases with increase in elevation. An altimeter or barometer is used for this purpose. This method is not highly accurate and is therefore restricted to situations where high accuracy is not required.

3) *Trigonometric Leveling*. The elevation difference between two points may be calculated by measuring the horizontal or slope distance between the two points, in addition to the vertical or zenith angle to the line of sight. This method is explained in chapters 5 and 6.

4) *Photogrammetric Leveling.* The elevation difference is measured from photographs taken for the area under consideration by a camera mounted in an airplane. This technique is discussed in Chapter 11.

5) *GPS Leveling.* The elevation of a point, or the elevation difference between two points is measured from special signals received from orbiting satellites using equipment known as GPS receivers. This subject is discussed in Chapter 12.

6) *Differential Leveling*. This is the most commonly used method in leveling for its high degree of accuracy. It is performed using an instrument called a level and a leveling staff, and will be the subject of this chapter.

4.2 BASIC DEFINITIONS

Various terms are used in the process of leveling, and it is useful at this stage to define them. These include:

1 - *Vertical line*. The vertical line at a point is the line formed by a freely falling body or by the string of a plumb bob when the tip is located directly over the point.

2 - *Horizontal line*. The horizontal line at a point is the straight line perpendicular to the vertical line at that point. There is an infinite number of horizontal lines at each point.

3 - *Horizontal plane*. This is the plane that passes through all the horizontal lines at a particular point. It is perpendicular to the vertical line at this point.

4 - *Level surface*. A level surface is a continuously curved surface that is perpendicular to the direction of gravity at all its points, such as the Geoid or any surface parallel to it (see Figure 4.1).

5 - *Level line*. This is a line that lies on the level surface, and is, therefore, perpendicular to the direction of gravity at all its points. The level line that passes through a particular point is tangent to the horizontal line at this point (Figure 4.1).

6 - *Difference in elevation between two points*. This is the vertical distance between the two level surfaces passing through these two points.

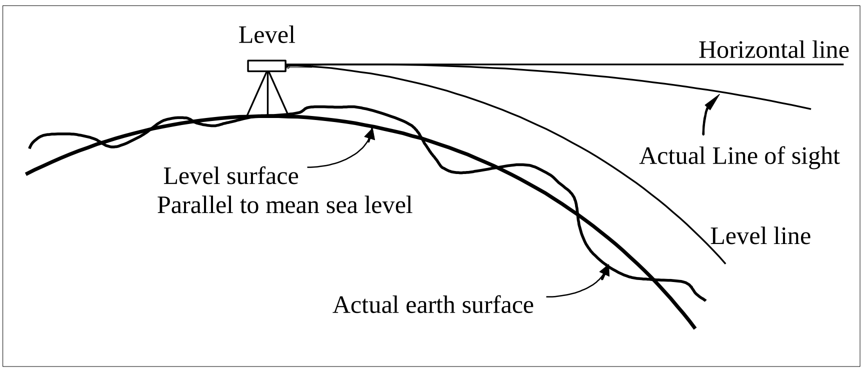


FIGURE 4.1: Relationship between the horizontal line, level line and the line of sight

7 - *Actual line of sight or collimation.* This is neither a horizontal nor level since it is affected by atmospheric refraction, which bends the line of sight downwards from the horizontal line (Figure 4.1).

8 - *Bench mark (BM).* This is a marked point whose elevation has been accurately measured. Benchmarks are established by the survey departments and made evenly distributed to enable surveyors to use them for measuring the elevations of nearby points without the need to start over from the elevation datum. They are well marked on the ground and have precise description to locate them easily.

9 - *Height of instrument*. This is the elevation of the line of collimation above the datum after setting up the level above a certain point.

4.3 BASIC PRINCIPLE OF A LEVEL

The basic instrument used in differential leveling to measure elevation or height differences is called a level. Although of many types and designs, a level consists essentially of:

a) A telescope for sighting, and
b) A leveling device for maintaining the line of sight in a horizontal position.

A level is set up so that the line of sight of the telescope is perpendicular to its vertical axis. If the vertical axis of the level is made to coincide with the direction of gravity, then the line of sight will be in a horizontal direction. Now, as the telescope is rotated around the vertical axis, the line of sight will move in a horizontal plane (Figure 4.2).

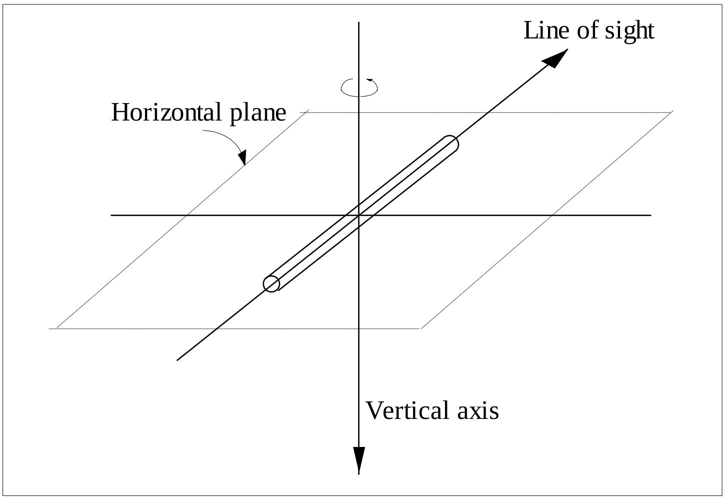


FIGURE 4.2: Basic principle of a level.

4.4 BUBBLE TUBE

A bubble tube, as shown in Figure 4.3 is used in most levels to establish a horizontal line. The bubble tube is a glass vial of uniform cross section. Its inside, upper surface is accurately ground in longitudinal section to the arc of a circle of specific radius. The tube is nearly filled with ether or some nonfreezing liquid, the remaining volume being a vapor space called the *bubble*. The buoyancy of the liquid lifts the bubble to a position symmetrical with the highest point in the tube. Since this highest point is on the arc of a circle that lies in a vertical plane, the tangent at that point will be truly horizontal.

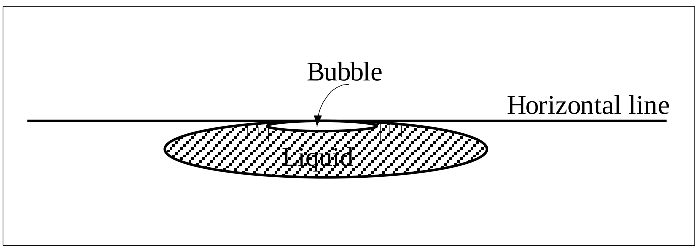


FIGURE 4.3: Cross section in a bubble tube.

A circular bubble vial, as shown in Figure 4.4, is used in many modern levels to approximately establish a horizontal plane.

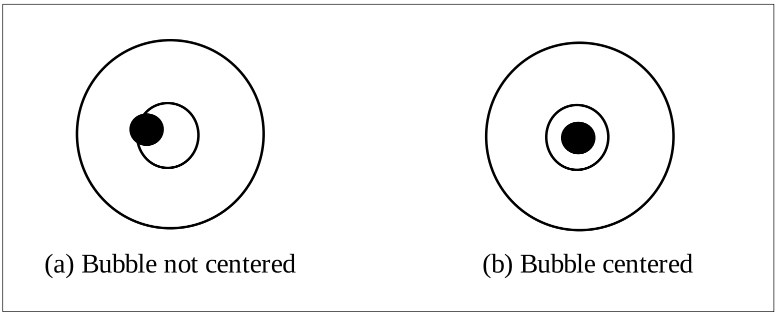


FIGURE 4.4: Centering the bubble.

By centering the bubble within the circular mark, the surface is approximately leveled. In some levels like the Tilting Dumpy Level, the horizontality of the plane is accurately controlled by another bubble that can be seen through an eyepiece near the telescope. This bubble consists of two separate halves. When the bubble is off-centered, a split image of the two ends of the bubble is seen through the viewing microscope, as shown in Figure 4.5a. When the bubble is correctly centered, the two images coincide to form a continuous U-shaped curve, as shown in Figure 4.5b.

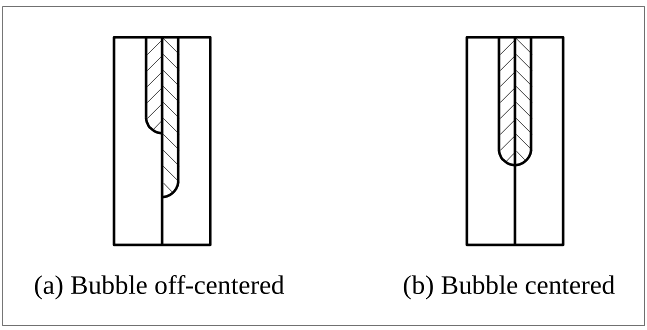


FIGURE 4.5: Auxiliary bubble tube.

4.5 EQUIPMENT USED IN DIFFERENTIAL LEVELING

The following types of equipment are used for differential leveling:

A) **The Level**. There are three basic types of level:

1) *The Dumpy Level*. In this type of instrument the line of sight defined by the center of the cross hairs and the optical center of the objective lens - the line of collimation - is fixed at right angles to the vertical axis of rotation of the instrument. The spirit level attached to the telescope should also be perpendicular to the vertical axis of the instrument and parallel to the line of collimation. The spirit level is brought to the horizontal plane using the three leveling screws on the tribrach. When level, the line of collimation should describe a true horizontal plane around the instrument. This type of level is mainly confined to construction sites or other cases where large numbers of level sights are required from a single instrument position.

2) *The Tilting Level*. In these instruments the telescope is hinged near the top of the vertical axis to allow a limited degree of movement with respect to the vertical axis. Like the dumpy level the spirit level is attached to the telescope. The vertical axis is made approximately vertical either by a ball and socket mounting or a three-screw tribrach. In both cases a small circular spirit level is used. The telescope is then leveled using the telescope spirit level by means of a tilting screw that tilts the telescope with respect to the vertical axis. It is important that this is done for each observation. The telescope bubble may be observed either by a mirror reflecting the bubble of the spirit level to the observer or by a split bubble prism. This allows the observer to see both ends of the bubble either in the telescope eyepiece or through a separate eyepiece. Leveling is achieved by making the two halves of the bubble coincide (Figure 4.5).

3) *The Automatic Level*. Many modern levels use a system of self-leveling compensators within the optical system of the telescope. This requires the observer to level the instrument within the working range of the compensator, which is usually about ±20' of the horizontal. The vertical axis may be supported by a three-screw or ball and socket mounting, the rough leveling being carried out by reference to a circular spirit level. The compensator, which consists of a system of fixed and suspended prisms, brings the line of sight to the horizontal plane even though the axis through the telescope is not truly horizontal. This type of level more normally gives an erect image as opposed to the inverted image of most other surveying telescopes. Figure 4.6a shows an example of an automatic level.

4) *The Electronic Digital Level*. This kind of level combines the merits of the automatic level with the fact that it is user friendly and easy to use (Figure 4.6b). All the user has to do is aim the staff, adjust the focus and then – with a single touch of a key – the level will accurately measure and record the staff reading on a display.

(a) Automatic level.

(b) Electronic digital level.

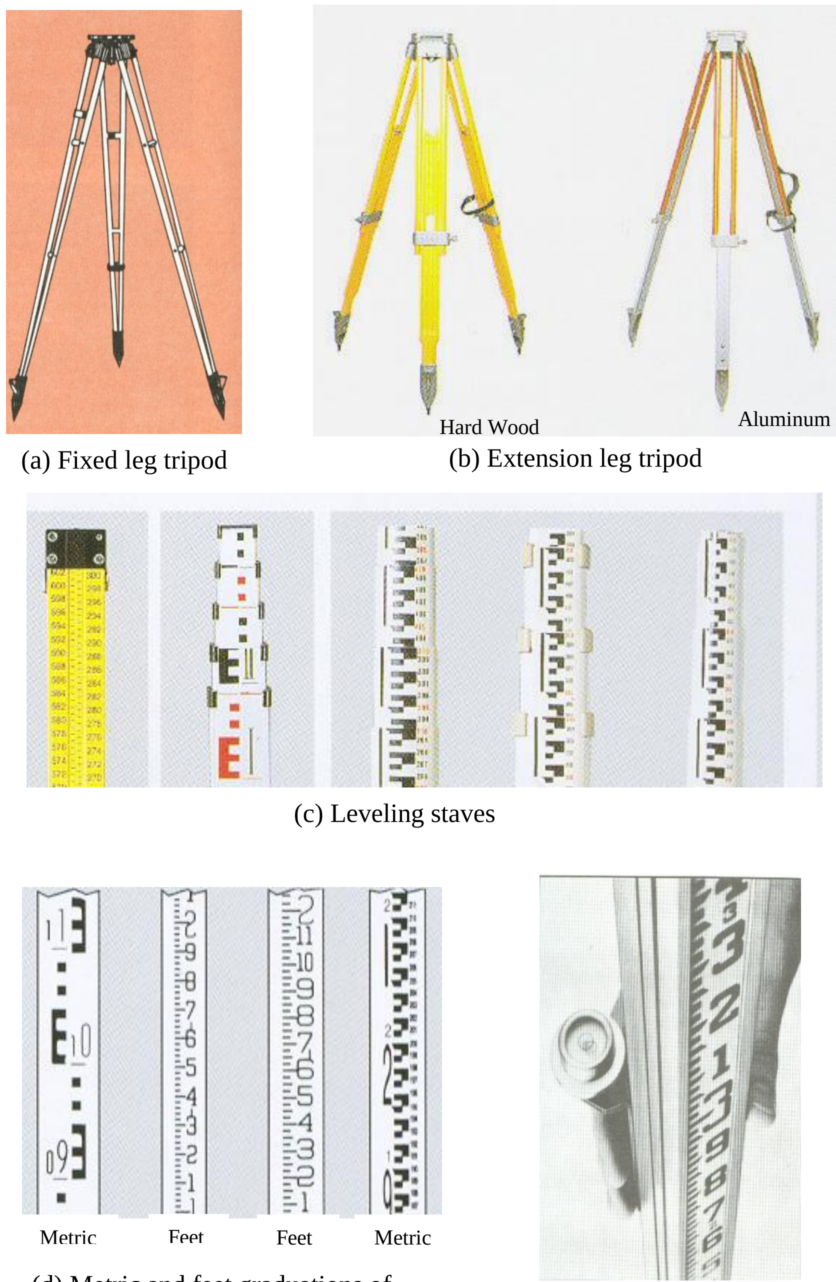
FIGURE 4.6: Automatic and electronic digital Levels.

B) **Tripod**. This is a three-legged stand used to support a level or other surveying instrument during field measurements. It consists of a head and three legs that are fitted with pointed metal shoes. The legs could be made of aluminum or hard wood, and are either with fixed length or adjustable length depending on the height of the user (Figure 4.7a & b). A survey instrument is usually secured to the tripod head by a threaded bolt.

C) **Level Rods (Staves)**. The level rod (staff) is used to measure the vertical distance between the point on which it is held and the line of collimation of the instrument, and is usually 3 - 5 m long. There are many different types of level staves (Figure 4.7c & d). These include:

1) The telescopic staff. This is composed of several sections that slide inside each other in a way similar to the radio antennae. It is made of either wood or metal. Currently, this is the most popular and widely used type of staves.
2) The folding staff. The most widely used type of this kind is the one composed of four folding sections into a 1-m long piece designed to fit easily into a car boot.
3) The one-piece staff. This is usually 1 - 3 m long and is difficult to transport in a car, and therefore it is suitable to be used in open rural areas.

D) **Rod Level**. This is a small device with a circular bubble tube mounted on a metal angle and attached to the edge of the level staff to ensure vertical standing over a point (Figure 4.7e).



(a) Fixed leg tripod

(b) Extension leg tripod

(c) Leveling staves

(d) Metric and feet graduations of leveling staves.

(e) Rod level

FIGURE 4.7: Tripods, leveling staves and rod levels.

4.6 SETTING UP THE LEVEL

The level should be mounted at a comfortable height on a rigid legged tripod making sure that it is firmly placed and the screws at the tripod head are finger tight. With practice, it should be possible to have the tripod head almost level before attaching the level. For a three-foot screws level, as in the case with most modern survey instruments, make sure that the three screws are in the middle of their paths. To level the instrument, bring the telescope parallel to one pair of foot screws (A and B in Figure 4.8), and by turning each screw either inwards or outwards, bring the small circular bubble to a position in line with the third foot screw C (Figure 4.8a). Now centralize the bubble with this third screw (Figure 4.8b).

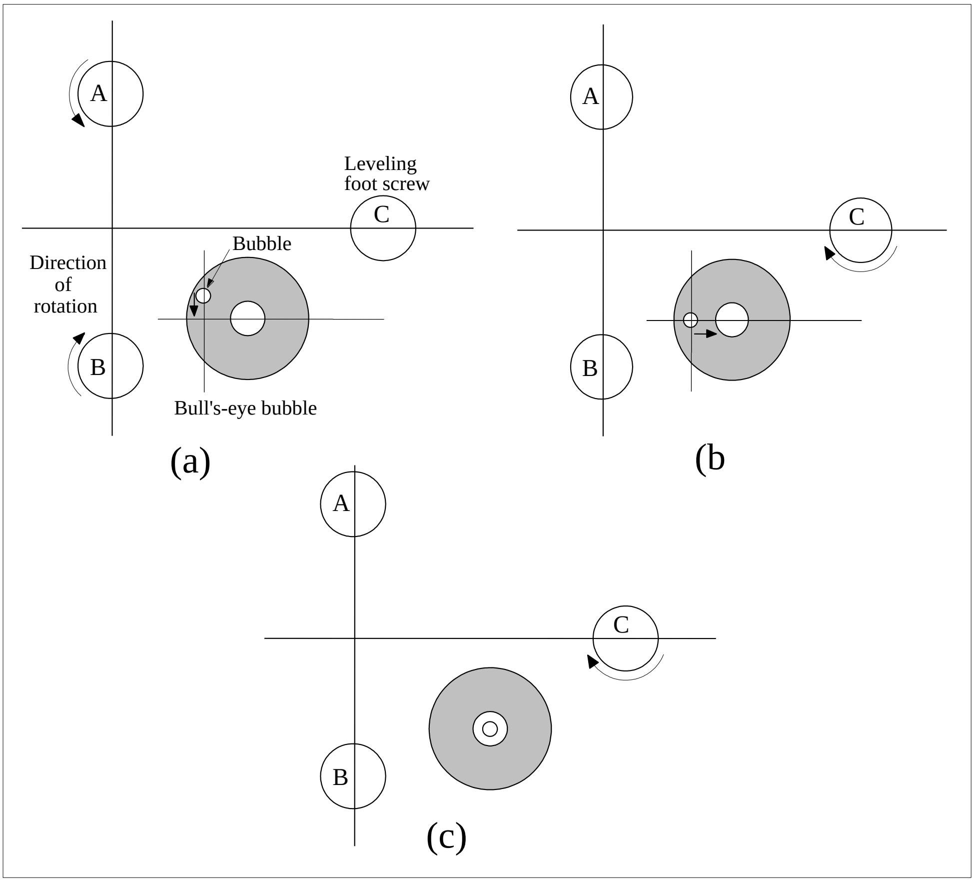


FIGURE 4.8: Leveling an instrument with three foot screws and a circular bubble tube.

Once the level has been roughly leveled using the circular bubble (Figure 4.8c), the cross-wires in the eyepiece should be focused preferably against a light background, until the lines are sharp and clear. The level should now be pointed at the staff and focused to make the staff graduations sharp. When using a dumpy or automatic level, the staff can now be read. If a tilting level is being used, the split level bubble must be brought into coincidence by turning the gradienter screw before the staff is read and this must be done before each reading. It is important to check the level bubble before every reading is taken, or in the case of an automatic level, that the compensator is operating correctly. This can be checked by tapping the instrument gently.

4.7 MEASURING ELEVATION DIFFERENCE USING A LEVEL

The basic operation in differential leveling is the determination of elevation difference between two points. Consider two points A & B as shown in Figure 4.9. Set up the level so that readings may be made on a staff held vertically at A, and then at B. Let point A be a benchmark whose reduced level (RL) or elevation = 520.43 m AMSL.

If the readings at A and B are 2.56 m and 0.93 m respectively (Figure 4.9a), then the difference in elevation between points A and B is equal to the vertical distance AC = 2.56 - 0.93 = 1.63 m. This positive value represents a rise of point B relative to A.

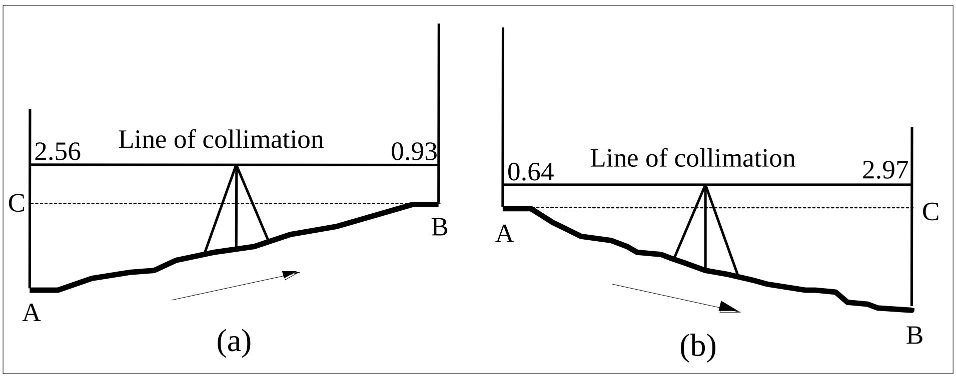


FIGURE 4.9: Principle of differential leveling.

If, on the other hand, the readings at A and B were 0.64 m and 2.97 m respectively (Figure 4.9b), then the difference in elevation between points A and B is equal to the vertical distance BC = 0.64 - 2.97 = -2.33 m. The negative sign indicates a fall of point B relative to A. Thus, we have for any two successive staff readings:

Second reading smaller than first reading represents a ***rise***.
Second reading greater than first reading represents a ***fall***.

If the actual elevation of one of the two points above an elevation datum is known, then the elevation of the second point can be calculated. For the cases of Figure 4.9, and given that the elevation of point A is 520.43 m AMSL, the elevation of point B can be calculated by one of the following two methods:

A) The Height of Instrument method:

1 - For Figure 4.9a:

Height of Instrument (HI)	= elevation of A + staff reading at A
	= 520.43 + 2.56 = 522.99 m AMSL
Elevation of B (or RL)	= HI - staff reading at B
	= 522.99 - 0.93 = 522.06 m AMSL

2 - For Figure 4.9b:

Height of Instrument (HI)	= elevation of A + staff reading at A
	= 520.43 + 0.64 = 521.07 m AMSL
Elevation of B (or RL)	= HI - staff reading at B
	= 521.07 - 2.97 = 518.10 m AMSL

B) The Rise and Fall method:

1 - For Figure 4.9a:

Elevation difference	= first reading at A - second reading at B
	= 2.56 - 0.93 = 1.63 m (positive $\Rightarrow$ rise)
Elevation of B (or RL)	= elevation of A + rise
	= 520.43 + 1.63 = 522.06 m AMSL

2 - For Figure 4.9b:
Elevation difference = first reading at A - second reading at B
= 0.64 - 2.97 =-2.33 m (negative ⇒ fall)
Elevation of B (or RL) = elevation of A + fall
= 520.43 + (-2.33)
= 520.43 - 2.33 = 518.10 m AMSL

It is obvious that the results agree in both methods. Now, let ΔH_{AB} represent the elevation difference between points A and B, H_A and H_B represent the elevations or reduced levels of points A and B respectively, and r_A and r_B represent the staff readings at A and B respectively, then the following equation always holds:

$$\Delta H_{AB} = H_B - H_A = r_A - r_B \qquad \text{(4.1)}$$

4.8 PROCEDURE IN DIFFERENTIAL LEVELING

The main purpose here is to provide reduced levels for a large number of points. These points could be located along the centerline of a construction project such as a highway, or in an area to produce a contour plan. Before explaining the general procedure, the following definitions need to be known:

1 - *Backsight (BS).* This is the first reading taken by the observer at every instrument station after setting up the level.

2- *Foresight (FS).* This is the last reading taken at every instrument station before moving the level.

3- *Intermediate Sight (IS).* This is any reading taken at an instrument station between the backsight and foresight readings.

4- *Turning Point (TP).* This a point at which both a foresight and a backsight are taken before moving the staff.

4.8.1 GENERAL PROCEDURE

This will be illustrated through an example, which is going to be about the production of a longitudinal section (profile) of a road (Figure 4.10).

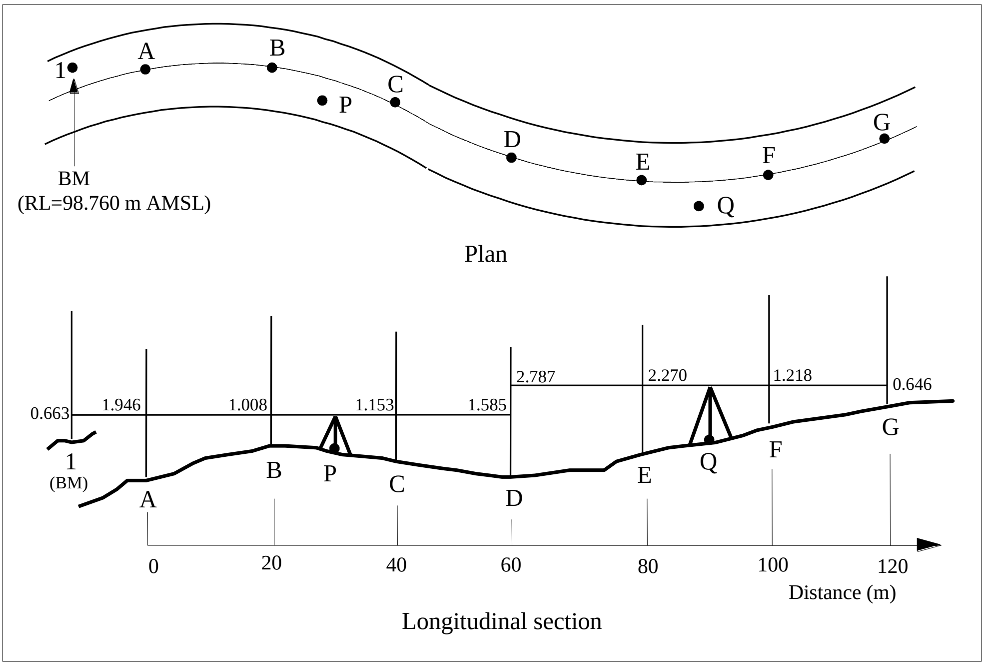


FIGURE 4.10: General procedure in differential leveling.

(1) After setting up the level at P, take the first (BS) reading on a nearby BM (point 1 in Figure 4.10).

(2) Take readings on the staff held vertically at points A, B and C respectively. These readings are called intermediate sights (IS).

(3) If the distance after point D, for example, becomes more than 100 m, or if the topography makes it difficult to take readings after this point from the same setup of the instrument, take the last reading at D. This reading is called foresight (FS). Point D is considered to be a turning or change point because the level position is changed at this stage.

(4) Move the level to point Q, and with the staff remaining at D, take a BS reading. Proceed taking intermediate sights at E & F, and a FS at G. If any further work is required, continue in accordance with the above procedure.

4.8.2 BOOKING AND CALCULATIONS

Staff readings are booked in a level field book that is mainly designed for this purpose. Table 4.1 shows a typical page from this book. The reduction of the readings is carried out in the same book. It can be done either by the height of instrument method or by the rise and fall method, whichever is more suitable. The following will explain the reduction of the readings using both of these methods on the data shown in Figure 4.10.

A) The Height of Instrument Method:

Table 4.2 shows the data of Figure 4.10. First of all, point numbers or names are booked in the left column (i.e. points 1, A, B through G). Each point is booked on a separate line. At point 1 (the bench mark), we took a BS of 0.663 m and therefore this value is written on the same line near point 1 but in the BS column. The elevation or reduced level of this point is known to be 98.760 m and is booked in the RL column. At points A, B & C we read IS of 1.946 m, 1.008 m & 1.153 m respectively. These values are booked in the IS column. The final reading taken from the first instrument setup is a FS of 1.585 m at point D and is therefore booked at the same line as point D in the FS column. Now, from the second setup of the instrument, a BS of 2.787 m was read at point D, and therefore is booked at the same line but in the BS column. The two IS readings of 2.270 m and 1.218 m taken at points E and F are then booked in the IS column. Finally, we book the last FS reading of 0.646 m taken at point G at the same line as point G, but in the FS column.

TABLE 4.2: Calculations using the height of instrument (HI) method.

Point	BS	IS	FS	HI	RL	Distance (m)	Notes
1	0.663			99.423	98.760	-	BM
A		1.946			97.477	0	Beginning of project
B		1.008			98.415	20	
C		1.153			98.270	40	
D	2.787		1.585	100.625	97.838	60	TP
E		2.270			98.355	80	
F		1.218			99.407	100	
G			0.646		99.979	120	
SUM	3.450	7.595	2.231				

TABLE 4.1: A typical page from a level field book

Project and Location: ....

Surveyor: .... Date: / /

Point #	BS	IS	FS	Rise	Fall	HI	RL (m)	Distance (m)	Remarks

The reduction of data is done in the following sequence:

1) Calculate the first height of instrument (HI). This is equal to the sum of the elevation of the BS point (point 1 or BM here) and the BS reading. In this case:
HI = 98.760 + 0.663 = 99.423 m

This value is entered in the same line as the BS reading, but in the HI column.

2) Calculate the RL of points A, B, C and D as follows:
RL of A = HI - IS at A = 99.423 - 1.946 = 97.477 m
RL of B = HI - IS at B = 99.423 - 1.008 = 98.415 m
RL of C = HI - IS at C = 99.423 - 1.153 = 98.270 m
RL of D = HI - FS at D = 99.423 - 1.585 = 97.838 m

These reduced levels are entered in the RL column in the corresponding lines.

3) Calculate the next height of instrument. This is equal to:
HI = RL of point D + BS reading at point D
= 97.838 + 2.787 = 100.625 m
This value is entered in the same line as point D, but in the HI column.

4) Calculate the RL of points E, F and G as follows:
RL of E = HI - IS at E = 100.625 - 2.270 = 98.355 m
RL of F = HI - IS at F = 100.625 - 1.218 = 99.407 m
RL of G = HI - FS at G = 100.625 - 0.646 = 99.979 m

These reduced levels are again entered in the RL column in the corresponding lines.

Note: The distance column in Table 4.2 indicates the cumulative distance between each point and the beginning point of the project.

Checks: The following checks on the booking and arithmetic calculations are performed:

1) Number of BS readings = number of FS readings = 2 $\Rightarrow$ OK

2) $\Sigma BS - \Sigma FS$ = RL of last point - RL of first point . . . . . . (4.2)
$\Sigma BS - \Sigma FS$ = 3.450 - 2.231 = 1.219
RL of last point - RL of first point = 99.979 - 98.760 = 1.219 $\Rightarrow$ OK

3) Sum of all reduced levels excluding the first reduced level = sum of the HI for each setup multiplied by the number of IS and FS readings taken from that setup - sum of IS - sum of FS . . . . (4.3)

For the data of Table 4.2
- Sum of all reduced levels excluding the reduced level of point 1 = 689.741
- Sum of HI multiplied by the number of IS and FS readings
  = 99.423 x 4 + 100.625 x 3 = 699.567
- ΣIS = 7.595
- ΣFS = 2.231

The right part of Equation (4.3) = 699.567 - 7.595 - 2.231 = 689.741
Left side = right side $\Rightarrow$ OK

The arithmetic check is now complete. This indicates that there are no mistakes in the calculations.

B) The Rise and Fall Method:

The booking of the staff readings is done in the same way as explained earlier. However, the reduction of the data is done as follows:

1) Calculate the elevation difference (ΔH) between points 1 and A. This is calculated as follows.

ΔH_{1A} = staff reading at 1 - staff reading at A
= 0.663 - 1.946 = -1.283 m

This value is negative which indicates a fall from 1 to A, and is entered in the fall column without a sign in the line of point A.
The reduced level of A = RL of 1 - fall
= 98.760 - 1.283 = 97.477
This RL of A is entered in the table in the RL column.

2) Calculate the elevation difference (ΔH) between points A and B. This is calculated as follows:
ΔH_{AB} = staff reading at A - staff reading at B
= 1.946 - 1.008 = +0.938
This value is positive which indicates a rise from A to B, and is entered in the rise column in the line of point B.
The reduced level of B = RL of A + Rise
= 97.477 + 0.938 = 98.415 m

This RL of B is entered in the table in the RL column.

3) The computations proceed in a similar manner until calculating the RL of point D. Next, we proceed with the calculations of point E using the BS at D (and not the FS), and so on until all the reduced levels are calculated. Table 4.3 shows the results of these computations.

TABLE 4.3: Calculations using the rise and fall method.

Point	BS	IS	FS	Rise	Fall	RL	Distance (m)	Notes
1	0.663					98.760	-	BM
A		1.946			1.283	97.477	0	Beginning of project
B		1.008		0.938		98.415	20	
C		1.153			0.145	98.270	40	
D	2.787		1.585		0.432	97.838	60	TP
E		2.270		0.517		98.355	80	
F		1.218		1.052		99.407	100	
G			0.646	0.572		99.979	120	
SUM	3.450	7.595	2.231	3.079	1.860			

Checks: The following checks on the booking and arithmetic calculations are performed:

1) Number of BS readings = number of FS readings = 2 $\Rightarrow$ OK

2) $\Sigma BS - \Sigma FS = \Sigma Rise - \Sigma Fall$

$= RL$ of last point $- RL$ of first point . . . . (4.4)

$\Sigma BS - \Sigma FS = 3.450 - 2.231 = 1.219$

$\Sigma Rise - \Sigma Fall = 3.079 - 1.860 = 1.219$ $\Rightarrow$ OK

$RL_{last} - RL_{1st} = 99.979 - 98.760 = 1.219$

The arithmetic check indicates that there are no mistakes in the reduction of the data.

4.9 GENERAL NOTES

1) The accuracy of the reduced levels does not depend only on correct calculations, but also on correct measurements (staff readings) and the correct booking of these measurements. For example, a staff reading of 2.38 m can be booked as 2.83 m and still all the arithmetic checks will indicate correct calculations, but the reduced levels will be incorrect. Therefore, to ensure accurate elevations of the level points, the fieldwork should start at a benchmark and close at another benchmark of known elevation. If no other benchmark is available, the surveyor should go back and close at the starting point.

2) If the purpose of leveling is to find out the elevation difference between two points, which are unseen from each other, or if they are far apart, no intermediate sights are necessary in this case. Only backsight and foresight readings are made.

3) The backsight and foresight distances should be approximately equal to avoid errors, which will result if the line of sight was not completely horizontal.

4) Turning points should be chosen on firm ground. On soft ground, a special triangular base is used.

5) If possible, staff readings should be made to the nearest mm at turning points and to the nearest cm at other points.

6) If the fieldwork starts at a point of an unknown elevation, but passes through a benchmark somewhere in the leveling chain, then the calculations are carried out as follows:

   a - If the benchmark is the last point, then the elevation of the first point is calculated from Equation (4.2) as follows:
   RL of first point = RL of last point (BM) + ΣFS - ΣBS . . . . . .(4.5)
   Now, that the RL of the first point is known, the computations are performed as explained earlier.

   b - If the benchmark is not the first point, neither the last point, we add the staff reading at this bench mark to the known BM elevation to get the HI. This HI is recorded in the table on the same line as the corresponding BS reading. Now, the computations are started from this known HI and continued until calculating the RL of the last point. The RL of the first point is then calculated using Equation (4.5). With the RL of the first point known, the computations are continued until reaching the BM, and with this, the table will be complete.

7) If the point whose elevation is to be calculated is the bottom of a bridge or a ceiling or the top of a barrier such as a wall or a column, the staff is held at this point in an upside-down position so that the zero of the staff will be at the point. The reading at this point is booked in the table with a negative sign and the calculations are carried out in the normal way as explained in the previous section.

EXAMPLE 4.1:

Using the data shown in Figure 4.11, do the required booking, and calculate the reduced levels of points A, B, C & D by both the height of instrument and rise and fall methods. Make the required arithmetic checks.

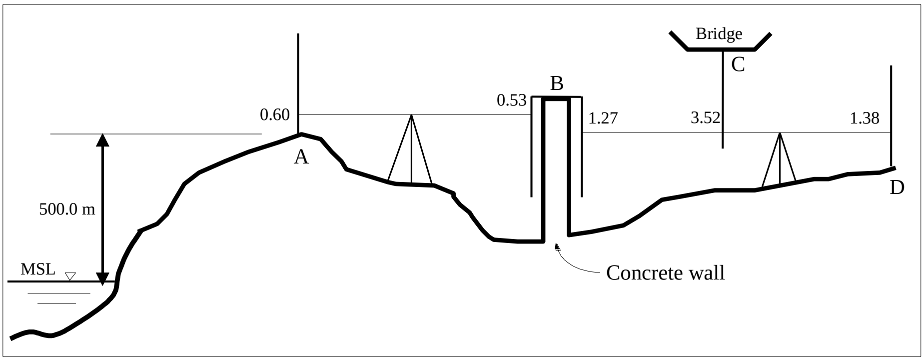


FIGURE 4.11: A leveling section with a bridge and a concrete wall obstacle.

SOLUTION:

Point	BS	IS	FS	Rise	Fall	HI	RL	Notes
A	0.60					500.60	500.00	BM
B	-1.27		-0.53	1.13		499.86	501.13	Top of concrete wall (TP)
C		-3.52		2.25			503.38	Bottom of bridge
D			1.38		4.90		498.48	
SUM	-0.67	-3.52	0.85	3.38	4.90			

Checks:

1) # of BS = # of FS = 2 ⇒ OK

2) $\Sigma BS - \Sigma FS = -0.67 - 0.85 = -1.52$

$\Sigma Rise - \Sigma Fall = 3.38 - 4.90 = -1.52$ ⇒ OK

$RL_{last} - RL_{1st} = 498.48 - 500.00 = -1.52$

3) ΣRL excluding first RL = 501.13 + 503.38 + 498.48 = 1502.99
Σ (HI x # of FS and IS) = 500.60 x 1 + 499.86 x 2
= 1500.32
ΣFS = 0.85
ΣIS = -3.52
1500.32 - 0.85 - (-3.52) = 1502.99 $\Rightarrow$ OK

4.10 ERRORS IN DIFFERENTIAL LEVELING

As in any other type of surveying work, errors in leveling can be divided into systematic errors, random errors and blunders.

a) Systematic errors:

There are two major sources of systematic errors in differential leveling. These are:

1) Inclination of the line of sight due to the earth's curvature and atmospheric refraction, and
2) Inclination of the line of sight due to maladjustment of the level.

1) Earth's curvature and atmospheric refraction:

As defined before, a level line is a curved line and is everywhere normal to the plumb line. However, a horizontal line of sight is perpendicular to the plumb line only at the point of observation. Hence, it should be carefully distinguished from a level line.

Because of atmospheric refraction, rays of light transmitted along the earth's surface are refracted or bent downward so that the actual line of sight is along a curve that is concave downward. This curve has a radius that is seven times the radius of the earth.

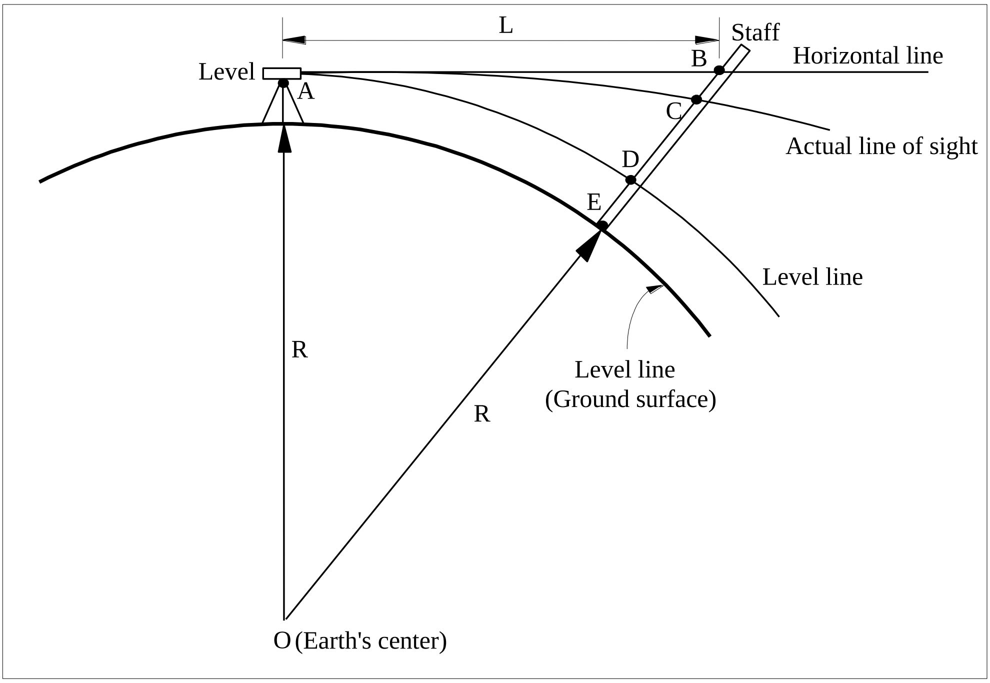


FIGURE 4.12: Effect of earth's curvature and atmospheric refraction on differential leveling.

In Figure 4.12,

BC = refraction of line of sight from horizontal
BD = Error due to earth's curvature
CD = Actual net error in the staff reading = BD - BC

In the triangle ABO,

$$(BD + DO)^2 = AB^2 + AO^2, \qquad AB = L, \qquad AO = DO \approx R$$

$$\Rightarrow \quad BD^2 + 2R \cdot BD + R^2 = AB^2 + R^2$$

$$BD^2 + 2R \cdot BD = L^2$$

Since BD is very small compared to R and L, BD^2 can be ignored.

$$\Rightarrow \quad 2R \cdot BD \approx L^2$$

$$\Rightarrow \quad BD \approx \frac{L^2}{2R}$$

Substituting R = 6365 km

$\Rightarrow \quad BD \approx 0.0786\ L^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.6)$

Where BD is in m

L is in km

$$\text{Refraction} = BC \approx \frac{BD}{7} \approx \frac{0.0786L^2}{7}$$

Actual error in staff reading $= CD = BD - BC$

$$\Rightarrow \quad CD = 0.0786L^2 - \frac{0.0786L^2}{7}$$

$$\Rightarrow \quad CD = 0.0673L^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.7)$$

Where CD is in m

L is in km

When $L = 1$ km $\Rightarrow CD \approx 0.07$ m = 7 cm

$L = 100$ m = 0.1 km $\Rightarrow CD \approx 0.001$ m = 1 mm

Therefore, to keep the effect of the earth's curvature and atmospheric refraction to a minimum, it is advisable that the distance between the level and the staff should not exceed 100 m.

2) Maladjustment of the level (collimation error):

When the line of sight of a level is not perfectly parallel to the axis of the level bubble due to maladjustment of the level, the line of sight will be actually inclined from the horizontal even though the level bubble is perfectly centered. This kind of error can be completely eliminated by balancing the BS and FS distances. Figure 4.13 illustrates the geometry of this source of error.

Let a = Actual BS reading with staff held at point A,

m = Correct BS reading at A if the line of sight was perfectly horizontal,

ε_1 = Reading error at A due to maladjustment of the level,

b = Actual FS reading with staff held at B,

n = Correct FS reading at B if the line of sight was perfectly horizontal,

ε_2 = Reading error at B due to maladjustment of the level, and

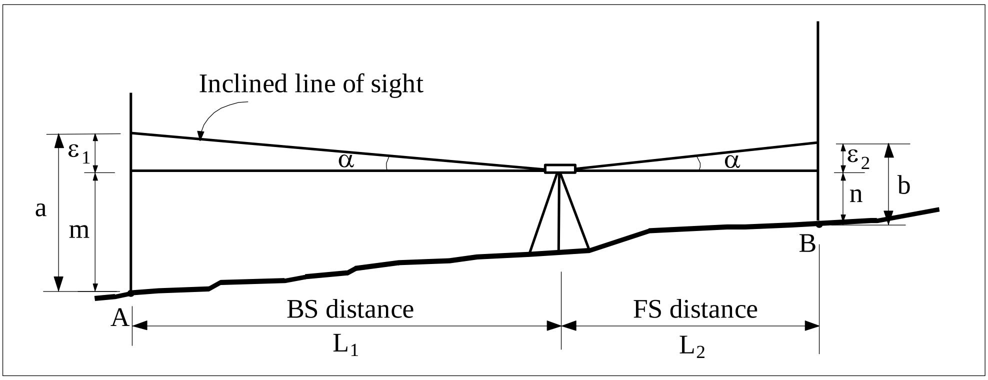


FIGURE 4.13: Error due to the maladjustment of the level.

α = Angle of inclination of the line of sight from the horizontal. It is positive if above the horizontal and negative if below the horizontal.

Then,

$$\left.\begin{aligned} \varepsilon_1 &= L_1 \cdot \tan\alpha \\ \varepsilon_2 &= L_2 \cdot \tan\alpha \end{aligned}\right. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.8)$$

Correct elevation difference (Δh) = m - n

$$= (a - \varepsilon_1) - (b - \varepsilon_2)$$

$$= (a - b) - (\varepsilon_1 - \varepsilon_2) \qquad \ldots\ldots\ldots\ldots (4.9)$$

Substitute equations (4.8) into (4.9),

$$\Rightarrow \quad \Delta h = (a - b) - \tan\alpha\,(L_1 - L_2) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4.10)$$

It is seen for Equation (4.10) that if $L_1 = L_2$, then $\varepsilon_1 = \varepsilon_2$, and the correct elevation difference will be equal to the difference between the two actual readings a & b, and will be free of error caused by the maladjustment of the level.

EXAMPLE 4.2:

To check a level for the existence of collimation error, the level was set up mid-way between points A and B and the following two staff readings were taken: 1.92 m at A and 1.40 at B. The level was then moved to another position and the readings in Figure 4.14 were taken. Is there a collimation error? If the answer is yes, then calculate the angle of inclination of the line of sight from the horizontal, as well as the correct readings that should have been taken at A and B in the second setup if there was no collimation error.

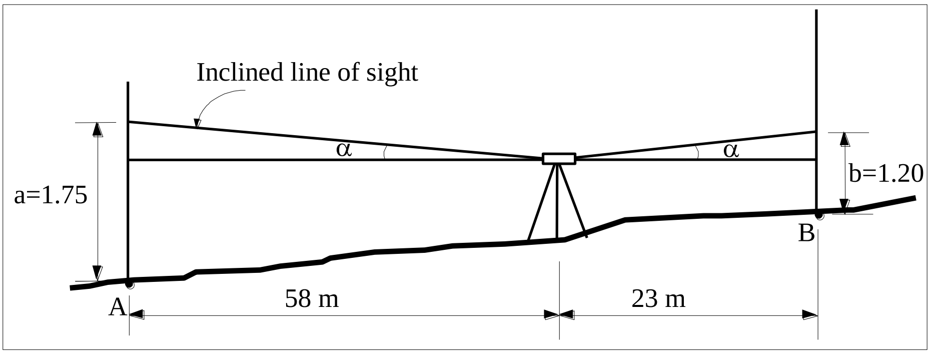


FIGURE 4.14: Checking a level for the existence of collimation error.

SOLUTION:

First setup: ΔH_1 (correct) $= 1.92 - 1.40 = 0.52$ m

Second setup: $\Delta H_2 = 1.75 - 1.20 = 0.55$ m

$\Delta H_1 \neq \Delta H_2 \Rightarrow$ There is a collimation error

From Equation (4.10)

$0.52 = (1.75 - 1.20) - \tan\alpha\,(58 - 23)$

$\Rightarrow \alpha = 0° 2' 57''$

Correct reading at A (m) $= 1.75 - 58 \tan\alpha = 1.70$ m

Correct reading at B (m) $= 1.20 - 23 \tan\alpha = 1.18$ m

Check:

$\Delta H = 1.70 - 1.18 = 0.52 \text{ m} = \Delta H_1 \Rightarrow$ OK

b) Random errors:

The principal sources of random errors that affect the accuracy of leveling results are:

1) The staff not held plumb
2) The bubble of the level not perfectly centered
3) The incorrect reading of the staff
4) The instability of turning points
5) Wind. Wind may vibrate the level and the staff and make it difficult to keep the bubble centered and to read the staff correctly.

c) Blunders or mistakes:

Blunders commonly made in leveling include the following:

1) Misreading the staff especially when the marks on the staff are obscured by a tree, fence and so on.
2) Not setting the staff on the same point for a FS and the subsequent BS readings.
3) Recording or booking of data. Examples of this kind of error will be reading 2.58 m as 2.85 m, or booking a FS in the BS column and vice versa. Page checks should always be made before leaving the field.

4.11 RECIPROCAL LEVELING

When a line of levels crosses a wide body of water (e.g., a river) or a ravine, it becomes impossible to balance the BS and FS distances, and it might be necessary to take sights at distances much longer than ordinarily permissible (greater than 100 m). Under such circumstances, errors due to earth's curvature, atmospheric refraction and the inclination of the line of sight become particularly significant. A special procedure called reciprocal leveling is used to overcome this problem and obtain the best results.

In Figure 4.15, the elevation difference between the two points A and B is to be determined. Using reciprocal leveling, the procedure is described as follows:

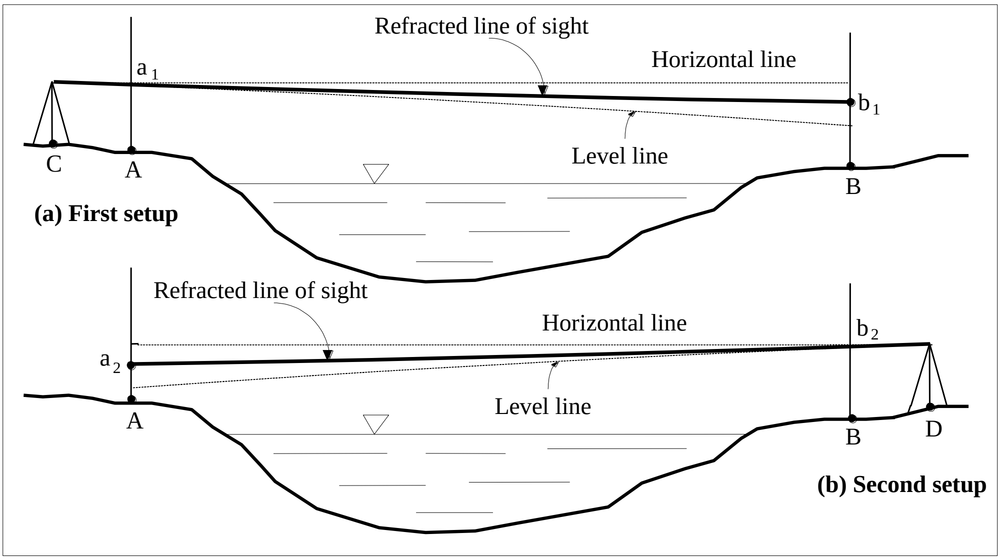


FIGURE 4.15: Reciprocal leveling.

1) Set up the level at point C (Figure 4.15a), 2 to 3 m from A and take the readings a_1 at A and b_1 at B. Calculate the first elevation difference:

$$\Delta H_1 = a_1 - b_1$$

2) Move the level to point D (Figure 4.15b) so that the distance AC = BD. Take the two readings a_2 at A and b_2 at B. Calculate the second elevation difference:

$$\Delta H_2 = a_2 - b_2$$

3) Calculate the correct elevation difference (ΔH) as follows:

$$\Delta H = \frac{\Delta H_1 + \Delta H_2}{2} = \frac{(a_1 - b_1) + (a_2 - b_2)}{2} \qquad \text{...........(4.11)}$$

EXAMPLE 4.3:

The elevation of point A in Figure 4.15 is 917.34 m. From a setup on the left bank, the BS reading at A was 1.44 m and the FS reading at B was 1.90 m. At the second setup (on the right bank) of the level, the BS reading at A was 1.80 m and the FS reading at B was 2.34 m. Find the elevation of point B.

SOLUTION:

Using Equation (4.11),

$$\Delta H_{AB} = \frac{(1.44 - 1.90) + (1.80 - 2.34)}{2} = -0.50 \text{ m}$$

The elevation of point B (H_B) $= H_A + \Delta H_{AB}$

$= 917.34 + (-0.50) = 916.84$ m

4.12 CLOSURE ERROR

For the case where leveling starts at a known BM and ends at the same or at another BM, the calculated elevation for the end station must be equal to the elevation of the known station if the leveling is free of errors. However, this case rarely occurs and a closure error results.

In Figure 4.16, leveling starts at BM1 and ends at BM2 with n_i and Δh_i referring to the number of level setups and elevation difference between consecutive stations. Assume that the known elevation of BM2 is (h) and the calculated elevation from leveling is (h'), then the closure error (ε) is:

$$\varepsilon = h' - h \qquad \text{(4.12)}$$

Corrections for closure error can be distributed to the line sections by proportion. Assuming that all the staff readings along a level line are made with the same care and accuracy, the number of setups needed for each of the line sections can be used for proportioning. Therefore:

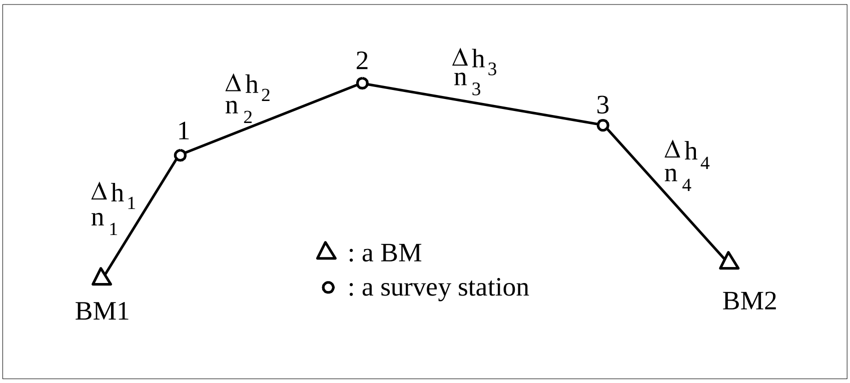


FIGURE 4.16: A level line which starts at a BM and closes at a different BM.

Closure correction for $\Delta h_i = -\dfrac{n_i}{\Sigma n_j}(\varepsilon)$ . . . .(4.13)

Corrected Δh_i = measured Δh_i + closure correction for Δh_i . . . .(4.14)

4.13 CLASSES AND ACCURACY OF LEVELING

Leveling might be divided primarily into two main classes:

1) Precise Leveling. This type of leveling is very accurate and is used mainly for the establishment of benchmarks, for extensive engineering projects, for regional crustal movement investigation, and for any other project, which requires high accuracy. For this purpose, special high precision levels and leveling rods, as well as, field procedures are used.

2) Ordinary Leveling. This type of leveling does not require very high accuracy, and is therefore, suitable for small engineering project, for topographic mapping, for making longitudinal and cross-sections, and for other projects for which medium accuracy is sufficient.

The allowable closure error in differential leveling is normally given in the form: $\varepsilon = \pm k\sqrt{D}$ mm where k is a constant determined by the type of leveling and D is the total leveled distance in kilometers. The value of k in ordinary leveling may vary from 10 to 30 mm depending on the type of ground being leveled, the accuracy required and the type of instrument being used. For precise leveling, the value of k may vary from 2 to 5 mm.

4.14 APPLICATIONS OF LEVELING

Apart from the general problem of determining elevation difference between two points, which has been fully dealt with in the previous sections, the main uses of leveling are:

1) The making of longitudinal sections (profiles)
2) The making of cross-sections
3) Contouring, and
4) Setting out levels.

4.14.1 LONGITUDINAL SECTIONS (PROFILES)

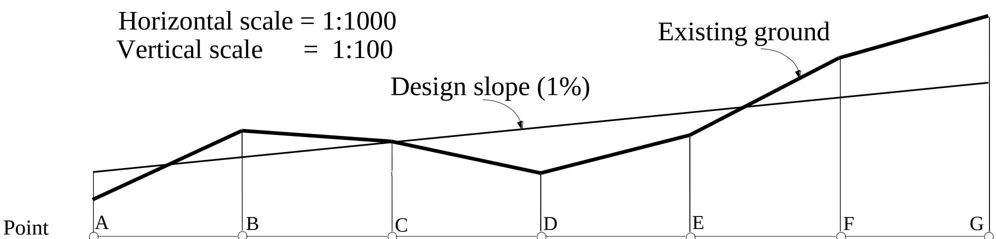
An example of such a section is given in Figure 4.10. The objective here is to reproduce on paper the existing ground profile along a particular line such as the center line of an existing or proposed work like the center line of a railway, road, canal, sewer or water main. As a general guide, levels are taken at:

1 - Every 20 m, 50 m or 100 m depending on the topography
2 - Points at which gradient changes, and
3 - Street intersections.

Staff readings to 0.01 m accuracy are generally adequate. The sections are usually plotted to a distorted scale, a common one for roadwork being 1/1000 horizontal and 1/100 vertical. This is due to the fact that the horizontal distance represented on the horizontal axis is much larger than the elevation variations of profile points. Figure 4.17 shows an example of a plotted profile with points connected by straight lines.

To avoid the build up of errors, the following points should be kept in mind when leveling to produce a longitudinal section:

1 - Start the work at a benchmark (BM) and make use of nearby benchmarks.
2 - Try to keep BS and FS distances approximately equal. This will minimize errors resulting from earth's curvature, atmospheric refraction and maladjustment of the level.



Point	A	B	C	D	E	F	G
Distance	0.0	20.0	40.0	60.0	80.0	100.0	120.0
Ground Elevation	97.48	98.42	98.27	97.84	98.36	99.41	99.98
Design Elevation	97.87	98.07	98.27	98.47	98.67	98.87	99.07
Cut		0.35	0.00			0.54	0.91
Fill	0.39		0.00	0.63	0.31		

FIGURE 4.17: A longitudinal section (profile) of the data of Table 4.2.

3 - Make all changes (TP) on firm ground to make checks if required later.
4 - Take the final FS on a BM or close back to the starting point.
5 - Do not work with the staff extended in high wind.

4.14.2 CROSS-SECTIONS

Some engineering works require that cross sections be taken at right angles to the centerline of a proposed or existing project such as a road. The width of these sections must be sufficient to cover the proposed works, e.g. 15 m either side of the centerline for a normal road. The longitudinal spacing of the sections depends on the nature of the ground, but should be constant if earthworks are to be computed. A spacing of 20 m is common.

The centerline is first set, and then perpendiculars are erected by the methods discussed earlier (see Chapter 3). If all the staff readings can be taken by a single setup of the level, a neat way to book the readings is as follows (Figure 4.18).

It is common to plot cross-sections to a normal, i.e. undistorted scale. A scale of 1/50 or 1/100 for both horizontal and vertical axes is good for this purpose.

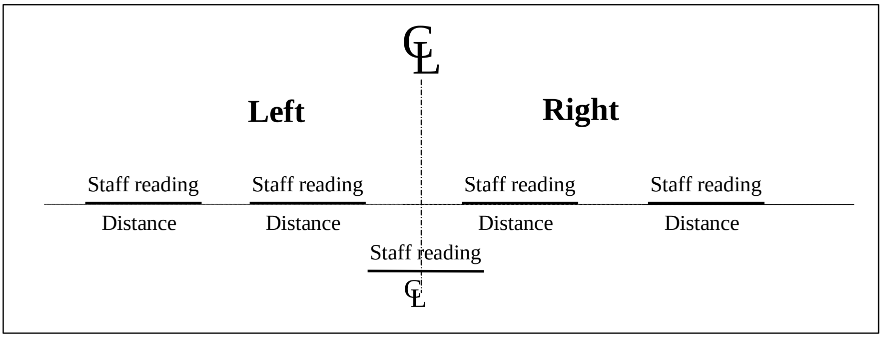


FIGURE 4.18: Booking leveling data for making cross-sections.

4.14.3 CONTOURING

A *contour* is an imaginary line connecting points on the ground that have the same elevation. It may be considered as the trace of the intersection of a level surface with the ground. The shoreline of a body of still water is a typical example of a contour. The vertical distance or elevation difference between two successive contours is called *contour interval*. Figure 4.19 shows an example of contours that have a contour interval of 20 m.

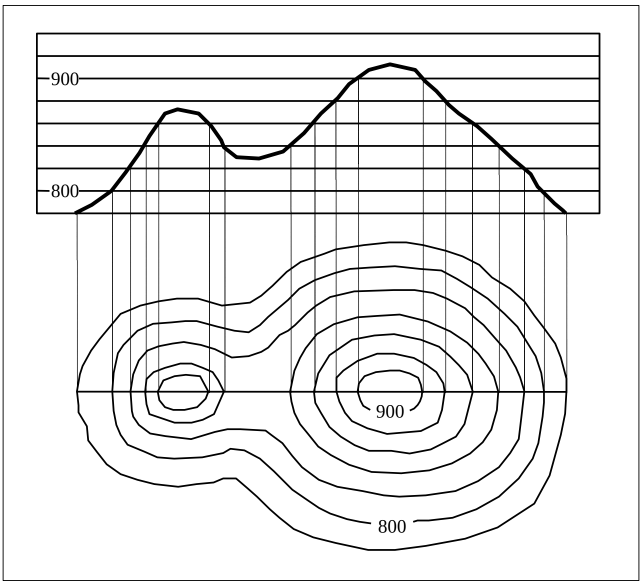


FIGURE 4.19: Contours and ground profile.

Characteristics of Contours:

The principal characteristics of contours are:

1) Contours spaced closely together represent a steep slope. When spaced far apart, they indicate a gentle slope (Figure 4.20).

2) Contours are perpendicular to the direction of the steepest slope (Figure 4.20).

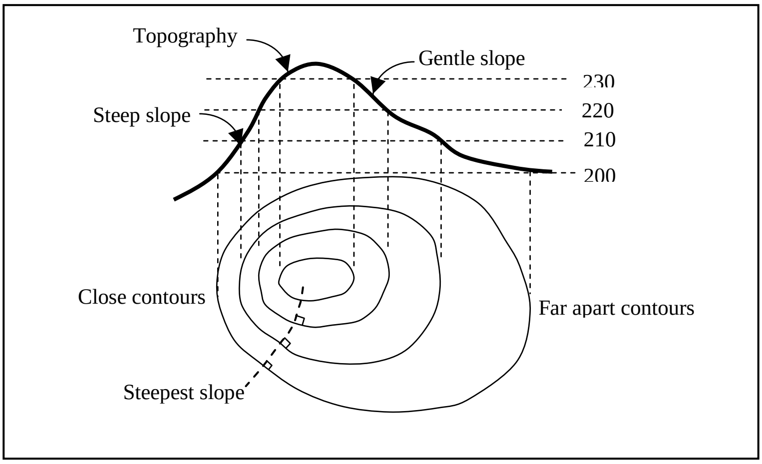


FIGURE 4.20: Relationship between contour spacing and topography slope.

3) Contours of different values do not cross each other nor do they merge except in rare situations where there is a cave or a vertically standing surface such as a wall. Figure 4.21 shows these two exceptions.

4) Contours that portray summits (such as a hill) or depressions (such as a bottom of a lake) are closed lines.

5) A single contour cannot split into two contours of the same value as itself, and it must make a closed circuit although not within the area covered by the contour plan.

6) Irregular contours represent a rough and uneven terrain.

Figure 4.22 shows examples of contours for different types of relief.

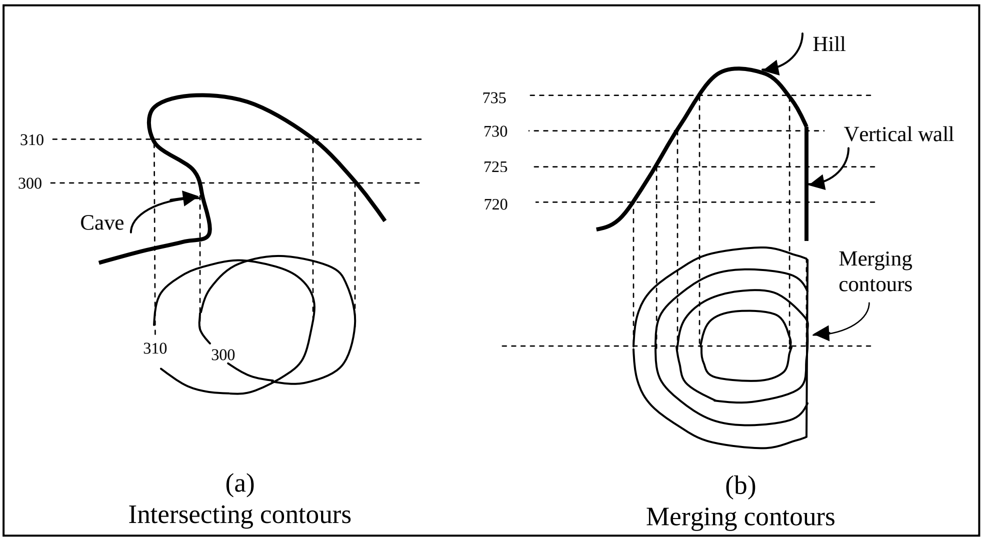


FIGURE 4.21: Examples of intersecting and merging contours.

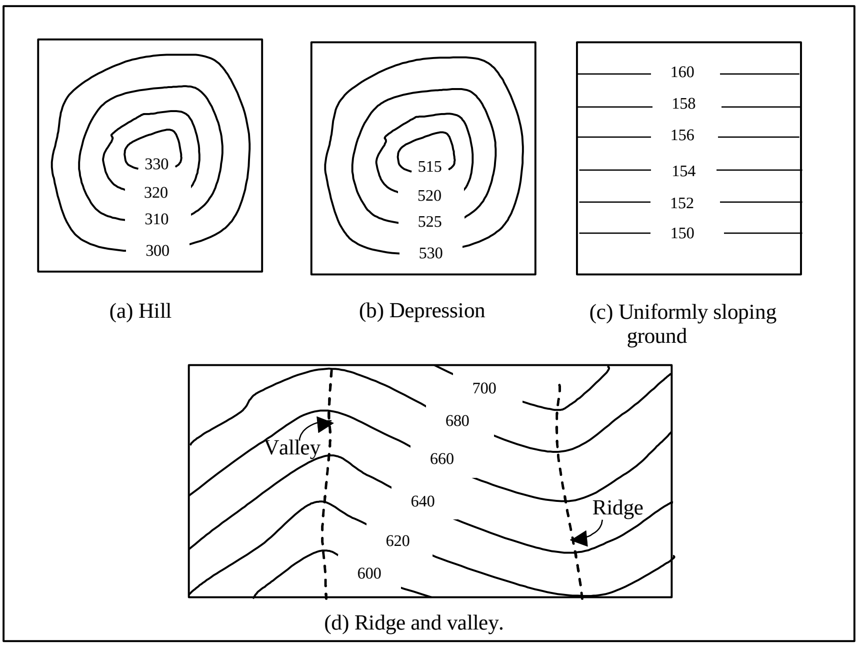


FIGURE 4.22: Examples of contours for different types of terrain.

Factors affecting the choice of contour intervals:

Several factors affect the choice of a contour interval. These include:

1) The scale of the contour plan. As the scale of the contour map becomes smaller, the distance between the contours becomes also smaller, and as a result, the map becomes crowded with lines and difficult to read. To prevent so, the contour interval is made larger.

2) The importance and purpose for which the plan is to be used. When more details are needed in the contour plan for computation of earth volumes and for design purposes, a smaller contour interval should be used. However, if the contour plan aims merely to give an idea about the shape of the relief, a larger contour interval can be used.

3) Accuracy, time and cost of the contour plan. As higher accuracy is needed, a smaller contour interval should be chosen. As a result, the project will be more costly and will require longer time to be accomplished.

4) The topography of the ground. Steep ground requires a larger contour interval to increase the distance between contours, while a small contour interval can be used for flat terrain.

5) The area for which the contour plan is to be made. As the area to be covered becomes larger, a smaller scale is usually chosen to draw the map. As a result, a larger contour interval is used.

Methods of Contouring:

The following are probably the most common methods for contouring:

1) *Griding*. This method is most suitable for flat terrain, especially on comparatively small sites. Rectangles or squares of 5 to 20 m a side are usually set out on the ground in the form of a grid, and levels (staff readings) are taken at the corners (Figure 4.23).

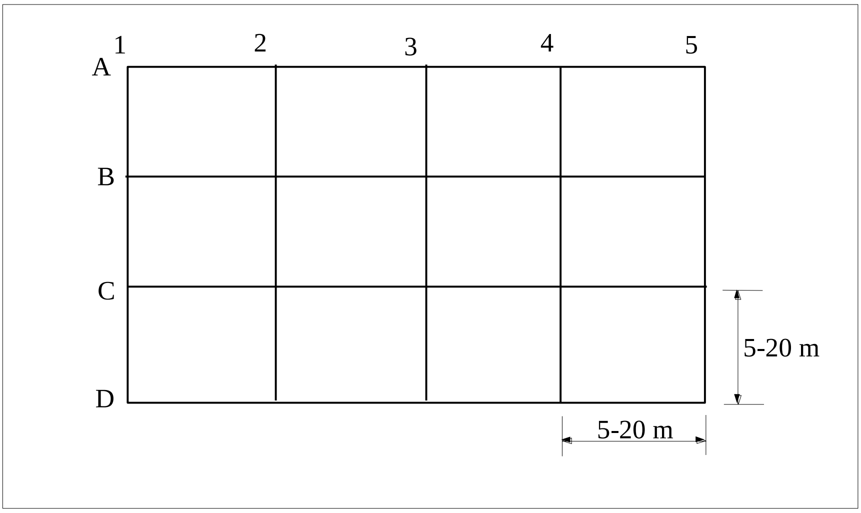


FIGURE 4.23: Griding.

In the booking process, corners are given numbers according to their coordinates in the grid, such as B3 or C5. The reduced levels of these corners are plotted on a plan, which has been girded in the same manner according to some suitable scale. The required contour lines are then plotted by a process called linear interpolation. To illustrate this process, let us assume that the elevations of points D4, D5 and C5 are 13 m, 10 m and 12 m respectively. Let us also assume that the distance between corners D4 and D5 is 12 m and the distance between corners C5 and D5 is 10 m. Now, if the contour line whose elevation is 11 m, is at distance x from corner D5 in the direction of D4, then from Figure 4.24,

$$\frac{x}{12} = \frac{1}{3} \Rightarrow x = \frac{12}{3} = 4 \text{ m}$$

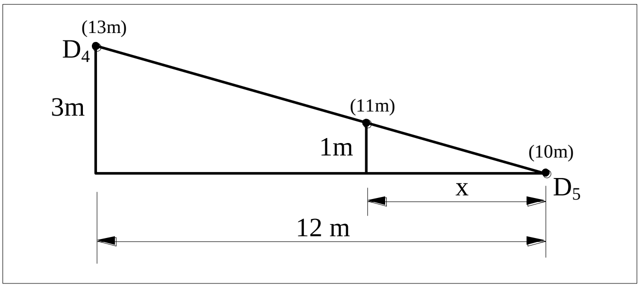


FIGURE 4.24: Linear interpolation.

This means that contour 11 m is 4 m from corner D5 in the direction of D4. By the same way, contour 11 m is found to be midway between corners C5 and D5. Contour 12 m is at 8 m from D5 in the direction of D4. Figure 4.25 shows the 11 m and 12 m contours drawn by a smooth line.

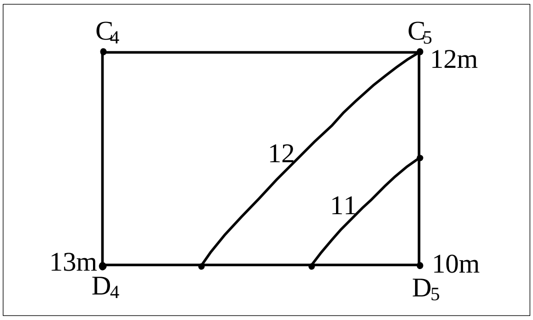


FIGURE 4.25: Plotting contour lines.

Notes:

a) The grid does not need to be made of regular squares or rectangles only. A grid of triangles or any other shapes could also be used. Also, an irregular grid can be used as long as it can be set up on the ground and plotted on a map.

b) The method of linear interpolation explained above is only one of the simplest methods used for contouring. Other several methods are available and can be used. It is beyond the scope of this book to talk about methods of contouring, and the reader can refer to specialized references about the subject.

2) *Radiating Lines*. Rays are set out on the ground from a central point such as the top of a small hill, the directions of these rays being known. Levels are taken along these rays at measured distances from the center (Figure 4.26). Again, linear interpolation is used to give the contour lines.

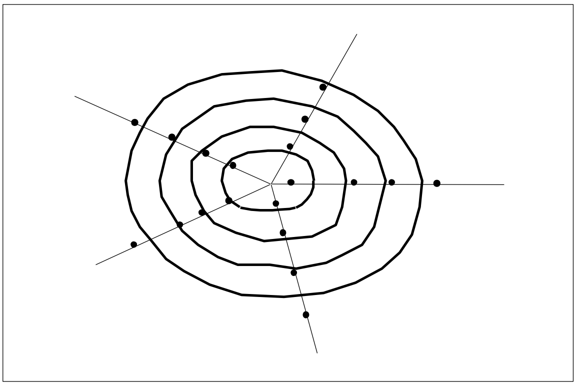


FIGURE 4.26: Contouring by radiating lines.

3) *Cross-Sections Method*. In this method, cross-sections are made on a line or a traverse inside the area for which the contour plan is to be made (Figure 4.27). Levels are then taken at points on the cross-sections where the topography changes. Again contour lines are drawn using the method of linear interpolation as explained earlier.

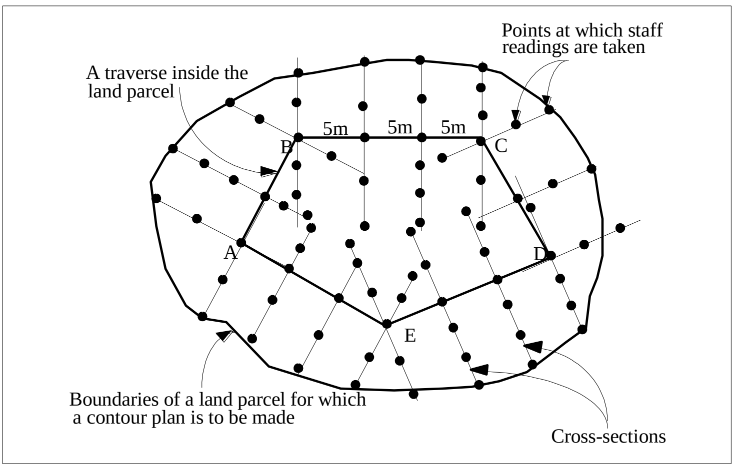


FIGURE 4.27: Contouring by the cross-sections method.

4) *Contouring Using Electronic Total Stations (see Chapter7: Section 7.2.8).* In this method, the coordinates and heights of representative points where topography changes are measured using total stations. These points are then plotted and contour lines are drawn using the method of linear interpolation as explained earlier.

4.14.4 SETTING OUT LEVELS

One of the basic applications of leveling is setting out sight rails, which enable the excavation operators to cut out earth to an even gradient, and enable the pipe-layer to lay the pipes according to this gradient. The following example illustrates the process:
Consider a length of a sewer being laid from manhole A, with an invert level of 30.02 m, to manhole B, 60 m away, and the gradient from A to B being 1 in 100 and falling from A to B (Figure 4.28). Thus, if two rails are fixed over stations A and B about 1 m above ground level, and each a fixed height above the invert level, then an eye sighting from rail A to rail B will be sighting down

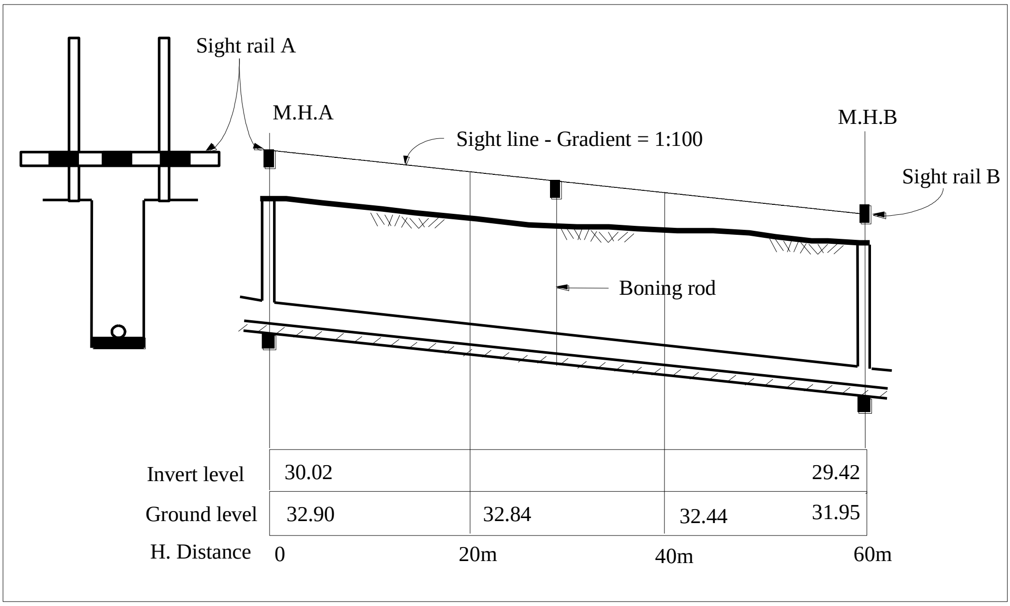


FIGURE 4.28: Setting out levels for laying out a sewage pipe.

a gradient equal to that of the proposed sewer (i.e., 1 in 100 here). In this example, a convenient height above the invert level is taken to be 3.75 m. If while digging, a boning rod (looking like the capital letter **T** with a sight bar across it) of this length is held vertically somewhere between points A & B so that its sight bar just touches the line of sight between sight rails A and B, it would give at its lower end a point on the sewer invert line.

To fix these sight rails for use with a 3.75 m long boning rod, we drive two posts on either side of the manholes and nail the rails between these at the following levels:

Sight rail A, RL = 30.02 + 3.75 = 33.77m
Distance AB = 60 m
Fall = 60 x 1/100 = 0.60 m
Invert level at B = 30.02 - 0.60 = 29.42 m
$\Rightarrow$ Sight rail height at B, RL = 29.42 + 3.75 = 33.17 m

If a level set up nearby has a height of collimation of 34.85 m, then the staff is moved up and down the posts at MH_A until a reading of 34.85 - 33.77 = 1.08 m is obtained. Pencil marks are made on each post and the black and white sight rail is nailed in position as shown in Figure 4.28. For rail B, the staff reading would be 34.85 - 33.17 = 1.68 m.

PROBLEMS

4.1 The following staff readings were taken during a leveling work: 1.26, 0.82, (1.79), 2.24, (1.25), X, -1.38, (1.42), 0.03, (1.19), 1.35, (2.37). Find the missing reading X, given that the height of the first and last points is 720.00 m AMSL and that the readings in brackets are foresights.

4.2 The staff readings observed with a level were: 2.565, 2.305, 2.115, 3.725, 3.565, 2.570, 1.905, 1.675, 1.565, 1.475, 3.725, 3.250, 3.105 and 1.250 m.

The level was moved after the fourth, ninth and twelfth readings. The first reading was taken on a BM of RL = 100.000 m and a standard deviation of ± 0.010 m. Calculate the difference of levels between the first and last points by both the height of instrument and the rise & fall methods. Check the accuracy of the arithmetic calculations. Also calculate the standard deviation of the RL of the last point given that the standard deviation of a single staff reading = ± 0.005 m.

4.3 A leveling section is run downhill from A to B. The 4m long staff readings taken in order are as follows: 0.53, 1.82, 2.97, 3.75, 1.02, 2.15, 3.36, 3.80, 1.09, 2.70 and 3.94. Given that point B is 500.00 m AMSL, tabulate these readings and calculate the reduced levels (RL) of all the section points and perform the required arithmetic checks.

4.4 A page of a field book is reproduced in the next table with some readings missing. Each station is compared with the one immediately preceding it. Complete the page with all arithmetic checks filling all X marks. (Note: staff readings are in ft).

Point	BS	IS	FS	Rise	Fall	RL	Notes
1	3.65					X	BM
2		X		2.75		X	
3		2.83				X	
4		3.64				X	
5	X		7.42			X	TP
6		12.41			7.32	X	
7		4.32				X	
8		3.00				X	
9		-6.17				X	
10	X		X			108.26	TP
11			X		1.32	X	
Sum	17.66				25.93		

4.5 A level set up in a position 30 m from peg A and 60 m from peg B reads 1.914 m on a staff held at A and 2.237 m on a staff held at B. The bubble has been carefully brought to the center of its run before each reading. It is known that the reduced levels of the tops of the pegs A and B are 87.575 m and 87.280 m respectively. Calculate:

a. The collimation error.
b. The readings that would have been obtained had there been no collimation error.

4.6 The first and last points of a longitudinal section were not given. For this purpose, you started leveling at point A with RL = 625.13 m AMSL and closed at point B with RL = 628.80 m AMSL. The readings are: 2.34, 1.15, 2.86, 0.97, 1.99, 2.68, 3.06, 2.24, 1.16, 1.48, 1.48, 2.90, 2.31, 2.85, 1.62, 0.89, 1.94, 0.92, 2.08, 1.88, 1.12. In each of the fourth and sixth positions of the level, you took two intermediate sights. In the fifth position, you took only one intermediate sight.

a. Tabulate the readings and calculate the RL. How accurate is the work.
b. The longitudinal section starts at the fourth point of the leveling and contains 9 points of equal 30 m distance separations. Draw the

longitudinal section to scales 1:1000 for distances and 1:100 for elevations. Show the road centerline on the same section if the first point has a RL = 627.00 m AMSL and goes upwards at a slope of 1%.

4.7 The following table gives the RL of some grid points. If you know that the grid is comprised of 3m side squares, and that point A1 is located at the top left corner of the grid, and point E1 is at the bottom left corner of the grid, draw the grid at a scale of 1:100, and show all the contour lines at 1 m interval.

Point #	RL (m)	Point #	RL (m)	Point #	RL (m)
A1	19	B5	17	D4	15
A2	20	C1	19	D5	14
A3	20	C2	20	E1	16
A4	19	C3	20	E2	15
A5	17	C4	18	E3	14
B1	20	C5	17	E4	13
B2	23	D1	18	E5	12
B3	23	D2	17		
B4	20	D3	16		

4.8 Proof that the approach discussed in section 4.11 about reciprocal leveling gives an elevation difference, which is free of errors due to the earth's curvature, atmospheric refraction and maladjustment of the level.

4.9 The adjustment of a tilting level was checked by taking the following readings on a vertical staff held in turn at stations X & Y that are 60 m apart:

LEVEL POSITION	STAFF READING	
	at X	at Y
Midway between X and Y	1.514	1.968
90 m from X on XY	2.025	2.439

Comment on these readings. This level was then used without making any adjustments to establish two sight rails at A and B, 90 m apart, for the setting out of a sewer which had a gradient of 1 : 120 falling from A to B. A backsight of 1.06 m was taken on a BM that has a RL of 81.46 m above datum, 60 m away from the level. What staff readings were needed to locate sight rails at A and B, which were 45 m & 75 m respectively away from the level, given that the invert level at A was 78.58 m above datum and that a boning rod of length 3.50 m was to be used?

4.10 In Figure 4.29, a level loop starts and ends at the same BM100 whose elevation is 732.456 m AMSL. Check for closure error and make any required corrections.

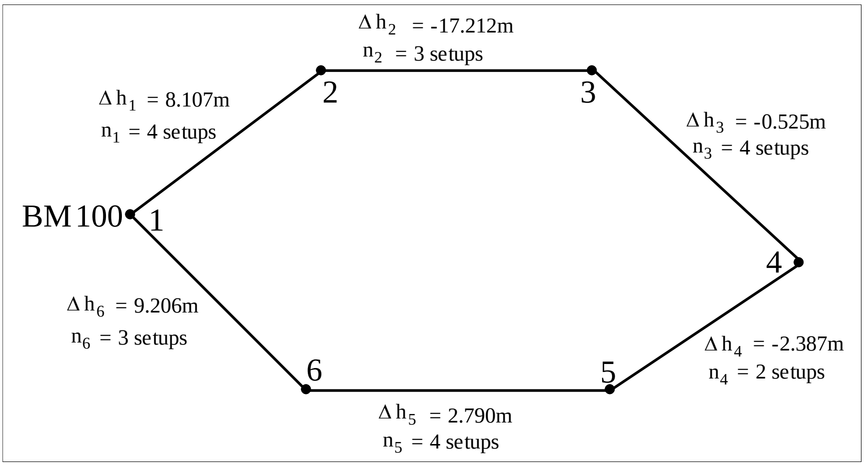


FIGURE 4.29: A level loop.