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Case Study

Inventory, transportation, service quality and the location of distribution centers

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Abstract

A crucial question in the design of efficient logistics systems is the identification of locations for distribution centers (DCs). However, the optimization of these location decisions requires careful attention to the inherent trade-offs among facility costs, inventory costs, transportation costs, and customer responsiveness. This paper presents a modeling approach that provides such an integrated view, and illustrates how it works in the context of a specific example involving the distribution of finished vehicles by an automotive manufacturer. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Inventory analysis; Location analysis; Weighting method; Multi-objective optimization

1. Introduction

US manufacturers and retailers are increasing their focus on logistics systems, looking for ways to reduce costs and improve customer responsiveness (providing a desired product where and when the customer wants it). The goal of cost reduction provides motivation for centralization of inventories (Eppen, 1979; Stulman, 1987; Chang and Lin, 1991). On the other hand, the goal of customer responsiveness provides motivation for having goods as near to the final consumer as possible. Thus, there is a basic conflict between these objectives, and locating distribution centers

(DCs) is a critical decision in finding an effective balance between them. Location decisions for DCs also affect transportation costs. This paper describes a modeling approach that provides an integrated view of inventory costs, transportation costs, and service levels when making DC location decisions.

2. Background

Thomas (1997) has provided the data plotted in Fig. 1, which show inventory and freight transportation costs for the US economy in 1985, and through the 1990s. Freight transportation costs have been nearly constant (at about 6% of GNP) over the entire period of the data. Inventory costs fell significantly between 1985 and 1992, as

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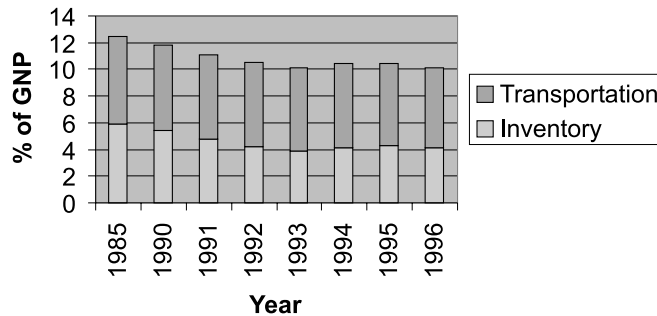


Fig. 1. Physical distribution (inventory and transportation) costs in the US.

companies implemented just-in-time systems and the economy grew steadily. However, since 1992 there has been almost no change in inventory costs, and total physical distribution costs seem to have reached a plateau. Further improvements are likely to require simultaneous attention to both inventory and transportation cost elements.

In addition to concern with total inventory and transportation costs, measures of service quality, or customer responsiveness must also be considered in the design of distribution systems. For example, General Motors is instituting a “customer express delivery” system in an attempt to provide better availability of a wide range of vehicles to customers, while reducing the amount of inventory held on dealer lots.

With the customer express delivery program, new vehicles would be held at DCs and be made available to the dealers on order, using a 24-hour delivery standard. A pilot test in Florida started in 1994 (Wall Street Journal, 1995), and has since been expanded to include areas in Maryland and California. The locations of the DCs which will hold the vehicle inventory is a critical set of decisions, and GM’s efforts to address this problem have provided a major motivation to the work described here.

However, re-examination of DC locations and inventory holding policies is by no means limited to the automotive industry. For example, Bowman (1996) describes the efforts of Best Buy (a large consumer electronics retailer) to reconfigure their DCs, and emphasizes customer responsiveness as a critical factor. He also notes that the

desire to provide just-in-time delivery to customers, using smaller, more frequent shipments, means that outbound transportation costs from the DC are relatively high. Thus, there is an additional incentive to locate near major customers.

Bucko (1996) describes how The Gap (a large clothing retailer) is reconfiguring their distribution operations. Their suppliers now ship initial orders directly to retail outlets, and the DCs receive only items intended for use as replenishment stock. This changes the role of the DCs. They now handle only about 50% of the flow of goods, and must be able to replenish stocks of specific items at individual retail stores very quickly. As a result, The Gap is re-evaluating the number and locations of the DCs.

There is a clear need to evaluate trade-offs among inventory costs, transportation costs, and customer responsiveness. The purpose of this paper is to focus on the integration of discrete choice location models, inventory analysis and multi-objective techniques to model the overall logistics impacts of locating DCs. As a first step in the analysis, an approximate inventory cost function is proposed that can be embedded in a fixed-charge facility location model. This allows the decisions on the optimal number and location of DCs to be directly tied to inventory cost implications. As a second step, the fixed-charge location model is extended to incorporate multiple objectives (minimizing cost and maximizing service coverage). The third step shows how this integrated model can be used to explore important trade-offs in the DC location decisions.

3. Modeling inventory costs

Fig. 2 provides a simple illustration of product flow from plants to retail outlets through DCs. Retailers demand products from DCs to which they are assigned in response to customer demand. DCs backorder excess demand. In parallel to these actions, orders are placed for the same products at the plants. That is, the inventory policy adopted at the DCs is continuous review with one-for-one replacement.

For a given number of retail outlets, inventory levels can be determined to provide a given level-of-service. Consider a single product configuration or group of configurations that for inventory planning purposes can be analyzed together and for which demand levels are assumed to be independent. That is, a temporary shortage of one product does not increase the demand for related products.

Assume that there are n retail locations assigned to the DC, each with a Poisson demand process whose mean rate is λ_i , where $1 \leq i \leq n$. Therefore, the demand at the DC follows a Poisson distribution with a mean rate:

$$A = \sum_{i=1}^n \lambda_i. \tag{1}$$

Assume the DC has s units of a product in inventory, and orders one unit from the plant each

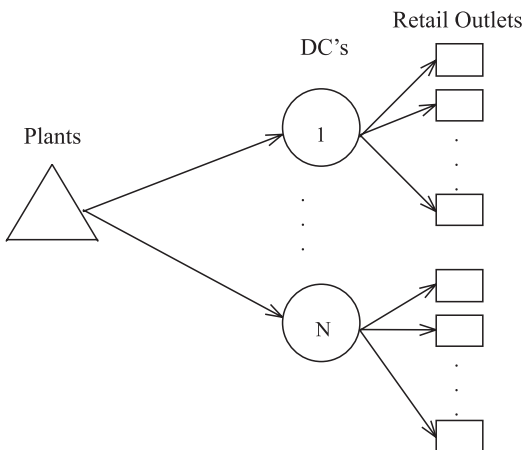


Fig. 2. A basic DC system.

time one unit is sent to a retail outlet. Let μ and σ^2 represent the mean delivery time and its variance for a product shipment from plant to DC.

Stockout rate, or the percentage of demand that cannot be satisfied from on-hand inventory, is an important level-of-service measure in inventory systems. In the presence of uncertain demand, an amount of safety stock will be carried to reduce stockout rates. Palm's Theorem (Feeney and Sherbrooke, 1966) states that if demand is Poisson distributed and replacements are made on a one-for-one basis, the total number of units in replenishment (on order) is also Poisson distributed. If the demand rate at the DC is λ , and the average replenishment time is μ , then the number of units on order, m , is Poisson distributed with parameter $\lambda\mu$. If the established stock level is s , then the probability of a stockout is simply $\Pr(m > s)$:

$$\Pr(m > s) = \sum_{k=s+1}^{\infty} \frac{e^{-A\mu}(A\mu)^k}{k!}. \tag{2}$$

Eq. (2) can be used to find the minimum inventory necessary for a maximum stockout rate. If r is the desired stockout rate, then find s_r , the smallest value of s such that Eq. (2) is less than or equal to r . Inventory savings from consolidation result from reductions in safety stock, as discussed below.

Nozick and Turnquist (1998) show that for a given stockout rate the safety stock held at each DC varies with the square root of the number of DCs. This is consistent with results provided by other authors like Eppen (1979). Nozick and Turnquist (1998) also show that for a given stockout rate and total demand, safety stock can be accurately approximated with a linear function as long as the number of DCs is relatively large. This provides an effective way to incorporate inventory costs into location models. They also show that if safety stock is calculated based on an equal allocation of demand to each DC, the result is an upper bound on actual safety stock required for any other demand assignments to DCs. This means that a conservative estimate of safety stock requirements can be determined from the number of DCs, without specifying exact locations and demand volumes.

In order to illustrate these concepts, suppose an auto manufacturer sells 700 products or new vehicle “configurations” in the continental US of which 200 have yearly demand of 8000 units, 225 have yearly demand of 6000 units, and 275 have yearly demand of 4000 units. Total annual demand for all products is therefore 4,050,000 new vehicles. Fig. 3 shows the expected safety stock for different numbers of DCs, given yearly volumes and a 5% stockout rate. Notice that safety stock is relatively linear in the range of 15–50 DCs.

A linear regression equation is estimated to predict safety stock requirements for each of the three demand volumes shown in Fig. 3 using data for the 15–50 DCs range. The equations are then aggregated using the number of configurations with each demand volume, yielding the following regression relationship between safety stock and number of DCs:

$$s = 58,836 + 2140 * N. \tag{3}$$

If average vehicle price is assumed to be \$15 000, and yearly holding cost is 22%, the inventory cost equation is as follows:

$$\$194,158,800 + \$7,063,523 * N \tag{4}$$

implying that a safety stock annual inventory cost is slightly over \$7 million for each additional DC.

Clearly, parameter values in (4) reflect assumptions concerning product prices, number of products, annual demand, etc. However, the idea is that if total inventory costs can be represented by a linear function of the number of DCs to be located, inventory costs can be embedded directly into a location model.

4. A location model

The fixed-charge facility location model can be specified as follows (Daskin, 1995):

$$\text{Minimize } \sum_j f_j X_j + \alpha \sum_{i,j} h_i d_{ij} Y_{ij} \tag{5}$$

subject to

$$\sum_j Y_{ij} = 1 \quad \forall i, \tag{6}$$

$$Y_{ij} \leq X_j \quad \forall i, j, \tag{7}$$

$$X_j \in (0, 1) \quad \forall j, \tag{8}$$

$$Y_{ij} \in (0, 1) \quad \forall i, j, \tag{9}$$

where f_j is the fixed cost of creating a facility at candidate site j , h_i the demand at location i , d_{ij} the

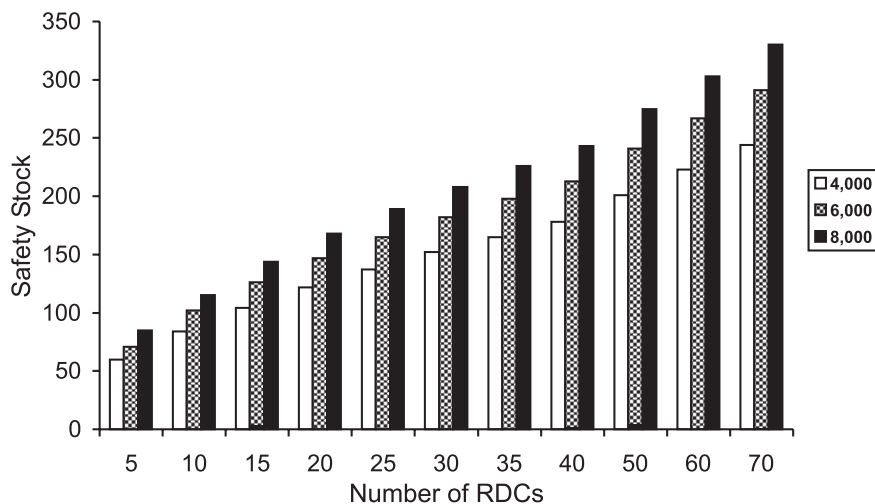


Fig. 3. Safety stock for different number of DCs and annual demand level.

distance from demand location i to candidate site j , α the cost per unit distance per unit demand,

$$x_j = \begin{cases} 1 & \text{if a facility is located at candidate site } j, \\ 0 & \text{otherwise,} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{if demands at } i \text{ are served by} \\ & \text{a facility at candidate site } j, \\ 0 & \text{otherwise.} \end{cases}$$

The linear approximation to safety stock requirements is incorporated into the fixed-charge coefficient, f_j , for the candidate sites. The “slope” parameter from the inventory cost equation (4) becomes part of f_j , because it reflects a constant increment in inventory safety stock needed for an additional DC. The intercept term in (4) is independent of the number or location of facilities, so it plays no role in the optimization.

The model specified by (5)–(9) minimizes both inventory costs and transportation costs. Solution procedures for the fixed-charge model include a variety of greedy heuristics including the add, drop, and exchange heuristics, Lagrangian relaxation, and branch and bound (Mirchandani and Francis, 1990; Daskin, 1995; Dresner, 1995). A hybrid heuristic using a combination of a greedy-add and an improvement algorithm is used in this paper, as discussed by Daskin (1995).

Integration of inventory costs into the location model is an important step for overall cost minimization, but it still does not deal with the competing objective of providing a high level of customer responsiveness in the distribution system. Stockout rate indirectly reflects customer demands, but the distance between DC and retail outlet may be too large to meet time-based service standards.

The objective of providing fast, reliable, delivery of products to retail outlets may be met by operating a large number of DCs conveniently located, implying large facility development and operating costs, and higher inventory costs. Thus, there is a fundamental trade-off between customer responsiveness and costs when designing a DC system.

A mathematical model to maximize coverage ensures that a proportion of demand that is within

a specified “coverage” distance of a DC will be met. Delivery is then guaranteed for the set of retail outlets within a certain radius (e.g., 200 miles) of the DC. The objective of maximizing the proportion of total demand covered by a set of N facilities was first described by Church and ReVelle (1974). An equivalent model which minimizes uncovered demand was formulated by Hillsman (1984). This formulation facilitates the integration of coverage maximization and cost minimization. Defining the following variable:

$$q_{ij} = \begin{cases} 1 & \text{if a facility located at candidate} \\ & \text{site } j \text{ cannot cover demand at } i, \\ 0 & \text{otherwise,} \end{cases}$$

the minimization of uncovered demand can be expressed as follows:

$$\text{Minimize } \sum_{i,j} h_i q_{ij} Y_{ij} \quad (10)$$

subject to

$$\sum_j Y_{ij} = 1 \quad \forall i, \quad (11)$$

$$\sum_j x_j = N, \quad (12)$$

$$Y_{ij} \leq X_j \quad \forall i, j, \quad (13)$$

$$Y_{ij} \in (0, 1) \quad \forall i, j, \quad (14)$$

$$X_j \in (0, 1) \quad \forall j. \quad (15)$$

Constraints (11) and (13)–(15) in this model are identical to constraints (6)–(9) in the fixed-charge model, allowing integration of the total cost objective and the coverage objective. More specifically

$$\text{Minimize } \sum_j f_j X_j + \sum_{ij} \{Wh_i q_{ij} + \alpha h_i d_{ij}\} Y_{ij} \quad (16)$$

subject to

$$\sum_j Y_{ij} = 1 \quad \forall i, \quad (17)$$

$$Y_{ij} \leq X_j \quad \forall i, j, \quad (18)$$

$$X_j \in (0, 1) \quad \forall j, \quad (19)$$

$$Y_{ij} \in (0, 1) \quad \forall i, j. \quad (20)$$

where W is the weight given to the objective of minimizing uncovered demand. Notice that the number of DCs is endogenous to the model, in the fixed-charge facility location model. All demand will be served but the uncovered demand will be served at a lower level-of-service.

If W is very large then the model given by Eqs. (16)–(20) is equivalent to minimizing uncovered demand. This will result in the location of a large number of DCs because the constraint that limits the number of facilities to locate has been removed. If W is small, then the model is equivalent to minimizing total cost. By varying the value of the weight, W , a variety of trade-off solutions can be identified. As the model given by Eqs. (16)–(20) has the same structure as the fixed-

charge facility location model, standard algorithms for solving that problem can be used.

In any given application of the model, the percentage coverage is determined by the trade-off with total cost. If a pre-specified percentage of coverage must be guaranteed, then an alternative formulation (and solution method) may be used, as described by Nozick (2000).

5. Application of the methodology

An automotive manufacturer serving the continental US through discrete demand areas is used as an illustration. For this analysis, a set of 698 demand areas (defined as an aggregation of counties) are used, all of which also represent potential DC locations. Demand within each demand area is assumed to occur at the centroid of the area as shown in Fig. 4, where the size of the circles represents volume of demand. Centroid to centroid distances have been calculated using available electronic representation of the highway network (Bureau of Transportation Statistics, 1996).

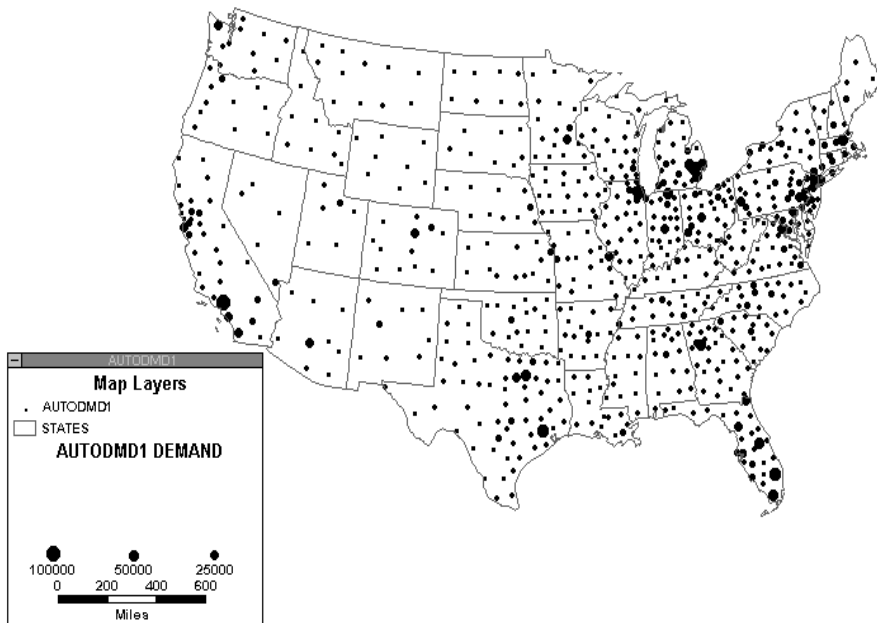


Fig. 4. Demand centroids and relative demand volumes.

Assume that DC construction costs are \$10 million with an expected life of 30 years, and that capital costs to the manufacturer are 15% per year implying an amortized annual facility cost of \$1.5 million. Further, assume that the per-mile cost for delivering vehicles from the DCs to dealerships is 60 cents.

The problem specified by (16)–(20) can be solved for varying values of the weighting parameter, W , to identify a family of possible distribution system designs. A similar sort of multi-objective location model is discussed by ReVelle and LaPorte (1996). They describe a slightly different model formulation and discuss possible solution procedures, but do not solve the problem or provide illustrations. ReVelle and LaPorte (1996) also note that a weighting method for finding the efficient frontier may not identify all efficient solutions, because the feasible region of the optimization problem is non-convex.

Fig. 5 illustrates the efficient frontier for the automotive example. At one extreme, the weighting parameter W , set to zero, generates a minimum cost solution with 23 DCs. This solution has a total annual cost of \$641 million, and covers 87% of demand within 200 miles of a DC. A map of the solution is shown in Fig. 6.

At the other extreme, a very large value of W covers 100% of demand within 200 miles. However, this solution requires 64 DCs, and has an annual cost of \$912 million. A map of this solution is provided in Fig. 7. Due to the widely dispersed demand locations in central and western states, a large number of additional DCs is necessary to provide complete coverage. Fig. 8 shows a solution which covers 94% of demand at a cost of \$672 million with a total number of 29 DCs. This is a good trade-off solution because in practical terms it is as far “down and to the right” as possible (Fig. 5). Notice that the locations selected for the minimum cost solution (23 DCs) are included in the 29 DC solution.

Fig. 9 illustrates the cost composition for each of the solutions shown in Fig. 5. In the minimum cost solution, transportation and inventory costs are nearly equal. As the number of DCs increases, transportation costs decrease (because the DCs are closer to final demand points), but inventory costs increase at a faster rate. The primary cost trade-offs seems to be between transportation and inventory costs, fixed facility costs being relatively less significant. Had the inventory costs been omitted from the optimization, the solutions would have failed to capture a major element of the cost analysis.

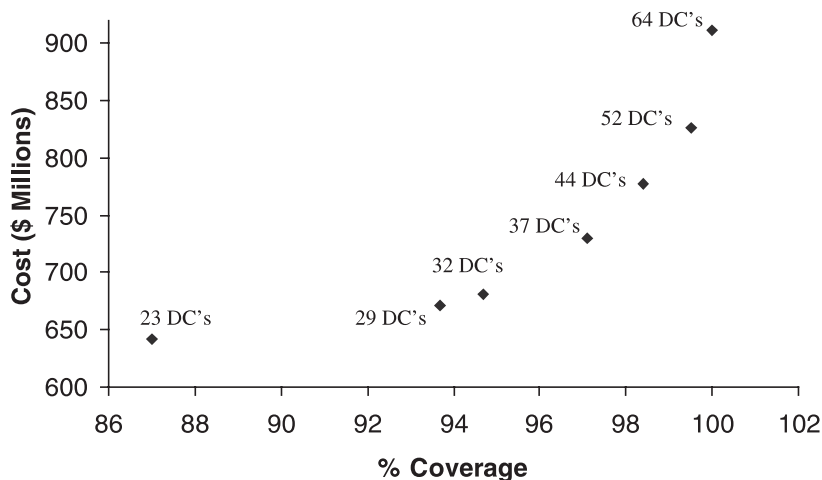


Fig. 5. “Trade-off” solutions (radius of coverage = 200 miles).

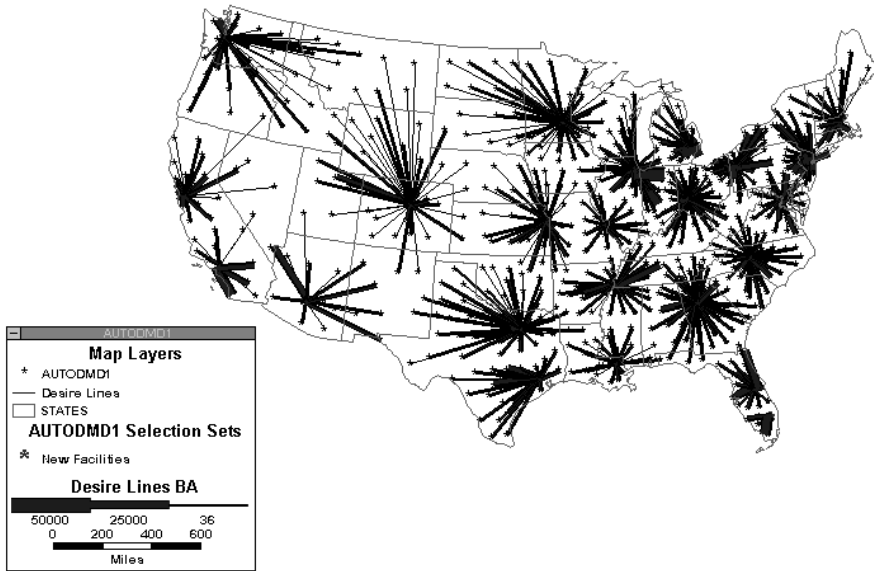


Fig. 6. Minimum total cost solution (23 DCs).

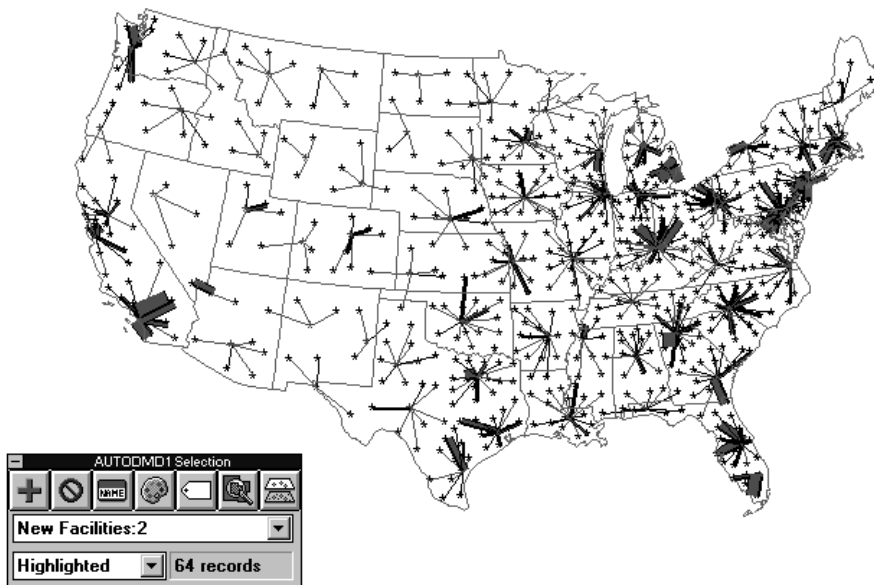


Fig. 7. Maximum coverage solution (64 DCs).

6. Conclusions

This paper has developed an analysis procedure for the location of DCs that integrates fa-

cility costs, inventory costs, transportation costs and service responsiveness. That procedure integrates ideas in queuing theory, discrete choice location analysis and multi-objective

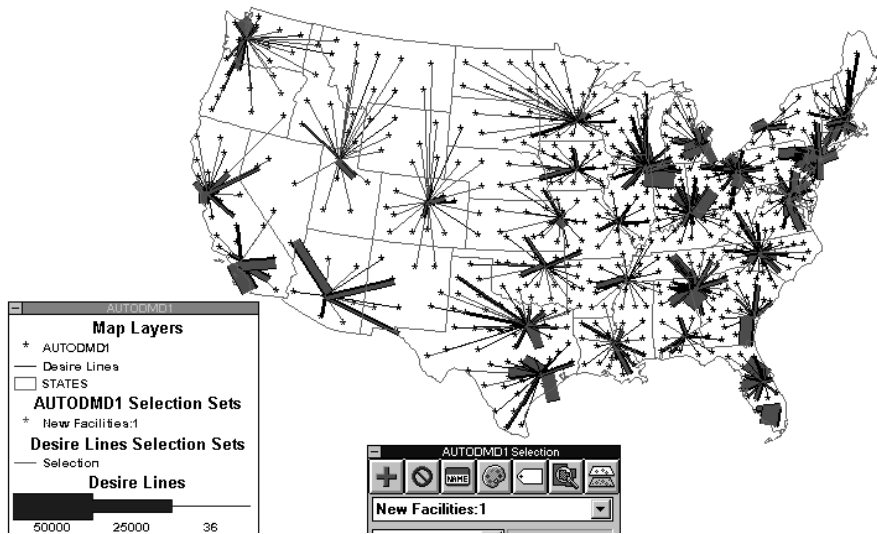


Fig. 8. Compromise solution (29 DCs).

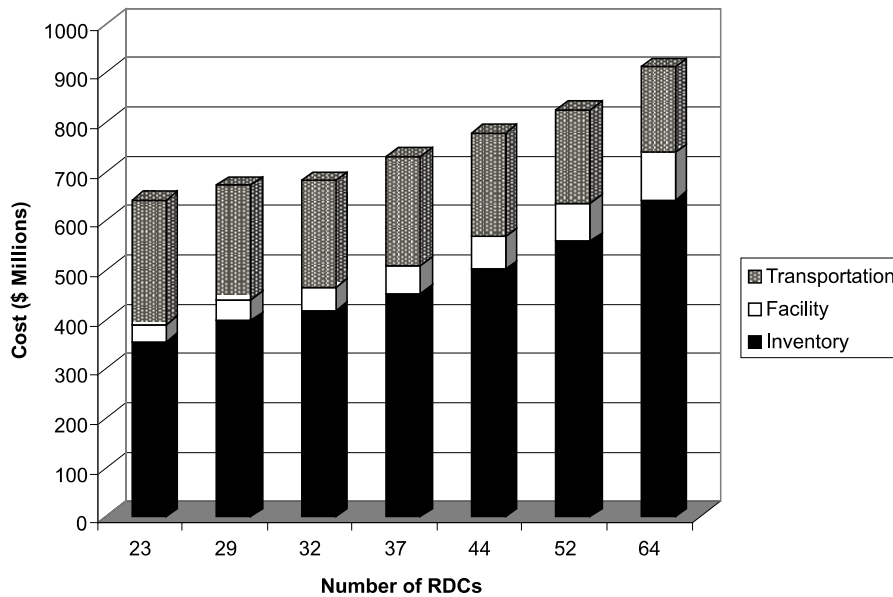


Fig. 9. Cost composition of efficient solutions.

decision-making. Using this procedure, decision-makers can easily understand the service-cost trade-offs that are available, so that optimal location decisions can be reached. An application to the US automotive industry is discussed in detail.

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