Supply Chain Management (6th Edition)

Chapter 11
Managing Economies of Scale in the Supply Chain: Cycle Inventory

Role of Cycle Inventory in a Supply Chain

- Lot, or batch size: quantity that a supply chain stage either produces or orders at a given time
- Cycle inventory: average inventory that builds up in the supply chain because a supply chain stage either produces or purchases in lots that are larger than those demanded by the customer
  - Q = lot or batch size of an order
  - D = demand per unit time
- Inventory profile: plot of the inventory level over time (Fig. 10.1)
- Cycle inventory = Q/2 (depends directly on lot size)
- Average flow time = Avg inventory / Avg flow rate
- Average flow time from cycle inventory = Q/(2D)

Inventory Profile

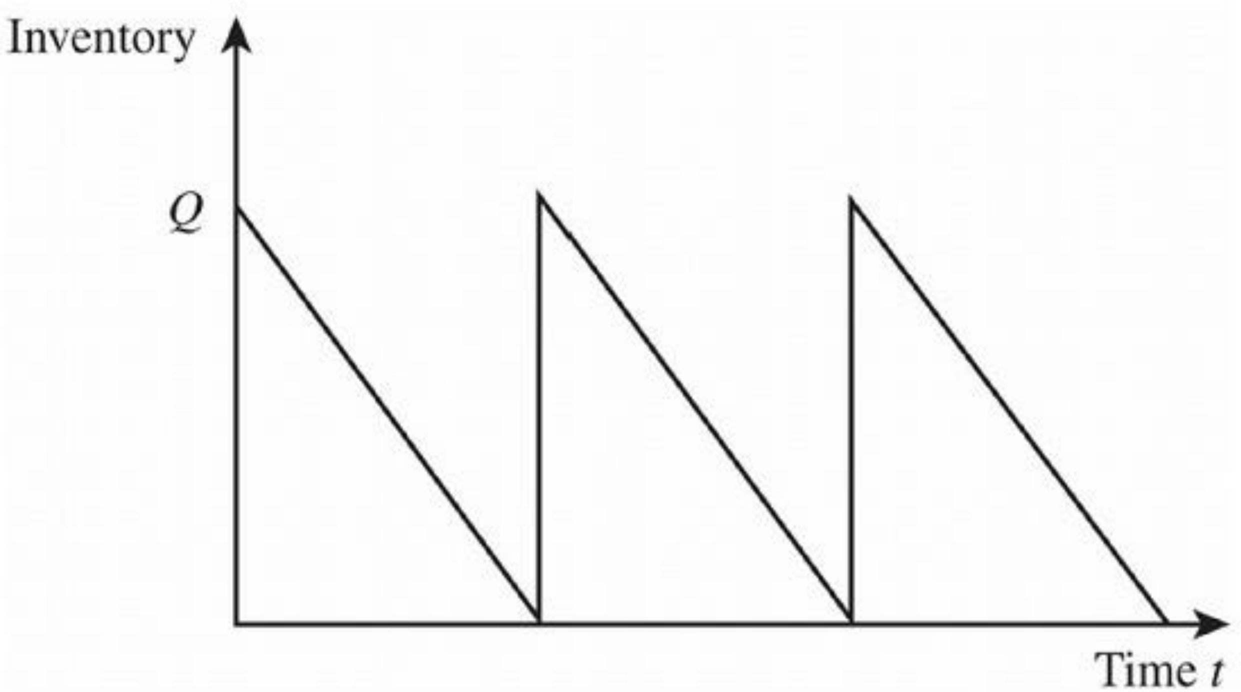


Figure 11-1

Role of Cycle Inventory in a Supply Chain

Q = 1000 units

D = 100 units/day

Cycle inventory = Q/2 = 1000/2 = 500 = Avg inventory level from cycle inventory

Avg flow time = Q/2D = 1000/(2)(100) = 5 days

- Cycle inventory adds 5 days to the time a unit spends in the supply chain
- Lower cycle inventory is better because:
  - Average flow time is lower
  - Working capital requirements are lower (less unneeded parts and space)
  - Lower inventory holding costs

Role of Cycle Inventory in a Supply Chain

- Cycle inventory is held primarily to take advantage of economies of scale in the supply chain
- Supply chain costs influenced by lot size:
  - Material cost = C
  - Fixed ordering cost = S
  - Holding cost = H = hC (h = cost of holding $1 in inventory for one year)
- Primary role of cycle inventory is to allow different stages to purchase product in lot sizes that minimize the sum of material, ordering, and holding costs
- Ideally, cycle inventory decisions should consider costs across the entire supply chain, but in practice, each stage generally makes its own supply chain decisions – increases total cycle inventory and total costs in the supply chain

Economies of Scale to Exploit Fixed Costs

- How do you decide whether to go shopping at a convenience store or at Sam's Club?
- Lot sizing for a single product (EOQ)
- Aggregating multiple products in a single order
- Lot sizing with multiple products or customers
  - Lots are ordered and delivered independently for each product
  - Lots are ordered and delivered jointly for all products
  - Lots are ordered and delivered jointly for a subset of products

Economies of Scale to Exploit Fixed Costs

- Lot sizing for a single product (EOQ)
  - D = Annual demand of the product
  - S = Fixed cost incurred per order
  - C = Cost per unit
  - H = Holding cost per year as a fraction of product cost
- Basic assumptions
  - Demand is steady at D units per unit time
  - No shortages are allowed
  - Replenishment lead time is fixed

Economies of Scale to Exploit Fixed Costs

Annual demand = D

Annual material cost = C*D

Number of orders per year = D/Q

Annual order cost = (D/Q)*S

Annual holding cost = (Q/2)*H = (Q/2)hC

Total annual cost = TC = CD + (D/Q)S + (Q/2)hC

Figure 10.2 shows variation in different costs for different lot sizes

Lot Sizing for a Single Product

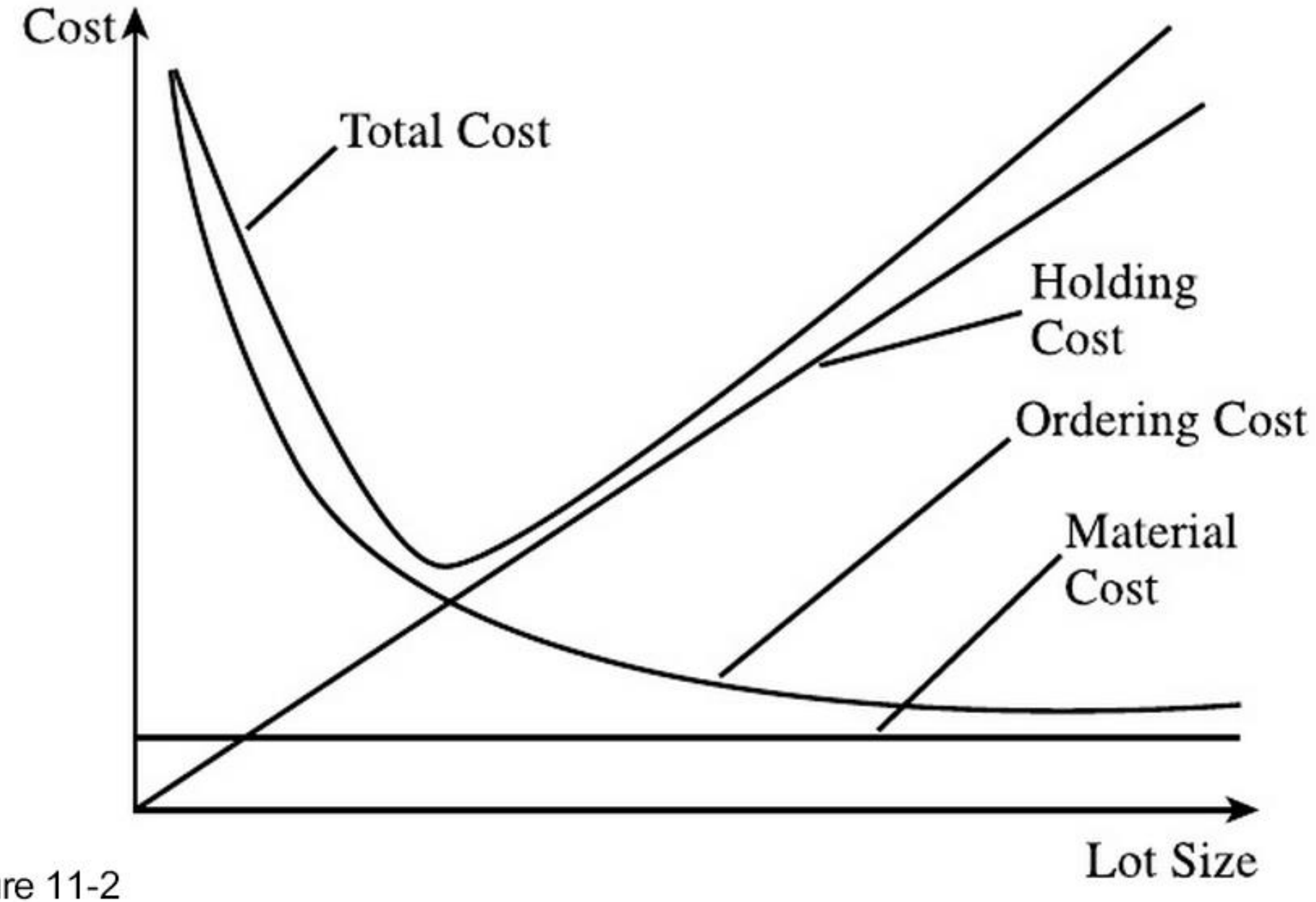


Figure 11-2

Fixed Costs: Optimal Lot Size and Reorder Interval (EOQ)

D: Annual demand

S: Setup or Order Cost

C: Cost per unit

h: Holding cost per year as a fraction of product cost

H: Holding cost per unit per year

Q: Lot Size

T: Reorder interval=Q*/D

Material cost is constant and therefore is not considered in this model

$$H = hC$$

$$Q^* = \sqrt{\frac{2DS}{hC}}$$

$$n^* = \sqrt{\frac{DhC}{2S}}$$

$$TC(Q^*) = \sqrt{2DShC} \quad (Verify)$$

Example 10.1

Demand, D = 12,000 computers per year

d = 1000 computers/month

Unit cost, C = $500

Holding cost fraction, h = 0.2

Fixed cost, S = $4,000/order

Q* = Sqrt[(2)(12000)(4000)/(0.2)(500)] = 980 computers

Cycle inventory = Q/2 = 490

Avg. Flow time = Q/2d = 980/(2)(1000) = 0.49 month

Reorder interval, T = 0.98 month

Example 10.1 (continued)

Annual ordering and holding cost =

$= (12000/980)(4000) + (980/2)(0.2)(500) = \$97,980$

Suppose lot size is reduced to Q=200, which would reduce flow time:

Annual ordering and holding cost =

$= (12000/200)(4000) + (200/2)(0.2)(500) = \$250,000$

To make it economically feasible to reduce lot size, the fixed cost associated with each lot would have to be reduced

Key Point

Total ordering and holding costs are relatively stable around the economic order quantity. A firm is often better served by ordering a convenient lot size close to the EOQ rather than the precise EOQ.

Example 10.1 (continued)

If demand at Best Buy increases to 4,000 computers a month (demand has increased by a factor of 4), the EOQ formula shows that the optimal lot size doubles and the number of orders placed per year also doubles. In contrast, average flow time decreases by a factor of 2. In other words, as demand increases, cycle inventory measured in terms of days (or months) of demand should reduce if the lot-sizing decision is made optimally. This observation can be stated as the following Key Point:

Key Point

If demand increases by a factor of k, the optimal lot size increases by a factor of $\sqrt{k}$. The number of orders placed per year should also increase by a factor of $\sqrt{k}$. Flow time attributed to cycle inventory should decrease by a factor of $\sqrt{k}$.

Example 10.2

If desired lot size = Q* = 200 units, what would S have to be?

D = 12000 units

C = $500

h = 0.2

Use EOQ equation and solve for S:

$S = [hC(Q^*)^2]/2D = [(0.2)(500)(200)^2]/(2)(12000) = \$166.67$

Key Point

To reduce the optimal lot size by a factor of k, the fixed order cost S must be reduced by a factor of k^2.

Lot Sizing with Capacity Constraint

In our discussion so far we have assumed that the economic order quantity for a retailer will fit on the truck. In reality the truck has a limited capacity, say K. If the economic order quantity Q is more than the K, the retailer will have to pay for more than one truck. In this case, the optimal

order quantity is obtained by comparing the cost of ordering K units (a full truck) and Q units ($\lceil Q/K \rceil$ trucks). If the setup cost S arises primarily from the cost of a truck, it is never optimal to order more than one truck. In this case, the optimal order size is the minimum of the EOQ and the truck capacity (K).

Aggregating Multiple Products in a Single Order

- Transportation is a significant contributor to the fixed cost per order
- Can possibly combine shipments of different products from the same supplier
  - same overall fixed cost
  - shared over more than one product
  - effective fixed cost is reduced for each product
  - lot size for each product can be reduced
- Can also have a single delivery coming from multiple suppliers or a single truck delivering to multiple retailers
- Aggregating across products, retailers, or suppliers in a single order allows for a reduction in lot size for individual products because fixed ordering and transportation costs are now spread across multiple products, retailers, or suppliers

Example: Aggregating Multiple Products in a Single Order

- Suppose there are 4 computer products in the previous example: Deskpro, Litepro, Medpro, and Heavpro
- Assume demand for each is 1000 units per month
- If each product is ordered separately:
  - Q* = 980 units for each product
  - Total cycle inventory = 4(Q/2) = (4)(980)/2 = 1960 units
- Aggregate orders of all four products:
  - Combined Q* = 1960 units
  - For each product: Q* = 1960/4 = 490
  - Cycle inventory for each product is reduced to 490/2 = 245
  - Total cycle inventory = 1960/2 = 980 units
  - Average flow time, inventory holding costs will be reduced

Lot Sizing with Multiple Products or Customers

- In practice, the fixed ordering cost is dependent at least in part on the variety associated with an order of multiple models
  - A portion of the cost is related to transportation (independent of variety)
  - A portion of the cost is related to loading and receiving (**not** independent of variety)
- Two scenarios:
  - Lots are ordered and delivered independently for each product
  - Lots are ordered and delivered jointly for all three models

Lot Sizing with Multiple Products

- Demand per year
  - D_L = 12,000; D_M = 1,200; D_H = 120
- Common transportation cost, S = \$4,000
- Product specific order cost
  - s_L = \$1,000; s_M = \$1,000; s_H = \$1,000
- Holding cost, h = 0.2
- Unit cost
  - C_L = \$500; C_M = \$500; C_H = \$500

Delivery Options

- No Aggregation: Each product ordered separately
- Complete Aggregation: All products delivered on each truck

No Aggregation: Order Each Product Independently (Ex.11-3)

	Litepro	*Medpro*	*Heavypro*
Demand per year	**12,000**	**1,200**	**120**
Fixed cost / order	**$5,000**	**$5,000**	**$5,000**
Optimal order size	**1,095**	**346**	**110**
Order frequency	**11.0 / year**	**3.5 / year**	**1.1 / year**
Annual cost	**$109,544**	**$34,642**	**$10,954**

Total cost = $155,140

LOTS ARE ORDERED AND DELIVERED JOINTLY FOR ALL THREE MODELS Given that all three models are included each time an order is placed, the combined fixed order cost per order is given by

$$S^* = S + s_L + s_M + s_H$$

The next step is to identify the optimal ordering frequency. Let n be the number of orders placed per year. We then have

$$\text{Annual order cost} = S^* n$$

$$\text{Annual holding cost} = \frac{D_L h C_L}{2n} + \frac{D_M h C_M}{2n} + \frac{D_H h C_H}{2n}$$

The total annual cost is thus given by

$$\text{Total annual cost} = \frac{D_L h C_L}{2n} + \frac{D_M h C_M}{2n} + \frac{D_H h C_H}{2n} + S^* n$$

The optimal order frequency minimizes the total annual cost and is obtained by taking the first derivative of the total cost with respect to n and setting it equal to 0. This results in the optimal order frequency n^*, where

$$n^* = \sqrt{\frac{D_L h C_L + D_M h C_M + D_H h C_H}{2S^*}} \qquad \textbf{(11.7)}$$

Equation 11.7 can be generalized to the case in which there are k items consolidated on a single order, as follows:

$$n^* = \sqrt{\frac{\sum_{i=1}^{k} D_i h C_i}{2S^*}} \qquad \textbf{(11.8)}$$

Aggregation: Order All Products Jointly (Ex.11-4)

$S^* = S + s_L + s_M + s_H = 4000+1000+1000+1000 = \$7000$

$n^* = Sqrt[(D_L hC_L + D_M hC_M + D_H hC_H)/2S^*]$

$= 9.75$

$Q_L = D_L/n^* = 12000/9.75 = 1230$

$Q_M = D_M/n^* = 1200/9.75 = 123$

$Q_H = D_H/n^* = 120/9.75 = 12.3$

Cycle inventory = Q/2

Average flow time = (Q/2)/(weekly demand)

Complete Aggregation: Order All Products Jointly

	Litepro	*Medpro*	*Heavypro*
Demand per year	12,000	1,200	120
Order frequency	9.75/year	9.75/year	9.75/year
Optimal order size	1,230	123	12.3
Annual holding cost	$61,512	$6,151	$615

Annual order cost = 9.75 × $7,000 = $68,250

Annual total cost = $136,528

EXAMPLE 11-5 Aggregation with Capacity Constraint

W.W. Grainger sources from hundreds of suppliers and is considering the aggregation of inbound shipments to lower costs. Truckload shipping costs $500 per truck along with $100 per pickup. Average annual demand from each supplier is 10,000 units. Each unit costs $50 and Grainger incurs a holding cost of 20 percent. What is the optimal order frequency and order size if Grainger decides to aggregate four suppliers per truck? What is the optimal order size and frequency if each truck has a capacity of 2,500 units?

Analysis:

In this case, W.W. Grainger has the following inputs:

Demand per product, $D_i = 10{,}000$

Holding cost, $h = 0.2$

Unit cost per product, $C_i = \$50$

Common order cost, $S = \$500$

Supplier-specific order cost, $s_i = \$100$

The combined order cost from four suppliers is given by

$$S^* = S + s_1 + s_2 + s_3 + s_4 = \$900 \text{ per order}$$

From Equation 11.8, the optimal order frequency is

$$n^* = \sqrt{\frac{\sum_{i=1}^{4} D_i h C_i}{2S^*}} = \sqrt{\frac{4 \times 10{,}000 \times 0.2 \times 50}{2 \times 900}} = 14.91$$

It is thus optimal for Grainger to order 14.91 times per year. The annual ordering cost per supplier is

$$\text{Annual order cost} = 14.91 \times \frac{900}{4} = \$3{,}355$$

The quantity ordered from each supplier is $Q = 10{,}000/14.91 = 671$ units per order. The annual holding cost per supplier is

$$\text{Annual holding cost per supplier} = \frac{hC_iQ}{2} = 0.2 \times 50 \times \frac{671}{2} = \$3{,}355$$

This policy, however, requires a total capacity per truck of $4 \times 671 = 2{,}684$ units. Given a truck capacity of 2,500 units, the order frequency must be increased to ensure that the order quantity from each supplier is $2{,}500/4 = 625$. Thus, W.W. Grainger should increase the order frequency to $10{,}000/625 = 16$. The limited truck capacity results in an optimal order frequency of 16 orders per year instead of 14.91 orders per year when truck capacity was ignored. The limited truck capacity will increase the annual order cost per supplier to \$3,600 and decrease the annual holding cost per supplier to \$3,125.

The main advantage of ordering all products jointly is that the system is easy to administer and implement. The disadvantage is that it is not selective enough in combining the particular models that should be ordered together. If product-specific order costs are high and products vary significantly in terms of their sales, it is possible to lower costs by being selective about the products being aggregated in a joint order.

LOTS ARE ORDERED AND DELIVERED JOINTLY FOR A SELECTED SUBSET OF THE PRODUCTS

We first describe the procedure in general and then apply it to the specific example. Assume that the products are indexed by i, where i varies from 1 to l (assuming a total of l products). Each product i has an annual demand D_i, a unit cost C_i, and a product-specific order cost s_i. The common order cost is S.

Step 1: As a first step, identify the most frequently ordered product, assuming each product is ordered independently. In this case, a fixed cost of $S + s_i$ is allocated to each product. For each product i (using Equation 11.6), evaluate the ordering frequency:

$$\overline{n}_i = \sqrt{\frac{hC_iD_i}{2(S + s_i)}}$$

This is the frequency at which product i would be ordered if it were the only product being ordered (in which case a fixed cost of $S + s_i$ would be incurred per order). Let $\overline{n}$ be the frequency of the most frequently ordered product, i^*; that is, $\overline{n}_{i^*}$ is the maximum among all $\overline{n}_i$ ($\overline{n} = \overline{n}_{i^*} = \max\{\overline{n}_i, i = 1, \ldots, l\}$). The most frequently ordered product is i^*, which is included each time an order is placed.

Step 2: For all products $i \neq i^*$, evaluate the ordering frequency:

$$\overline{\overline{n}}_i = \sqrt{\frac{hC_iD_i}{2s_i}}$$

$\overline{\overline{n}}_i$ represents the desired order frequency if product i incurs the product-specific fixed cost s_i only each time it is ordered.

Step 3: Our goal is to include each product $i \neq i^*$ with the most frequently ordered product i^* after an integer number of orders. For all $i \neq i^*$, evaluate the frequency of product i relative to the most frequently ordered product i^* to be m_i, where

$$m_i = \lceil \bar{n}/\bar{\bar{n}}_i \rceil$$

In this case, $\lceil\ \rceil$ is the operation that rounds a fraction up to the closest integer. Product i is included with the most frequently ordered product i^* every m_i orders. Given that the most frequently ordered product i^* is included in every order, $m_{i^*} = 1$.

Step 4: Having decided the ordering frequency of each product i, recalculate the ordering frequency of the most frequently ordered product i^* to be n, where

$$n = \sqrt{\frac{\sum_{i=1}^{l} hC_i m_i D_i}{2(S + \sum_{i=1}^{l} s_i / m_i)}} \qquad \textbf{(11.9)}$$

Note that n is a better ordering frequency for the most frequently ordered product i^* than $\bar{n}$ because it takes into account the fact that each of the other products i is included with i^* every m_i orders.

Step 5: For each product, evaluate an order frequency of $n_i = n/m_i$ and the total cost of such an ordering policy. The total annual cost is given by

$$TC = nS + \sum_{i=1}^{l} n_i s_i + \sum_{i=1}^{l} \left(\frac{D_i}{2n_i}\right) hC_i$$

This procedure results in *tailored aggregation*, with higher-demand products ordered more frequently and lower-demand products ordered less frequently. Example 11-6 (see worksheet *Example 11-6*) considers tailored aggregation for the Best Buy ordering decision in Example 11-3.

EXAMPLE 11-6 Lot Sizes Ordered and Delivered Jointly for a Selected Subset That Varies by Order

Consider the Best Buy data in Example 11-3. Product managers have decided to order jointly, but to be selective about which models they include in each order. Evaluate the ordering policy and costs using the procedure discussed previously.

Analysis:

Recall that $S = \$4,000$, $s_L = \$1,000$, $s_M = \$1,000$, $s_H = \$1,000$. Applying Step 1, we obtain

$$\bar{n}_L = \sqrt{\frac{hC_LD_L}{2(S + s_L)}} = 11.0,\ \bar{n}_M = \sqrt{\frac{hC_MD_M}{2(S + s_M)}} = 3.5,\ \bar{n}_H = \sqrt{\frac{hC_HD_H}{2(S + s_H)}} = 1.1$$

Clearly, Litepro is the most frequently ordered model. Thus, we set $\bar{n} = 11.0$.

We now apply Step 2 to evaluate the frequency with which Medpro and Heavypro are included with Litepro in the order. We first obtain

$$\bar{\bar{n}}_M = \sqrt{\frac{hC_MD_M}{2s_M}} = 7.7 \quad \text{and} \quad \bar{\bar{n}}_H = \sqrt{\frac{hC_HD_H}{2s_H}} = 2.4$$

Next, we apply Step 3 to evaluate

$$m_M = \left\lceil \frac{\bar{n}}{\bar{\bar{n}}_M} \right\rceil = \left\lceil \frac{11.0}{7.7} \right\rceil = 2 \quad \text{and} \quad m_H = \left\lceil \frac{\bar{n}}{\bar{\bar{n}}_H} \right\rceil = \left\lceil \frac{11.0}{2.4} \right\rceil = 5$$

Thus, Medpro is included in every second order and Heavypro is included in every fifth order (Litepro, the most frequently ordered model, is included in every order). Now that we have decided on the ordering frequency of each model, apply Step 4 (Equation 11.9) to recalculate the ordering frequency of the most frequently ordered model as

$$n = \sqrt{\frac{hC_L m_L D_L + hC_M m_M D_M + hC_H m_M D_H}{2(S + s_L/m_L + s_M/m_M + s_H/m_H)}} = 11.47$$

Thus, the Litepro is ordered 11.47 times per year. Next, we apply Step 5 to obtain an ordering frequency for each product:

$$n_L = 11.47\text{/year},\ n_M = 11.47/2 = 5.74/\text{year, and } n_H = 11.47/5 = 2.29\text{/year}$$

The ordering policies and resulting costs for the three products are shown in Table 11-3.

The annual holding cost of this policy is $65,383.50. The annual order cost is given by

$$nS + n_L s_L + n_M s_M + n_H s_H = \$65{,}383.50$$

The total annual cost is thus equal to $130,767. Tailored aggregation results in a cost reduction of $5,761 (about 4 percent) compared with the joint ordering of all models. The cost reduction results because each model-specific fixed cost of $1,000 is not incurred with every order.

TABLE 11-3 Lot Sizes and Costs for Ordering Policy Using Heuristic

	Litepro	Medpro	Heavypro
Demand per year (*D*)	12,000	1,200	120
Order frequency (*n*)	11.47/year	5.74/year	2.29/year
Order size (*D*/*n*)	1,046	209	52
Cycle inventory	523	104.5	26
Annual holding cost	$52,307	$10,461	$2,615
Average flow time	2.27 weeks	4.53 weeks	11.35 weeks

Lessons from Aggregation

- Aggregation allows firm to lower lot size without increasing cost
- Complete aggregation is effective if product specific fixed cost is a small fraction of joint fixed cost

Economies of Scale to Exploit Quantity Discounts

- All-unit quantity discounts
- Marginal unit quantity discounts
- Why quantity discounts?
  - Coordination in the supply chain
  - Price discrimination to maximize supplier profits

Quantity Discounts

- Lot size based
  - All units
  - Marginal unit
- Volume based

- *How should buyer react?*
- *What are appropriate discounting schemes?*

All-Unit Quantity Discounts

- Pricing schedule has specified quantity break points $q_0, q_1, \ldots, q_r$, where $q_0 = 0$
- If an order is placed that is at least as large as q_i but smaller than q_{i+1}, then each unit has an average unit cost of C_i
- The unit cost generally decreases as the quantity increases, i.e., $C_0 > C_1 > \ldots > C_r$
- The objective for the company (a retailer in our example) is to decide on a lot size that will minimize the sum of material, order, and holding costs

All-Unit Quantity Discount Procedure (different from what is in the textbook)

Step 1: Calculate the EOQ for the lowest price. If it is feasible (i.e., this order quantity is in the range for that price), then stop. This is the optimal lot size. Calculate TC for this lot size.

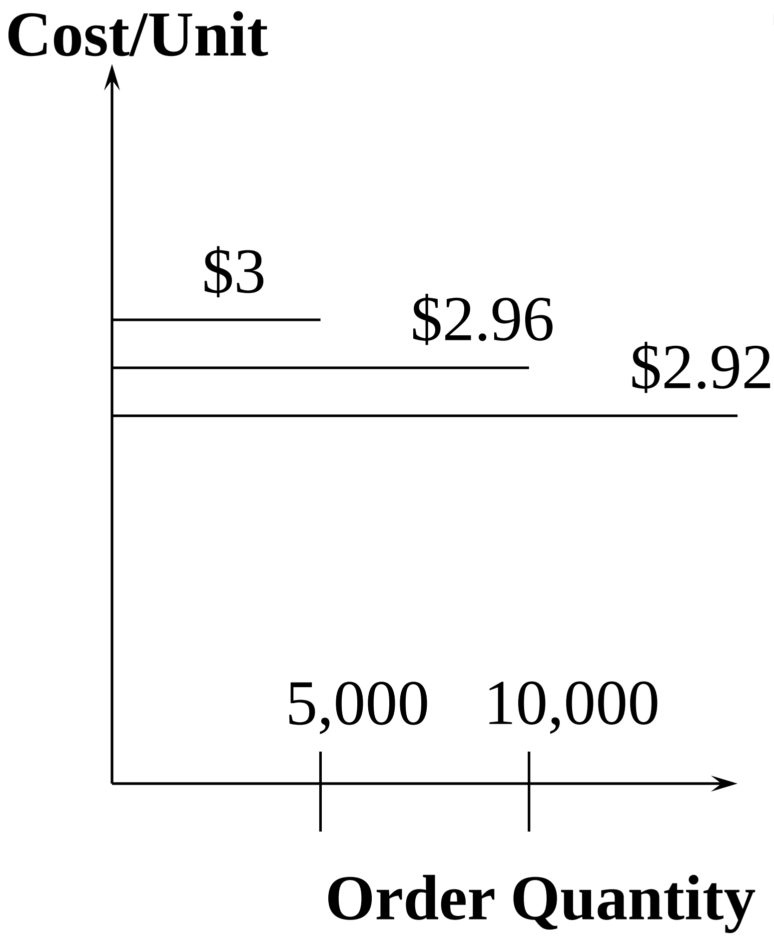
Step 2: If the EOQ is not feasible, calculate the TC for this price and the smallest quantity for that price.

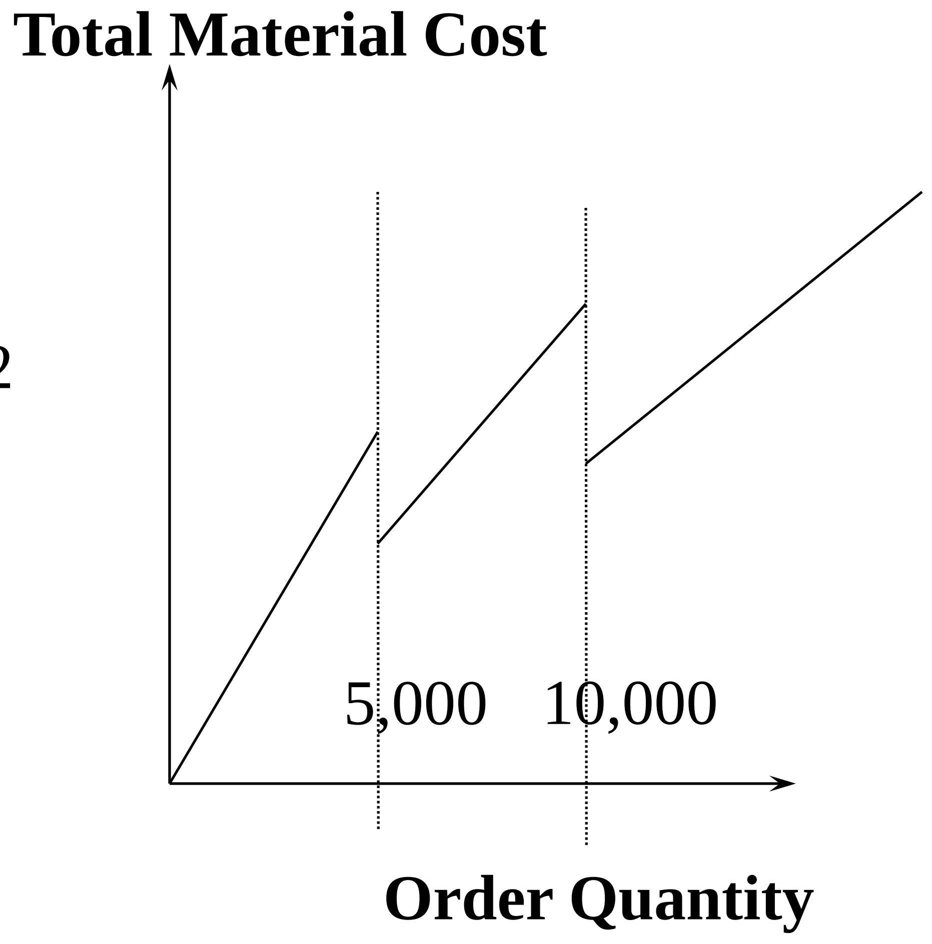
Step 3: Calculate the EOQ for the next lowest price. If it is feasible, stop and calculate the TC for that quantity and price.

Step 4: Compare the TC for Steps 2 and 3. Choose the quantity corresponding to the lowest TC.

Step 5: If the EOQ in Step 3 is not feasible, repeat Steps 2, 3, and 4 until a feasible EOQ is found.

All-Unit Quantity Discounts: Example





All-Unit Quantity Discount: Example

Order quantity	Unit Price
0-5000	\$3.00
5001-10000	\$2.96
Over 10000	\$2.92

$q0 = 0$, $q1 = 5000$, $q2 = 10000$

$C0 = \$3.00$, $C1 = \$2.96$, $C2 = \$2.92$

$D = 120000$ units/year, $S = \$100$/lot, $h = 0.2$

All-Unit Quantity Discount: Example

Step 1: Calculate Q2* = Sqrt[(2DS)/hC2]

= Sqrt[(2)(120000)(100)/(0.2)(2.92)] = 6410

Not feasible (6410 < 10001)

Calculate TC2 using C2 = $2.92 and q2 = 10001

TC2 = (120000/10001)(100)+(10001/2)(0.2)(2.92)+(120000)(2.92)

= $354,520

Step 2: Calculate Q1* = Sqrt[(2DS)/hC1]

=Sqrt[(2)(120000)(100)/(0.2)(2.96)] = 6367

Feasible (5000<6367≤10000) ➔ **Stop**

TC1 = (120000/6367)(100)+(6367/2)(0.2)(2.96)+(120000)(2.96)

= $358,969

TC2 < TC1 ➔ <u>The optimal order quantity Q* is q2 = 10001</u>

All-Unit Quantity Discounts

- Suppose fixed order cost were reduced to $4
  - Without discount, Q* would be reduced to 1265 units
  - With discount, optimal lot size would still be 10001 units
- What is the effect of such a discount schedule?
  - Retailers are encouraged to increase the size of their orders
  - Average inventory (cycle inventory) in the supply chain is increased
  - Average flow time is increased
  - Is an all-unit quantity discount an advantage in the supply chain?

Marginal Unit Quantity Discounts

Marginal (or incremental) unit quantity discounts are also referred to as *multi-block tariffs*. In this case, the pricing schedule contains specified break points $q_0, q_1, \ldots, q_r$. It is not the *average*

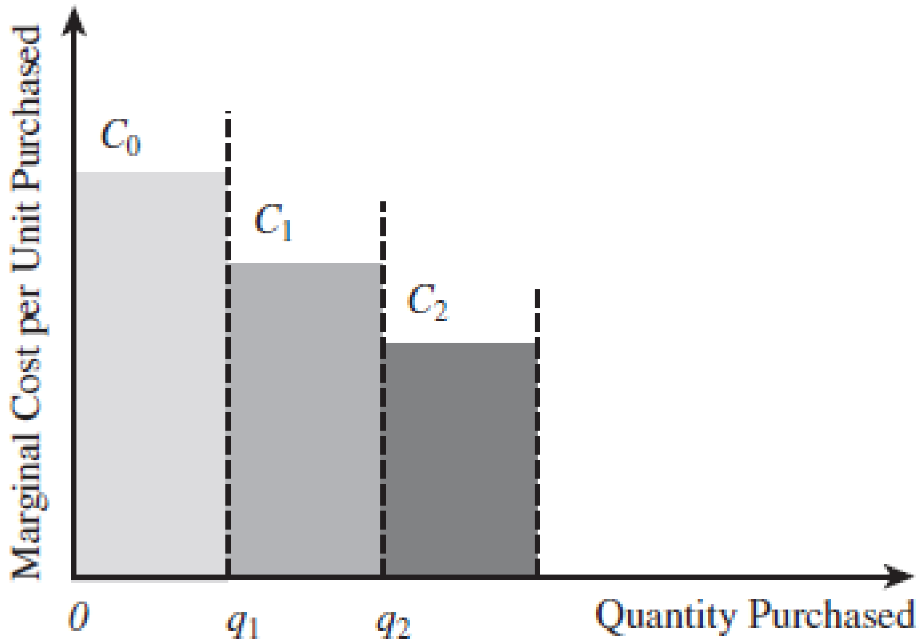


FIGURE 11-4 Marginal Unit Cost with Marginal Unit Quantity Discount

cost of a unit but the *marginal cost* of a unit that decreases at a breakpoint (in contrast to the all unit discount scheme). If an order of size q is placed, the first $q_1 - q_0$ units are priced at C_0, the next $q_2 - q_1$ are priced at C_1, and, in general, $q_{i+1} - q_i$ units are priced at C_i. The marginal cost per unit varies with the quantity purchased, as shown in Figure 11-4.

Faced with such a pricing schedule, the retailer's objective is to decide on a lot size that maximizes profits or, equivalently, minimizes material, order, and holding costs.

The solution procedure discussed here evaluates the optimal lot size for each marginal price C_i (this forces a lot size between q_i and q_{i+1}) and then settles on the lot size that minimizes the overall cost. A more streamlined procedure has been provided by Hu and Munson (2002).

For each value of i, $0 \le i \le r$, let V_i be the cost of ordering q_i units. Define $V_0 = 0$ and V_i for $0 \le i \le r$ as follows:

$$V_i = C_0(q_1 - q_0) + C_1(q_2 - q_1) + \cdots + C_{i-1}(q_i - q_{i-1}) \qquad (11.12)$$

For each value of i, $0 \le i \le r - 1$, consider an order of size Q in the range q_i to q_{i+1} units; that is, $q_{i+1} \ge Q \ge q_i$. The material cost of each order of size Q is given by $V_i + (Q - q_i)C_i$. The various costs associated with such an order are as follows:

$$\text{Annual order cost} = \left(\frac{D}{Q}\right)S$$

$$\text{Annual holding cost} = [V_i + (Q - q_i)C_i]h/2$$

$$\text{Annual material cost} = \frac{D}{Q}[V_i + (Q - q_i)C_i]$$

$$V_i = \sum_{j=0}^{i-1} C_j(q_{j+1} - q_j)$$

The total annual cost is the sum of the three costs and is given by

$$\text{Total annual cost} = \left(\frac{D}{Q}\right)S + [V_i + (Q - q_i)C_i]h/2 + \frac{D}{Q}[V_i + (Q - q_i)C_i]$$

The optimal lot size for price C_i is obtained by taking the first derivative of the total cost with respect to the lot size and setting it equal to 0. This results in the following optimal lot size:

$$\text{Optimal lot size for price } C_i \text{ is } Q_i = \sqrt{\frac{2D(S + V_i - q_iC_i)}{hC_i}} \qquad (11.13)$$

Observe that the optimal lot size is obtained using a formula very much like the EOQ formula (Equation 11.5), except that the presence of the quantity discount has the effect of raising the

fixed cost per order by $V_i - q_iC_i$ (from S to $S + V_i - q_iC_i$). The overall optimal lot size is obtained as follows:

Step 1: Evaluate the optimal lot size using Equation 11.13 for each price C_i.

Step 2: We next select the order quantity Q_i^* for each price C_i. There are three possible cases for Q_i:

1. If $q_i \leq Q_i \leq q_{i+1}$ then set $Q_i^* = Q_i$
2. If $Q_i < q_i$ then set $Q_i^* = q_i$
3. If $Q_i > q_{i+1}$ then set $Q_i^* = q_{i+1}$

Step 3: Calculate the total annual cost of ordering Q_i^* units as follows:

$$TC_i = \left(\frac{D}{Q_i^*}\right)S + [V_i + (Q_i^* - q_i)C_i]h/2 + \frac{D}{Q_i^*}[V_i + (Q_i^* - q_i)C_i] \qquad \textbf{(11.14)}$$

Step 4: Over all i, select order size Q_i^* with the lowest cost TC_i.

EXAMPLE 11-8 Marginal Unit Quantity Discounts

Let us return to DO from Example 11-7. Assume that the manufacturer uses the following marginal unit discount pricing schedule:

Order Quantity	Marginal Unit Price
0–5,000	$3.00
5,000–10,000	$2.96
Over 10,000	$2.92

This implies that if an order is placed for 7,000 bottles, the first 5,000 are at a unit cost of $3.00, with the remaining 2,000 at a unit cost of $2.96. Evaluate the number of bottles that DO should order in each lot.

Analysis:

In this case, we have

$$q_0 = O, q_1 = 5{,}000, q_2 = 10{,}000$$
$$C_0 = \$3.00, C_1 = \$2.96, C_2 = \$2.92$$
$$D = 120{,}000\,/\,\text{year}, S = \$100\,/\,\text{lot}, h = 0.2$$

Using Equation 11.12, we obtain

$$V_0 = 0; V_1 = 3 \times (5{,}000 - 0) = \$15{,}000$$
$$V_2 = 3 \times (5{,}000 - 0) + 2.96 \times (10{,}000 - 5{,}000) = \$29{,}800$$

Using Step 1 and Equation 11.13, we obtain

$$Q_0 = \sqrt{\frac{2D(S + V_0 - q_0C_0)}{hC_0}} = 6,325$$

$$Q_1 = \sqrt{\frac{2D(S + V_1 - q_1C_1)}{hC_1}} = 11,028$$

$$Q_2 = \sqrt{\frac{2D(S + V_2 - q_2C_2)}{hC_2}} = 16,961$$

In Step 2, we set $Q_0^* = q_1 = 5,000$ because $Q_0 = 6,325 > q_1 = 5,000$. Similarly, we obtain $Q_1^* = q_2 = 10,000$ (because $Q_1 = 11,028 > q_2 = 10,000$) and $Q_2^* = Q_2 = 16,961$.

In Step 3, we obtain the total cost for $i = 0, 1, 2$ using Equation 11.14 to be

$$TC_0 = \left(\frac{D}{Q_0^*}\right)S + [V_0 + (Q_0^* - q_0)C_0]\ h/2 + \frac{D}{Q_0^*}[V_0 + (Q_0^* - q_0)C_0] = \$363,900$$

$$TC_1 = \left(\frac{D}{Q_1^*}\right)S + [V_1 + (Q_1^* - q_1)C_1]\ h/2 + \frac{D}{Q_1^*}[V_1 + (Q_1^* - q_1)C_1] = \$361,780$$

$$TC_2 = \left(\frac{D}{Q_2^*}\right)S + [V_2 + (Q_2^* - q_2)C_2]\ h/2 + \frac{D}{Q_2^*}[V_2 + (Q_2^* - q_2)C_2] = \$360,365$$

Observe that the lowest cost is for $i = 2$. Thus, it is optimal for DO to order in lots of $Q_2^* = 16,961$ bottles. This is much larger than the optimal lot size of 6,325 when the manufacturer does not offer any discount.

Why Quantity Discounts?

- Coordination in the supply chain
  - Commodity products
  - Products with demand curve
    - 2-part tariffs
    - Volume discounts

COORDINATION TO INCREASE TOTAL SUPPLY CHAIN PROFITS A supply chain is *coordinated* if the decisions the retailer and supplier make maximize total supply chain profits. In reality, each stage in a supply chain may have a separate owner and thus attempt to maximize its own profits. For example, each stage of a supply chain is likely to make lot-sizing decisions with an objective of minimizing its own overall costs. The result of this independent decision making can be a lack of coordination in a supply chain because actions that maximize retailer profits may not maximize supply chain profits. In this section, we discuss how a manufacturer may use appropriate quantity discounts to ensure that total supply chain profits are maximized even if the retailer is acting to maximize its own profits.

Quantity discounts for commodity products. Economists have argued that for commodity products such as milk, a competitive market exists and prices are driven down to the products' marginal cost. In this case, the market sets the price and the firm's objective is to lower costs in order to increase profits. Consider, for example, the online retailer DO, discussed earlier. It can be argued that it sells a commodity product. In this supply chain, both the manufacturer and DO incur costs related to each order placed by DO. Assume that the manufacturer has a fixed

cost S_M, a unit cost C_M, and a holding cost h_M. The manufacturer incurs fixed costs related to order setup and fulfillment (S_M) and holding costs ($h_M C_M$) as it builds up inventory to replenish the order. Assume that the retailer has a fixed cost S_R, a unit cost C_R, and a holding cost h_R. Thus, DO incurs fixed costs (S_R) for each order it places and holding costs ($h_R C_R$) for the inventory it holds as it slowly sells an order. Even though both parties incur costs associated with the lot-sizing decision made by DO, the retailer makes its lot-sizing decisions based solely on minimizing its local costs. This results in lot-sizing decisions that are locally optimal but do not maximize the supply chain surplus. We illustrate this idea in Example 11-9 (see spreadsheet *Chapter11-quantity discounts* worksheet *Example 11-9*).

EXAMPLE 11-9 The Impact of Locally Optimal Lot Sizes on a Supply Chain

Demand for vitamins is 10,000 bottles per month. DO incurs a fixed order placement, transportation, and receiving cost of $100 each time it places an order for vitamins with the manufacturer. DO incurs a holding cost of 20 percent. The manufacturer charges $3 for each bottle of vitamins purchased. Evaluate the optimal lot size for DO.

Each time DO places an order, the manufacturer must process, pack, and ship the order. The manufacturer has a line packing bottles at a steady rate that matches demand. The manufacturer incurs a fixed-order filling cost of $250, production cost of $2 per bottle, and a holding cost of 20 percent. What is the annual fulfillment and holding cost incurred by the manufacturer as a result of DO's ordering policy?

Analysis:

In this case, we have

$$D = 120{,}000\,/\,\text{year},\ S_R = \$100\,/\,\text{lot},\ h_R = 0.2,\ C_R = \$3$$
$$S_M = \$250\,/\,\text{lot},\ h_M = 0.2,\ C_M = \$2$$

Using the EOQ formula (Equation 11.5), we obtain the optimal lot size and annual cost for DO to be:

$$Q_R = \sqrt{\frac{2DS_R}{h_R C_R}} = \sqrt{\frac{2 \times 120{,}000 \times 100}{0.2 \times 3}} = 6{,}325$$

$$\text{Annual cost for DO} = \left(\frac{D}{Q_R}\right)S_R + \left(\frac{Q_R}{2}\right)h_R C_R = \$3{,}795$$

If DO orders in lots sizes of $Q_R = 6{,}325$, the annual cost incurred by the manufacturer is obtained to be:

$$\text{Annual cost for manufacturer} = \left(\frac{D}{Q_R}\right)S_M + \left(\frac{Q_R}{2}\right)h_M C_M = \$6{,}008$$

The annual supply chain cost (manufacturer + DO) is thus \$6,008 + \$3,795 = \$9,803.

In Example 11-9, DO picks the lot size of 6,325 with an objective of minimizing only its own costs. From a supply chain perspective, the optimal lot size should account for the fact that both DO and the manufacturer incur costs associated with each replenishment lot. If we assume that the manufacturer produces at a rate that matches demand (as assumed in Example 11-9), the total supply chain cost of using a lot size Q is obtained as follows:

$$\text{Annual cost for DO and manufacturer} = \left(\frac{D}{Q}\right)S_R + \left(\frac{Q}{2}\right)h_R C_R + \left(\frac{D}{Q}\right)S_M + \left(\frac{Q}{2}\right)h_M C_M$$

The optimal lot size (Q^*) for the supply chain is obtained by taking the first derivative of the total cost with respect to Q and setting it equal to 0 as follows (see worksheet *Example 11-9*):

$$Q^* = \sqrt{\frac{2D(S_R + S_M)}{h_R C_R + h_M C_M}} = 9{,}165$$

If DO orders in lots of $Q^* = 9{,}165$ units, the total costs for DO and the manufacturer are as follows:

$$\text{Annual cost for DO} = \left(\frac{D}{Q^*}\right)S_R + \left(\frac{Q^*}{2}\right)h_R C_R = \$4{,}059$$

$$\text{Annual cost for manufacturer} = \left(\frac{D}{Q^*}\right)S_M + \left(\frac{Q^*}{2}\right)h_M C_M = \$5{,}106$$

Observe that if DO orders a lot size of 9,165 units, the supply chain cost decreases to $9,165 (from $9,803 when DO ordered its own optimal lot size of 6,325). There is thus an opportunity for the supply chain to save $638. The challenge, however, is that ordering in lots of 9,165 bottles raises the cost for DO by $264 per year from $3,795 to $4,059 (even though it reduces overall supply chain costs). The manufacturer's costs, in contrast, go down by $902 from $6,008 to $5,106 per year. Thus, the manufacturer must offer DO a suitable incentive for DO to raise its lot size. A lot-size–based quantity discount is an appropriate incentive in this case. Example 11-10 (see worksheet *Example 11-10*) provides details of how the manufacturer can design a suitable quantity discount that gets DO to order in lots of 9,165 units even though DO is optimizing its own profits (and not total supply chain profits).

EXAMPLE 11-10 Designing a Suitable Lot-Size–Based Quantity Discount

Consider the data from Example 11-9. Design a suitable quantity discount that gets DO to order in lots of 9,165 units when it aims to minimize only its own total costs.

Analysis:

Recall that ordering in lots of 9,165 units instead of 6,325 increases annual ordering and holding costs for DO by $264. Thus, the manufacturer needs to offer an incentive of at least $264 per year to DO in terms of decreased material cost if DO orders in lots of 9,165 units. Decreasing material cost by $264/year from sales of 120,000 units implies that material cost must be decreased from $3/unit to $3 – 264/120,000 = $2.9978/unit if DO orders in lots of 9,165.

Thus, the appropriate quantity discount is for the manufacturer to charge $3 if DO orders in lots that are smaller than 9,165 units and discount the price to $2.9978 for orders of 9,165 or more.

Observe that offering a lot-size–based discount in this case decreases total supply chain cost. It does, however, increase the lot size the retailer purchases and thus increases cycle inventory in the supply chain.

Key Point

For commodity products for which price is set by the market, manufacturers with large fixed costs per lot can use lot-size–based quantity discounts to maximize total supply chain profits. Lot-size–based discounts, however, increase cycle inventory in the supply chain.

Our discussion on coordination for commodity products highlights the important link between the lot-size–based quantity discount offered and the order costs incurred by the manufacturer. As the manufacturer works on lowering order or setup cost, the discount it offers to retailers should change. For a low enough setup or order cost, the manufacturer gains little from using a lot-size–based quantity discount. In Example 11-9, discussed earlier, if the manufacturer lowers its fixed cost per order from $250 to $100, the total supply chain costs are close to the minimum without quantity discounts even if DO is trying to minimize its cost. Thus, if its fixed order costs are lowered to $100, it makes sense for the manufacturer to eliminate all quantity discounts. In most companies, however, marketing and sales design quantity discounts, whereas operations works on reducing setup or order cost. As a result, changes in pricing do not always occur in response to setup cost reduction in manufacturing. It is important that the two functions coordinate these activities.

Quantity discounts for products for which the firm has market power. Now, consider the scenario in which the manufacturer has invented a new vitamin pill, Vitaherb, which is derived from herbal ingredients and has other properties highly valued in the market. Few competitors have a similar product, so it can be argued that the price at which the retailer DO sells Vitaherb influences demand. Assume that the annual demand faced by DO is given by the demand curve 360,000 – 60,000p, where p is the price at which DO sells Vitaherb. The manufacturer incurs a production cost of $C_M = \$2$ per bottle of Vitaherb sold. The manufacturer must decide on the price C_R to charge DO, and DO in turn must decide on the price p to charge the customer. The profit at DO ($Prof_R$) and the manufacturer ($Prof_M$) as a result of this policy is given by

$$Prof_R = (p - C_R)(360{,}000 - 60{,}000\,p);\ Prof_M = (C_R - C_M)(360{,}000 - 60{,}000\,p)$$

DO picks the price p to maximize $Prof_R$. Taking the first derivative with respect to p and setting it to 0, we obtain the following relationship between p and C_R

$$p = 3 + \frac{C_R}{2} \qquad \textbf{(11.15)}$$

Given that the manufacturer is aware that DO is aiming to optimize its own profits, the manufacturer is able to use the relationship between p and C_R to obtains its own profits to be

$$Prof_M = (C_R - C_M)\left(360{,}000 - 60{,}000\left(3 + \frac{C_R}{2}\right)\right) = (C_R - 2)(180{,}000 - 30{,}000\,C_R)$$

The manufacturer picks its price C_R to maximize $Prof_M$. Taking the first derivative of $Prof_M$ with respect C_R to and setting it to 0 we obtain $C_R = \$4$. Substituting back into Equation 11.15, we obtain $p = \$5$. Thus, when DO and the manufacturer make their pricing decisions independently, it is optimal for the manufacturer to charge a wholesale price of $C_R = \$4$ and for DO to charge a retail price of $p = \$5$. The total market demand in this case is $360{,}000 - 60{,}000p = 60{,}000$ bottles. DO makes a profit of $Prof_R = (5 - 4)(360{,}000 - [60{,}000 \times 5]) = \$60{,}000$ and the manufacturer makes a profit of $Prof_M = (4 - 2)(360{,}000 - [60{,}000 \times 5]) = \$120{,}000$ (see worksheet 2-*stage*).

Now, consider the case in which the two stages coordinate their pricing decisions with a goal of maximizing the supply chain profit $Prof_{SC}$, which is given by

$$Prof_{SC} = (p - C_M)(360{,}000 - 60{,}000p)$$

The optimal retail price is obtained by setting the first derivative of $Prof_{SC}$ with respect to p to 0. We thus obtain the coordinated retail price to be

$$p = 3 + \frac{C_M}{2} = 3 + \frac{2}{2} = \$4$$

If the two stages coordinate pricing and DO prices at $p = \$4$, market demand is $360{,}000 - 60{,}000p = 120{,}000$ bottles. The total supply chain profit if the two stages coordinate is $Prof_{SC} = (\$4 - \$2) \times 120{,}000 = \$240{,}000$. As a result of each stage settings its price independently, the supply chain thus loses \$60,000 in profit. This phenomenon is referred to as *double marginalization*. Double marginalization leads to a loss in profit because the supply chain margin is divided between two stages, but each stage makes its pricing decision considering only its own local profits.

Given that independent pricing decisions lower supply chain profits, it is important to consider pricing schemes that may help recover some of these profits even when each stage of the supply chain continues to act independently. We propose two pricing schemes that the manufacturer may use to achieve the coordinated solution and maximize supply chain profits even though DO acts in a way that maximizes its own profit.

1. ***Two-part tariff:*** In this case, the manufacturer charges its entire profit as an up-front franchise fee *ff* (which could be anywhere between the noncoordinated manufacturer profit $Prof_M$ and the difference between the coordinated supply chain profit and the noncoordinated retailer profit, $Prof_{SC} - Prof_R$) and then sells to the retailer at cost; that is, the manufacturer sets its wholesale price $C_R = C_M$. This pricing scheme is referred to as a *two-part tariff* because the manufacturer sets both the franchise fee and the wholesale price. The retail pricing decision is thus based on maximizing its profits $(p - C_M)(360{,}000 - 60{,}000p) - ff$. Under the two-part tariff, the franchise fee *ff* is paid up front and is thus a fixed cost that does not change with the retail price p. The retailer DO is thus effectively maximizing the coordinated supply chain profits $Prof_{SC} = (p - C_M)(360{,}000 - 60{,}000p)$. Taking the first derivative with respect to p and setting it equal to 0, the optimal coordinated retail price p is evaluated to be

$$p = 3 + \frac{C_M}{2}$$

In the case of DO, recall that total supply chain profit when the two stages coordinate is $Prof_{SC}$ = \$240,000 with DO charging the customer \$4 per bottle of Vitaherb. The profit made by DO when the two stages do not coordinate is $Prof_R$ = \$60,000. One option available to the manufacturer is to construct a two-part tariff by which DO is charged an upfront fee of $ff = Prof_{SC} - Prof_R$ = \$180,000 (see worksheet 2-*part-tariff*) and material cost of $C_R = C_M$ = \$2 per bottle. DO maximizes its profit if it prices the vitamins at $p = 3 + C_M / 2 = 3 + 2/2$ = \$4 per bottle. It has annual sales of $360{,}000 - 60{,}000p = 120{,}000$ and profits of \$60,000. The manufacturer makes a profit of \$180,000, which it charges up front. Observe that the use of a two-part tariff has increased supply chain profits from \$180,000 to \$240,000 even though the retailer DO has made a locally optimal pricing decision given the two-part tariff. A similar result can be obtained as long as the manufacturer sets the up-front fee ff to be any value between \$120,000 and \$180,000 with a wholesale price of $C_R = C_M = 2$.

2. *Volume-based quantity discount:* Observe that the two-part tariff is really a volume-based quantity discount whereby the retailer DO pays a lower average unit cost as it purchases larger quantities each year (the franchise fee ff is amortized over more units). This observation can be made explicit by designing a volume-based discount scheme that gets the retailer DO to purchase and sell the quantity sold when the two stages coordinate their actions.

Recall that the coordinated solution results in a retail price of $p = 3 + C_M/2 = 3 + 2/2 = 4$. This retail price results in total demand of $d^{coord} = 360{,}000 - (60{,}000 \times 4) = 120{,}000$. The objective of the manufacturer is to design a volume-based discounting scheme that gets the retailer DO to buy (and sell) $d^{coord} = 120{,}000$ units each year. The pricing scheme must be such that retailer gets a profit of at least \$60,000, and the manufacturer gets a profit of at least \$120,000 (these are the profits that DO and the manufacturer made when their actions were not coordinated).

Several such pricing schemes can be designed. One such scheme is for the manufacturer to charge a wholesale price of $C_R = \$4$ per bottle (this is the same wholesale price that is optimal when the two stages are not coordinated) for annual sales below $d^{coord} = 120{,}000$ units, and to charge $C_R = \$3.50$ (any value between \$3.00 and \$3.50 will work) if sales reach 120,000 or more (see worksheet *Volume Discount*). It is then optimal for DO to order 120,000 units in the year and price them at $p = \$4$ per bottle to the customers (to ensure that they are all sold). The total profit earned by DO $(360{,}000 - 60{,}000p) \times (p - C_R) = \$60{,}000$. The total profit earned by the manufacturer is $120{,}000 \times (C_R - \$2) = 180{,}000$ when $C_R = \$3.50$. The total supply chain profit is \$240,000, which is higher than the \$180,000 that the supply chain earned when actions were not coordinated.

If the manufacturer charges \$3.00 (instead of \$3.50) for sales of 120,000 units or more, it is still optimal for DO to order 120,000 units in the year and price them at $p = \$4$ per bottle. The only difference is that the total profit earned by DO now increases to \$120,000, whereas that for the manufacturer now drops to \$120,000. The total supply chain profits remain at \$240,000. The price that the manufacturer is able to charge (between \$3.00 and \$3.50) for sales of 120,000 or more will depend on the relative bargaining power of the two parties.

At this stage, we have seen that even in the absence of inventory-related costs, quantity discounts play a role in supply chain coordination and improved supply chain profits. Unless the manufacturer has large fixed costs associated with each lot, the discount schemes that are optimal are volume based and not lot-size based. It can be shown that even in the presence of large fixed costs for the manufacturer, a two-part tariff or volume-based discount, with the manufacturer passing on some of the fixed cost to the retailer, optimally coordinates the supply chain and maximizes profits given the assumption that customer demand decreases when the retailer increases price.

A key distinction between lot-size–based and volume discounts is that lot-size discounts are based on the quantity purchased per lot, not the rate of purchase. Volume discounts, in contrast, are based on the rate of purchase or volume purchased on average per specified time period (say, a month, quarter, or year). Lot-size–based discounts tend to raise the cycle inventory in the supply chain by encouraging retailers to increase the size of each lot. Volume-based discounts, in contrast, are compatible with small lots that reduce cycle inventory. Lot-size–based discounts make sense only when the manufacturer incurs high fixed cost per order. In all other instances, it is better to have volume-based discounts.

Key Point

For products for which the firm has market power, two-part tariffs or volume-based quantity discounts can be used to achieve coordination in the supply chain and maximize supply chain profits.

Key Point

For products for which a firm has market power, lot-size–based discounts are not optimal for the supply chain even in the presence of inventory costs. In such a setting, either a two-part tariff or a volume-based discount, with the supplier passing on some of its fixed cost to the retailer, is needed for the supply chain to be coordinated and maximize profits.

Lessons from Discounting Schemes

- Lot size based discounts increase lot size and cycle inventory in the supply chain
- Lot size based discounts are justified to achieve coordination for commodity products
- Volume based discounts with some fixed cost passed on to retailer are more effective in general
  - Volume based discounts are better over rolling horizon

11.6 SHORT-TERM DISCOUNTING: TRADE PROMOTIONS

Manufacturers use *trade promotions* to offer a discounted price to retailers and set a time period over which the discount is effective. For example, a manufacturer of canned soup may offer a price discount of 10 percent for the shipping period December 15 to January 25. For all purchases within the specified time horizon, retailers get a 10 percent discount. In some cases, the manufacturer may require specific actions from the retailer, such as displays, advertising, promotion, and so on, to qualify for the trade promotion. Trade promotions are quite common in the consumer packaged-goods industry, with manufacturers promoting different products at different times of the year.

The goal of trade promotions is to influence retailers to act in a way that helps the manufacturer achieve its objectives. The following are a few of the key goals (from the manufacturer's perspective) of a trade promotion (see Blattberg and Neslin [1990] for more details):

1. Induce retailers to use price discounts, displays, or advertising to spur sales.
2. Shift inventory from the manufacturer to the retailer and the customer.
3. Defend a brand against competition.

Although these may be the manufacturer's objectives, it is not clear that they are always achieved as the result of a trade promotion. Our goal in this section is to investigate the impact of a trade promotion on the behavior of the retailer and the performance of the entire supply chain. The key to understanding this impact is to focus on how a retailer reacts to a trade promotion that a manufacturer offers. In response to a trade promotion, the retailer has the following options:

1. Pass through some or all of the promotion to customers to spur sales.
2. Pass through very little of the promotion to customers but purchase in greater quantity during the promotion period to exploit the temporary reduction in price.

The first action lowers the price of the product for the end customer, leading to increased purchases and, thus, increased sales for the entire supply chain. The second action does not increase purchases by the customer, but increases the amount purchased and held at the retailer. As a result, the cycle inventory and flow time within the supply chain increase.

A *forward buy* occurs when a retailer purchases in the promotional period for sales in future periods. A forward buy helps reduce the retailer's future cost of goods for product sold after the promotion ends. Although a forward buy is often the retailer's appropriate response to a price promotion, it can decrease supply chain profits because it results in higher demand variability, with a resulting increase in inventory and flow times within the supply chain.

Our objective in this section is to understand a retailer's optimal response when faced with a trade promotion. We identify the factors affecting the forward buy and quantify the size of a forward buy by the retailer. We also identify factors that influence the amount of the promotion that a retailer passes on to the customer.

We first illustrate the impact of a trade promotion on forward buying behavior of the retailer. Consider a Cub Foods supermarket selling chicken noodle soup manufactured by the Campbell Soup Company. Customer demand for chicken noodle soup is D cans per year. Campbell charges $\$C$ per can. Cub Foods incurs a holding cost of h (per dollar of inventory held for a year). Using the EOQ formula (Equation 11.5), Cub Foods normally orders in the following lot sizes:

$$Q^* = \sqrt{\frac{2DS}{hC}}$$

Campbell announces that it is offering a discount of $\$d$ per can for the coming four-week period. Cub Foods must decide how much to order at the discounted price compared with the lot size of Q^* that it normally orders. Let Q^d be the lot size ordered at the discounted price.

The costs the retailer must consider when making this decision are material cost, holding cost, and order cost. Increasing the lot size Q^d lowers the material cost for Cub Foods because it purchases more cans (for sale now and in the future) at the discounted price. Increasing the lot size Q^d increases the holding cost because inventories increase. Increasing the lot size Q^d lowers the order cost for Cub Foods because some orders that would otherwise have been placed are now not necessary. Cub Foods' goal is to make the trade-off that minimizes the total cost.

The inventory pattern when a lot size of Q^d is followed by lot sizes of Q^* is shown in Figure 11-5. The objective is to identify Q^d that minimizes the total cost (material cost + ordering cost + holding cost) over the time interval during which the quantity Q^d (ordered during the promotion period) is consumed.

The precise analysis in this case is complex, so we present a result that holds under some restrictions (see Silver, Pyke, and Petersen [1998] for a more detailed discussion). The first key

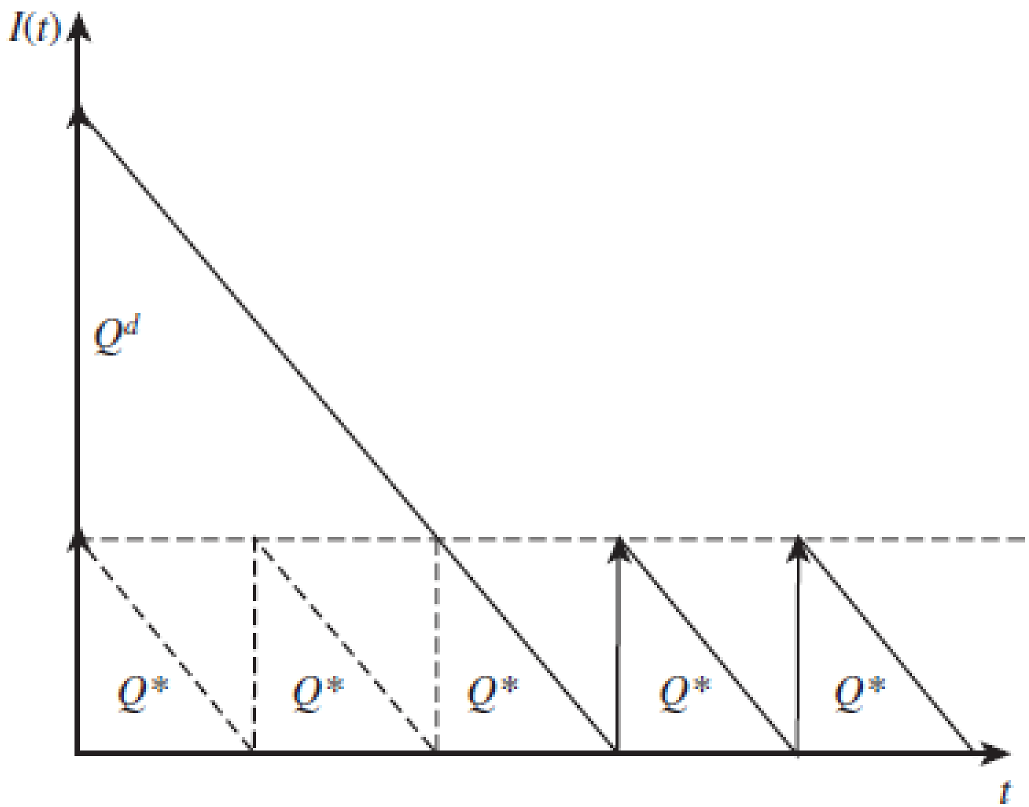


FIGURE 11-5 Inventory Profile for Forward Buying

assumption is that the discount is offered once, with no future discounts. The second key assumption is that the retailer takes no action (such as passing on part of the trade promotion) to influence customer demand. The customer demand thus remains unchanged. The third key assumption is that we analyze a period over which the demand is an integer multiple of Q^*. With these assumptions, the optimal order quantity at the discounted price is given by

$$Q^d = \frac{dD}{(C - d)h} + \frac{CQ^*}{C - d} \quad (11.16)$$

In practice, retailers are often aware of the timing of the next promotion. If the demand until the next anticipated trade promotion is Q_1, it is optimal for the retailer to order $min\{Q^d, Q_1\}$ Observe that the quantity Q^d ordered as a result of the promotion is larger than the regular order quantity Q^*. The forward buy in this case is given by

$$\text{Forward buy} = Q^d - Q^*$$

Even for relatively small discounts, the order size increases by a large quantity, as illustrated in Example 11-11 (see spreadsheet *Chapter11-examples11-12*).

EXAMPLE 11-11 Impact of Trade Promotions on Lot Sizes

DO is a retailer that sells Vitaherb, a popular vitamin diet supplement. Demand for Vitaherb is 120,000 bottles per year. The manufacturer currently charges $3 for each bottle, and DO incurs a holding cost of 20 percent. DO currently orders in lots of $Q^* = 6{,}325$ bottles. The manufacturer has offered a discount of $0.15 for all bottles purchased by retailers over the coming month. How many bottles of Vitaherb should DO order given the promotion?

Analysis:

In the absence of any promotion, DO orders in lot sizes of $Q^* = 6{,}325$ bottles. Given a monthly demand of $D = 10{,}000$ bottles, DO normally orders every 0.6325 months. In the absence of the trade promotion we have the following:

$$\text{Cycle inventory at DO} = Q^*/2 = 6{,}325/2 = 3{,}162.50 \text{ bottles}$$

$$\text{Average flow time} = Q^*/2D = 6{,}325/(2D) = 0.3162 \text{ months}$$

The optimal lot size during the promotion is obtained using Equation 11.15 and is given by

$$Q^d = \frac{dD}{(C-d)h} + \frac{CQ^*}{C-d} = \frac{0.15 \times 120{,}000}{(3.00 - 0.15) \times 0.20} + \frac{3 \times 6{,}325}{3.00 - 0.15} = 38{,}236$$

During the promotion, DO should place an order for a lot size of 38,236. In other words, DO places an order for 3.8236 months' worth of demand. In the presence of the trade promotion we have

$$\text{Cycle inventory at DO} = Q^d/2 = 38{,}236/2 = 19{,}118 \text{ bottles}$$

$$\text{Average flow time} = Q^d/(2D) = 38{,}236/(20{,}000) = 1.9118 \text{ months}$$

In this case, the forward buy is given by

$$\text{Forward buy} = Q^d - Q^* = 38{,}236 - 6{,}325 = 31{,}911 \text{ bottles}$$

As a result of this forward buy, DO will not place any order for the next 3.8236 months (without a forward buy, DO would have placed another 31,912/6,325 = 5.05 orders for 6,325 bottles each during this period). Observe that a 5 percent discount causes the lot size to increase by more than 500 percent.

As the example illustrates, forward buying as a result of trade promotions leads to a significant increase in the quantity ordered by the retailer. The large order is then followed by a period of small orders to compensate for the inventory built up at the retailer. The fluctuation in orders as a result of trade promotions is one of the major contributors to the bullwhip effect discussed in Chapter 10. The retailer can justify the forward buying during a trade promotion because it decreases its total cost. In contrast, the manufacturer can justify this action only as a competitive necessity (to counter a competitor's promotion) or if it has either inadvertently built up a lot of excess inventory or the forward buy allows the manufacturer to smooth demand by shifting it from peak- to low-demand periods. In practice, manufacturers often build up inventory in anticipation of planned promotions. During the trade promotion, this inventory shifts to the retailer, primarily as a forward buy. If the forward buy during trade promotions is a significant fraction of total sales, manufacturers end up reducing the revenues they earn from sales because most of the product is sold at a discount. The increase in inventory and the decrease in revenues often lead to a reduction in manufacturer as well as total supply chain profits as a result of trade promotions (see Blattberg and Neslin [1990] for more details).

Key Point

Trade promotions lead to a significant increase in lot size and cycle inventory because of forward buying by the retailer. This generally results in reduced supply chain profits unless the trade promotion reduces demand fluctuations.

Now, let us consider the extent to which the retailer may find it optimal to pass through some of the discount to the end customer to spur sales. As Example 11-12 shows, it is not optimal for the retailer to pass through the entire discount to the customer. In other words, it is optimal for the retailer to capture part of the promotion and pass through only part of it to the customer.

EXAMPLE 11-12 How Much of a Discount Should the Retailer Pass Through?

Assume that DO faces a demand curve for Vitaherb of $300,000 - 60,000p$. The normal price charged by the manufacturer to the retailer is $C_R = \$3$ per bottle. Ignoring all inventory-related costs, evaluate the optimal response of DO to a discount of $0.15 per unit.

Analysis:

The profits for DO, the retailer, are given as follows:

$$Prof_R = (300{,}000 - 60{,}000\,p)\,p - (300{,}000 - 60{,}000\,p)C_R$$

The retailer prices to maximize profits, and the optimal retail price is obtained by setting the first derivative of retailer profits with respect to p to 0. This implies that

$$300{,}000 - 120{,}000p + 60{,}000\,C_R = 0$$

or

$$p = (300{,}000 + 60{,}000\,C_R)/120{,}000 \qquad \textbf{(11.17)}$$

Substituting $C_R = \$3$ into Equation 11.17, we obtain a retail price of $p = \$4$. As a result, the customer demand at the retailer in the absence of the promotion is

$$D_R = 30{,}000 - 60{,}000\,p = 60{,}000$$

During the promotion, the manufacturer offers a discount of \$0.15, resulting in a price to the retailer of $C_R = \$2.85$. Substituting into Equation 11.17, the optimal price set by DO is

$$p = (300{,}000 + 60{,}000 \times 2.85)/120{,}000 = \$3.925$$

Observe that the retailer's optimal response is to pass through only \$0.075 of the \$0.15 discount to the customer. The retailer does not pass through the entire discount. At the discounted price, DO experiences a demand of

$$D_R = 300{,}000 - 60{,}000\,p = 64{,}500$$

This represents an increase of 7.5 percent in demand relative to the base case. It is optimal here for DO to pass on half the trade promotion discount to the customers. This action results in a 7.5 percent increase in customer demand.

From Examples 11-11 and 11-12, observe that the increase in customer demand resulting from a trade promotion (7.5 percent of demand in Example 11-12) is small relative to the increased purchase by the retailer due to forward buying (500 percent from Example 11-11). The impact of the increase in customer demand may be further dampened by customer behavior. For many products, such as detergent and toothpaste, most of the increase in customer purchases is a forward buy by the customer; customers are unlikely to start brushing their teeth more frequently simply because they have purchased a lot of toothpaste. For such products, a trade promotion does not truly increase demand.

Key Point

Faced with a short-term discount, it is optimal for retailers to pass through only a fraction of the discount to the customer, keeping the rest for themselves. Simultaneously, it is optimal for retailers to increase the purchase lot size and forward buy for future periods. Thus, trade promotions often lead to an increase of cycle inventory in a supply chain without a significant increase in customer demand.

Manufacturers have always struggled with the fact that retailers pass along only a small fraction of a trade discount to the customer. In a study conducted by Kurt Salmon and Associates (1993), almost a quarter of all distributor inventories in the dry-grocery supply chain could be attributed to forward buying.

Estimating Cycle Inventory-Related Costs in Practice

- Inventory holding cost
  - Cost of capital
  - Obsolescence cost
  - Handling cost
  - Occupancy cost
  - Miscellaneous costs
- Order cost
  - Buyer time
  - Transportation costs
  - Receiving costs
  - Other costs

Levers to Reduce Lot Sizes Without Hurting Costs

- Cycle Inventory Reduction
  - Reduce transfer and production lot sizes
    - Aggregate fixed costs across multiple products, supply points, or delivery points
  - Are quantity discounts consistent with manufacturing and logistics operations?
    - Volume discounts on rolling horizon
    - Two-part tariff
  - Are trade promotions essential?
    - EDLP (every day low pricing) eg. P&G and WalMart
    - Based on sell-thru rather than sell-in