Investment Portfolio Management Course

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An-Najah National University

Summer Semester 19/20

Chapter 7: Capital Asset Pricing and Arbitrage Pricing Theory

Capital Market Line (CML)

- Previously, we discussed how the risk-free asset could be combined with a risky portfolio to create a capital allocation line (CAL).
- In this section, we discuss a specific CAL that uses the market portfolio as the optimal risky portfolio and is known as the capital market line (CML).

Passive and Active Portfolios

- After constructing the portfolio and finding the optimal weights of the ORP, the next decision is the Security Selection.
- Based on your belief of the degree of market efficiency, your approach of security selection is determined.
  - If the market is an **information ally efficient market**, the price in the market is an unbiased estimate of all future discounted cash flows and investors cannot expect to earn a return that is greater than the required rate of return for that asset.
- Portfolios that are based on the assumption of unbiased market prices are referred to as **passive portfolios**.

Optimal Risky Portfolio “P^{o}”

Stocks	**75%**
Bonds	**25%**

Passive and Active Portfolios

- Passive portfolios most commonly replicate and track market indexes, which are passively constructed on the basis of market prices and market capitalizations.
  - Examples of market indexes: S&P 500 Index, Nikkei 300, CAC 40
- Passive portfolios based on market indexes are called index funds

Market Efficiency

Stock	Current Price	Expected One Year Price	Expected Return	Required Return
ABC	$10	$12.5	25%	25%
XYZ	$15	$18	20%	20%
WZY	$30	$34	13%	13%

Passive and Active Portfolios

"Don't look for the needle in the haystack. Just buy the haystack!"

John Bogle, from the "*Little Book of Common Sense Investing: The Only Way to Guarantee Your Fair Share of Stock Market Returns*"

Don't try to beat the market – invest in index funds that track performance of broad asset categories that match your risk aversion/tolerance.

Active Investing

- On the other hand, if investor have more confidence in their own ability to estimate cash flows, growth rates, and discount rates and therefore, determine whether an asset is fairly valued, then they practice **active investing.**
- In an **actively managed portfolio**, assets that are undervalued will have a positive weight.
- This style of investing is called active investment management, and the portfolios are referred to as **active Portfolios**
  - Open-end mutual funds and hedge funds

Active Management View of World

What is the “Market”?

- The **optimal risky portfolio and the capital market line** depend on the definition of the market. So **what is the market**?
- **Theoretically,** the **market** includes all risky assets or anything that has value, which includes stocks, bonds, real estate, and even human capital.
- **Practically, the market is defined quite narrowly.**
  - A local or regional stock market index is used as a proxy for the market because of **active trading** and because a local or regional market is most **visible to the local investors**.

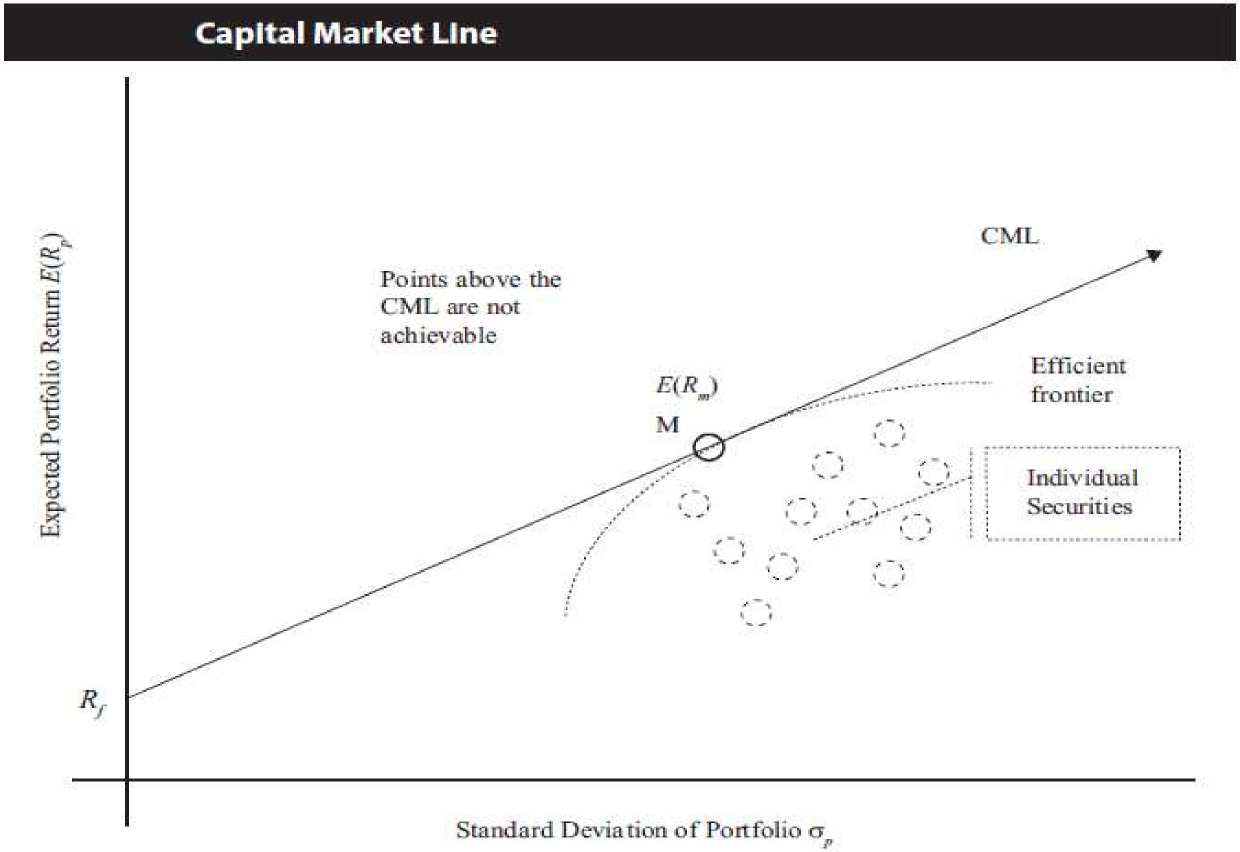
What is the “Market”?

- For our analysis, we use the S&P 500 index.
- Therefore, “market” return and the “market” risk premium refer to US equity return and the US equity risk premium.

The Capital Market Line (CML)

- A **capital allocation line (CAL)** includes all possible combinations of the risk-free asset and an investor's optimal risky portfolio.
- The **capital market line (CML)** is a special case of the capital allocation line, where the risky portfolio is the market portfolio.
  - The **S&P 500** is a proxy of the market portfolio, which is **the optimal risky portfolio.**
- Therefore, the expected return on the risky portfolio is the expected market return, expressed as $E(R_m)$.

CML



Capital Market Line
Expected Portfolio Return $E(R_p)$
Points above the CML are not achievable
CML
Efficient frontier
$E(R_m)$
M
Individual Securities
R_f
Standard Deviation of Portfolio σ_p

CML

- Risk and return characteristics of the portfolio represented by the CML are:

$$E(R_p) = w_1 R_f + (1 - w_1)E(R_m)$$

$$\sigma_p = \sqrt{\cancel{w_1^2 \sigma_f^2} + (1 - w_1)^2 \sigma_m^2 + \cancel{2w_1(1 - w_1)\mathrm{Cov}(R_f, R_m)}}$$

$$\sigma_p = (1 - w_1)\sigma_m$$

CML

- By substitution, we can express $E(R_p)$ in terms of σ_p

$$E(R_p) = R_f + \left(\frac{E(R_m) - R_f}{\sigma_m}\right) \times \sigma_p$$

Example

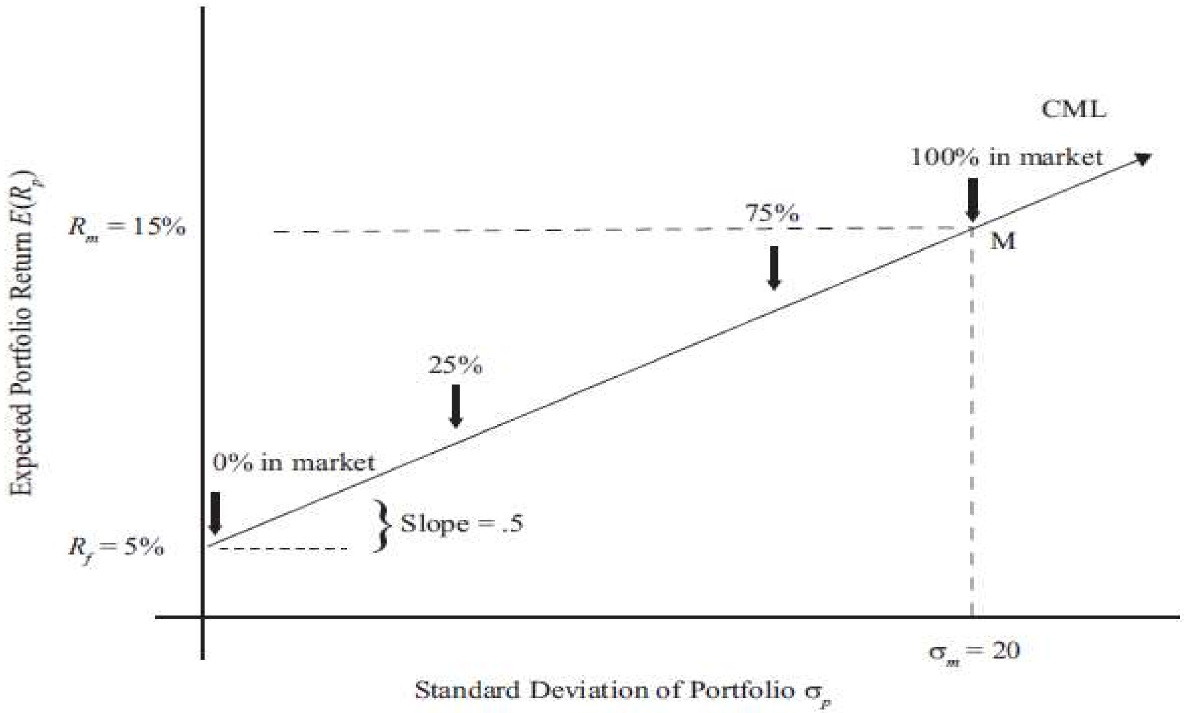
Mr. Miles is a first time investor and wants to build a portfolio using only US T-bills and an index fund that closely tracks the S&P 500 Index. The T-bills have a return of 5 percent. The S&P 500 has a standard deviation of 20 percent and an expected return of 15 percent.

1 Draw the CML and mark the points where the investment in the market is 0 percent, 25 percent, 75 percent, and 100 percent.

2 Mr. Miles is also interested in determining the exact risk and return at each point.

Asset	Expected Return	Standard Deviation
Index Fund	15%	20%
T-bills	5%	0%

Solution



CML
100% in market
75%
M
25%
0% in market
Slope = .5
$R_m = 15\%$
$R_f = 5\%$
$\sigma_m = 20$
Expected Portfolio Return $E(R_p)$
Standard Deviation of Portfolio σ_p

Cont’d

Return with 0 percent invested in the market = 5 percent, which is the risk-free return.

Standard deviation with 0 percent invested in the market = 0 percent because T-bills are not risky.

Return with 25 percent invested in the market = $(0.75 \times 5\%) + (0.25 \times 15\%) = 7.5\%$.

Standard deviation with 25 percent invested in the market = $0.25 \times 20\% = 5\%$.

Return with 75 percent invested in the market = $(0.25 \times 5\%) + (0.75 \times 15\%) = 12.50\%$.

Standard deviation with 75 percent invested in the market = $0.75 \times 20\% = 15\%$.

Return with 100 percent invested in the market = 15 percent, which is the return on the S&P 500.

Standard deviation with 100 percent invested in the market = 20 percent, which is the risk of the S&P 500.

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CAPM and Its Development Led To…

Harry Markowitz: wrote about mean-variance efficient frontiers in 1950s

Nobel Prize in 1990

Pricing individual stocks leads to Capital Asset Pricing Model (CAPM)

Developed by William Sharpe (1964) & John Lintner (1965)

Sharpe won Nobel Prize in 1990

Capital Asset Pricing Model (CAPM)

- The **CAPM** is a centerpiece of modern financial economics, which was proposed by William Sharpe, who was awarded the 1990 Nobel Prize for economics
- It is an "equilibrium" model derived using principles of diversification and some simplified assumptions for the behavior of investors and the market condition.
  - The market equilibrium refers to a condition in which for all securities, market prices are established to balance the demand of buyers and the supply of sellers. These prices are called equilibrium prices

CAPM Assumptions

Market Assumptions	Investor Assumptions
All investors are price takers	Investors plan for the same (single-period) horizon
All information relevant to security analysis is free and publicly available.	Investors are efficient users of analytical methods → investors have homogeneous expectations.
All securities are publicly owned and traded.	Investors are rational, mean-variance optimizers.
No taxes on investment returns.	
No transaction costs.	
Lending and borrowing at the same risk-free rate are unlimited.	

CAPM

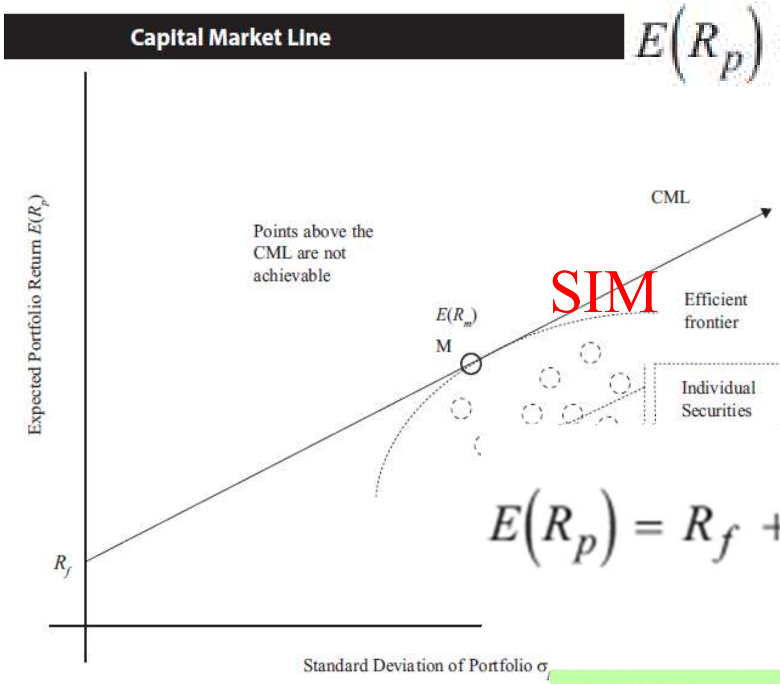
$$E(R_i) = R_f + \beta_i \overbrace{[E(R_m) - R_f]}^{\text{Market Risk Premium}}$$

Expected Return on security i

Risk Free Rate of Return

Beta Coefficient

From CML to SML



$$E(R_p) = R_f + \left(\frac{E(R_m) - R_f}{\sigma_m}\right) \times \sigma_p$$

SIM

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sum_{i=1}^{N} w_i^2 \sigma_e^2$$

$$E(R_p) = R_f + \left(\frac{E(R_m) - R_f}{\cancel{\sigma_m}}\right) \times \beta_p \cancel{\sigma_m} + \cancel{\sum_{i=1}^{N} w_i \sigma_e}$$

CAPM Equation

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

CAPM

$$E(R_i) = R_f + \beta_i \underbrace{[E(R_m) - R_f]}_{\text{Market Risk Premium}}$$

Expected Return on security i

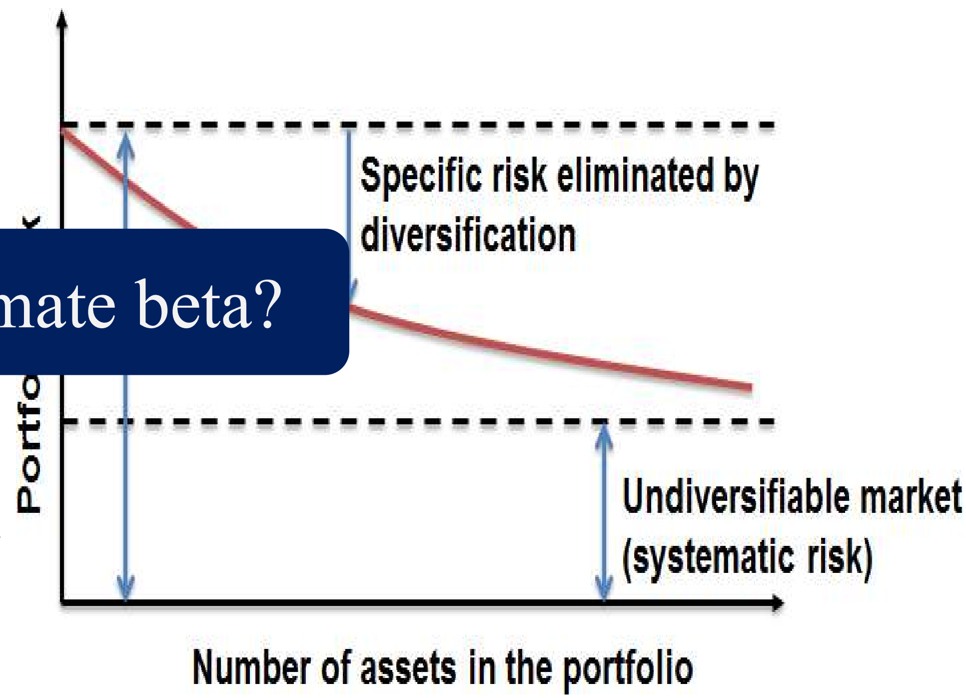
Risk Free Rate of Return

Beta Coefficient

How to Estimate beta?

Interpreting Beta

- Beta is a measure of the **systematic risk** associated with a particular stock, asset, or portfolio.
  - A beta of **more than 1** indicates an asset that has **amplified** the return of the whole market (posi
  - A beta **close to ze** asset that provides a more **stable return** than the market as a whole.
  - A **negative beta** would signify an asset whose performance is **counter-cyclical**, i.e., offsets the overall market experience.



How to estimate beta?

What is the Meaning of beta?

- **Beta** is a measure of how sensitive an asset's return is to the market as a whole. It beta captures an asset's systematic risk, or the portion of an asset's risk that cannot be eliminated by diversification.

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i\sigma_m}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$$

- The **market's beta** can be found as:

$$\beta_i = \frac{\rho_{i,m}\sigma_i}{\sigma_m} = \frac{\rho_{m,m}\sigma_m}{\sigma_m} = 1$$

Beta and its Interpretation

SELECTED BETAS AND ASSOCIATED INTERPRETATIONS

Beta	Comment	Interpretation
2.0	Move in same direction as the market	Twice as responsive as the market
1.0		Same response as the market
0.5		One half as responsive as the market
0.0		Unaffected by market movement
−0.5	Move in opposite direction of the market	One-half as responsive as the market
−1.0		Same response as the market
−2.0		Twice as responsive as the market

Example

1 Suppose the risk-free rate is 3 percent, the expected return on the market portfolio is 13 percent, and its standard deviation is 23 percent. An Indian company, Bajaj Auto, has a standard deviation of 50 percent but is uncorrelated with the market. Calculate Bajaj Auto's beta and expected return.

2 Suppose the risk-free rate is 3 percent, the expected return on the market portfolio is 13 percent, and its standard deviation is 23 percent. A German company, Mueller Metals, has a standard deviation of 50 percent and a correlation of 0.65 with the market. Calculate Mueller Metal's beta and expected return.

Answer

- 1)

$$\beta_i = \frac{\rho_{i,m}\sigma_i}{\sigma_m} = \frac{0.0 \times 0.50}{0.23} = 0$$

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] = 0.03 + 0 \times (0.13 - 0.03) = 0.03 = 3.0\%$$

- 2)

$$\beta_i = \frac{\rho_{i,m}\sigma_i}{\sigma_m} = \frac{0.65 \times 0.50}{0.23} = 1.41$$

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] = 0.03 + 1.41 \times (0.13 - 0.03) = 0.171 = 17.1\%$$

Example

- **You have gathered the following information concerning a particular investment and conditions in the market.**

U.S T-Bill rate	3.50%
Market rate of Return	12%
Beta of Investment	1.65

According to the Capital Asset Pricing Model, the required return for this investment is

A) 17.53%.
B) 11.48%.
C) 13.65%.
D) 15.50%.

CAPM Components

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

Treasury Yields

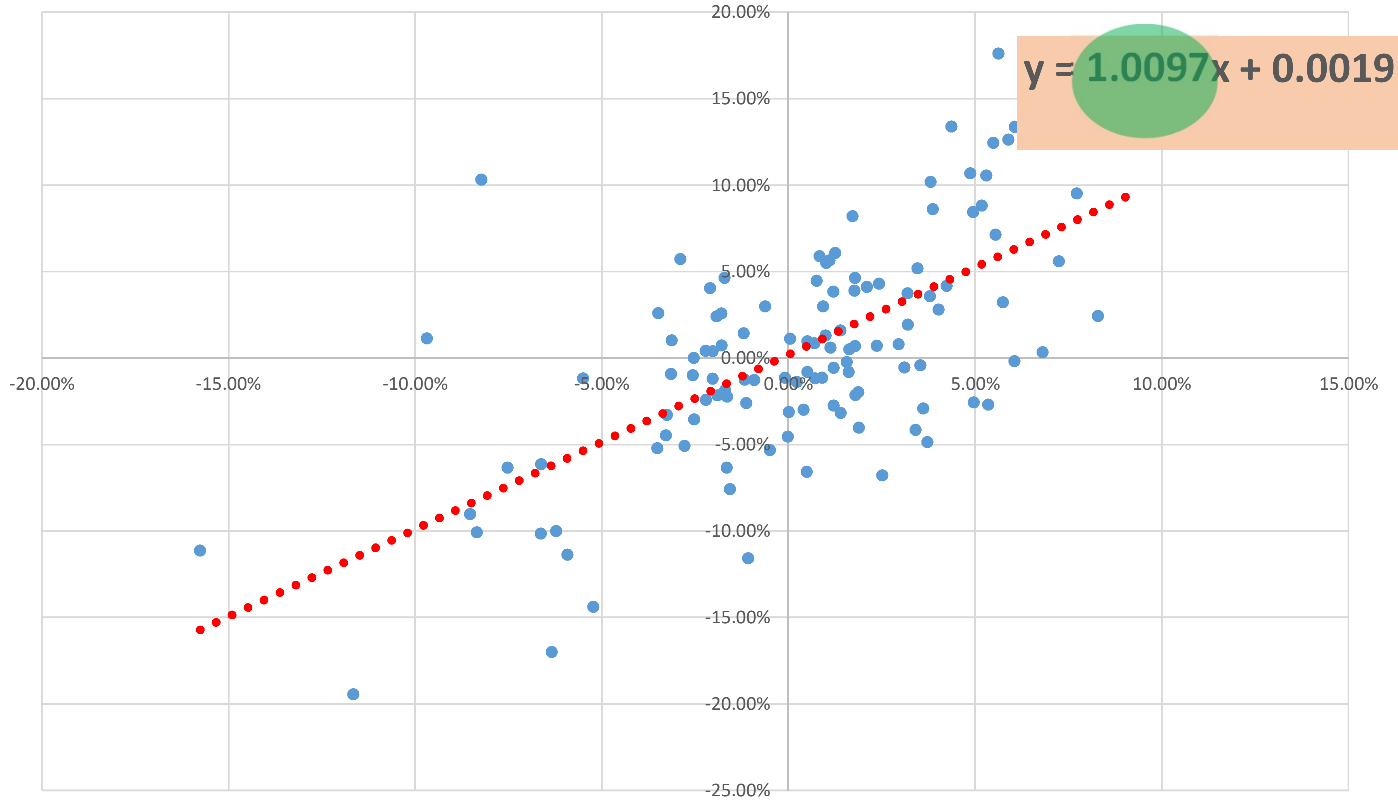
NAME	COUPON	PRICE	YIELD	1 MONTH	1 YEAR	TIME (EDT)
GB3:GOV **3 Month**	0.00	0.19	0.19%	-18	-220	4/10/2020
GB6:GOV **6 Month**	0.00	0.22	0.22%	-16	-221	4/10/2020
GB12:GOV **12 Month**	0.00	0.20	0.20%	-15	-219	4/10/2020
GT2:GOV **2 Year**	0.38	100.29	0.23%	-30	-213	4/10/2020
GT5:GOV **5 Year**	0.50	100.47	0.40%	-30	-191	4/10/2020
GT10:GOV **10 Year**	1.50	107.41	0.72%	-15	-178	4/10/2020
GT30:GOV **30 Year**	2.00	116.09	1.34%	-5	-158	4/10/2020

https://www.bloomberg.com/markets/rates-bonds/government-bonds/us

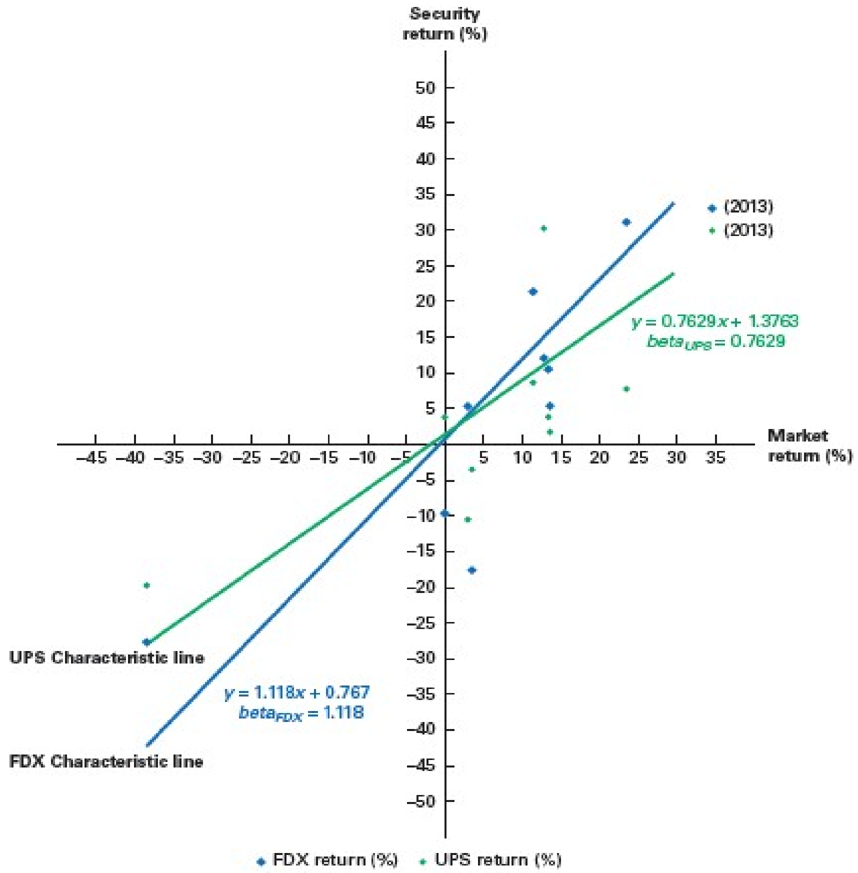
Previously: SCL

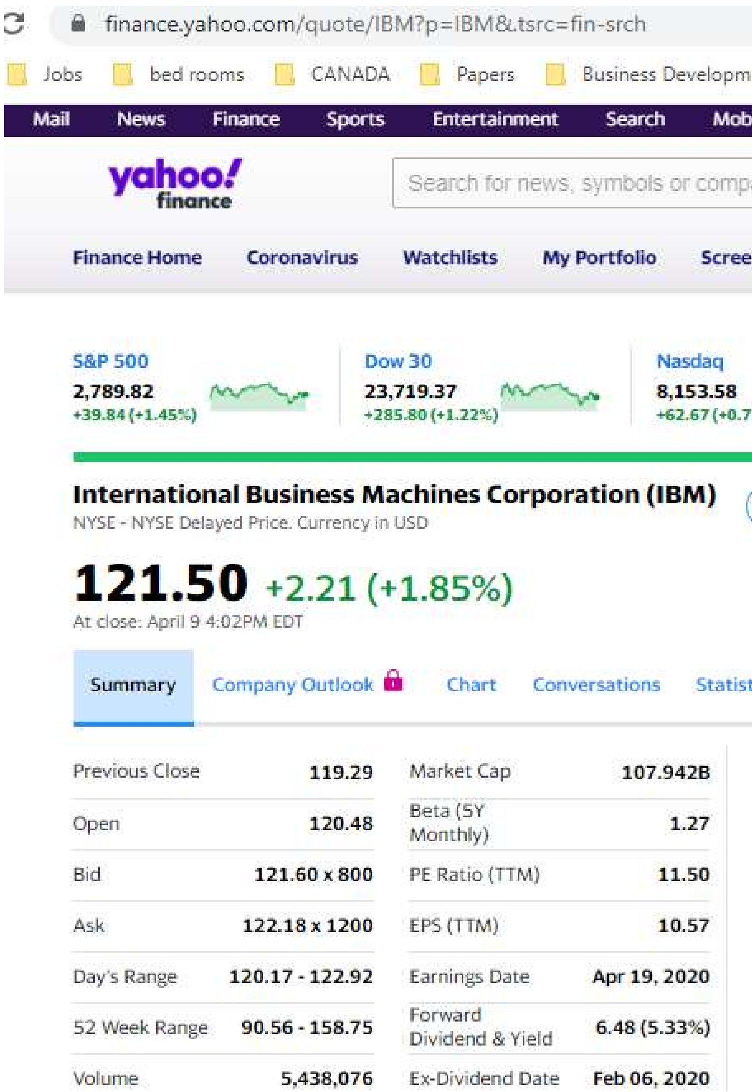
$$R_i = \alpha_i + \beta_i R_M + e_i$$

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2}$$



Where Do Betas Come From?





Beta Coefficients

	Beta, β	
Company	**Yahoo!**	**Morningstar**
ExxonMobil	0.88	1.02
IBM	0.88	0.64
Starbucks	0.82	0.76
Walmart	0.82	0.46
Microsoft	0.86	1.07
Harley-Davidson	0.93	0.64
eBay	0.91	1.08
Tesla	1.40	0.54
Southwest Airlines	0.99	1.20
Yahoo!	1.75	2.02

Source: finance.yahoo.com and www.morningstar.com. Accessed November 2, 2015.

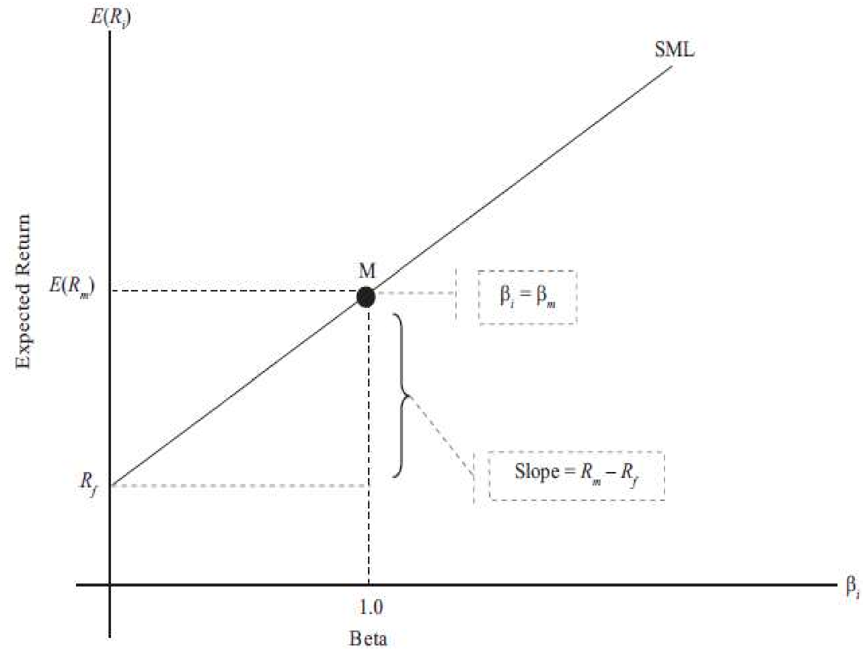
Example

- **How to calculate expected returns for a security using CAPM?**
  - Calculate the expected return for a security, given:
    - $R_f = 5\%$
    - Std. dev. of security = 40%
    - Security correlation with market = 0.80
    - Std. dev. of market = 20%
    - $R_m = 10\%$

$$E(R_i) = 5\% + 1.6 \times (10\% - 5\%) = 13\%$$

SML

- The **security market line** (SML) is a graphical representation of the capital asset pricing model with beta, reflecting systematic risk, on the *x*-axis and expected return on the *y*-axis.



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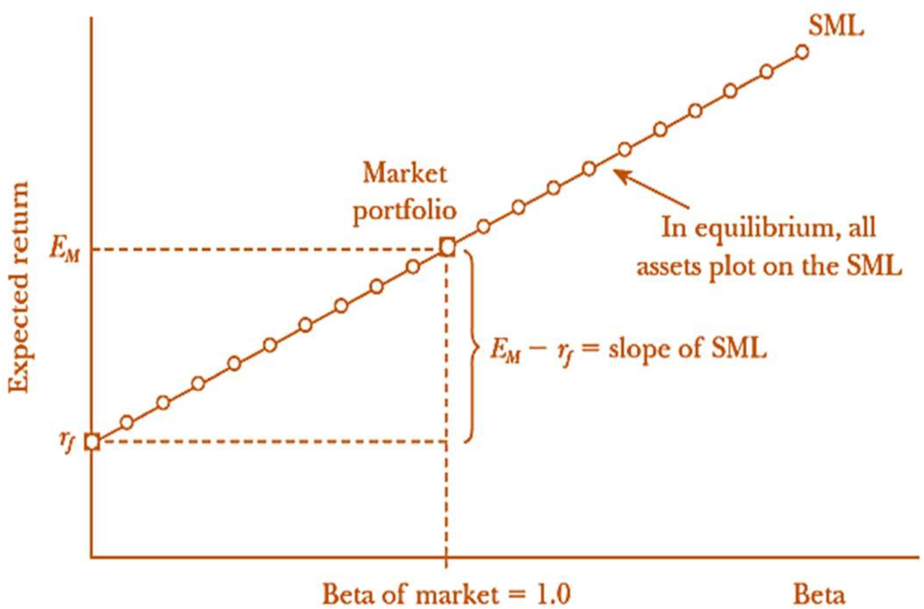
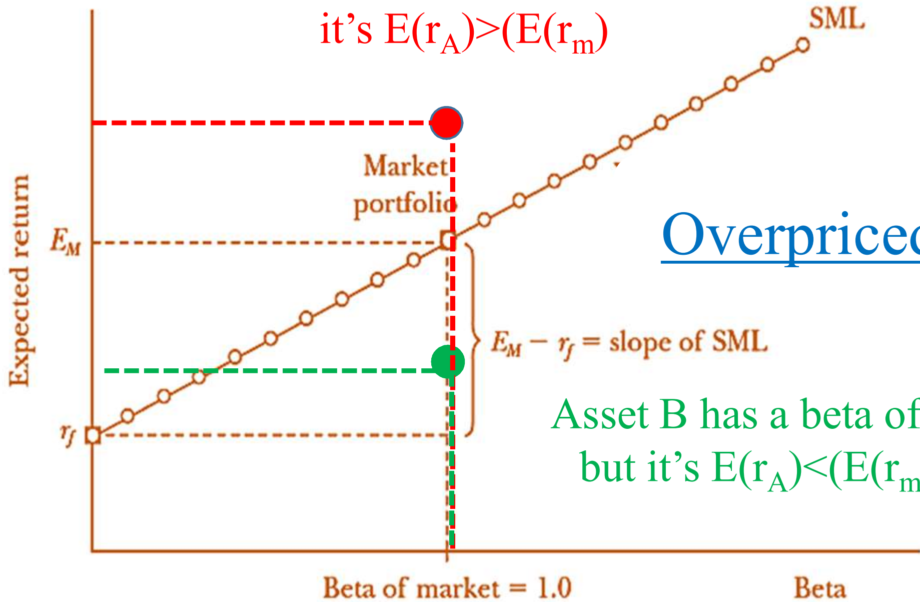


SML and Mispricing

Individual assets are on the SML if the CAPM is correct!

Underpriced assets

Asset A has a beta of 1, but it's $E(r_A) > (E(r_m)$



Overpriced assets

Asset B has a beta of 1, but it's $E(r_A) < (E(r_m)$

Portfolio Beta

- Consider we formed a portfolio of two assets; asset 1 and asset 2 with a weight of w_i in Security 1 and the balance in Security 2.

- The return for the two securities and return of the portfolio can be written as

$$E(R_1) = R_f + \beta_1\left[E(R_m) - R_f\right]$$

$$E(R_2) = R_f + \beta_2\left[E(R_m) - R_f\right]$$

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2)$$

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2) = w_1 R_f + w_1\beta_1\left[E(R_m) - R_f\right] + w_2 R_f + w_2\beta_2\left[E(R_m) - R_f\right]$$

$$E(R_p) = R_f + (w_1\beta_1 + w_2\beta_2)\left[E(R_m) - R_f\right]$$

Portfolio Beta

$$\beta_p = \sum_{i=1}^{n} w_i \beta_i; \sum_{i=1}^{n} w_i = 1$$

The portfolio's return given by the CAPM is

$$E(R_p) = R_f + \beta_p[E(R_m) - R_f]$$

Example

- Suppose you have the following information:

Security	Amount Invested	Expected Return	Beta	Weight
Stock A	$1,000	8%	0.80	=1,000/1,0000= 10%
Stock B	2,000	12	0.95	=2,000/10,000=20%
Stock C	3,000	15	1.10	=3,000/10,000=30%
Stock D	4,000	18	1.40	=4,000/10000=40%

What is the **expected return** on this portfolio? What is the **beta of this portfolio**? Does this portfolio have more or less systematic risk than an average a

$$\begin{aligned}\beta_P &= 0.10 \times \beta_A + 0.20 \times \beta_B + 0.30 \times \beta_C + 0.40 \times \beta_D \\ &= 0.10 \times 0.80 + 0.20 \times 0.95 + 0.30 \times 1.10 + 0.40 \times 1.40 \\ &= 1.16\end{aligned}$$

Example

- You invest 20 percent of your money in the risk-free asset, 30 percent in the market portfolio, and 50 percent in RedHat, a US stock that has a beta of 2.0. Given that the risk-free rate is 4 percent and the market return is 16 percent, **what are the portfolio's beta and expected return?**

Solution:

The beta of the risk-free asset = 0, the beta of the market = 1, and the beta of RedHat is 2.0. The portfolio beta is

$$\beta_p = w_1\beta_1 + w_2\beta_2 + w_3\beta_3 = (0.20 \times 0.0) + (0.30 \times 1.0) + (0.50 \times 2.0) = 1.30$$

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] = 0.04 + 1.30 \times (0.16 - 0.04) = 0.196 = 19.6\%$$

The portfolio beta is 1.30, and its expected return is 19.6 percent.

Example

1 Suppose the risk-free rate is 3 percent, the expected return on the market portfolio is 13 percent, and its standard deviation is 23 percent. An Indian company, Bajaj Auto, has a standard deviation of 50 percent but is uncorrelated with the market. Calculate Bajaj Auto's beta and expected return.

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Answer

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$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] = 0.03 + 0 \times (0.13 - 0.03) = 0.03 = 3.0\%$$

- 2)

$$\beta_i = \frac{\rho_{i,m}\sigma_i}{\sigma_m} = \frac{0.65 \times 0.50}{0.23} = 1.41$$

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] = 0.03 + 1.41 \times (0.13 - 0.03) = 0.171 = 17.1\%$$

Applications of the CAPM

- The CAPM is not only important from a theoretical perspective but is also used extensively in practice.
- it is important to understand that the CAPM and the SML are functions that give an indication of what the return in the market *should* be, given a certain level of risk. The actual

  1. ***Estimate of Expected Return***
  2. ***Security Selection***
  3. *Portfolio Performance Evaluation*

Application of the CAPM to Capital Budgeting: Example

- You are a consultant to a large manufacturing corporation considering a project with the following net after-tax cash flows (in millions of dollars):

Years from Now	After-Tax CF
0	−20
1–9	10
10	20

- The project's beta is 1.7. Assuming r_f = 9% and $E(r_M)$ =19%, what is the net present value of the project? What is the highest possible beta estimate for the project before its NPV becomes negative?

Remember!

Time	0	1	2	3	4	5	6	7	8	9	10
Cash Flows	($20)	$10	$10	$10	$10	$10	$10	$10	$10	$10	$20

$$NPV = -C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_T}{(1+r)^T}$$

$-C_0 = Initial\ Investment$

$C = Cash\ Flow$

$r = Discount\ Rate$

$T = Time$

Wait, We need the discount rate !

Discount Rate

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

$$E(R_i) = 0.09 + 1.7 * (0.19 - 0.09) = 0.26$$

NPV

NPV=$-20 + PVOA($10,9,0.26)+ PV($20,10,0.26)

$$PVA_N = PMT \times \left[\frac{1 - \frac{1}{(1+i)^N}}{i} \right] \qquad PV = \frac{FV_N}{(1+i)^N}$$

NPV=-$20+ $33.66+$1.98 = $15.64

Rate Making Case

- **Example**

Suppose shareholder equity invested in a utility is $100 million, and the equity beta is 0.6. If the T-bill rate is 6%, and the market risk premium is 8%, What is the fair annual profit?

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

$E(R_i)$= 0.06+0.60*(0.08)= 0.108

Fair annual profit= $E(R_i)$ * equity investment

= 0.108*$100mn =$10.8mn

2. Security Selection

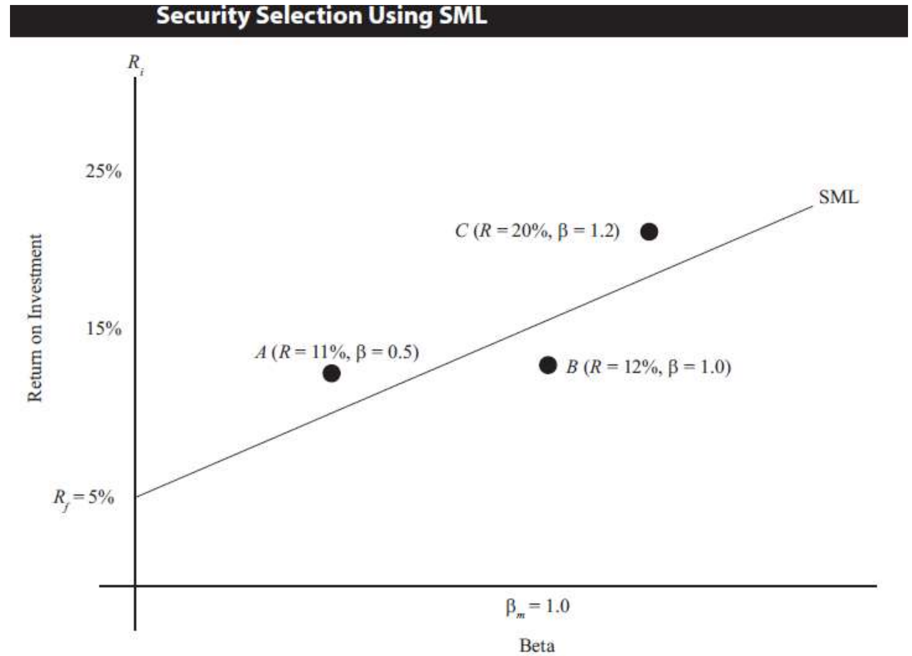
- The CAPM is an effective tool for determining whether an asset is undervalued or overvalued and whether an investor should buy or sell the asset.
- Potential investors can plot a security's expected return and beta against the SML and use this relationship to decide whether the security is overvalued or undervalued in the market.

2. Security Selection

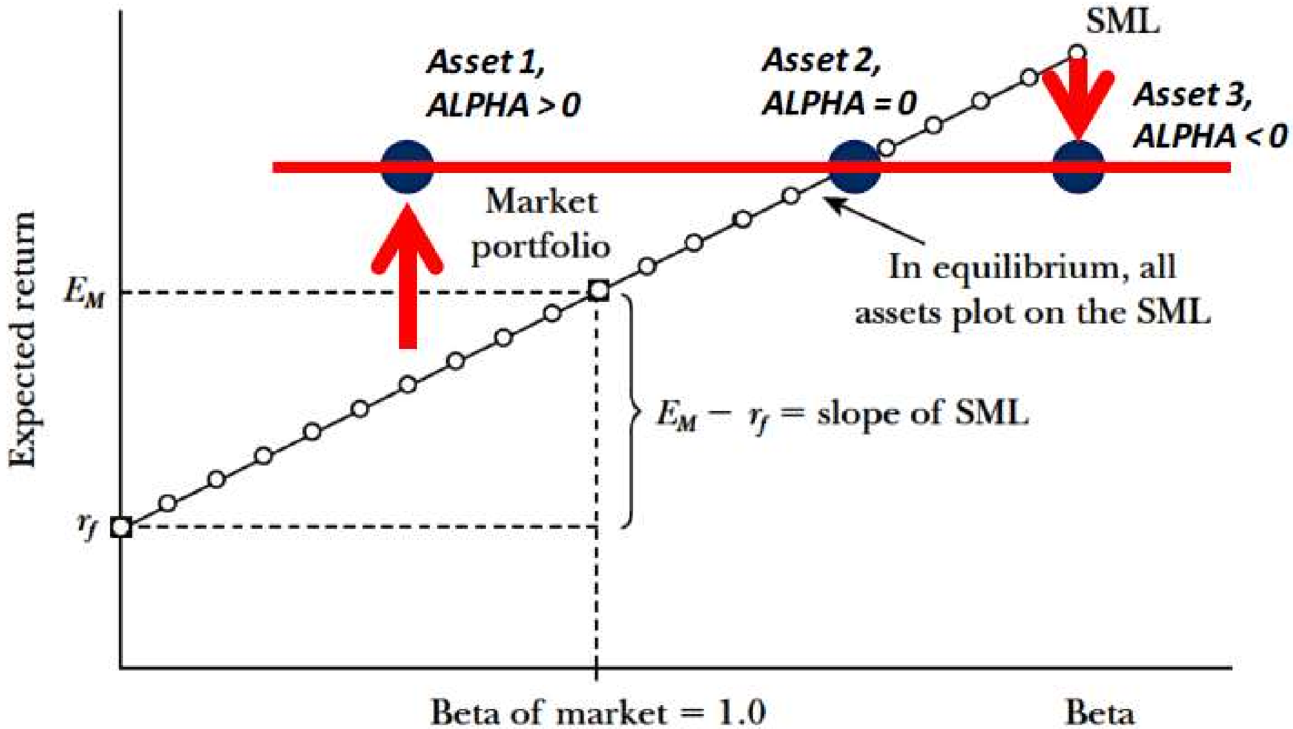
$$\alpha_i = R_i - [R_f + \beta_i (R_m - R_f)]$$

- All securities that re market view are points directly on the SML (i.e., properly valued).

- Alpha: The abnormal rate of return on a security in excess of what would be predicted by an equilibrium model such as the CAPM.



CAPM & Alpha



SML
Asset 1,
ALPHA > 0
Asset 2,
ALPHA = 0
Asset 3,
ALPHA < 0
Market
portfolio
In equilibrium, all
assets plot on the SML
$E_M - r_f$ = slope of SML
Expected return
E_M
r_f
Beta of market = 1.0
Beta

Example

A. Stock XYZ has an actual return of 12% and beta = 1. Stock ABC has actual return of 13% with a beta of 1.5. The market's expected return is 11% and $r_f = 5\%$. According to the CAPM, which stock is a better buy? What is the alpha of each stock? Plot the SML and the two stocks. Show the alphas of each on the graph.

Risk Free rate of	5%
$E(R_m)$	11%

Stock	Actual Return	Expected Return	Alpha
XYZ	12%	11%	1%
ABC	13%	14%	-1%

Stock	Beta	Expected Return	
XYZ	1	11%	=+Risk_Free_rate_of_Return+beta_XYZ*(E_Rm-Risk_Free_rate_of_Return)
ABC	1.5	14%	=+Risk_Free_rate_of_Return+Beta_ABC*(E_Rm-Risk_Free_rate_of_Return)

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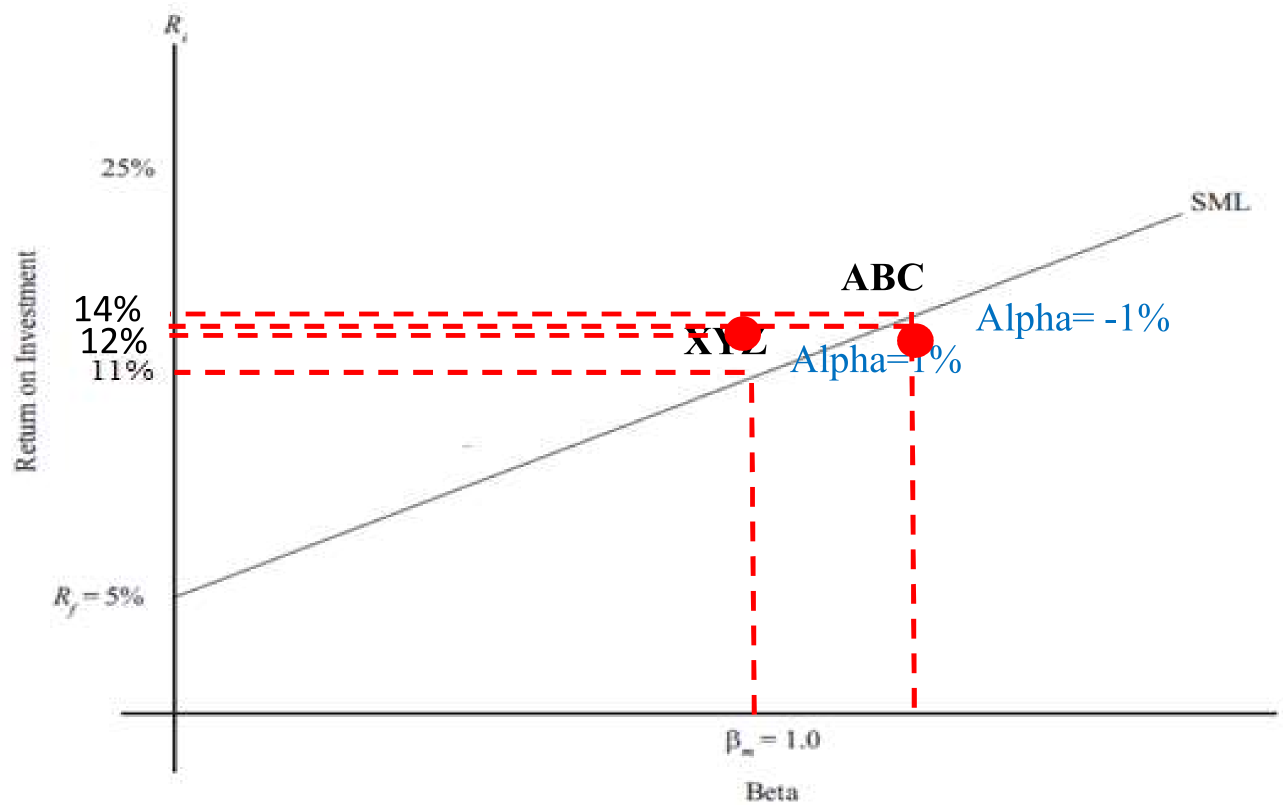

R_i
25%
SML
ABC
14%
12%
11%
XYZ
Alpha= -1%
Alpha=1%
Return on Investment
$R_f = 5\%$
$\beta_m = 1.0$
Beta

Cont'd

- *B*. The risk-free rate is 8% and the expected return on the market portfolio is 16%. A firm considers a project with an estimated beta of 1.3. What is the required rate of return on the project? If the IRR of the project is 19%, what is the project alpha?

$E(R_i)$= 0.08+1.3*(0.16-0.08)= 0.184

Alpha=IRR-$E(R_i)$=0.19-0.184= +0.006

Multifactor Models

- A multi-factor model is a financial model that employs **multiple factors** in
- its calculations to explain asset prices.
  - These models introduce **uncertainty** stemming from multiple sources.
  - CAPM, on the other hand, limits risk to one source – **covariance** with the market portfolio.
- Multifactor models can be used to calculate the **required rate** of return for portfolios as well as individual stocks.
  - CAPM uses just one factor to determine the required return – the **market factor**.
- The market factor can be split up even further into different **macroeconomic** factors.
  - These may include inflation, interest rates, business cycle uncertainty, etc.

Multifactor Models

- A factor can be defined as a **variable** which explains the expected return of an asset.
- A factor beta is a measure of the **sensitivity** of a given asset to a specific factor.
- The bigger the factor, the more sensitive the asset is to that factor. A multifactor appears as follows:

$$R_i = E(R_i) + \beta_{i1} F_1 + \beta_{i2} F_2 + \cdots + \beta_{ik} F_k + ei$$

- Where
  - R_i = Rate of return on stock i
  - $E(R_i)$ = Expected return on stock I
  - β_{ik} = Sensitivity of the stock's return to a one unit change in factor k
  - F_k = Macroeconomic factor k
  - e_I = the firm specific portion of the stock's return unexplained by macro factors

The Arbitrage Pricing Theory

- The APT theory describes expected returns as a linear function of exposures to common macroeconomic risk factors.

  - $E(R_i) = R_F + \beta_{i1} RP_1 + \beta_{i2} RP_2 + \beta_{i3} RP_3 + \cdots + \beta_{iK} RP_K$

- *Where* RP_j represents the risk premium attached to risk factor j.

Assumptions of the APT model

- ✓Well-diversified portfolios can be formed;
- ✓No arbitrage opportunities exist; and
- ✓Returns follow a k-factor process.

➢Note: Both CAPM and APT describe equilibrium expected returns for assets. CAPM can be considered a special case of the APT in which there is **only one risk factor – the market factor**.

Example

- The following data exists for asset A:
  - Risk-free rate = 3%
  - GDP factor beta = 0.40
  - Consumer sentiment factor beta = 0.20
  - GDP risk premium = 2%
  - Consumer sentiment risk premium = 1%
- Calculate the expected return for Asset A using a 2-factor APT model.

Solution

- $E(R_A) = 0.03 + 0.4 * 0.02 + 0.2 * 0.01 = 0.04 = 4\%$

The Fama-French Three-Factor Model

- A major weakness of the APT model is that it's silent on the issue of the appropriate risk factors for use.
- The FF three-factor model puts three factors forward :

1. **Size of firms**
   - The firm size factor, also known as **SMB** (small minus big) is equal to the difference in returns between portfolios of small and big firms $(R_s - R_b)$.
2. **Book-to-market values**
   - The book-to-market value factor, also known as **HML** (high minus low) is equal to the difference in returns between portfolios of high and low book-to-market firms $(R_H - R_L)$.
   - *Note: book-to-market value is book value per share divided by the stock price.*
3. **Excess return on the market**

The Fama-French Three-Factor Model

- **Why SMB and HML:**
  - Fama and French put forth the argument that returns are higher on small versus big firms as well as on high versus low book-to-market firms.
    - This argument has indeed been validated through historical analysis.
    - Fama and French contend that small firms are inherently riskier than big firms.
    - High book-to-market firms are undervalued compared to low book-to-market firms.
  - The equation for the Fama-French three-factor model is:

$$E(r_G) = r_f + \beta_M[E(r_M) - r_f] + \beta_{SMB} E(r_{SMB}) + \beta_{HML} E(r_{HML})$$