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# **Chapter 8**

## **Mode Choice**

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**TRANSPORTATION PLANNING**

# Contents

- *Basic Concepts in Mode Choice*

- *Types of Mode Choice Models*

- *Logit Model*

# BASIC CONCEPTS IN MODE CHOICE

- **Mode Choice is aspect of demand analysis process that determines the number (or %) of trips between zones made by modes (auto, bus..)**
- **Selection of a mode is a complex process that depends on factors such as the income, availability of transit/auto ownership, and the relative advantages of modes (tt, cost, safety,..)**
- **Models attempt to replicate the relevant characteristics of the traveler, the trans. system, and the trip, so that a realistic estimate of the no. of trips by each mode for each zonal pair is obtained**

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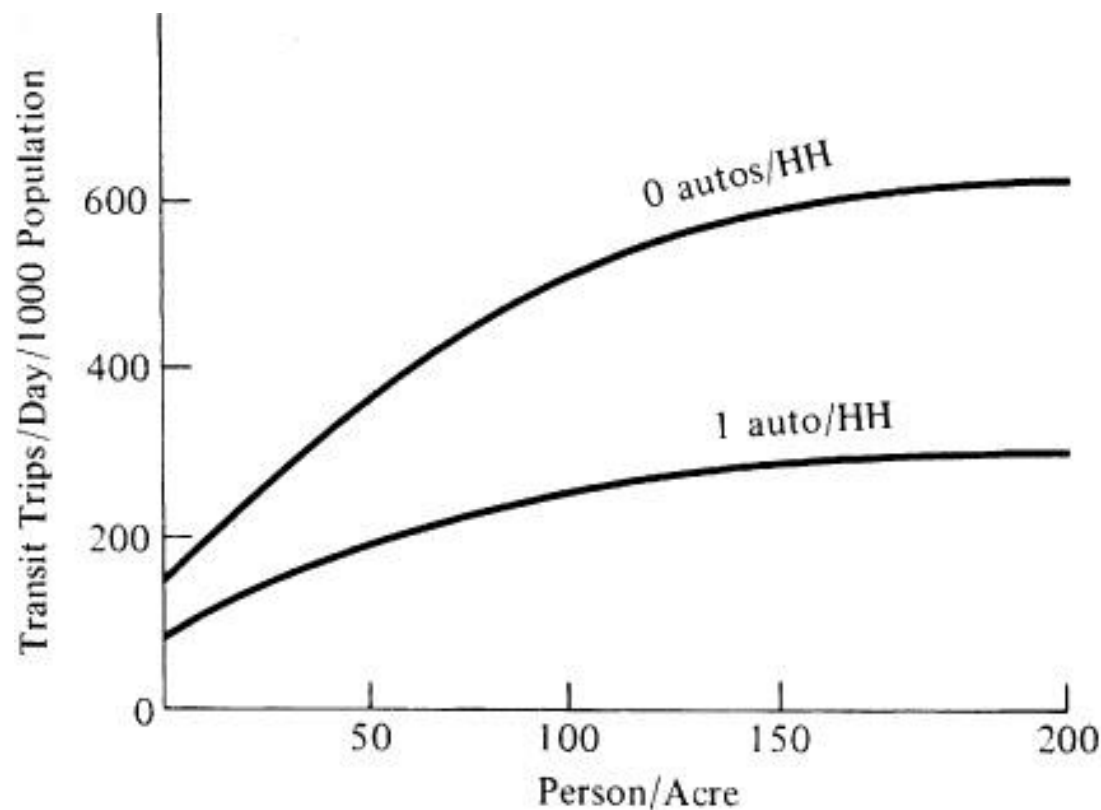
# TYPES OF MODE CHOICE MODELS

- Mode choice calculations typically involve distinguishing trip interchanges as either auto or transit.
- Depending on the level of detail required, three types of transit estimating procedures are used:
  - (1) direct generation of transit trips,
  - (2) use of trip end models, and
  - (3) trip interchange modal split models.
- Disaggregate (Logit) mode choice models

# MODE CHOICE MODELS

## Direct Generation Models

- Transit trips can be generated directly, by estimating either total person trips or auto driver trips.
- Figure 12.8 illustrates the relationship between transit trips per day per 1000 population and persons per acre versus auto ownership.
- As density of population increases, it can be expected that transit riding will also increase for a given level of auto ownership.



**Figure 12.8** Number of Transit Trips by Population Density and Automobile Ownership per Household

### Example 12.7 Estimating Mode Choice by Direct Trip Generation

Determine the number of transit trips per day in a zone which has 5000 people living on 50 acres. The auto ownership is 40% of zero autos per household and 60% of one auto per household.

**Solution:** Calculate the number of persons per acre:  $5000 / 50 = 100$ . Then determine the number of transit trips per day per 1000 persons (from Figure 12.8) to calculate the total of all transit trips per day for the zone.

Zero autos / HH: 510 trips / day / 1000 population

One auto / HH: 250 trips / day / 1000 population

$$\begin{aligned}\text{Total Transit Trips: } & (0.40)(510)(5) + (0.60)(250)(5) = \\ & 1020 + 750 = 1770 \text{ transit trips per day}\end{aligned}$$

# MODE CHOICE MODELS

## Direct Generation Models

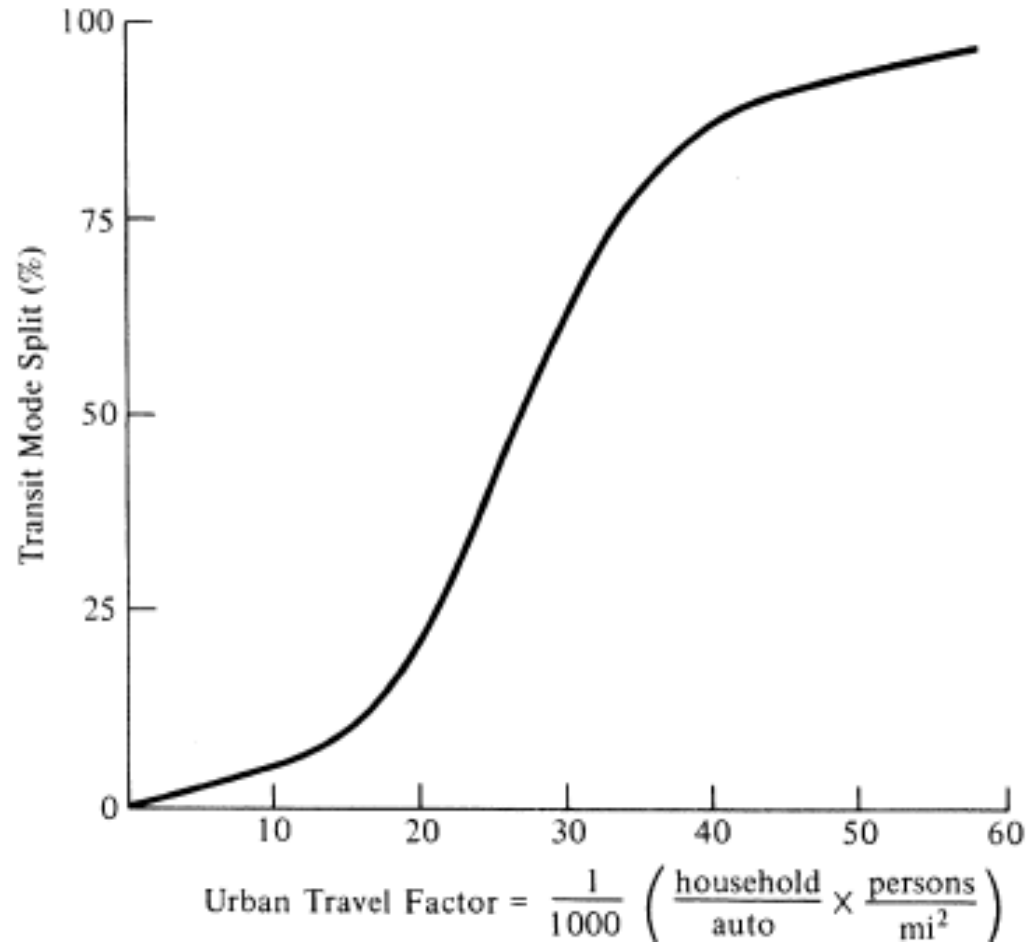
- This method assumes that the attributes of the system are not relevant.
- Factors such as travel time, cost, and convenience are not considered.
- These so-called “pre-trip distribution” models apply when transit service is poor and riders are “captive,” or when transit service is excellent and “choice” clearly favors transit.
- When highway and transit modes “compete” for auto riders, then system factors are considered.



# MODE CHOICE MODELS

## Trip End Models

- To determine the no. or % of persons trips that will use transit, estimates are made prior to the trip distribution phase based on land-use or socioeconomic characteristics of the zone.
- This method does not include quality of service.
- The model of Figure 12.9, is based on factors: households/auto and persons/mile<sup>2</sup>.
- The product of these factors is the urban travel factor (UTF).
- %of transit trips increase as UTF increases



**Figure 12.9** Transit Mode Split versus Urban Travel Factor

# MODE CHOICE MODELS

## Trip End Models

- The procedure is as follows:
  1. Generate total person trip productions and attractions by trip purpose.
  2. Compute the urban travel factor.
  3. Determine the percentage of these trips by transit using a mode choice curve.
  4. Apply auto occupancy factors.
  5. Distribute transit and auto trips separately.

### Example 12.8 Estimating Trip Productions by Transit

The total number of productions in a zone is 10,000 trips/day. The number of households per auto is 1.80, and residential density is 15,000 persons/square mile. Determine the percent of residents who can be expected to use transit.

Solution: Compute the urban travel factor.

$$\begin{aligned}\text{UTF} &= \frac{1}{1000} \left( \frac{\text{household}}{\text{auto}} \right) \left( \frac{\text{persons}}{\text{mi}^2} \right) \\ &= \frac{1}{1000} \times 1.80 \times 15,000 = 27.0\end{aligned}$$

Enter Figure 12.9. Transit mode split = 45%.

# MODE CHOICE MODELS

## Trip Interchange Models

- In this method, system level-of-service variables are considered, including relative travel time, relative travel cost, economic status of the trip maker, and relative travel service.
- An example of this procedure is illustrated using the QRS method which takes account of service parameters in estimating mode choice.
- The QRS method is based on the following relationship:

# MODE CHOICE MODELS

## Trip Interchange Models

$$MS_a = \frac{I_{ijt}^{-b}}{I_{ija}^{-b} + I_{ijt}^{-b}} \times 100 \text{ or } \frac{I_{ija}^b}{I_{ijt}^b + I_{ija}^b} \times 100 \quad (12.6)$$

$$MS_t = (1 - MS_a) \times 100 \quad (12.7)$$

where

$MS_t$  = proportion of trips between zone  $i$  and  $j$  using transit

$MS_a$  = proportion of trips between zone  $i$  and  $j$  using auto

$I_{ijm}$  = a value referred to as the *impedance* of travel of mode  $m$ , between  $i$  and  $j$ , which is a measure of the total cost of the trip. [*Impedance* = (in-vehicle time min) + (2.5 × excess time min) + (3 × trip cost, \$/ income earned/min).]

$b$  = an exponent, which depends on trip purpose

$m = t$  for transit mode;  $a$  for auto mode

# MODE CHOICE MODELS

## Trip Interchange Models

- **In-vehicle time is time spent traveling in vehicle**
- **Excess time is time spent but not in vehicle (inc. waiting for bus and walking to station)**
- **The impedance value is determined for each zone pair and represents a measure of the expenditure required to make the trip by either auto or transit.**
- **Data required for estimating mode choice:**

# MODE CHOICE MODELS

## Trip Interchange Models

- (1) distance between zones by auto and transit,
- (2) transit fare,
- (3) out-of-pocket auto cost,
- (4) parking cost,
- (5) highway and transit speed,
- (6) exponent values,  $b$ ,
- (7) median income, and
- (8) excess time (inc. time to walk to a transit vehicle and time waiting or transferring).



### Example 12.9 Computing Mode Choice Using the QRS Model

To illustrate the application of the QRS method, assume that the data shown in Table 12.21 have been developed for travel between a suburban zone *S* and a downtown zone *D*. Determine the percent of work trips by auto and transit. An exponent value of 2.0 is used for work travel. Median income is \$24,000 per year.

**Table 12.21** Travel Data Between Two Zones, *S* and *D*

|               | <i>Auto</i>           | <i>Transit</i> |
|---------------|-----------------------|----------------|
| Distance      | 10 mi                 | 8 mi           |
| Cost per mile | \$0.15                | \$0.10         |
| Excess time   | 5 min                 | 8 min          |
| Parking cost  | \$1.50 (or 0.75/trip) | —              |
| Speed         | 30 mi/h               | 20 mi/h        |

Solution: Use Eq. 12.6.

$$MS_a = \frac{I_{ija}^b}{I_{ijt}^b + I_{ija}^b}$$

$$\begin{aligned} I_{SDa} &= \left( \frac{10}{30} \times 60 \right) + (2.5 \times 5) + \left\{ \frac{3 \times [(1.50/2) + 0.15 \times 10]}{24,000/120,000} \right\} \\ &= 20 + 12.5 + 33.75 \\ &= 66.25 \text{ equivalent min} \end{aligned}$$

$$\begin{aligned} I_{SDt} &= \left( \frac{8}{20} \times 60 \right) + (2.5 \times 8) + \left[ \frac{3 \times (8 \times 0.10)}{24,000/120,000} \right] = 24 + 20 + 12 \\ &= 56 \text{ equivalent min} \end{aligned}$$

$$MS_a = \frac{(56)^2}{(56)^2 + (66.25)^2} \times 100 = 41.6\%$$

$$MS_t = (1 - 0.416) \times 100 = 58.4\%$$

Thus, the mode choice of travel by transit between zones  $S$  and  $D$  is 68.4%, and by highway the value is 41.6%. These percentages are applied to the estimated trip distribution values to determine the number of trips by each mode. If for example, the number of work trips between zones  $S$  and  $D$  was computed to be 500, then the number by auto would be  $500 \times 0.416 = 208$ , and by transit, the number of trips would be  $500 \times 0.584 = 292$ .

# MODE CHOICE MODELS

## Logit Model

- An alternative approach used in transportation demand analysis is to consider the relative utility of each mode as a summation of each modal attribute.
- Then the choice of a mode is expressed as a probability distribution.
- For example, assume that the utility of each mode is:

# MODE CHOICE MODELS

## Logit Model

$$U_x = \sum_{i=1}^n a_i X_i \quad (12.8)$$

where

$U_x$  = utility of mode  $x$

$n$  = number of attributes

$X_i$  = attribute value (time, cost, and so forth)

$a_i$  = coefficient value for attributes  $i$  (negative, since the values are disutilities)

- If two modes, auto (A) and transit (T), are being considered, the probability of selecting the auto mode A can be written as

$$P(A) = \frac{e^{U_A}}{e^{U_A} + e^{U_T}} \quad (12.9)$$

# MODE CHOICE MODELS

## Logit Model

- This form is called the logit model
- With the definition of utility, the probability that a traveler will choose some alternative, say  $x$ , is equal to the probability that the given alternative's utility is greater than the utility of all other possible alternatives.

# MODE CHOICE MODELS

## Logit Model

- This implies that with:
  - the basic probability, and
  - the utility expression, then
- A probabilistic choice model can be derived
- The coefficients in the utility function ( $a_i$ 's) can be estimated with data collected from traveler surveys, along the same lines as was done for the coefficients in the trip generation models.

# MODE CHOICE MODELS

## Logit Model

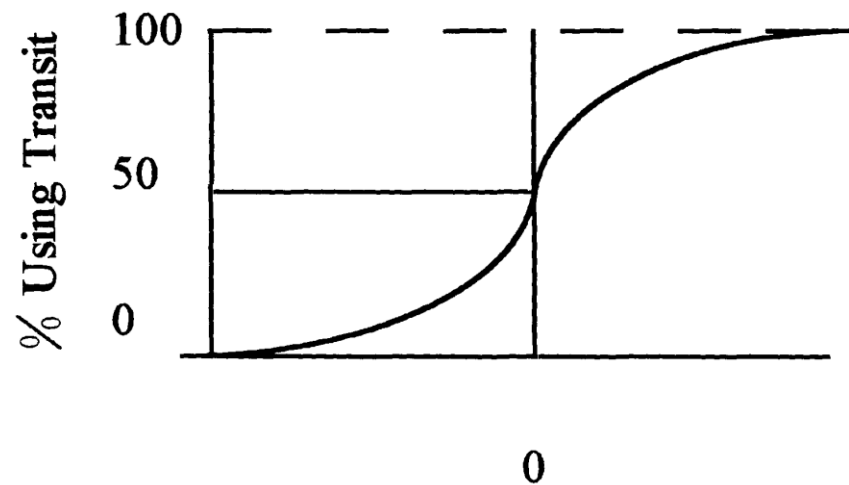
- The coefficients that comprise the specifiable portion of utility ( $a_i$ 's in Eq. 12.8) are estimated by the method of maximum likelihood (see Appendix B in Ch. 6).



# MODE CHOICE MODELS

## Logit Model

- The logit model as illustrated in Figure 12.10 and provides a convenient way to compute mode choice.



Utility Differences between Modes

# MODE CHOICE MODELS

## Logit Model

- Choice models are utilized within the urban transportation planning process, for discrete choice modeling and to directly estimate travel demand.
- This can be applied to:
  - Mode Choice
  - Destination Choice
  - Combined Mode/Destination Choice
  - Route Choice

### Example 12.10 Use of Logit Model to Compute Mode Choice

The utility functions for auto and transit are as follows.

$$\text{Auto: } U_A = -0.46 - 0.35T_1 - 0.08T_2 - 0.005C$$

$$\text{Transit: } U_T = -0.07 - 0.05T_1 - 0.15T_2 - 0.005C$$

where

$T_1$  = total travel time (minutes)

$T_2$  = waiting time (minutes)

$C$  = cost (cents)

The travel characteristics between two zones are as follows:

|       | <i>Auto</i> | <i>Transit</i> |
|-------|-------------|----------------|
| $T_1$ | 20          | 30             |
| $T_2$ | 8           | 6              |
| $C$   | 320         | 100            |

**Solution:** Use the logit model to determine the percent of travel in the zone by auto and transit.

$$U_x = \sum_{i=1}^n a_i x_i$$

$$U_A = -0.46 - (0.35 \times 20) - (0.08 \times 8) - (0.005 \times 320) = -9.70$$

$$U_B = -0.07 - (0.35 \times 30) - (0.08 \times 6) - (0.005 \times 100) = -11.55$$

Using Eq.12.9 yields

$$P_A = \frac{e^{U_A}}{e^{U_A} + e^{U_T}} = \frac{e^{-9.70}}{e^{-9.7} + e^{-11.55}} = 0.86$$

$$P_T = \frac{e^{U_T}}{e^{U_A} + e^{U_T}} = \frac{e^{-11.55}}{e^{-9.7} + e^{-11.55}} = 0.14$$

# Example

A simple work-mode–choice model is estimated from data in a small urban area to determine the probabilities of individual travelers selecting various modes. The mode choices include automobile drive-alone ( $DL$ ), automobile shared-ride ( $SR$ ), and bus ( $B$ ), and the utility functions are estimated as

$$U_{DL} = 2.2 - 0.2(\text{cost}_{DL}) - 0.03(\text{travel time}_{DL})$$

$$U_{SR} = 0.8 - 0.2(\text{cost}_{SR}) - 0.03(\text{travel time}_{SR})$$

$$U_B = -0.2(\text{cost}_B) - 0.01(\text{travel time}_B)$$

where cost is in dollars and time is in minutes. Between a residential area and an industrial complex, 4000 workers

## Example..

(generating vehicle-based trips) depart for work during the peak hour.

For all workers, the cost of driving an automobile is \$6.00 with a travel time of 20 minutes, and the bus fare is \$1.00 with a travel time of 25 minutes. If the shared-ride option always consists of two travelers sharing costs equally, how many workers will take each mode?

# Solution

Substitution of cost and travel time values into the utility expressions gives

$$U_{DL} = 2.2 - 0.2(6) - 0.03(20) = 0.4$$

$$U_{SR} = 0.8 - 0.2(3) - 0.03(20) = -0.4$$

$$U_B = -0.2(1.0) - 0.01(25) = -0.45$$

Substituting these values into the prob. eqn. yields:

## Solution..

$$P_{DL} = \frac{e^{0.4}}{e^{0.4} + e^{-0.4} + e^{-0.45}} = \frac{1.492}{1.492 + 0.670 + 0.638} = \frac{1.492}{2.80} = 0.533$$

$$P_{SR} = \frac{0.670}{2.80} = 0.239$$

$$P_B = \frac{0.638}{2.80} = 0.228$$

Multiplying these probabilities by 4000 (the total number of workers departing in the peak hour) gives 2132 workers driving alone, 956 sharing a ride, and 912 using a bus.



# MODE CHOICE MODELS

## Logit Model

### Borrowing Utility Functions from Other Sources

- If a utility function such as that shown in Eq. 12.9 is not available (not derived from survey data), then the coefficients for the function may be borrowed from another source
- To the extent that the selection of a mode is governed by its in-vehicle travel time, out-of-vehicle travel time, and cost, a utility function may be written as:

# MODE CHOICE MODELS

## Logit Model

### Borrowing Utility Functions from Other Sources

$$\text{Utility}_i = b (\text{IVTT}) + c (\text{OVTT}) + d (\text{COST}) \quad (12.10)$$

where

$\text{Utility}_i$  = utility function for mode  $i$

IVTT = in-vehicle travel time (min)

OVTT = out-of-vehicle travel time (min)

COST = out-of-pocket cost (cents)

- The following approach for calibrating the coefficients  $b$ ,  $c$ , and  $d$  in Eq. 12.10 are
- based on methods published by TRB:

# MODE CHOICE MODELS

## Logit Model

### Borrowing Utility Functions from Other Sources

- In-vehicle travel time (IVTT) coef. (b)=0.025
- Out-of-vehicle travel time coef. (c)= 0.050  
(reflecting the observation that time waiting for a vehicle is perceived to be twice as great as time spent inside a moving vehicle)
- Cost coefficient d is computed as follows:

$$d = \frac{(b)(1248)}{(TVP)(AI)}$$

where

$TVP$  = the ratio of (value of one hour travel time)/(hourly employment rate).

In the absence of other data  $TVP = 0.30$

$AI$  = the average annual regional household income, (\$)

1248 is the factor that converts \$/yr to cents/min.

### Example 12.11 Borrowing Utility Coefficients from Other Sources

A transit authority wishes to determine the number of total travelers in a corridor that will shift from auto to a proposed new bus line. Since local data are unavailable, use of borrowed utility values is the only option. It is believed that the key factors in the decision to use transit will be time and cost. Average annual household income (AI) is \$60,000,  $TVP = 0.30$ , and waiting time is perceived to be twice as long as riding time. System times and cost values are as follows.

| <i>Variable</i> | <i>Bus</i> | <i>Auto</i> |
|-----------------|------------|-------------|
| IVIT (min)      | 30         | 20          |
| OVIT(min)       | 6          | 8           |
| Cost (cents)    | 100        | 320         |

Determine the proportion of persons who will use the new bus line.

**Solution:** Determine coefficients  $b$ ,  $c$ , and  $d$  based on these data.

$$b = -0.025$$

$$c = -0.050$$

$$d = \frac{(b)(1248)}{(TVP)(AI)} = \frac{(-0.025)(1248)}{(0.30)(\$60,000)} = -0.00173$$

$a_i = 0$  since the problem stated IVTT, OVTT, and COST sufficiently explain mode choice

The utility functions are:

$$\begin{aligned} U_{\text{auto}} &= b (\text{IVTT}) + c (\text{OVTT}) + d (\text{COST}) \\ &= -0.025(20) + -0.050(8) + -0.00173(320) = -1.454 \end{aligned}$$

$$\begin{aligned} U_{\text{bus}} &= b (\text{IVTT}) + c (\text{OVTT}) + d (\text{COST}) \\ &= -0.025(30) + -0.050(6) + -0.00173(100) = -1.223 \end{aligned}$$

The proportion of travelers using the bus is computed using Eq. 12.9.

$$P_{\text{bus}} = \frac{e^{U_{\text{bus}}}}{e^{U_{\text{bus}}} + e^{U_{\text{auto}}}} = \frac{e^{-1.223}}{e^{-1.223} + e^{-1.454}} = 0.557$$

Thus, this model predicts that 56% of travelers will use the new bus line.

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### Example 12.12 Adding a Mode-Specific Constant to the Utility Function

Referring to Example 12.11, upon inaugurating bus service, the percentage of travelers that use the new bus service is actually 65%. Follow-up surveys confirm that the coefficients  $b$ ,  $c$ , and  $d$  which were used to estimate potential bus service appear to have been correct. However, the surveys suggest that a further incentive (beyond time and cost) for using the bus is influenced by the availability of laptop outlets at each seat and a complimentary beverage service.

Given this added information, explain how to modify the utility function to reflect the influence of added amenities.

**Solution:** Because the coefficients  $b$ ,  $c$ , and  $d$  do not include the additional features that favor bus usage, a mode specific coefficient ( $a_i$ ) should be included in one of the utility functions. This term may either be a positive coefficient that is added to the bus utility function or a negative coefficient that is subtracted from the auto utility function. Using the former approach, simply add a constant value (which in this example is 0.3885) to the bus utility function order to yield the required 65% of travelers using the bus. The result is shown in the following calculation.

$$P_{\text{bus}} = \frac{e^{(U_{\text{bus}} + 0.3885)}}{e^{(U_{\text{bus}} + 0.3885)} + e^{U_{\text{auto}}}} = \frac{e^{(-1.223 + 0.3885)}}{e^{(-1.223 + 0.3885)} + e^{-1.454}} = 0.650$$

Thus, the bus utility function is rewritten and the auto utility function is unchanged, as follows.

$$\begin{aligned} U_{\text{bus}} &= a_{\text{bus}} + b (\text{IVTT}) + c (\text{OVTT}) + d (\text{COST}) \\ U_{\text{bus}} &= 0.3855 + -0.025(\text{IVTT}) + -0.050(\text{OVTT}) + -0.00173(\text{COST}) \end{aligned}$$



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# MODE CHOICE MODELS

## Logit Model - Calibrating Utility Functions with Survey Data

- A second approach to determine utility function coefficients is to calibrate the coefficients based on survey data using the method of maximum likelihood estimation
- Software packages such as SAS, ALOGIT, TransCAD are available that support maximum likelihood estimation and replace manual procedures presented here.

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# MODE CHOICE MODELS

## Logit Model - Calibrating Utility Functions with Survey Data

- To illustrate this process, a simple calibration of a utility function using survey data is shown
- in Example 12.14.
- For discussion of more complex cases, refer to Appendix B in Ch. 6.

### Example 12.14 Calibrating Utility Functions

A regional transportation agency wishes to calibrate a utility function that can be used with the logit model to predict modal choice between bus, auto, and rail. Survey data were obtained by interviewing seven people identified as persons A through G who reported the travel time for three modes they considered (car, bus, and rail) and the mode that they used. The results of the survey are shown in the following table. The agency has proposed to select a utility function of the form  $U = b$  (time).

Use the method of maximum likelihood estimation to calibrate this utility function for the parameter,  $b$ .

### Sample Interview Survey Data:

| <i>Respondent</i> | <i>Auto Time<br/>(min)</i> | <i>Bus Time<br/>(min)</i> | <i>Rail Time<br/>(min)</i> | <i>Mode Used</i> |
|-------------------|----------------------------|---------------------------|----------------------------|------------------|
| A                 | 10                         | 13                        | 15                         | Auto             |
| B                 | 12                         | 9                         | 8                          | Auto             |
| C                 | 35                         | 32                        | 20                         | Rail             |
| D                 | 45                         | 15                        | 44                         | Bus              |
| E                 | 60                         | 58                        | 64                         | Bus              |
| F                 | 70                         | 65                        | 60                         | Auto             |
| G                 | 25                         | 20                        | 15                         | Rail             |

**Solution:** The utility function is

$$U = b \text{ (IVTT)}$$

where

$b$  = a constant to be determined from the calibration process

IVTT = in-vehicle travel time (in minutes)

A maximum likelihood function may be used to derive model coefficients that replicate the observed data. For these data, a “perfect” function would predict that respondents A, B, and F would select auto; C and G would select rail; and D and E would select bus. For respondent A, the utility function is as shown, since A selected auto and not the bus or rail. Thus,

$$L_A = (P_{A-\text{auto}})$$

The probability that A will select a mode is computed using Eqs. 12.8 and 12.9. For example, the probability that respondent A will select auto, bus, and rail is

$$P_{A,\text{auto}} = \frac{e^{U_{1\text{auto}}}}{e^{U_{1\text{auto}}} + e^{U_{1\text{bus}}} + e^{U_{1\text{rail}}}} = \frac{e^{b_{10}}}{e^{b_{10}} + e^{b_{13}} + e^{b_{15}}}$$

$$P_{A,\text{bus}} = \frac{e^{U_{1\text{bus}}}}{e^{U_{1\text{auto}}} + e^{U_{1\text{bus}}} + e^{U_{1\text{rail}}}} = \frac{e^{b_{13}}}{e^{b_{10}} + e^{b_{13}} + e^{b_{15}}}$$

$$P_{A,\text{rail}} = \frac{e^{U_{1\text{rail}}}}{e^{U_{1\text{auto}}} + e^{U_{1\text{bus}}} + e^{U_{1\text{rail}}}} = \frac{e^{b_{15}}}{e^{b_{10}} + e^{b_{13}} + e^{b_{15}}}$$

Substitution of the appropriate equation into the expression for  $L_A$  yields the maximum likelihood function for respondent A.

$$L_A = \left( \frac{e^{b_{10}}}{e^{b_{10}} + e^{b_{13}} + e^{b_{15}}} \right)$$

For the entire data set, therefore, the maximum likelihood function may be computed as

$$L = (L_A)(L_B)(L_C)(L_D)(L_E)(L_F)(L_G)$$

Since  $b$  cannot be determined such that  $L$  is exactly equal to 1.0, the best possible result is to select a value of  $b$  such that  $L$  is as close to 1.0 as possible. Theoretically,  $L$  could be differentiated with respect to  $b$  and equated to zero. However, the nonlinear equations that result usually necessitate the use of specialized software to solve. Plot  $L$  versus  $b$  is as shown in Figure 12.11. The value of  $b = (-0.1504)$  maximizes  $L$ . Thus, the utility expression based on the data collected about user behavior is

$$U = (-0.1504) (\text{IVTT})$$

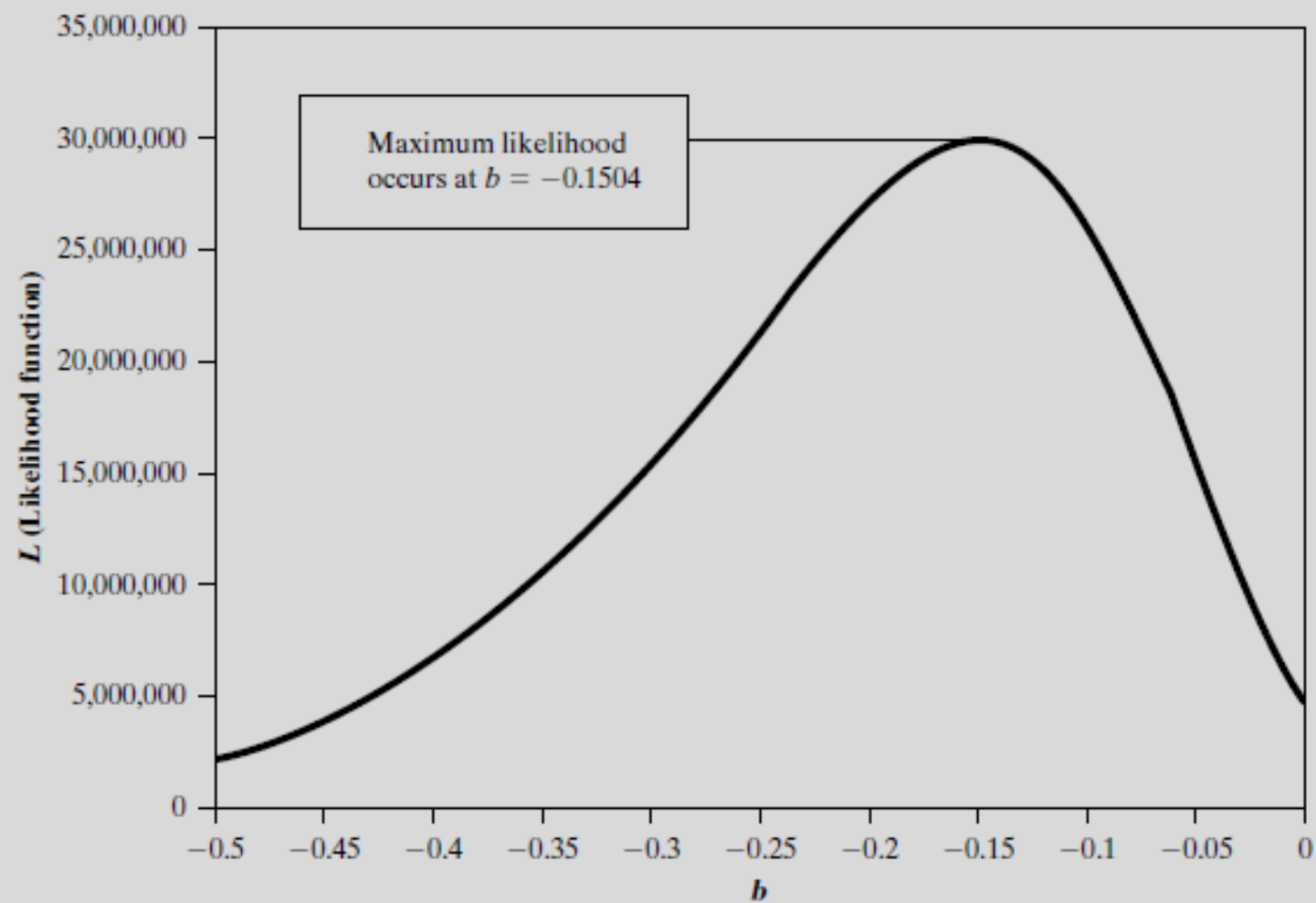


Figure 12.11 Plot of Maximum Likelihood Function versus  $b$



## **Example on Combined Mode/Destination Choice**

Consider a residential area and two shopping centers that are possible destinations. From 7:00 to 8:00 P.M. on Friday night, 900 vehicle-based shopping trips leave the residential area for the two shopping centers. A joint shopping-trip mode-destination choice logit model (choice of either auto or bus) is estimated, giving the following coefficients:

| Variable   | Auto<br>coefficient | Bus<br>coefficient |
|--|---------------------|--------------------|
| Auto constant  | 0.6                 | 0.0                |
| Travel time in minutes                                       | −0.3                | −0.3               |
| Commercial floor space<br>(in thousands of ft <sup>2</sup> ) | 0.012               | 0.012              |

Initial travel times to shopping centers 1 and 2 are as follows:

|   | By auto | By bus |
|---|---------|--------|
| Travel time to shopping center 1 (in minutes) | 8       | 14     |
| Travel time to shopping center 2 (in minutes) | 15      | 22     |

If shopping center 2 has 400,000 ft<sup>2</sup> of commercial floor space and shopping center 1 has 250,000 ft<sup>2</sup>, determine the distribution of Friday night shopping trips by destination and mode.

## ***SOLUTION***

Let  $U_{A1}$  be the utility of the auto mode to shopping center 1,  $U_{A2}$  the utility of the auto mode to shopping center 2, and  $U_{B1}$  and  $U_{B2}$  the utility of the bus mode to shopping centers 1 and 2, respectively. The utilities are

$$U_{A1} = 0.6 - 0.3(8) + 0.012(250) = 1.2$$

$$U_{B1} = -0.3(14) + 0.012(250) = -1.2$$

$$U_{A2} = 0.6 - 0.3(15) + 0.012(400) = 0.9$$

$$U_{B2} = -0.3(22) + 0.012(400) = -1.8$$

Substituting these values into the prob. eqn. gives

$$P_{A1} = \frac{3.32}{6.246} = 0.532$$

$$P_{B1} = \frac{0.301}{6.246} = 0.048$$

$$P_{A2} = \frac{2.46}{6.246} = 0.394$$

$$P_{B2} = \frac{0.165}{6.246} = 0.026$$

Multiplying these probabilities by the 900 trips gives 479 trips by auto to shopping center 1, 43 trips by bus to shopping center 1, 355 trips by auto to shopping center 2, and 23 trips by bus to shopping center 2.