Efficient Diversification

6

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Essentials of Investments,

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6.1 Diversification and Portfolio Risk

- Market/Systematic/Nondiversifiable Risk
  - Risk factors common to whole economy
- Unique/Firm-Specific/Nonsystematic/ Diversifiable Risk
  - Risk that can be eliminated by diversification

Figure 6.1 Risk as Function of Number of Stocks in Portfolio

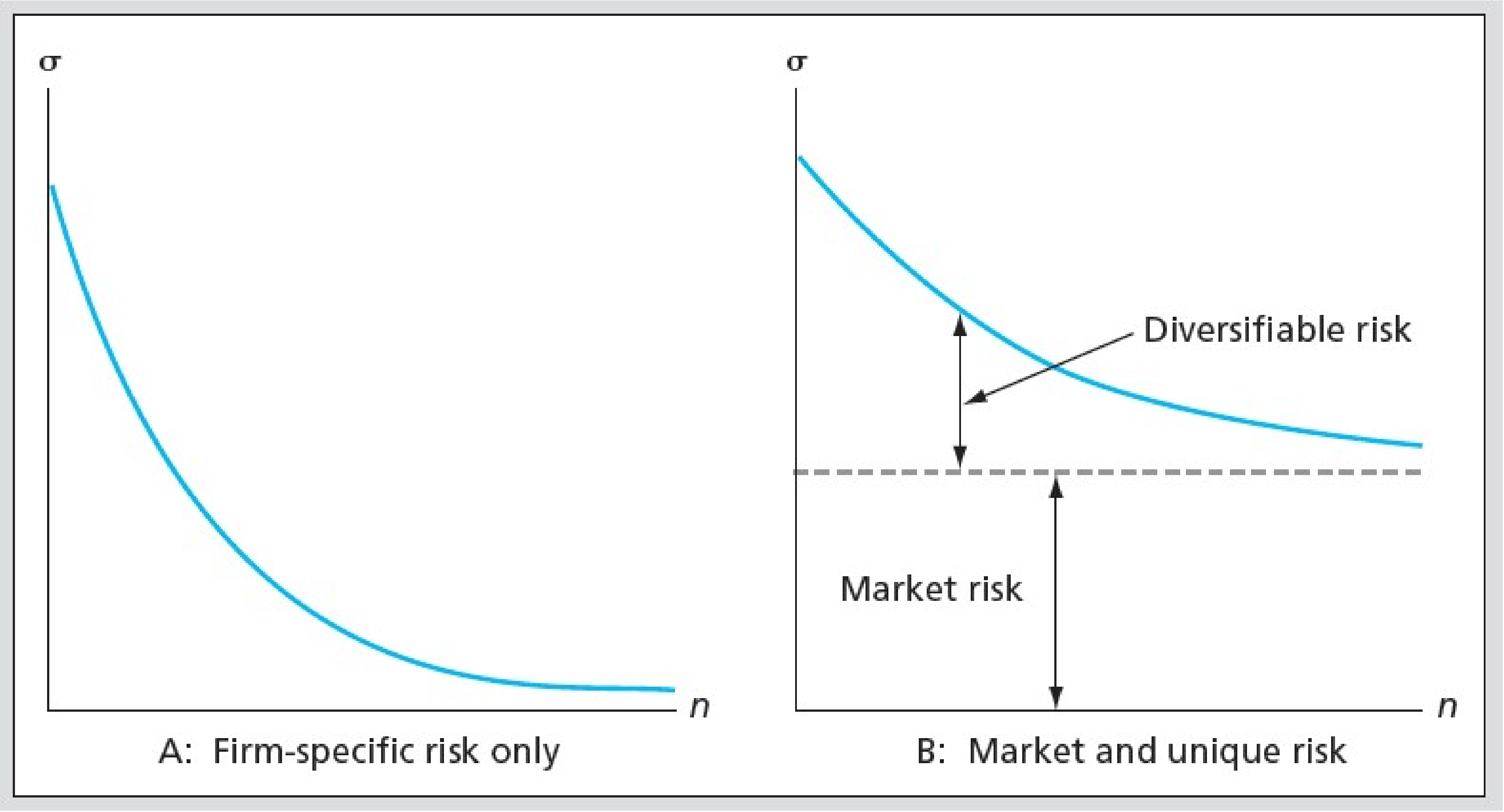
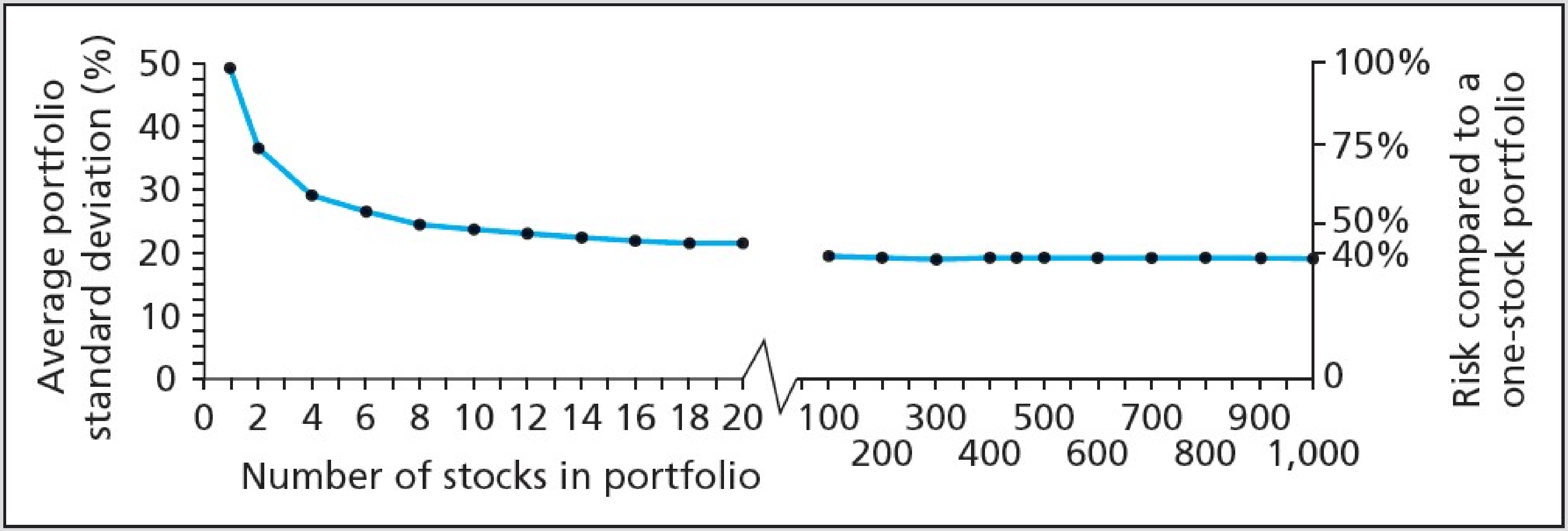


Figure 6.2 Risk versus Diversification



6.2 Asset Allocation with Two Risky Assets

- Covariance and Correlation
  - Portfolio risk depends on covariance between returns of assets
  - Expected return on two-security portfolio
    - $E(r_p) = W_1 r_1 + W_2 r_2$
    - $W_1 = \text{Proportion of funds in security 1}$
    - $W_2 = \text{Proportion of funds in security 2}$
    - $r_1 = \text{Expected return on security 1}$
    - $r_2 = \text{Expected return on security 2}$

6.2 Asset Allocation with Two Risky Assets

- Covariance Calculations

$$\text{Cov}(r_S, r_B) = \sum_{i=1}^{S} p(i)[r_S(i) - E(r_S)][r_B(i) - E(r_B)]$$

- Correlation Coefficient

$$\rho_{SB} = \frac{\text{Cov}(r_S, r_B)}{\sigma_S \times \sigma_B}$$

$$\text{Cov}(r_S, r_B) = \rho_{SB}\sigma_S\sigma_B$$

Spreadsheet 6.1 Capital Market Expectations

	A	B	C	D	E	F
1			Stock Fund		Bond Fund	
2	Scenario	Probability	Rate of Return	Col B x Col C	Rate of Return	Col B x Col E
3	Severe recession	.05	−37	−1.9	−9	−0.45
4	Mild recession	.25	−11	−2.8	15	3.8
5	Normal growth	.40	14	5.6	8	3.2
6	Boom	.30	30	9.0	−5	−1.5
7	Expected or Mean Return:		SUM:	10.0	SUM:	5.0

Spreadsheet 6.2 Variance of Returns

	A	B	C	D	E	F	G	H	I	J
1				Stock Fund			Bond Fund			
2				Deviation				Deviation		
3			Rate	from		Column B	Rate	from		Column B
4			of	Expected	Squared	×	of	Expected	Squared	×
5	Scenario	Prob.	Return	Return	Deviation	Column E	Return	Return	Deviation	Column I
6	Severe recession	.05	−37	−47	2209	110.45	−9	−14	196	9.80
7	Mild recession	.25	−11	−21	441	110.25	15	10	100	25.00
8	Normal growth	.40	14	4	16	6.40	8	3	9	3.60
9	Boom	.30	30	20	400	120.00	−5	−10	100	30.00
10				Variance = SUM		347.10			Variance:	68.40
11		Standard deviation = SQRT(Variance)				18.63			Std. Dev.:	8.27

Spreadsheet 6.3 Portfolio Performance

	A	B	C	D	E	F	G
1			Portfolio invested 40% in stock fund and 60% in bond fund				
2			Rate	Column B	Deviation from		Column B
3			of	×	Expected	Squared	×
4	Scenario	Probability	Return	Column C	Return	Deviation	Column F
5	Severe recession	.05	−20.2	−1.01	−27.2	739.84	36.99
6	Mild recession	.25	4.6	1.15	−2.4	5.76	1.44
7	Normal growth	.40	10.4	4.16	3.4	11.56	4.62
8	Boom	.30	9.0	2.70	2.0	4.00	1.20
9			Expected return:	7.00		Variance:	44.26
10						Standard deviation:	6.65

Spreadsheet 6.4 Return Covariance

	A	B	C	D	E	F
1			Deviation from Mean Return		Covariance	
2	Scenario	Probability	Stock Fund	Bond Fund	Product of Dev	Col B × Col E
3	Severe recession	.05	−47	−14	658	32.9
4	Mild recession	.25	−21	10	−210	−52.5
5	Normal growth	.40	4	3	12	4.8
6	Boom	.30	20	−10	−200	−60.0
7				Covariance =	SUM:	−74.8
8	Correlation coefficient = Covariance/(StdDev(stocks)*StdDev(bonds)) =					−0.49

6.2 Asset Allocation with Two Risky Assets

- Using Historical Data
  - Variability/covariability change slowly over time
  - Use realized returns to estimate
    - Cannot estimate averages precisely
  - Focus for risk on deviations of returns from average value

6.2 Asset Allocation with Two Risky Assets

- Three Rules
  - RoR: Weighted average of returns on components, with investment proportions as weights

$$r_P = w_B r_B + w_S r_S$$

  - ERR: Weighted average of expected returns on components, with portfolio proportions as weights

$$E(r_P) = w_B E(r_B) + w_S E(r_S)$$

  - Variance of RoR:

$$\sigma_P^2 = (w_B \sigma_B)^2 + (w_S \sigma_S)^2 + 2(w_B \sigma_B)(w_S \sigma_S)\rho_{BS}$$

6.2 Asset Allocation with Two Risky Assets

- Risk-Return Trade-Off
  - Investment opportunity set
    - Available portfolio risk-return combinations
- Mean-Variance Criterion
  - If $E(r_A) \geq E(r_B)$ and $\sigma_A \leq \sigma_B$
    - Portfolio *A* dominates portfolio *B*

Spreadsheet 6.5 Investment Opportunity Set

	A	B	C	D	E
1			**Input Data**		
2	$E(r_S)$	$E(r_B)$	σ_S	σ_B	ρ_{BS}
3	10	5	19	8	0.2
4	Portfolio Weights		Expected Return, $E(r_p)$		Std Dev
5	$w_S = 1 - w_B$	w_B	Col A*A3 + Col B*B3		(Equation 6.6)
6	−0.2	1.2	4.0		9.59
7	−0.1	1.1	4.5		8.62
8	0.0	1.0	5.0		8.00
9	0.0932	0.9068	5.5		7.804
10	0.1	0.9	5.5		7.81
11	0.2	0.8	6.0		8.07
12	0.3	0.7	6.5		8.75
13	0.4	0.6	7.0		9.77
14	0.5	0.5	7.5		11.02
15	0.6	0.4	8.0		12.44
16	0.7	0.3	8.5		13.98
17	0.8	0.2	9.0		15.60
18	0.9	0.1	9.5		17.28
19	1.0	0.0	10.0		19.00
20	1.1	−0.1	10.5		20.75
21	1.2	−0.2	11.0		22.53
22	Notes:				
23	1. Negative weights indicate short positions.				
24	2. The weights of the minimum-variance portfolio are computed using the formula in Footnote 1.				

Figure 6.3 Investment Opportunity Set

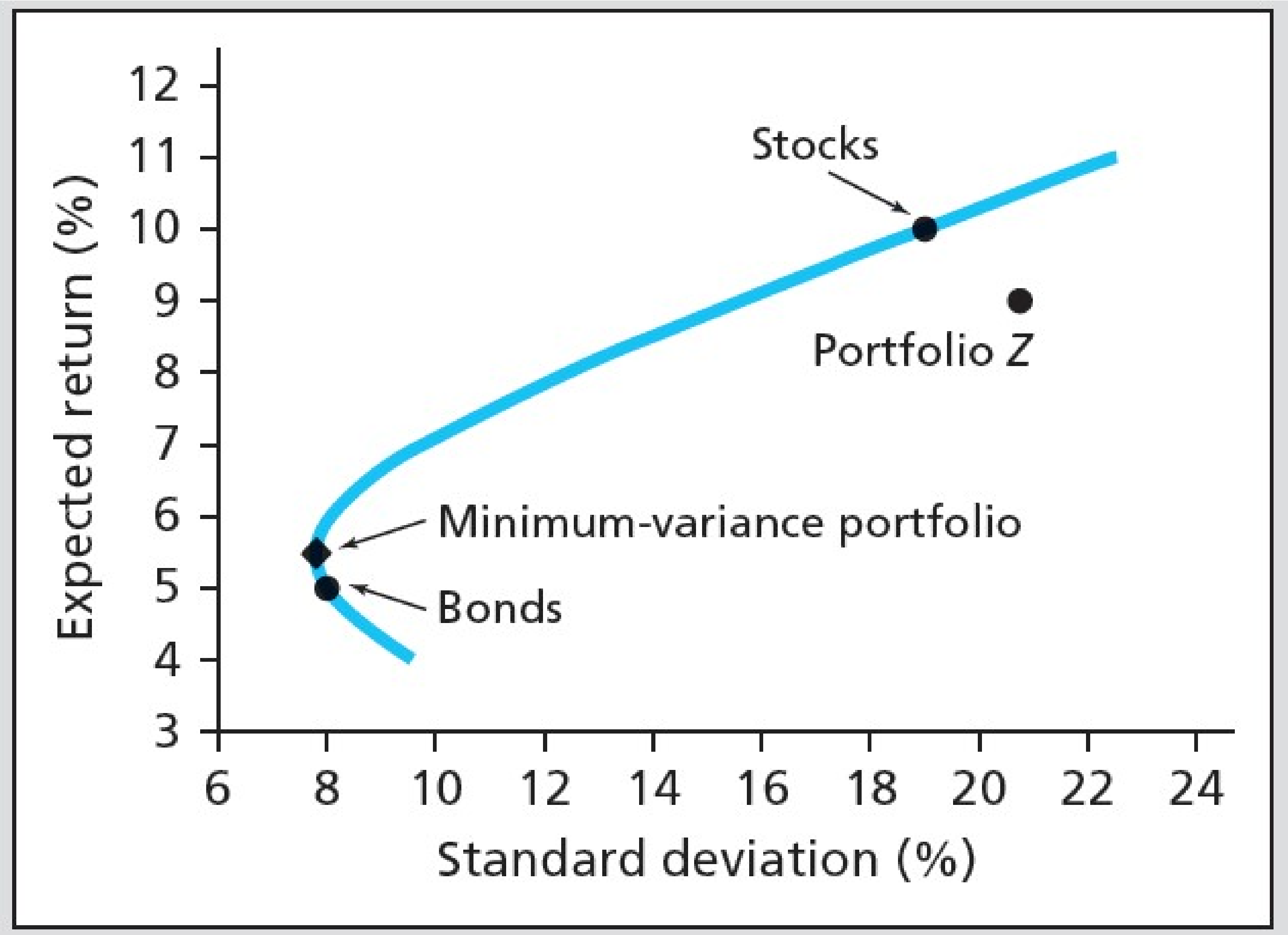
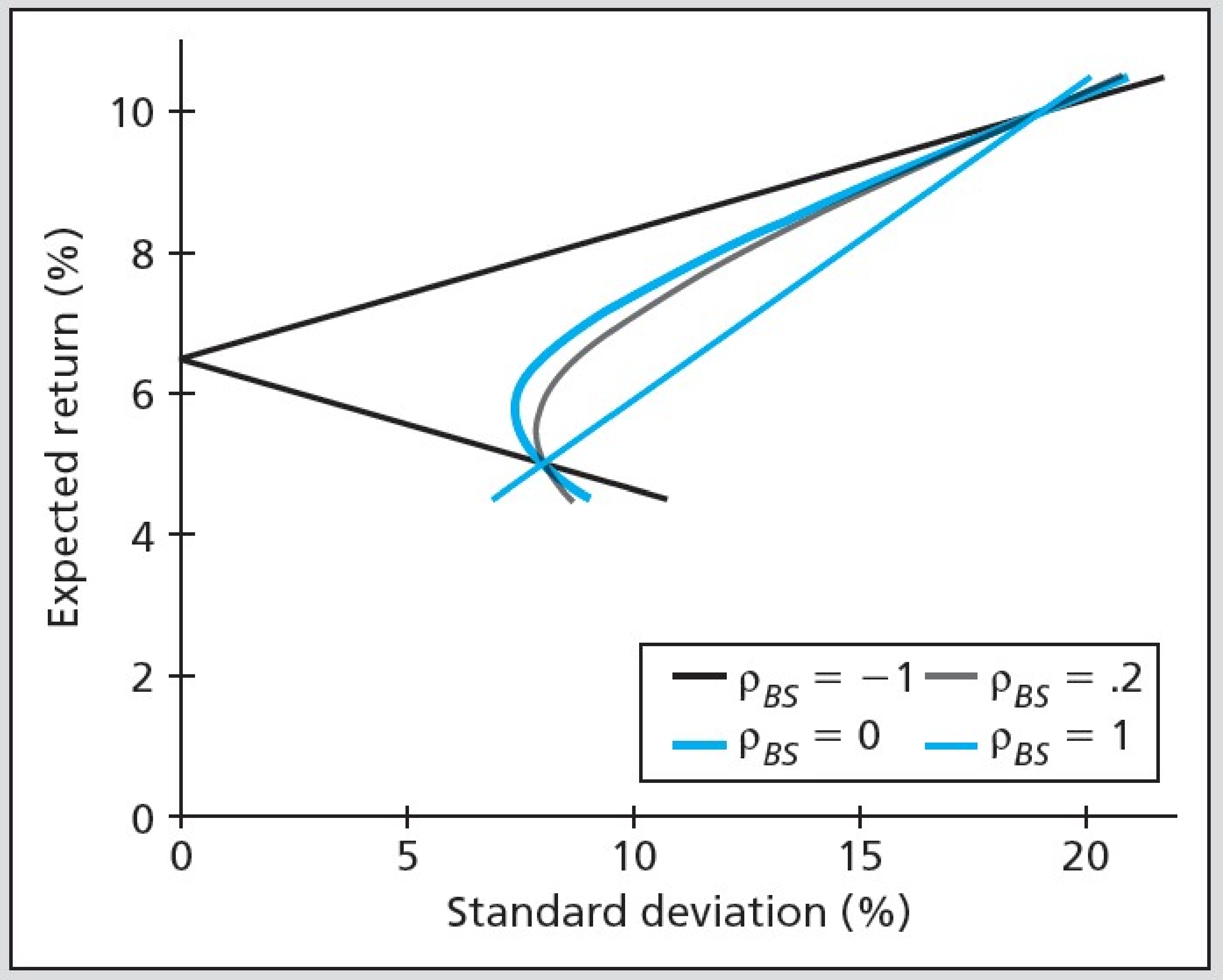


Figure 6.4 Opportunity Sets: Various Correlation Coefficients



Spreadsheet 6.6 Opportunity Set -Various Correlation Coefficients

	A	B	C	D	E	F	G
1		Input Data					
2	$E(r_S)$	$E(r_B)$	σ_S	σ_B			
3	10	5	19	8			
4							
5	Weights in Stocks	Portfolio Expected Return	Portfolio Standard Deviation[1] for Given Correlation, ρ				
6	w_S	$E(r_P)$ = Col A*A3 + (1 − Col A)*B3	−1	0	0.2	0.5	1
7	−0.1	4.5	10.70	9.00	8.62	8.02	6.90
8	0.0	5.0	8.00	8.00	8.00	8.00	8.00
9	0.1	5.5	5.30	7.45	7.81	8.31	9.10
10	0.2	6.0	2.60	7.44	8.07	8.93	10.20
11	0.3	6.5	0.10	7.99	8.75	9.79	11.30
12	0.4	7.0	2.80	8.99	9.77	10.83	12.40
13	0.6	8.0	8.20	11.84	12.44	13.29	14.60
14	0.8	9.0	13.60	15.28	15.60	16.06	16.80
15	1.0	10.0	19.00	19.00	19.00	19.00	19.00
16	1.1	10.5	21.70	20.92	20.75	20.51	20.10
17				Minimum-Variance Portfolio[2,3,4,5]			
18	$w_S(\min) = (\sigma_B{}^\wedge 2 - \sigma_B\sigma_S\rho)/(\sigma_S{}^\wedge 2 + \sigma_B{}^\wedge 2 - 2{*}\sigma_B\sigma_S\rho) =$		0.2963	0.1506	0.0923	−0.0440	−0.7273
19	$E(r_P) = w_S$ (min)*A3 + (1 − w_S (min))*B3 =		6.48	5.75	5.46	4.78	1.36
20		$\sigma_P =$	0.00	7.37	7.80	7.97	0.00

6.3 The Optimal Risky Portfolio with a Risk-Free Asset

- Slope of CAL is Sharpe Ratio of Risky Portfolio
  - $S_P = \dfrac{E(r_P) - r_f}{\sigma_P}$
- Optimal Risky Portfolio
  - Best combination of risky and safe assets to form portfolio

6.3 The Optimal Risky Portfolio with a Risk-Free Asset

- Calculating Optimal Risky Portfolio
  - Two risky assets

$$w_B = \frac{[E(r_B) - r_f]\sigma_S^2 - [E(r_s) - r_f]\sigma_B \sigma_S \rho_{BS}}{[E(r_B) - r_f]\sigma_S^2 + [E(r_s) - r_f]\sigma_B^2 - [E(r_B) - r_f + E(r_s) - r_f]\sigma_B \sigma_S \rho_{BS}}$$

$$w_S = 1 - w_B$$

Figure 6.5 Two Capital Allocation Lines

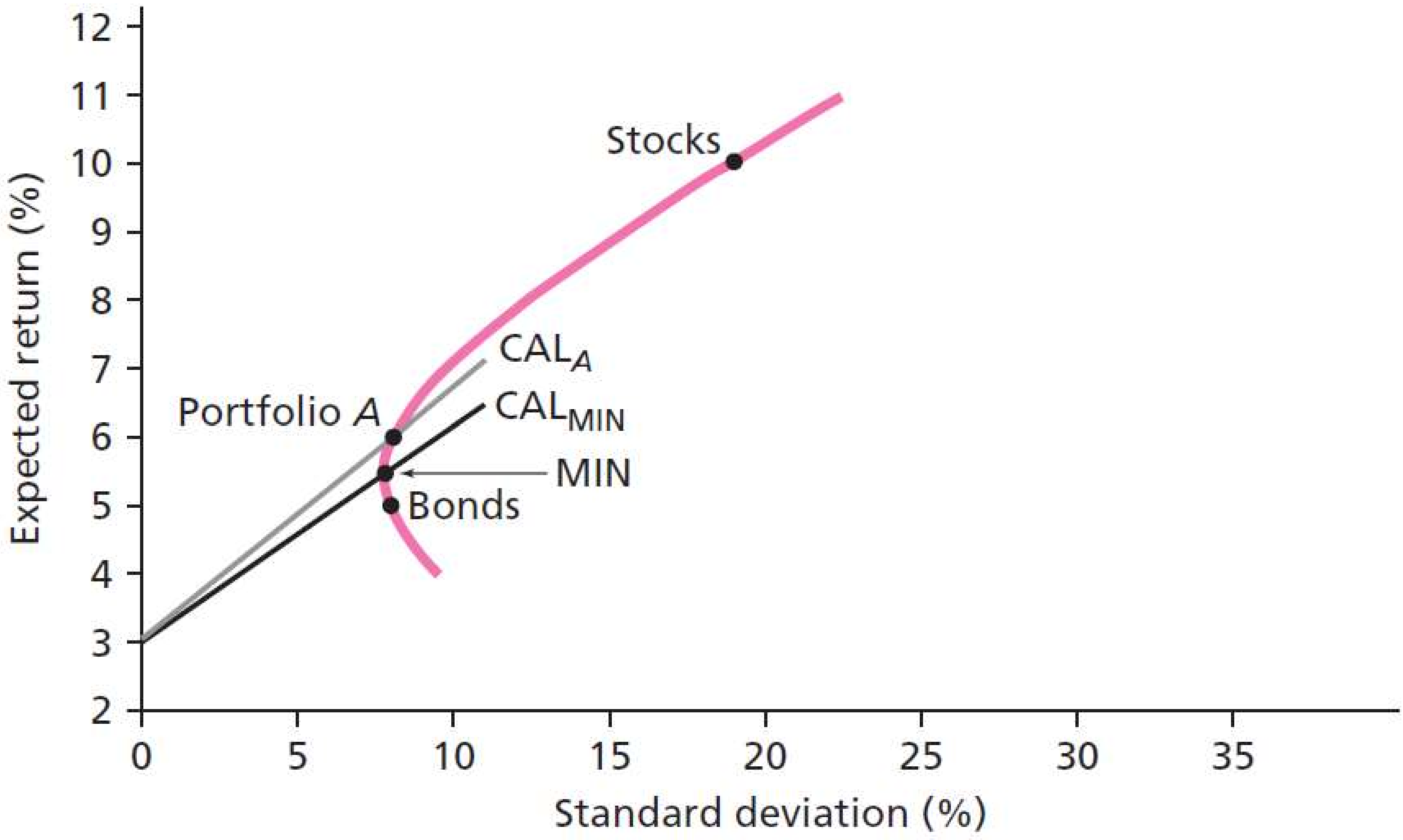


Figure 6.6 Bond, Stock and T-Bill Optimal Allocation

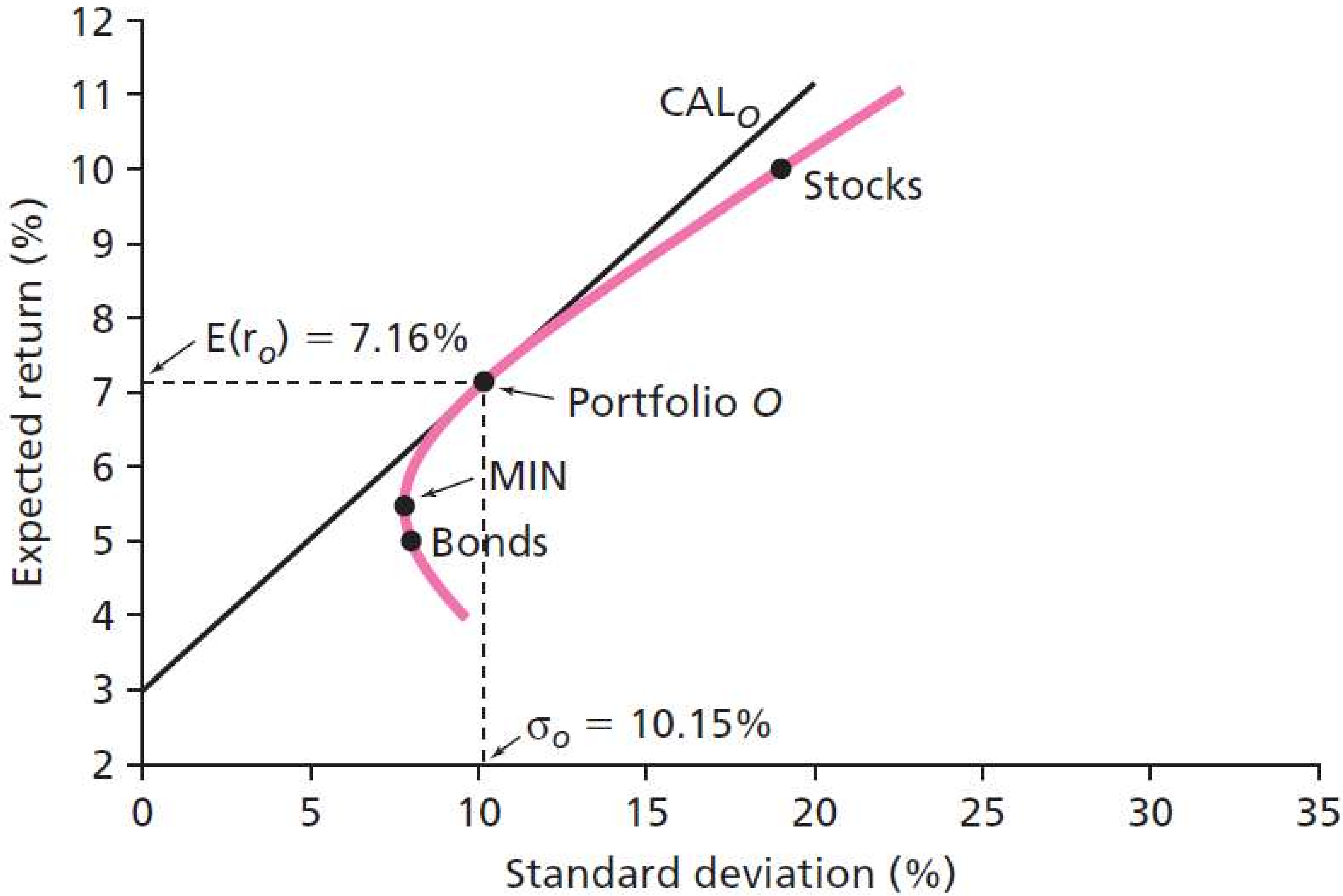


Figure 6.7 The Complete Portfolio

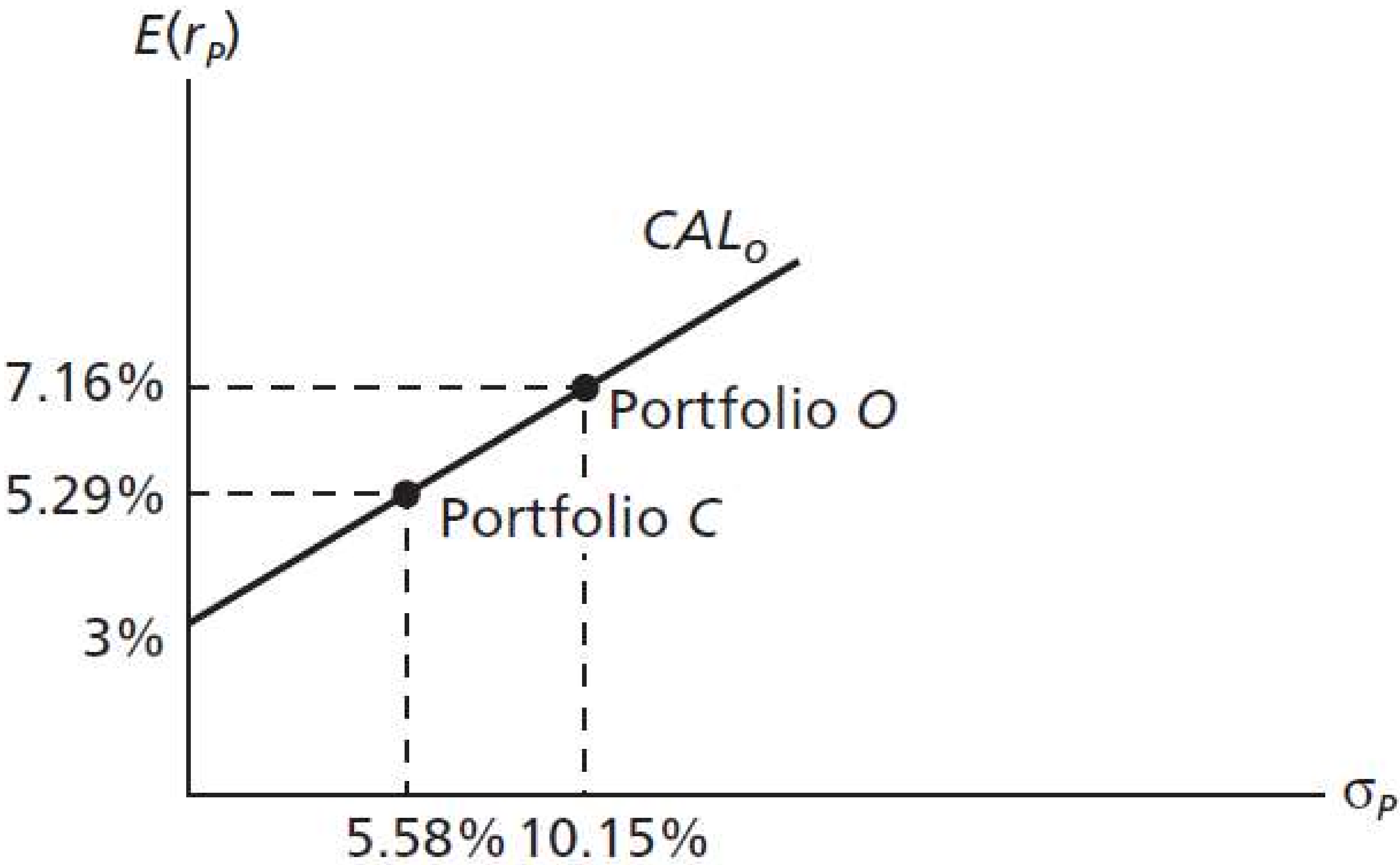
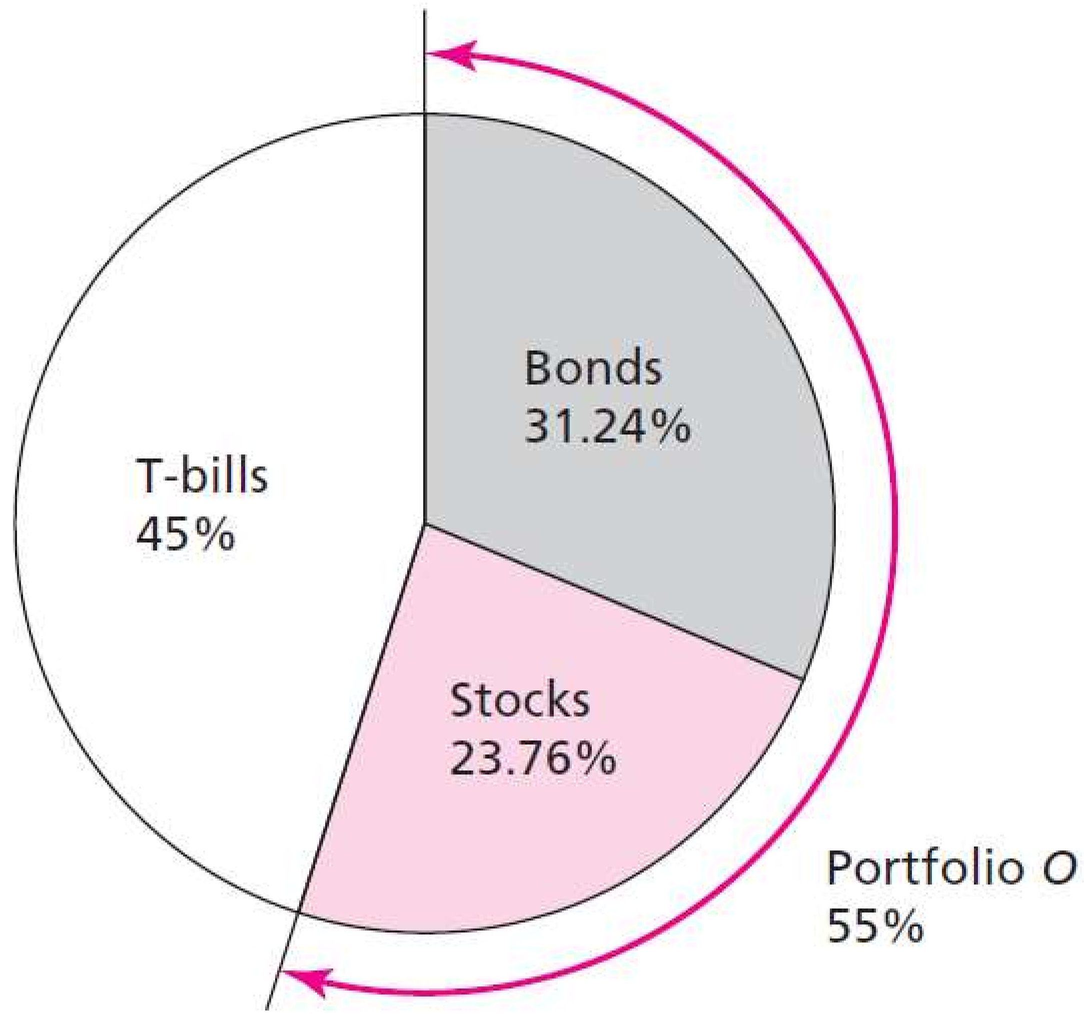


Figure 6.8 Portfolio Composition: Asset Allocation Solution



6.4 Efficient Diversification with Many Risky Assets

- Efficient Frontier of Risky Assets
  - Graph representing set of portfolios that maximizes expected return at each level of portfolio risk
    - Three methods
      - Maximize risk premium for any level standard deviation
      - Minimize standard deviation for any level risk premium
      - Maximize Sharpe ratio for any standard deviation or risk premium

Figure 6.9 Portfolios Constructed with Three Stocks

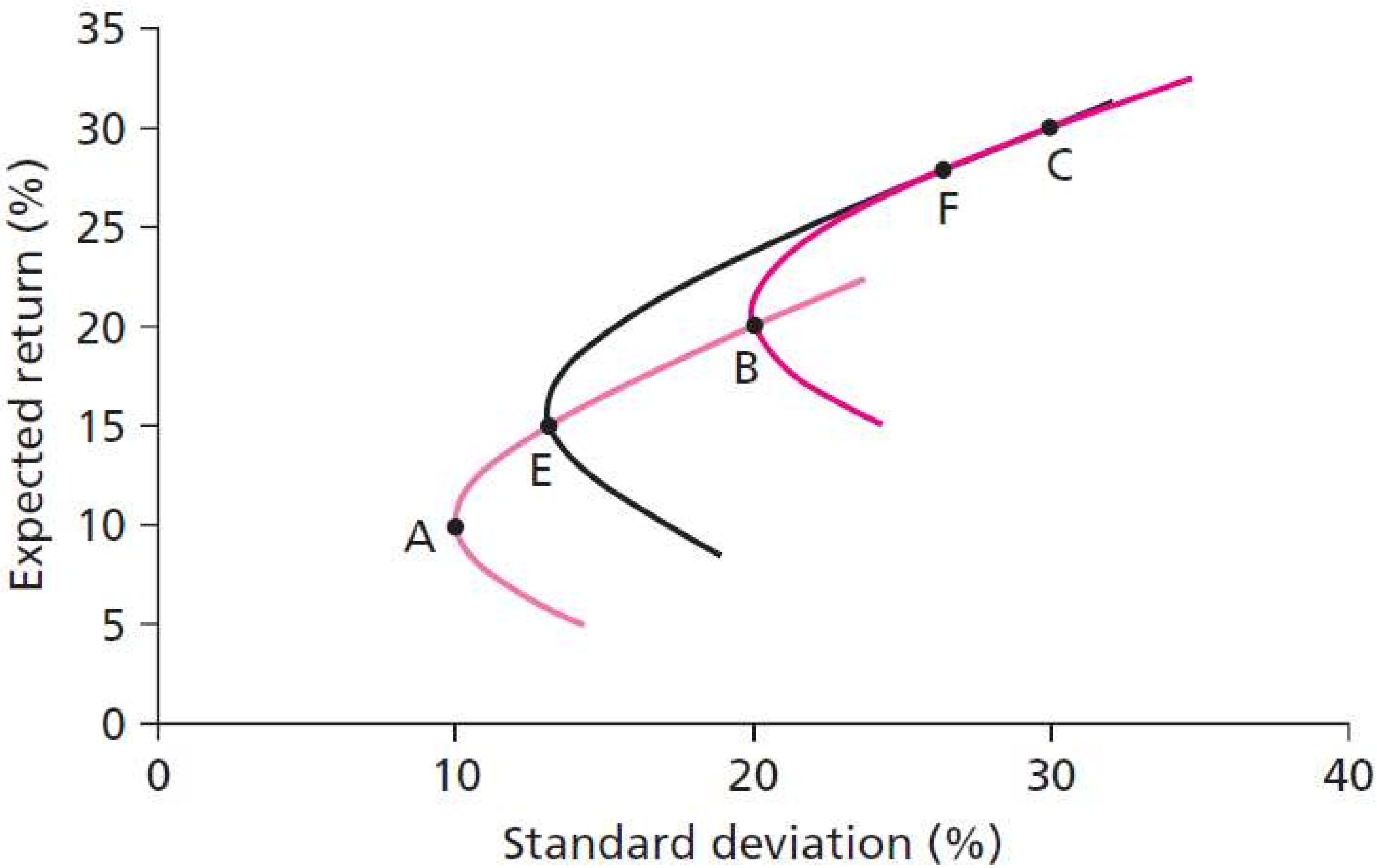
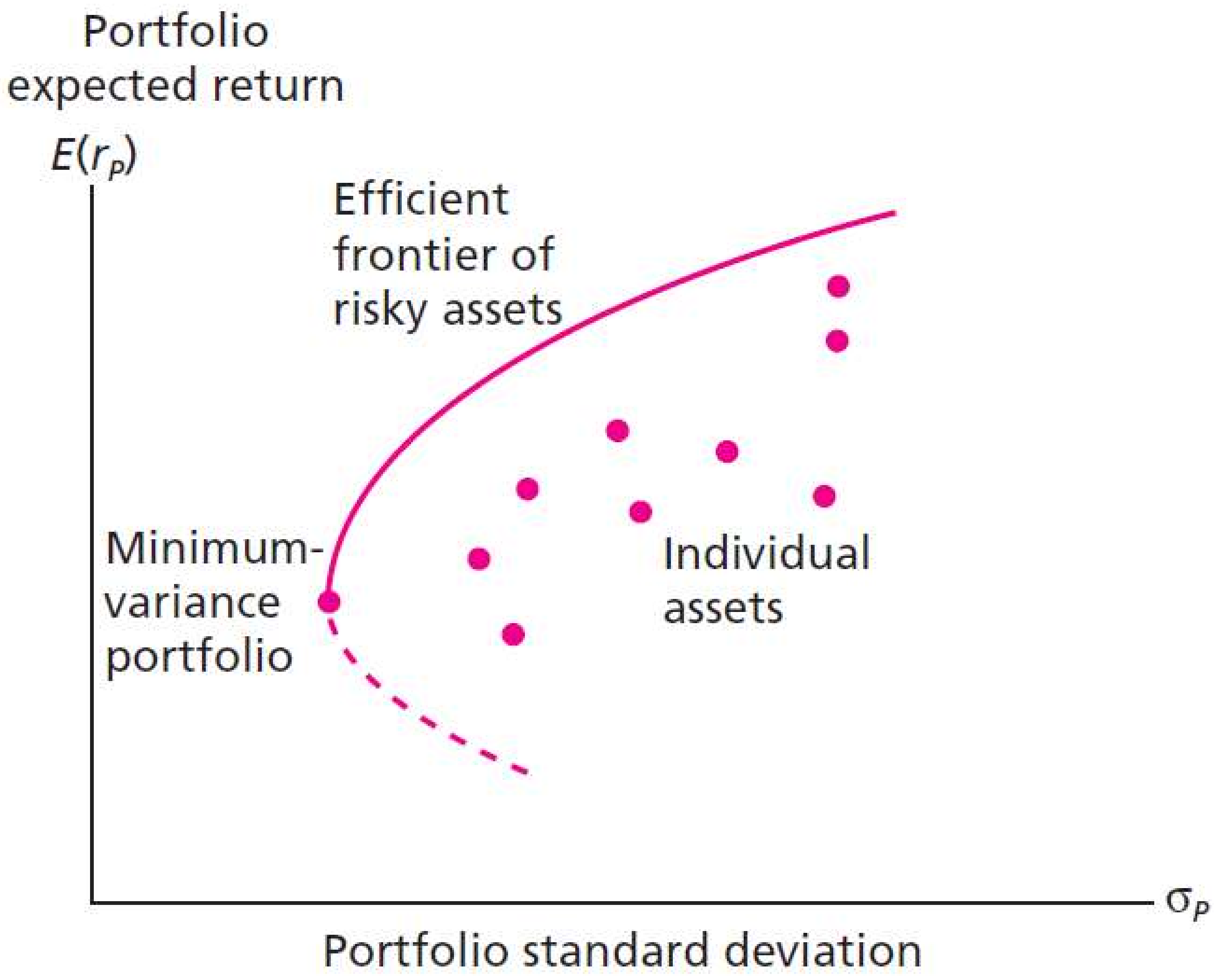


Figure 6.10 Efficient Frontier: Risky and Individual Assets



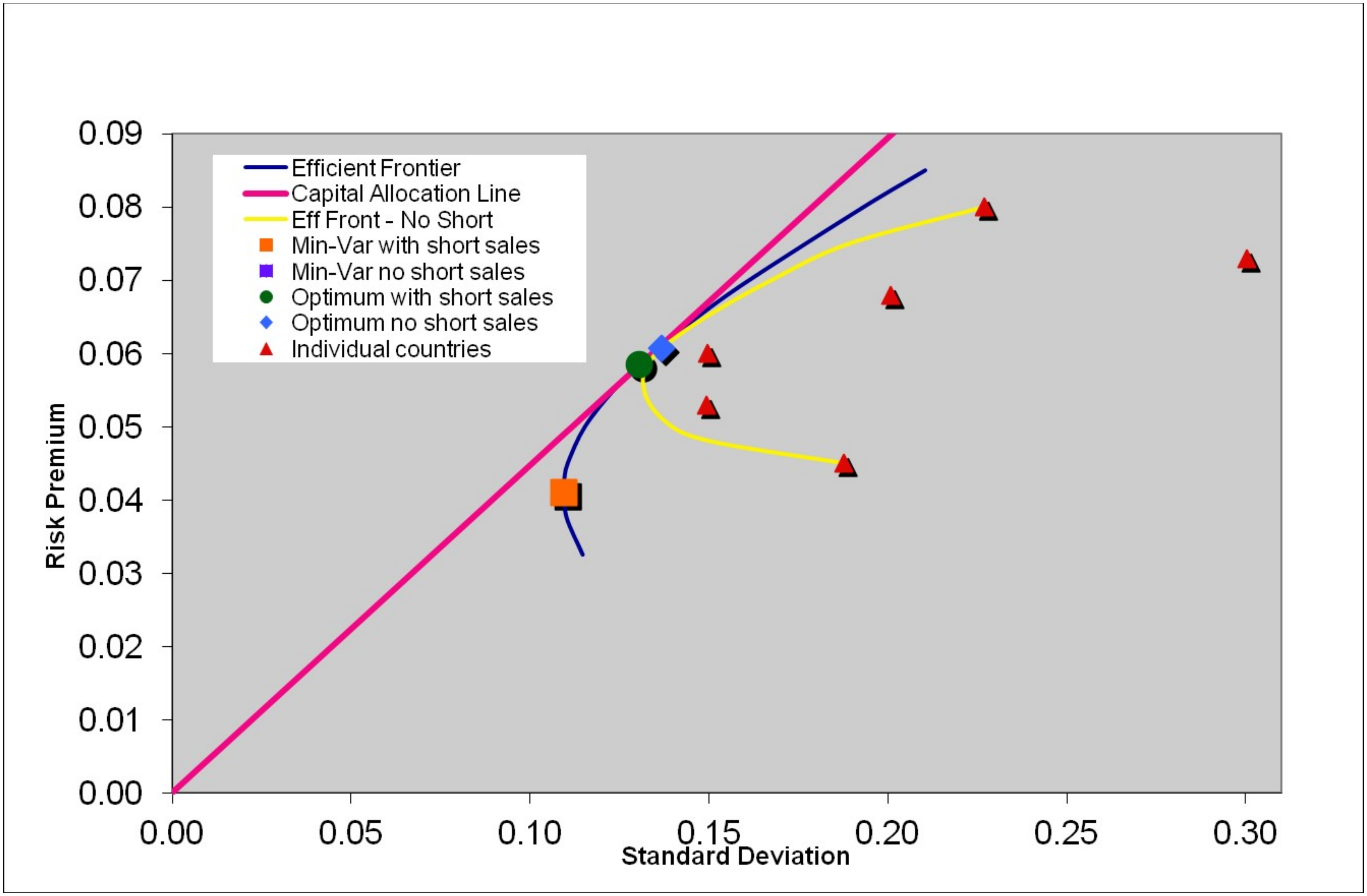
6.4 Efficient Diversification with Many Risky Assets

- Choosing Optimal Risky Portfolio
  - Optimal portfolio CAL tangent to efficient frontier
- Preferred Complete Portfolio and Separation Property
  - Separation property: implies portfolio choice, separated into two tasks
    - Determination of optimal risky portfolio
    - Personal choice of best mix of risky portfolio and risk-free asset

6.4 Efficient Diversification with Many Risky Assets

- Optimal Risky Portfolio: Illustration
  - Efficiently diversified global portfolio using stock market indices of six countries
  - Standard deviation and correlation estimated from historical data
  - Risk premium forecast generated from fundamental analysis

Figure 6.11 Efficient Frontiers/CAL: Table 6.1



Example: Portfolio Construction

Three Risky Assets

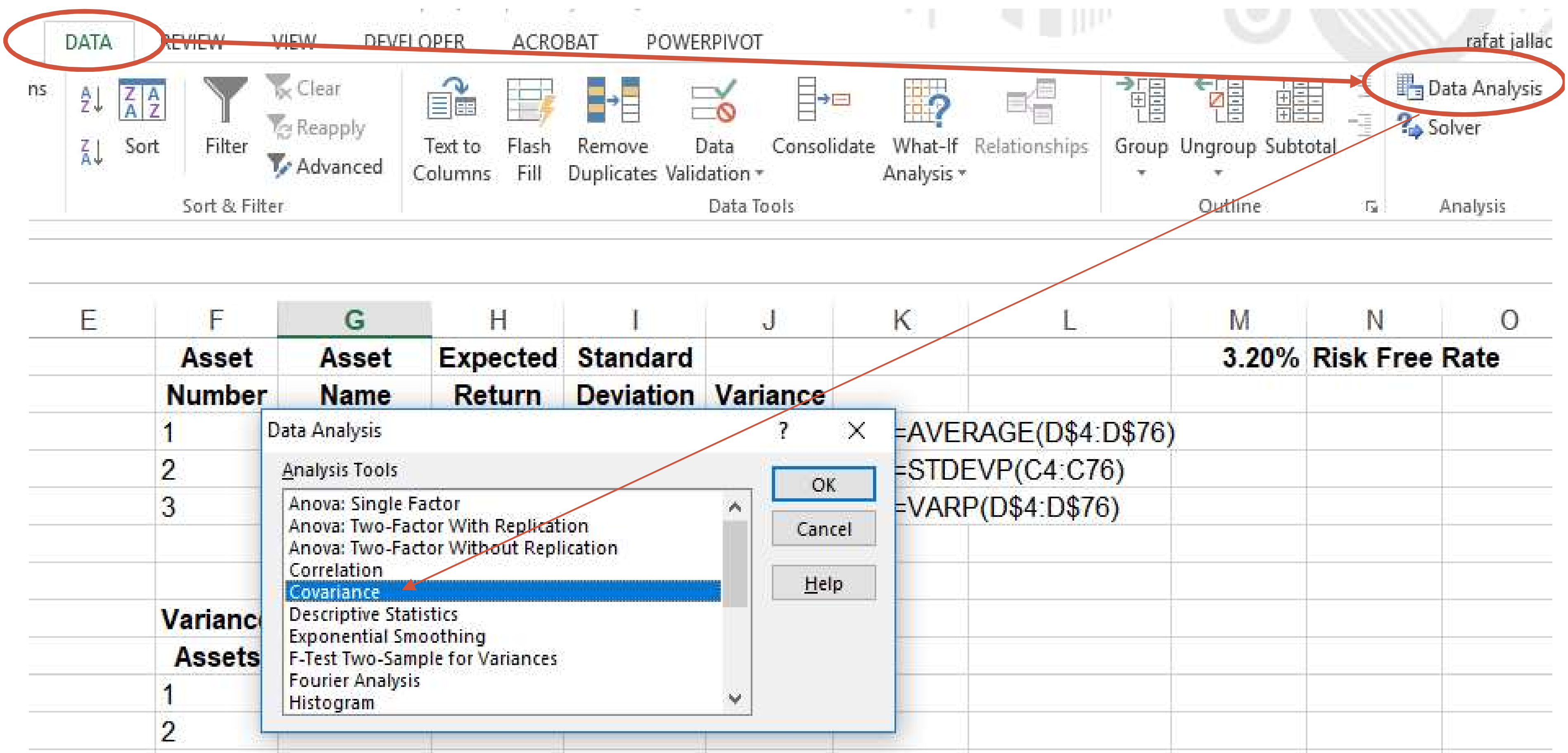
	A	B	C	D
1				
2		Asset 1	Asset 2	Asset 3
3	DATE	Lg Stock	Sm. Stock	Corporate
4	1926	11.62	0.28	7.37
5	1927	37.49	22.1	7.44
6	1928	43.61	39.69	2.84
7	1929	-8.42	-51.36	3.27
8	1930	-24.9	-38.15	7.98
9	1931	-43.34	-49.75	-1.85
10	1932	-8.19	-5.39	10.82
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

F	G	H	I	J	K	L
Asset Number	Asset Name	Expected Return	Standard Deviation	Variance		
1	Lg Stock	13.17	20.12	404.8	<--	=AVERAGE(D$4:D$76)
2	Sm. Stock	17.39	33.55	1,125.6	<--	=STDEVP(C4:C76)
3	Corporate	6.12	8.57	73.4	<--	=VARP(D$4:D$76)

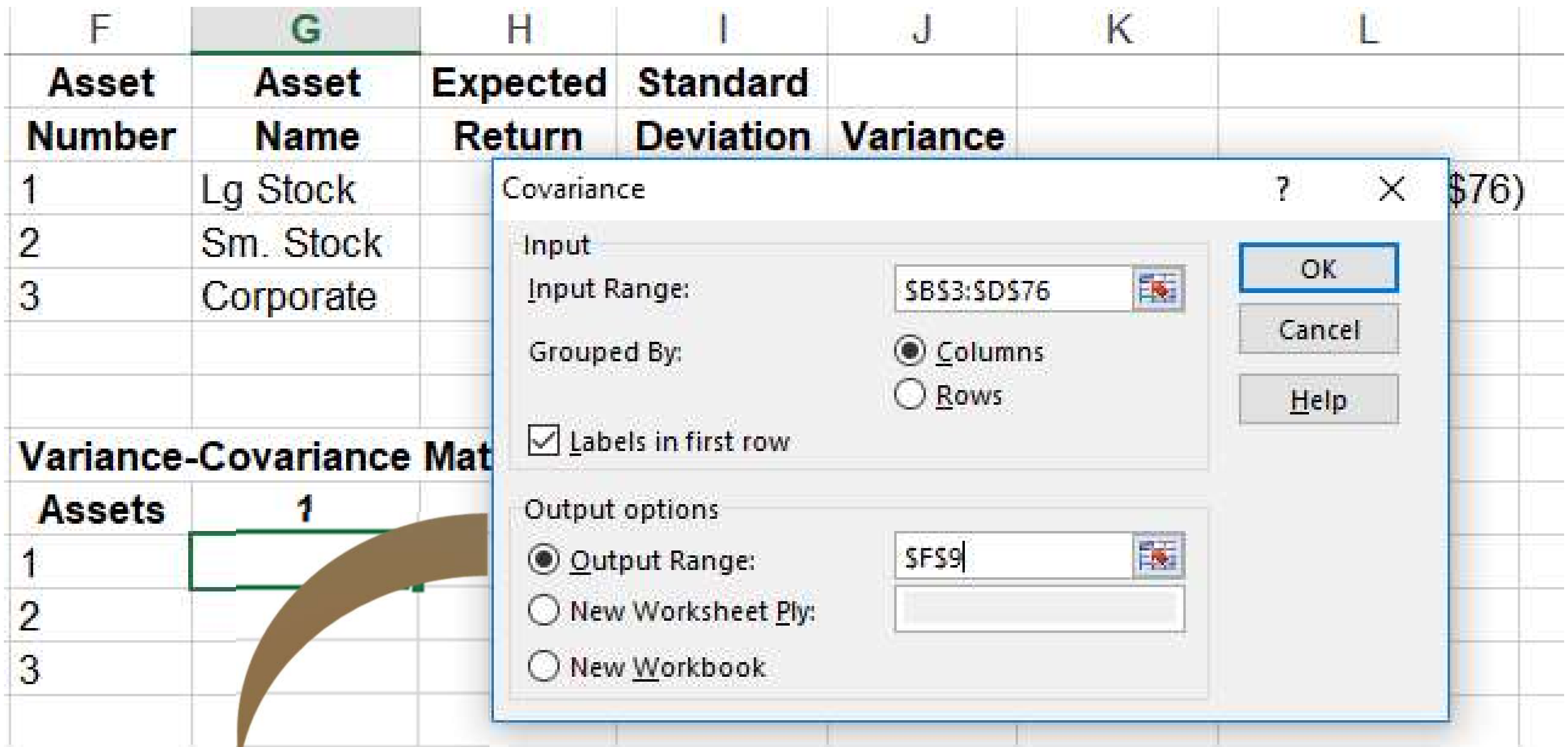
- Phase One: Investment Decision
- **Step1:** Formulate the Capital Market Expectations (CMEs)

Example: Portfolio Construction

- **Step 2: Construct the Variance Covariance Matrix**



Example: Portfolio Construction



Variance-Covariance Matrix			
	Lg Stock	*Sm. Stock*	*Corporate*
Lg Stock	404.84	535.41	45.61
Sm. Stock	535.41	1,125.64	31.53
Corporate	45.61	31.53	73.38

Example: Portfolio Construction

- Step 3: Portfolio Statistics (Expected return + standard deviation)

E	F	G	H	I	J	K	L
	Asset Number	**Asset Name**	**Expected Return**	**Standard Deviation**	**Variance**		
	1	Lg Stock	13.17	20.12	404.8	<--	=AVERAGE(D$4:D$76)
	2	Sm. Stock	17.39	33.55	1,125.6	<--	=STDEVP(C4:C76)
	3	Corporate	6.12	8.57	73.4	<--	=VARP(D$4:D$76)

Using Bordered variance covariance matrix

		0.00	0.00	1.00
	Variance-Covariance Matrix			
		Lg Stock	*Sm. Stock*	*Corporate*
0.00	Lg Stock	404.84	535.41	45.61
0.00	Sm. Stock	535.41	1,125.64	31.53
1.00	Corporate	45.61	31.53	73.38

Put Initial Weights

Portfolio Weights

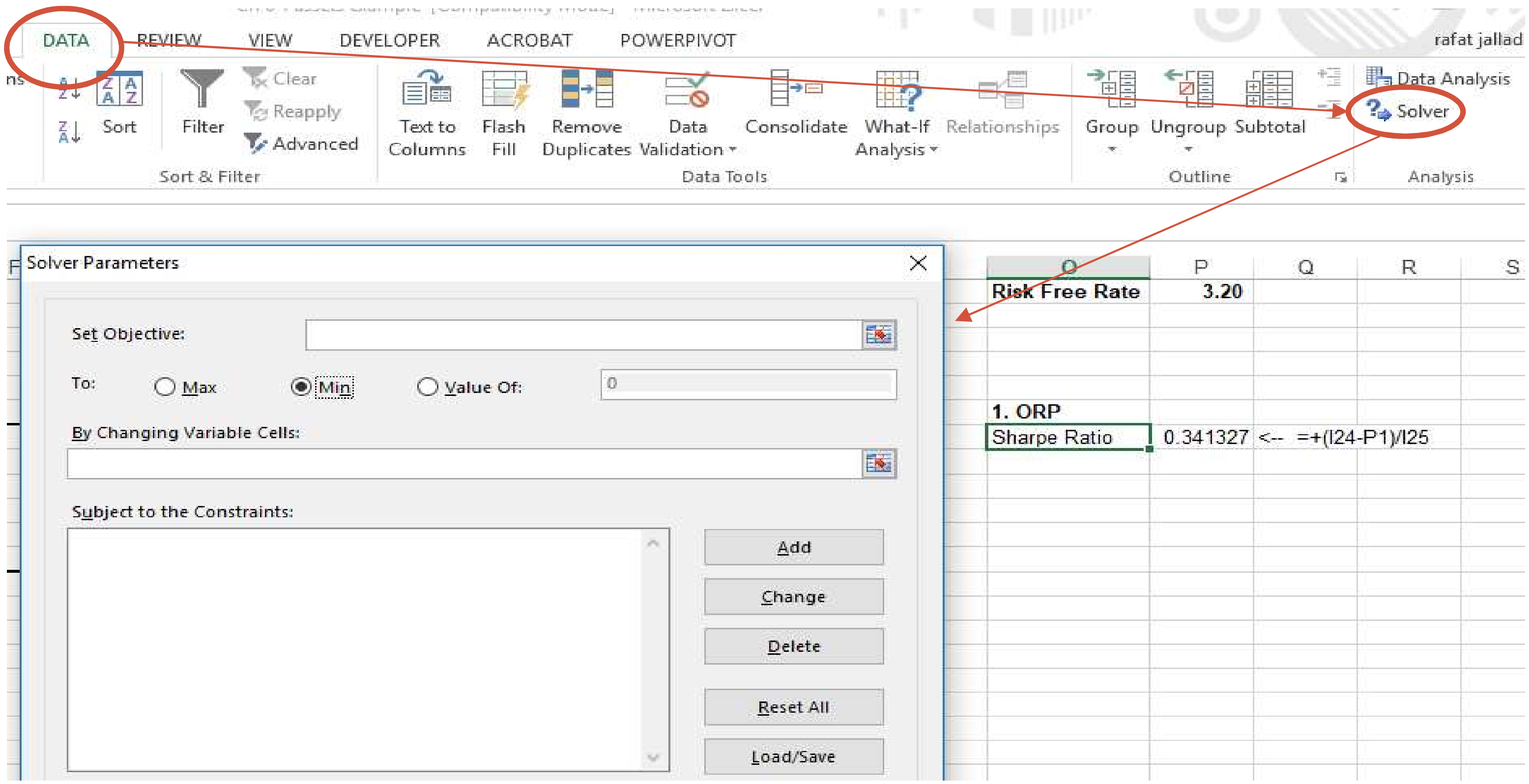
Asset	**Weight**		
Lg Stock	0.00		
Sm. Stock	0.00		
Corporate	1.00	<--	=1-SUM(G17:G18)
Total	**1.00**	<--	=SUM(G17:G19)

Portfolio Summary Statistics

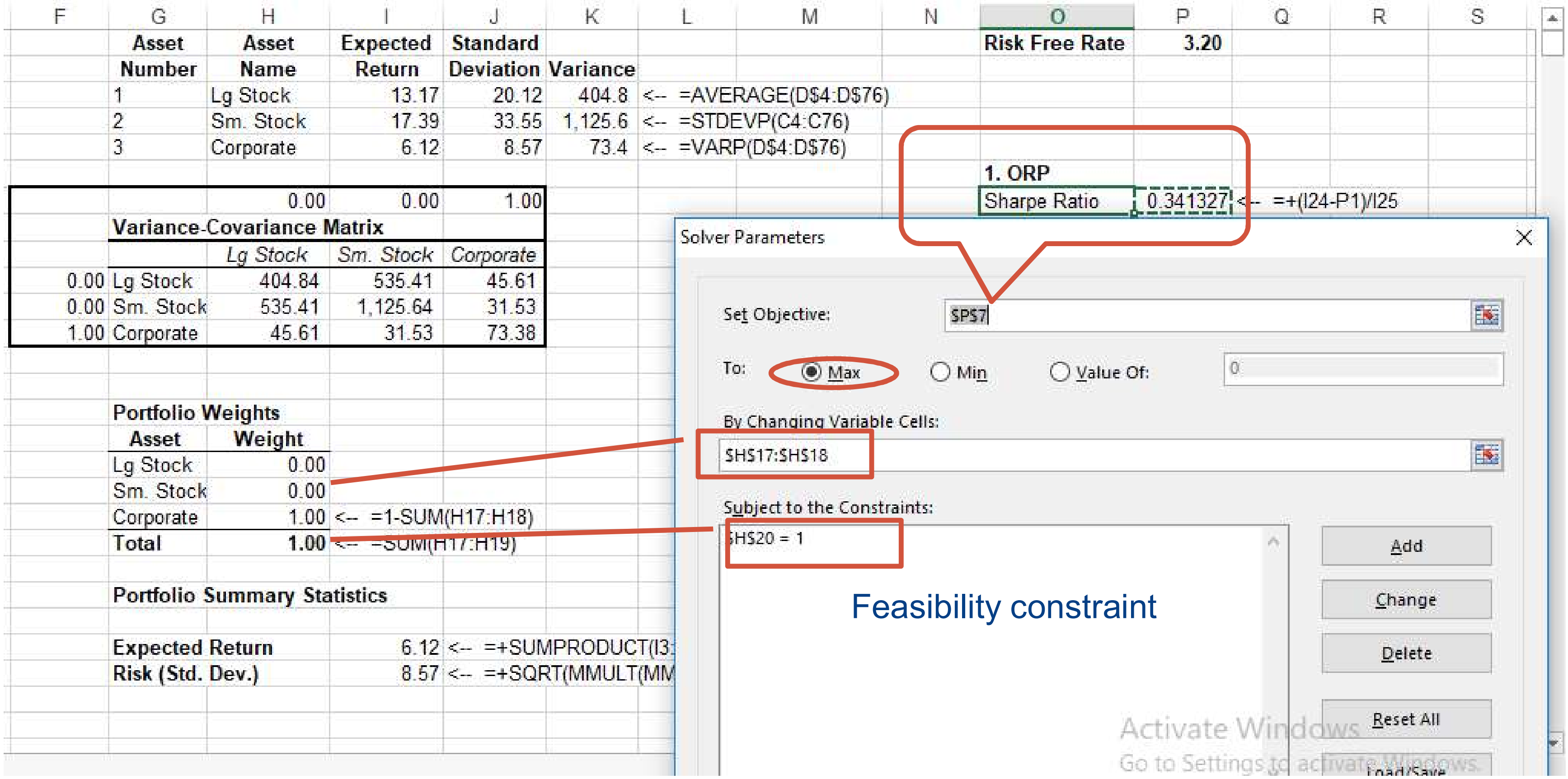
Expected Return	6.12	<--	=+SUMPRODUCT(H3:H5,G17:G19)
Risk (Std. Dev.)	8.57	<--	=+SQRT(MMULT(MMULT(G7:I7,G10:I12),E10:E12))

Example: Portfolio Construction

- Step 3: Identify the Optimal Risky portfolio (ORP) that maximize sharpe ratio



Example: Portfolio Construction



Example: Portfolio Construction

		0.33	0.09	0.58
	Variance-Covariance Matrix			
		Lg Stock	*Sm. Stock*	*Corporate*
0.33	Lg Stock	404.84	535.41	45.61
0.09	Sm. Stock	535.41	1,125.64	31.53
0.58	Corporate	45.61	31.53	73.38

1. ORP

Sharpe Ratio 0.548418 <-- =+(I24-P1)/I25

Portfolio Weights

Asset	Weight	
Lg Stock	0.33	
Sm. Stock	0.09	
Corporate	0.58	<-- =1-SUM(H17:H18)
Total	**1.00**	<-- =SUM(H17:H19)

Optimized Solution

Portfolio Summary Statistics

Expected Return	9.43	<-- =+SUMPRODUCT(I3:I5,H17:H19)
Risk (Std. Dev.)	11.36	<-- =+SQRT(MMULT(MMULT(H7:J7,H10:J12),F10:F12))

Example: Portfolio Construction

- Phase Two: Capital Allocation

O	P	Q	R	S
Risk Free Rate	**3.20**			
1. ORP				
Sharpe Ratio	0.548418	<-- =+(I24-P1)/I25		
2. Capital Allocation				
A	2			
Y*	13.7%	<-- =+(I24-P1)/(P11*I25*2)		
1-Y*	86.3%	<-- =1-P12		

6.5 A Single-Index Stock Market

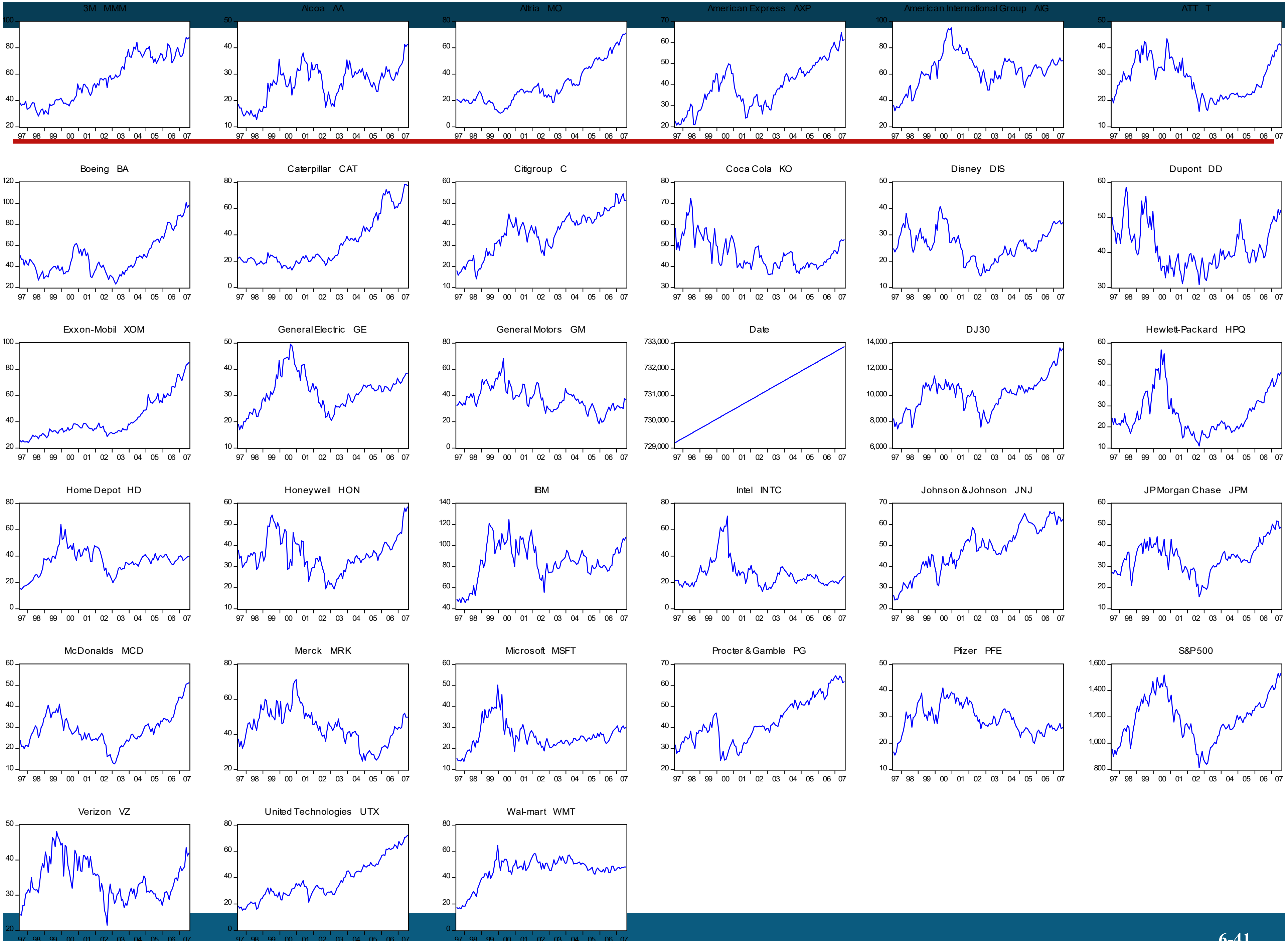
- Index model
  - Relates stock returns to returns on broad market index/firm-specific factors
- Excess return
  - RoR in excess of risk-free rate
- Beta
  - Sensitivity of security's returns to market factor
- Firm-specific or residual risk
  - Component of return variance independent of market factor
- Alpha
  - Stock's expected return beyond that induced by market index

Single Index Model (SIM)

- An Index Model is a Statistical model of security returns *(as opposed to an economic, equilibrium-based model).*
- A Single Index Model (SIM) specifies two sources of uncertainty for a security's return:
  - 1. **Systematic (macroeconomic) uncertainty** (which is assumed to be well represented by a single index of stock returns)
  - 2. **Unique (microeconomic) uncertainty** (which is represented by a security-specific random component)

SIM: Model's Components

- **1. The Basic Idea**
- A Casual Observation: Stocks tend to move together, driven by the same economic forces.


3M MMM
Alcoa AA
Altria MO
American Express AXP
American International Group AIG
ATT T
Boeing BA
Caterpillar CAT
Citigroup C
Coca Cola KO
Disney DIS
Dupont DD
Exxon-Mobil XOM
General Electric GE
General Motors GM
Date
DJ30
Hewlett-Packard HPQ
Home Depot HD
Honeywell HON
IBM
Intel INTC
Johnson & Johnson JNJ
JP Morgan Chase JPM
McDonalds MCD
Merck MRK
Microsoft MSFT
Procter & Gamble PG
Pfizer PFE
S&P500
Verizon VZ
United Technologies UTX
Wal-mart WMT
97 98 99 00 01 02 03 04 05 06 07

2. Formalizing the Basic Idea: The Return Generating Model

- Can always write the return of asset i as related linearly to a single common underlying factor (typically chosen to be a stock index):

$$R_i = \beta_i R_M + \alpha_i + e_i$$

$E[e_i] = 0$, $Cov[e_i, R_m] = 0$

$\beta_i R_M$: return from movements in overall market

β_i : security's responsiveness to market $\beta_i = Cov[R_i, R_m]/Var[R_m]$

α_i : stock's expected excess return if market factor is neutral, i.e. market-index excess return is zero

e_i : firm-specific risk

Formalizing the Basic Idea: The Return Generating Model

So, $\alpha_i + e_i$ is the return part independent of the index return,

$\beta_i R_m$ is the return part due to index fluctuations.

B. Expressing the First and Second Moments using the Model's Components

1. Mean return of security i: $E[r_i] = \alpha_i + \beta_i E[R_m]$

2. Variance of security i: $\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma^2[e_i]$

3. Covariance between return of security j and return of security i

$$\sigma_{ji} = \beta_j \beta_i \sigma_m^2$$

Formalizing the Basic Idea: The Return Generating Model

1. ***Expected Return,*** $E[r_j] = \alpha_j + \beta_j E[R_m]$, ***has 2 parts***

   a. Unique (asset specific): α_j

   b. Systematic (index driven): $\beta_j E[R_m]$

2. ***Variance,*** $\sigma_j^2 = \beta_j^2 \sigma_m^2 + \sigma^2[e_j]$, ***has similarly 2 parts:***

   a. Unique risk (asset specific): $\sigma^2[e_j]$

   b. Systematic risk (index driven): $\beta_j^2 \sigma_m^2$

Formalizing the Basic Idea: The Return Generating Model

3. Covariance between securities' returns is due to only the systematic source of risk:

$$\begin{aligned}\mathrm{Cov}[r_j, r_i] &= \mathrm{Cov}[\alpha_j + \beta_j R_m + e_i,\ \alpha_i + \beta_i R_m + e_i] \\ &= \mathrm{Cov}[\beta_j r_I ,\ \beta_i r_I] = \beta_j \beta_i \mathrm{Cov}[r_I , r_I] \\ &= \beta_j \beta_i \sigma^2\end{aligned}$$

D. Typically, the chosen index is a "Market Index"

- It "makes sense" to choose the entire stock market (a value-weighted portfolio) as a proxy to capture all macroeconomic fluctuations.
- In practice, take a portfolio, i.e., index, which proxies for the market.
- A popular choice is for the S&P500 index to be the index in the SIM.
- Then, the model states that $r_j = \alpha_j + \beta_j r_M + e_j$
  - where **r_M is the *random* return on the market proxy**.
  - This SIM is often referred to as the "Market Model."

Example

- You choose the S&P500 as your market proxy. You analyze the stock of General Electric (GE), and find (see later in the notes) that, using weekly returns, a

$$\alpha_j = -0.07\%,\ \beta_j = 1.44$$

If you expect the S&P500 to increase to 5% next week, then according to the market model, you expect the return on GE next week to be:

$$E[r_{GE}] = \alpha_{GE} + \beta_{GE} E[r_M] = -0.07\% + 1.44 \times 5\% = 7.13\%.$$

Why the Single Index Model is Useful?

- *A. The SIM Provides the Most Simple Tool to Quantify the Forces Driving Assets' Returns*
- *B. The SIM Helps us to Derive the Optimal Portfolio for Asset Allocation (the Tangent Portfolio T) by Reducing the Necessary Inputs to the Markowitz Portfolio Selection Procedure*

The SIM Provides the Most Simple Tool to Quantify the Forces Driving Assets' Returns

- We identified the portfolio *P*, used for the asset allocation, with the tangency portfolio *P*.
- To compute the weights of *P* , we need to describe all the risky assets in the portfolio selection model.
- This requires a large number of parameters.
- Usually these parameters are unknown, and have to be estimated.

1. With n risky assets, we need $2n + (n^2 - n)/2$ parameters:

n	expected returns $E[r_i]$
n	return standard deviations σ_i
$n(n-1)/2$	correlations (or covariances)

Example

$n = 2$	number of parameters = 2	+	2	+	1	=	5
$n = 8$	number of parameters = 8	+	8	+	28	=	44
$n = 100$	number of parameters = 100	+	100	+	4950	=	5150
$n = 1000$	number of parameters = 1000	+	1000	+	499500	=	501500

With large n:

Large estimation error,

Large data requirements (for monthly estimates, with n=1000, need at least 1000 months, i.e., more than 83 years of data)

Cont'd

- **2. Assuming the SIM is correctly specified, we only need the following parameters:**

n	α_i parameters
n	β_i parameters
n	$\sigma^2[e_i]$ parameters
1	$E[R_m]$
1	$\sigma^2[R_m]$

These $3n+2$ parameters generate all the $E[r_i]$, σ_j, and σ_{ji}.
We get the parameters by estimating the index model for each of the n securities.

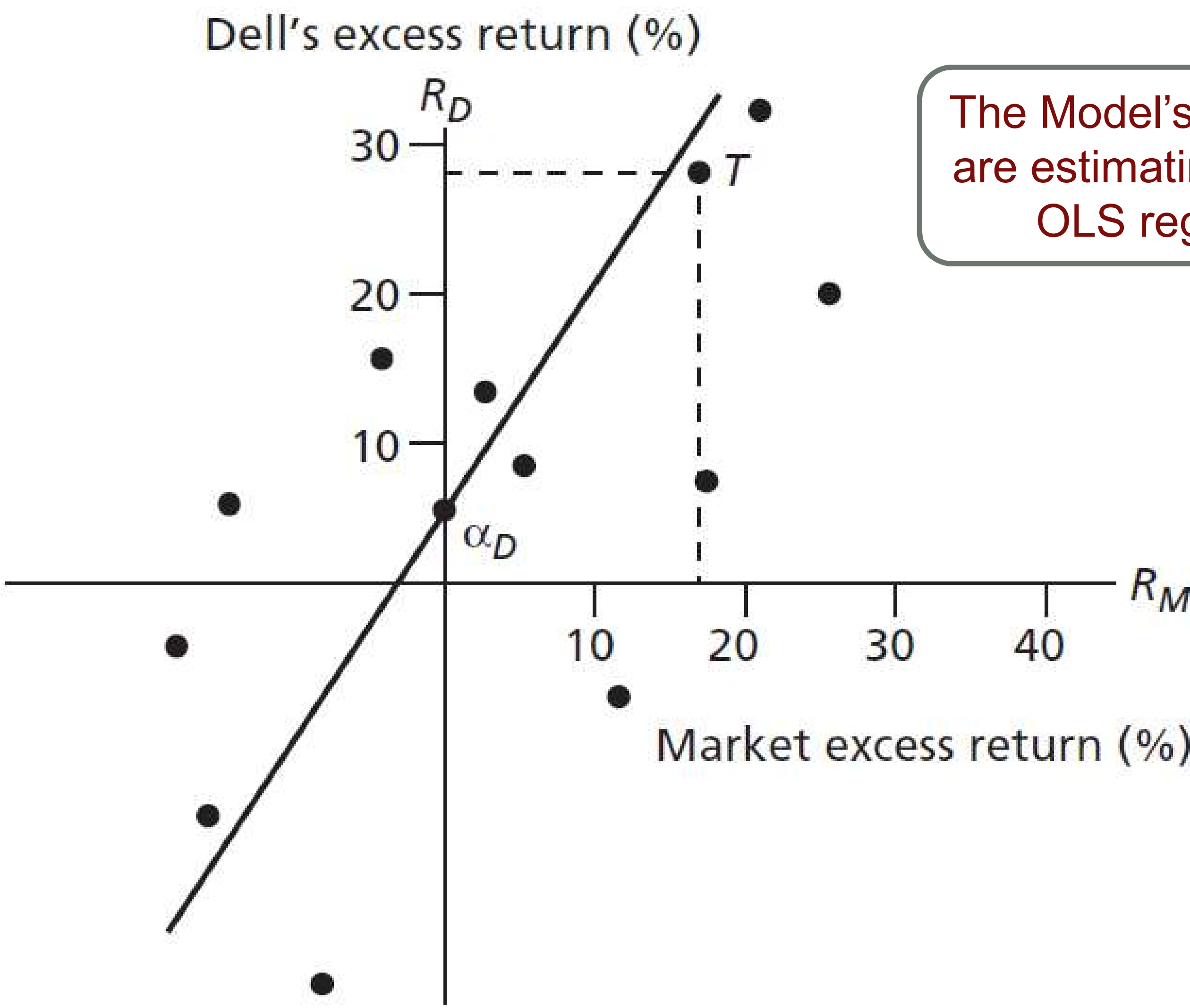
Example

With 100 stocks need 302 parameters. With 1000 need 3002.

6.5 A Single-Index Stock Market

- Statistical and Graphical Representation of Single-Index Model
  - Security Characteristic Line (SCL)
    - Plot of security's predicted excess return from excess return of market
  - Algebraic representation of regression line
    - $E(R_D | R_M) = \alpha_D + \beta_D R_M$

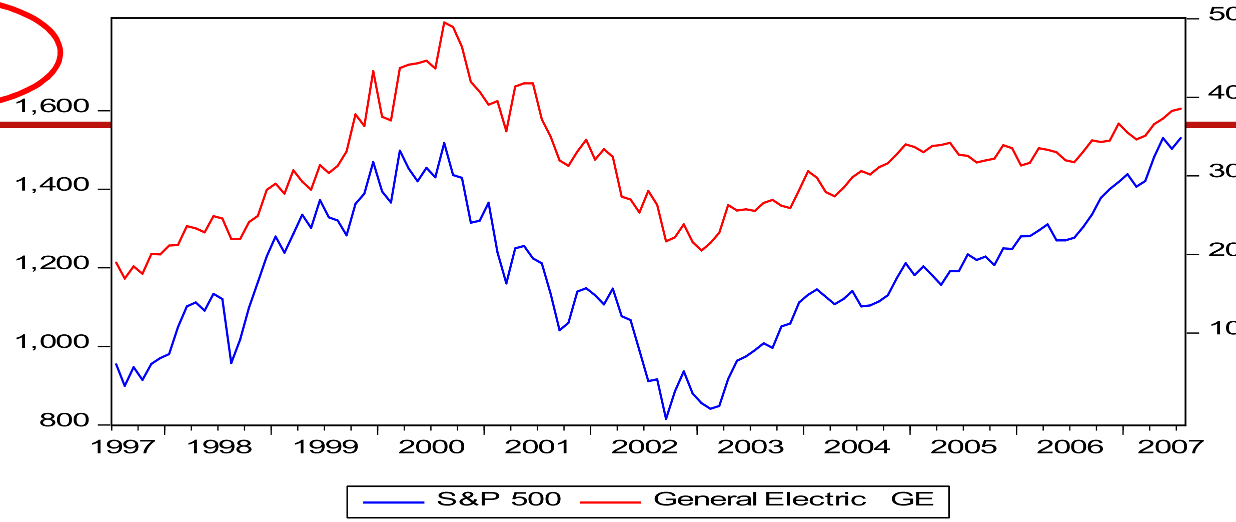
Figure 6.12 Scatter Diagram for Dell

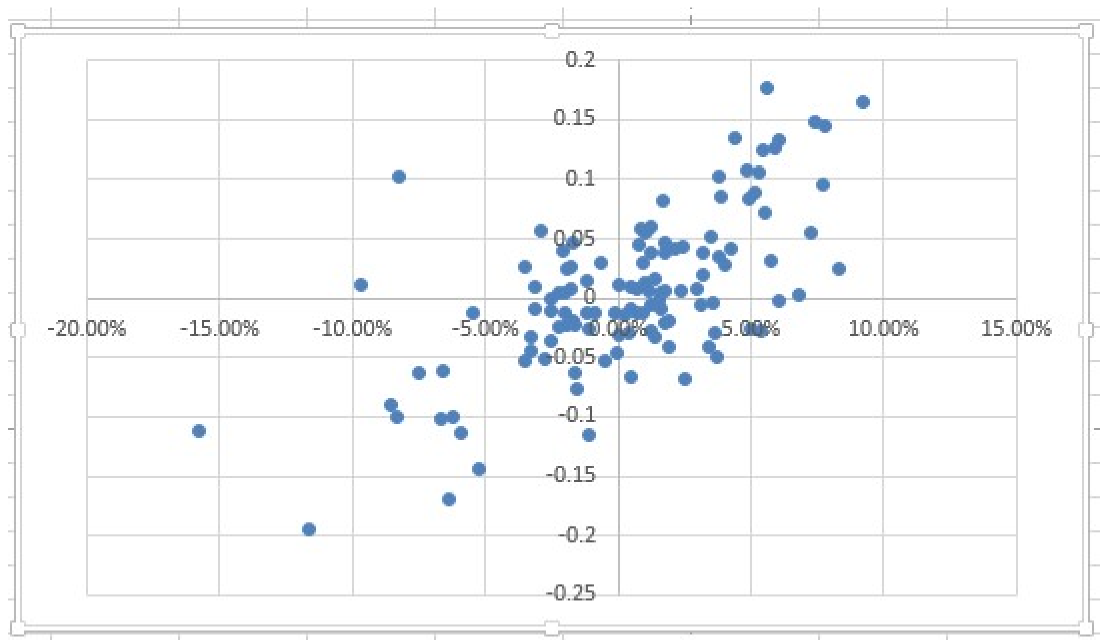


The Model's parameters are estimating using the OLS regression

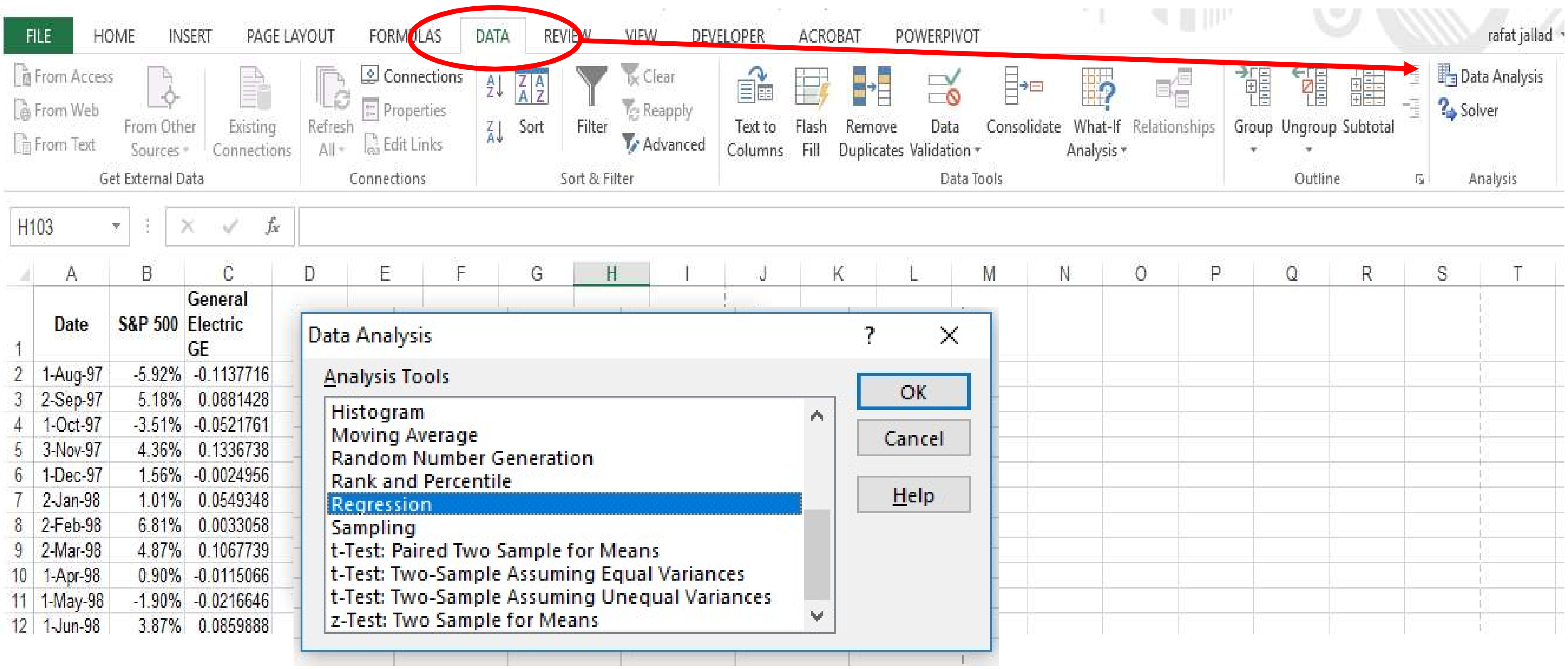
R_m R_i

	Date	S&P 500	General Electric GE
1			
2	1-Aug-97	-5.92%	-0.1137716
3	2-Sep-97	5.18%	0.0881428
4	1-Oct-97	-3.51%	-0.0521761
5	3-Nov-97	4.36%	0.1336738
6	1-Dec-97	1.56%	-0.0024956
7	2-Jan-98	1.01%	0.0549348
8	2-Feb-98	6.81%	0.0033058
9	2-Mar-98	4.87%	0.1067739
10	1-Apr-98	0.90%	-0.0115066
11	1-May-98	-1.90%	-0.0216646
12	1-Jun-98	3.87%	0.0859888
13	1-Jul-98	-1.17%	-0.0125381
14	3-Aug-98	-15.76%	-0.1113932
15	1-Sep-98	6.05%	-0.0018215
16	1-Oct-98	7.72%	0.0951444
17	2-Nov-98	5.74%	0.0322148
18	1-Dec-98	5.48%	0.1243603
19	4-Jan-99	4.02%	0.0279543
20	1-Feb-99	-3.28%	-0.0447494
21	1-Mar-99	3.81%	0.1016781
22	1-Apr-99	3.72%	-0.0486925
23	3-May-99	-2.53%	-0.035482
24	1-Jun-99	5.30%	0.1054313
25	1-Jul-99	-3.26%	-0.0330448
26	2-Aug-99	-0.63%	0.029853
27	1-Sep-99	-2.90%	0.0571584
28	1-Oct-99	6.07%	0.1334937



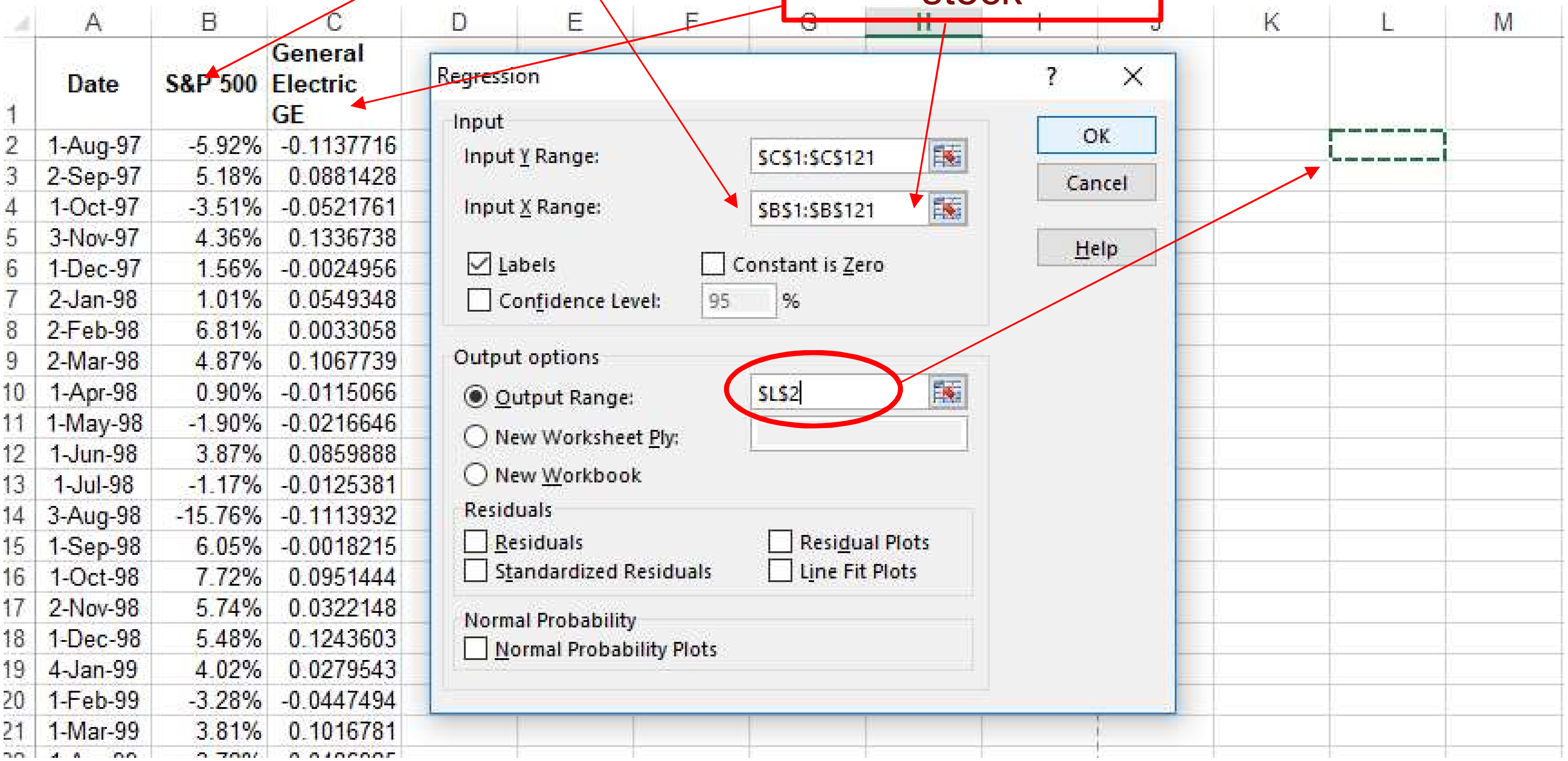


Excel Estimation



R_m: Return on Market Index (S&P)

R_i: Return on GE stock



	Date	S&P 500	General Electric GE
2	1-Aug-97	-5.92%	-0.1137716
3	2-Sep-97	5.18%	0.0881428
4	1-Oct-97	-3.51%	-0.0521761
5	3-Nov-97	4.36%	0.1336738
6	1-Dec-97	1.56%	-0.0024956
7	2-Jan-98	1.01%	0.0549348
8	2-Feb-98	6.81%	0.0033058
9	2-Mar-98	4.87%	0.1067739
10	1-Apr-98	0.90%	-0.0115066
11	1-May-98	-1.90%	-0.0216646
12	1-Jun-98	3.87%	0.0859888
13	1-Jul-98	-1.17%	-0.0125381
14	3-Aug-98	-15.76%	-0.1113932
15	1-Sep-98	6.05%	-0.0018215
16	1-Oct-98	7.72%	0.0951444
17	2-Nov-98	5.74%	0.0322148
18	1-Dec-98	5.48%	0.1243603
19	4-Jan-99	4.02%	0.0279543
20	1-Feb-99	-3.28%	-0.0447494
21	1-Mar-99	3.81%	0.1016781

Estimation Results

Date	S&P 500	General Electric GE
3-Apr-06	1.21%	-0.0056844
1-May-06	-3.14%	-0.0093445
1-Jun-06	0.01%	-0.0313775
3-Jul-06	0.51%	-0.0081582
1-Aug-06	2.11%	0.0410488
1-Sep-06	2.43%	0.0429123
2-Oct-06	3.10%	-0.0055193
1-Nov-06	1.63%	0.0049397
1-Dec-06	1.25%	0.0607269
3-Jan-07	1.40%	-0.0318719
1-Feb-07	-2.21%	-0.0242281
1-Mar-07	0.99%	0.0128997
2-Apr-07	4.24%	0.0415623
1-May-07	3.20%	0.0192131
1-Jun-07	-1.80%	0.0256662
2-Jul-07	1.79%	0.0067691
Varaiance	0.00187541	0.0043859

R^2=SS(Reg)/TSS

Systematic Risk	0.001912	<-- =B123*M19^2
Systematic Risk	0.001912	<-- =N13/M15
Unsystematic Risk	0.002495	<-- =N14/M14

Systematic risk=$\beta^2 * \sigma^2$

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.660225849
R Square	0.435898171
Adjusted R Square	0.431117647
Standard Error	0.049950679
Observations	120

ANOVA

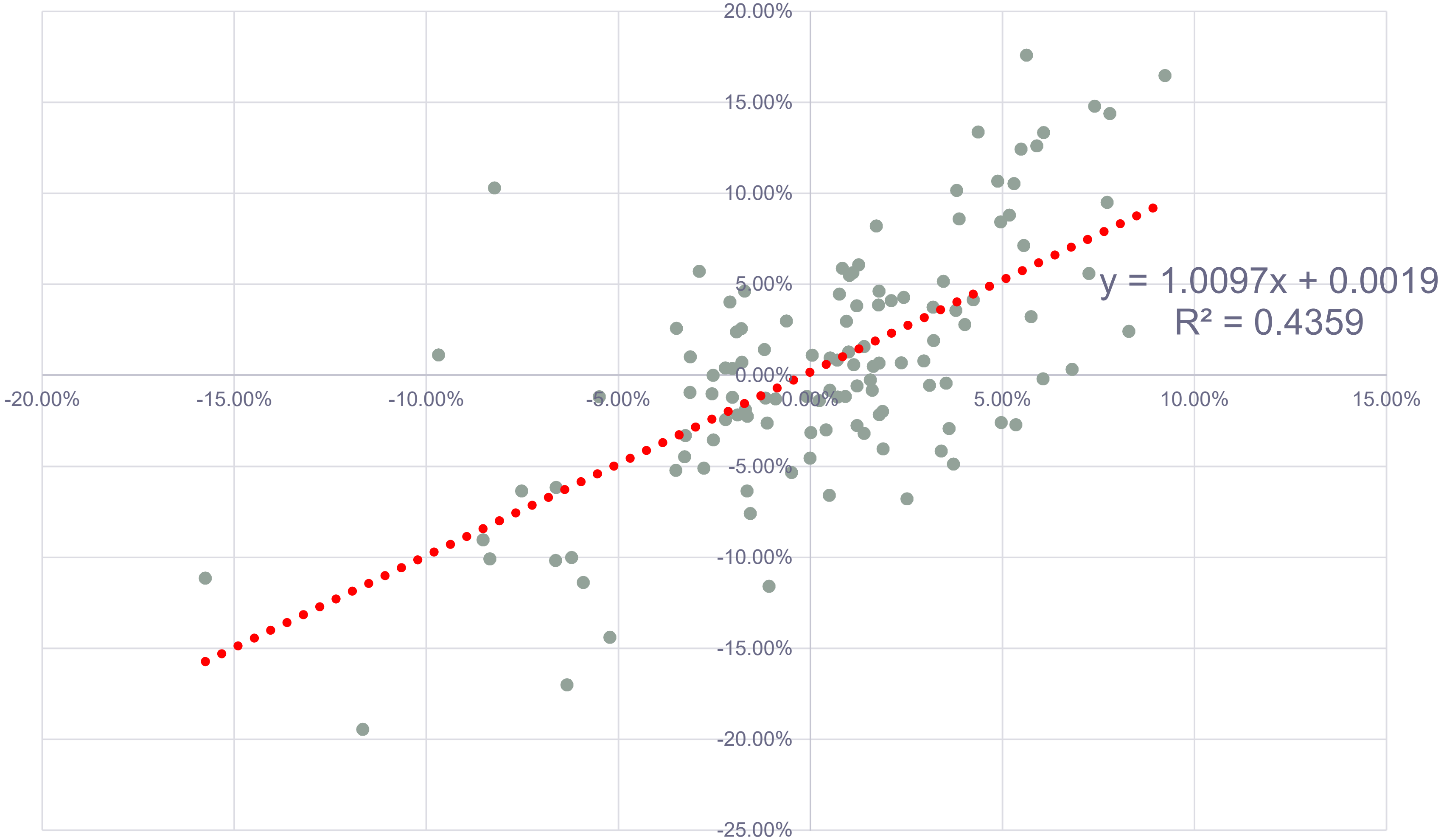
	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.227506	0.227506	91.18209	2.35E-16
Residual	118	0.294418	0.002495		
Total	119	0.521924			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0019	0.0046	0.4222	0.6737	(0.0071)	0.0110	(0.0071)	0.0110
S&P 500	1.0097	0.1057	9.5489	0.0000	0.8003	1.2190	0.8003	1.2190

Beta

Alpha

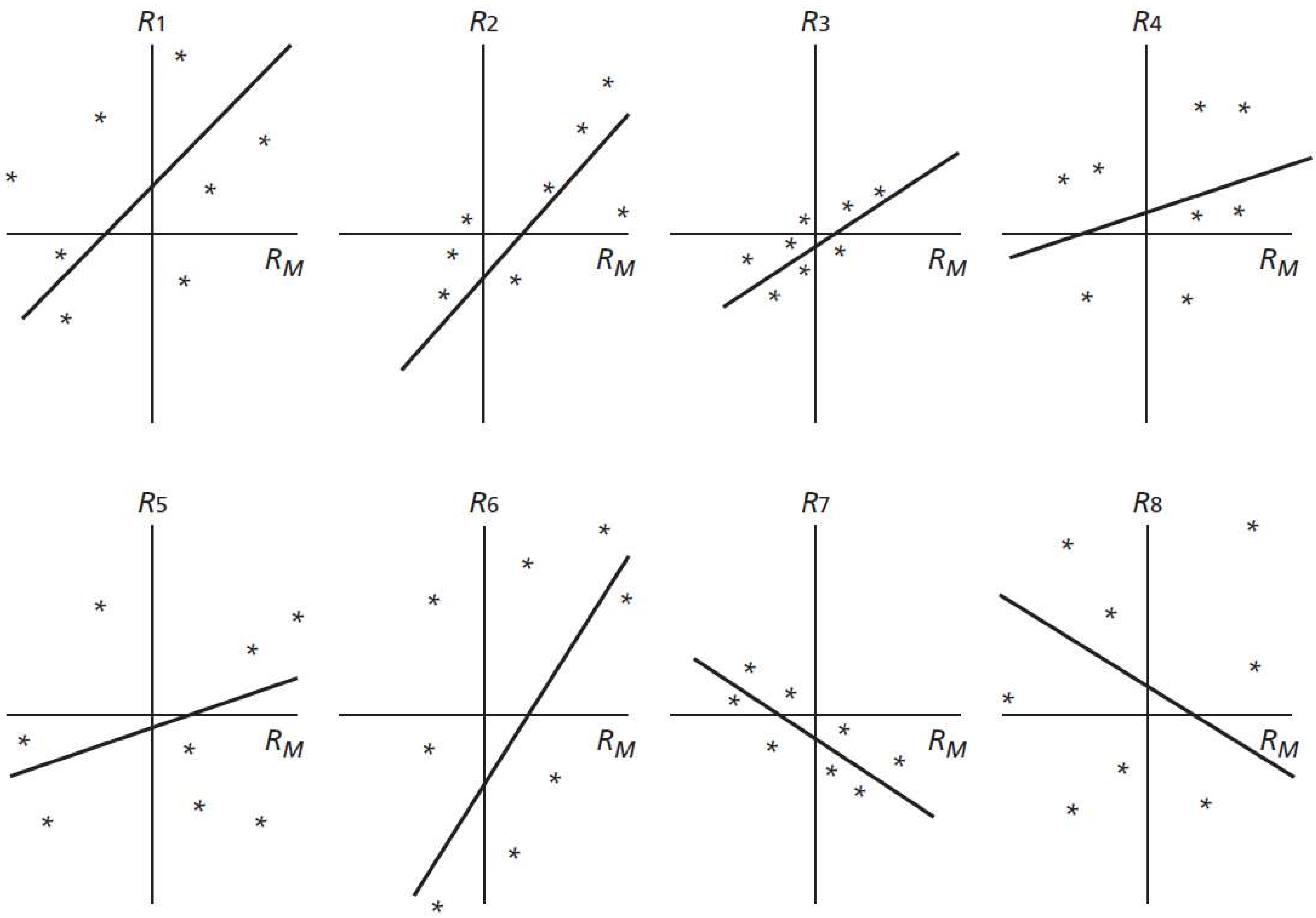
Characteristic Line



6.5 A Single-Index Stock Market

- Statistical and Graphical Representation of Single-Index Model
  - Ratio of systematic variance to total variance
    - $$\rho^2 = \frac{\text{Systematic (or explained) variance}}{\text{Total variance}} = \frac{\beta_D^2\sigma_M^2}{\sigma_D^2} = \frac{\beta_D^2\sigma_M^2}{\beta_D^2\sigma_M^2 + \sigma^2(e_D)}$$

Figure 6.13 Various Scatter Diagrams



A Detailed Example

- ***A. SIM for GE***
- How is the return on an individual stock (GE) driven by the return on an overall market index, *M,* measured by the S&P500 index?
- To answer this:
  - Collect historical data on r_{GE} and r_M
  - Run a simple linear regression of r_{GE} against r_M:

$$r_{GE} = \alpha_{GE} + \beta_{GE} r_M + e_{GE}$$

Example

The fitted regression line is called the ***Security Characteristic Line*** (SCL).

GE Equity BETA
Screen printed.

DG21 **Equity BETA**

HISTORICAL BETA

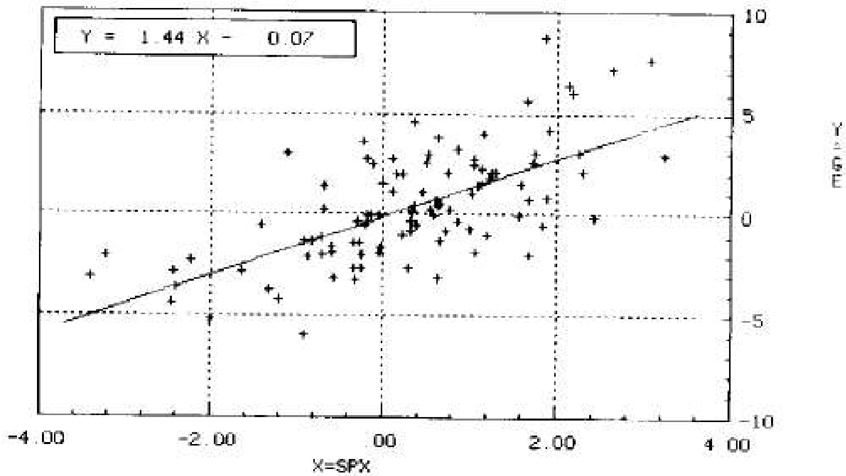
GE **US** GENERAL ELECTRIC CO.

Market SPX S&P 500 INDEX

Period W (D-W-M-Q-Y)
Range 2/18/94 To 2/ 9/96
T (T=Trade,B=Bid,A=Ask)

ADJ BETA	1.29
RAW BETA	1.44
Alpha (Intercept)	-.07
R2 (Correlation)	.44
Std Dev of Error	2.08
Std Error of Beta	.16
Number of Points	103

Adj beta = (0.67) * Raw Beta
+ (0.33) * 1.0



B. Using the SIM to Interpret the Risk of Investing in GE

The variance of GE is $\sigma_{GE}^2 = \beta_{GE}^2 \sigma_M^2 + \sigma^2[e_{GE}]$

Where $\sigma[e_{GE}]$ is the "Std. Dev. Of Error."

Over the sample, $\sigma_M = 1.28\%$ (from other data):

$$\sigma_{GE}^2 = \beta_{GE}^2 \sigma_M^2 + \sigma^2[e_{GE}]$$
$$= (1.44)^2 (1.28)^2 + (2.08)^2 = 3.40 + 4.33 = 7.73$$

$$R^2 = \frac{\beta_{GE}^2 \sigma_M^2}{\sigma_{GE}^2} = \frac{3.40}{7.73} = .44$$

Cont'd

- R^2 is called ***the coefficient of determination***, and it gives the fraction of the variance of the dependent variable (the return on GE) that is explained by movements in the independent variable (the return on the Market portfolio).
- Note that for portfolios, the coefficient of determination from a regression estimation can be used as a measure of diversification (0 min, 1 max).

Cont'd

$$\underset{}{r_{GE}} = \underbrace{\alpha_{GE}}_{\text{constant}} + \underbrace{\beta_{GE} r_M}_{\substack{\text{market-driven,} \\ \text{systematic}}} + \underbrace{e_{GE}}_{\substack{\text{non-market,} \\ \text{firm-specific}}}$$

$$\underbrace{\sigma_{GE}^2}_{\text{Total "risk"}} = \underbrace{\beta_{GE}^2 \sigma_M^2}_{\substack{\text{Market or} \\ \text{systematic risk}}} + \underbrace{\sigma^2(e_{GE})}_{\substack{\text{Non-market,} \\ \text{firm-specific risk}}}$$

$$\underbrace{7.73}_{\text{Total "risk"}} = \underbrace{3.40}_{\substack{\text{Market or} \\ \text{systematic risk}}} + \underbrace{4.33}_{\substack{\text{Non-market,} \\ \text{firm-specific risk}}}$$

Example

MSFT Equity BETA

DG21 Equity **BETA**

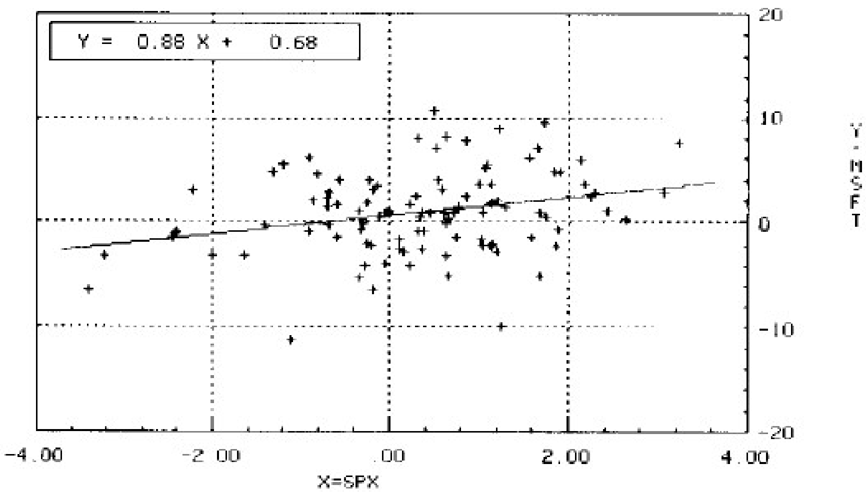
HISTORICAL BETA

MSFT **US** MICROSOFT CORP

Market SPX S&P 500 INDEX

Period W (D-W-M-Q-Y)
Range 2/18/94 To 2/ 9/96
T (T=Trade,B=Bid,A=Ask)

ADJ BETA	.92
RAW BETA	.88
Alpha (Intercept)	.68
R2 (Correlation)	.08
Std Dev of Error	3.81
Std Error of Beta	.30
Number of Points	103



Adj beta = (0.67) * Raw Beta
+ (0.33) * 1.0

The SIM and the Covariance between r_{GE} and r_{MSFT}

$$r_{GE} = \alpha_{GE} + \beta_{GE} r_M + e_{GE}$$

constant

firm-specific

$$r_{MSFT} = \alpha_{MSFT} + \beta_{MSFT} r_M + e_{MSFT}$$

Cont'd

- The *only common influence* driving GE and MSFT is the market return r_M, so can easily calculate the covariance and correlation:

$$Cov\left(r_{GE}, r_{MSFT}\right) = \beta_{GE}\beta_{MSFT}\sigma_M^2 = (.88)(1.44)(1.28)^2 = 2.08$$

$$Corr\left(r_{GE}, r_{MSFT}\right) = 2.08 / \sqrt{7.33 \times 15.79} = 0.19$$

6.5 A Single-Index Stock Market

- Diversification in Single-Index Security Market
  - In portfolio of n securities with weights $w_i \; (\sum_{i=1}^{n} w_i = 1)$
    - In securities with nonsystematic risk $\sigma_{e_i}^2$
    - Nonsystematic portion of portfolio return
      - $e_P = \sum_{i=1}^{n} w_i e_i$
    - Portfolio nonsystematic variance
      - $\sigma_{e_P}^2 = \sum_{i=1}^{n} w_i^2 \sigma_{e_i}^2$

6.7 Selected Problems

Problem 1

A three-asset portfolio has the following characteristics:

Asset	Expected Return	Standard Deviation	Weight
X	15%	22%	0.50
Y	10	8	0.40
Z	6	3	0.10

What is the expected return on this three-asset portfolio?

E(r) = **(0.5 x 15%) + (0.4 x 10%) + (0.1 x 6%)**

E(r) = **12.1%**

Problem 2

George Stephenson's current portfolio of $2.0 million is invested as follows:

Summary of Stephenson's Current Portfolio

	Value	Percent of Total	Expected Annual Return	Annual Standard Deviation
Short-term bonds	$ 200,000	10%	4.6%	1.6%
Domestic large-cap equities	600,000	30	12.4	19.5
Domestic small-cap equities	1,200,000	60	16.0	29.9
Total portfolio	$2,000,000	100%	13.8%	23.1%

Stephenson soon expects to receive an additional $2.0 million and plans to invest the entire amount in an index fund that best complements the current portfolio. Stephanie Coppa, CFA, is evaluating the four index funds shown in the following table for their ability to produce a portfolio that will meet two criteria relative to the current portfolio: (1) maintain or enhance expected return and (2) maintain or reduce volatility.

Each fund is invested in an asset class that is not substantially represented in the current portfolio.

Index Fund Characteristics

Index Fund	Expected Annual Return	Expected Annual Standard Deviation	Correlation of Returns with Current Portfolio
Fund A	15%	25%	+0.80
Fund B	11	22	+0.60
Fund C	16	25	+0.90

State which fund Coppa should recommend to Stephenson. Justify your choice by describing how your chosen fund *best* meets both of Stephenson's criteria. No calculations are required.

Criteria 1:

Eliminate Fund B

Criteria 2:

Choose Fund D

Lowest correlation, best chance of improving return per unit of risk ratio.

Problem 3

Abigail Grace has a $900,000 fully diversified portfolio. She subsequently inherits ABC Company common stock worth $100,000. Her financial advisor provided her with the following forecasted information:

Risk and Return Characteristics		
	Expected Monthly Returns	Standard Deviation of Monthly Returns
Original Portfolio	0.67%	2.37%
ABC Company	1.25	2.95

The correlation coefficient of ABC stock returns with the original portfolio returns is 0.40.

a. The inheritance changes Grace's overall portfolio and she is deciding whether to keep the ABC stock. Assuming Grace keeps the ABC stock, calculate the:

i. Expected return of her new portfolio which includes the ABC stock.

ii. Covariance of ABC stock returns with the original portfolio returns.

iii. Standard deviation of her new portfolio which includes the ABC stock.

a. **Subscript OP refers to the original portfolio, ABC to the new stock, and NP to the new portfolio.**

i. $E(r_{NP}) = w_{OP}\,E(r_{OP}) + w_{ABC}\,E(r_{ABC}) = (0.9 \times 0.67) + (0.1 \times 1.25) = \underline{0.728\%}$

ii $Cov = \rho \times \sigma_{OP} \times \sigma_{ABC} = 0.40 \times .0237 \times .0295 = .00027966 \cong 0.00028$

iii. $\sigma_{NP} = [w_{OP}^2\,\sigma_{OP}^2 + w_{ABC}^2\,\sigma_{ABC}^2 + 2\,w_{OP}\,w_{ABC}\,(Cov_{OP,\,ABC})]^{1/2}$

$= [(0.9^2 \times .0237^2) + (0.1^2 \times .0295^2) + (2 \times 0.9 \times 0.1 \times .00028)]^{1/2}$

$= 2.2673\% \cong \underline{2.27\%}$

Problem 3

Abigail Grace has a $900,000 fully diversified portfolio. She subsequently inherits ABC Company common stock worth $100,000. Her financial advisor provided her with the following forecasted information:

Risk and Return Characteristics

	Expected Monthly Returns	Standard Deviation of Monthly Returns
Original Portfolio	0.67%	2.37%
ABC Company	1.25	2.95

b. If Grace sells the ABC stock, she will invest the proceeds in risk-free government securities yielding 0.42 percent monthly. Assuming Grace sells the ABC stock and replaces it with the government securities, calculate the:

i. Expected return of her new portfolio which includes the government securities.
ii. Covariance of the government security returns with the original portfolio returns.
iii. Standard deviation of her new portfolio which includes the government securities.

b. **Subscript OP refers to the original portfolio, GS to government securities, and NP to the new portfolio.**

i. $E(r_{NP}) = w_{OP}\, E(r_{OP}) + w_{GS}\, E(r_{GS}) =$ $0 \times .0237 \times 0 = 0$

ii. $Cov = \rho \times \sigma_{OP} \times \sigma_{GS} =$ $(0.9 \times 0.67\%) + (0.1 \times 0.42\%) = 0.645\%$

iii. $\sigma_{NP} = [w_{OP}^2\, \sigma_{OP}^2 + w_{GS}^2\, \sigma_{GS}^2 + 2\, w_{OP}\, w_{GS}\, (Cov_{OP\,,\,GS})]^{1/2}$

$= [(0.9^2 \times 0.0237^2) + (0.1^2 \times 0) + (2 \times 0.9 \times 0.1 \times 0)]^{1/2}$

$= 0.9 \text{ x } 0.0237 = 2.133\% \cong 2.13\%$

Problem 3

Abigail Grace has a $900,000 fully diversified portfolio. She subsequently inherits ABC Company common stock worth $100,000. Her financial advisor provided her with the following forecasted information:

Risk and Return Characteristics

	Expected Monthly Returns	Standard Deviation of Monthly Returns
Original Portfolio	0.67%	2.37%
ABC Company	1.25	2.95

c. Determine whether the beta of her new portfolio which includes the government securities will be higher or lower than the beta of her original portfolio.

c. **β_{GS} = 0, so adding the risk-free government securities would result in a lower beta for the new portfolio.**

$$\beta_p = \sum_{i=1}^{n} W_i \beta_i$$

Problem 3

Abigail Grace has a $900,000 fully diversified portfolio. She subsequently inherits ABC Company common stock worth $100,000. Her financial advisor provided her with the following forecasted information:

Risk and Return Characteristics

	Expected Monthly Returns	Standard Deviation of Monthly Returns
Original Portfolio	0.67%	2.37%
ABC Company	1.25	2.95

d. Based on conversations with her husband, Grace is considering selling the $100,000 of ABC stock and acquiring $100,000 of XYZ Company common stock instead. XYZ stock has the same expected return and standard deviation as ABC stock. Her husband comments, "It doesn't matter whether you keep all of the ABC stock or replace it with $100,000 of XYZ stock." State whether her husband's comment is correct or incorrect. Justify your response.

d. **The comment is not correct. Although the respective standard deviations and expected returns for the two securities under consideration are equal, the** covariances **and** correlations **between each security and the original portfolio are unknown, making it impossible to draw the conclusion stated.**

Problem 3

Abigail Grace has a $900,000 fully diversified portfolio. She subsequently inherits ABC Company common stock worth $100,000. Her financial advisor provided her with the following forecasted information:

Risk and Return Characteristics

	Expected Monthly Returns	Standard Deviation of Monthly Returns
Original Portfolio	0.67%	2.37%
ABC Company	1.25	2.95

e. In a recent discussion with her financial adviser, Grace commented, "If I just don't lose money in my portfolio, I will be satisfied." She went on to say, "I am more afraid of losing money than I am concerned about achieving high returns." Describe *one* weakness of using standard deviation of returns as a risk measure for Grace.

- e. **Returns above expected contribute to risk as measured by the standard deviation but her statement indicates she is only concerned about returns sufficiently below expected to generate losses.**

- **However, as long as returns are normally distributed, usage of σ should be fine.**

Problem 4

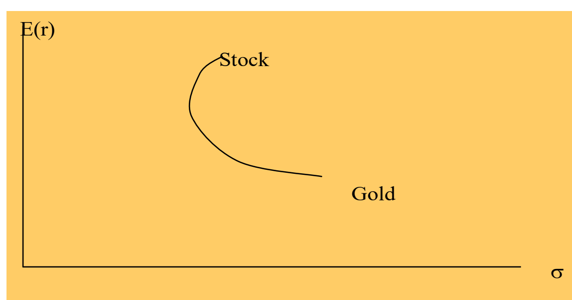
. Stocks offer an expected rate of return of 10% with a standard deviation of 20% and gold offers an expected return of 5% with a standard deviation of 25%.

a. In light of the apparent inferiority of gold to stocks with respect to both mean return and volatility, would anyone hold gold? If so, demonstrate graphically why one would do so.

b. How would you answer (*a*) if the correlation coefficient between gold and stocks were 1.0? Draw a graph illustrating why one would or would not hold gold. Could these expected returns, standard deviations, and correlation represent an equilibrium for the security market?

a. Although it appears that gold is dominated by stocks, gold can still be an attractive diversification asset. If the correlation between gold and stocks is sufficiently low, gold will be held as a component in the optimal portfolio.

b. If gold had a perfectly positive correlation with stocks, gold would not be a part of efficient portfolios. The set of risk/return combinations of stocks and gold would plot as a straight line with a negative slope. (See the following graph.)

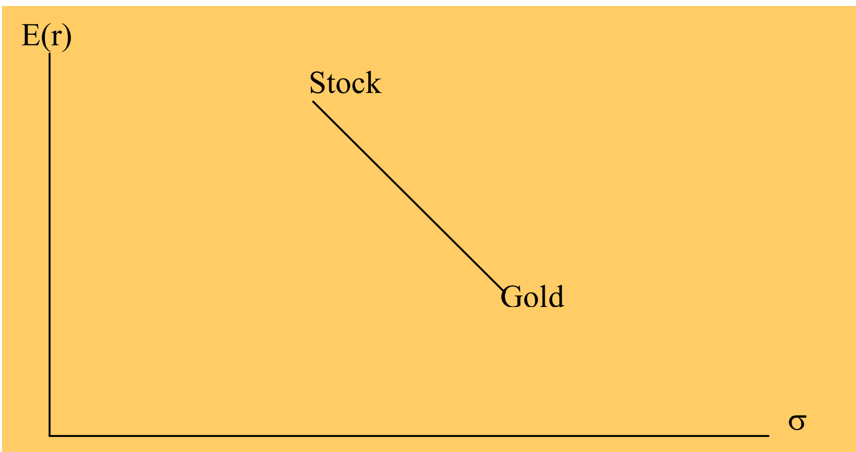


Problem 4

Stocks offer an expected rate of return of 10% with a standard deviation of 20% and gold offers an expected return of 5% with a standard deviation of 25%.

a. In light of the apparent inferiority of gold to stocks with respect to both mean return and volatility, would anyone hold gold? If so, demonstrate graphically why one would do so.

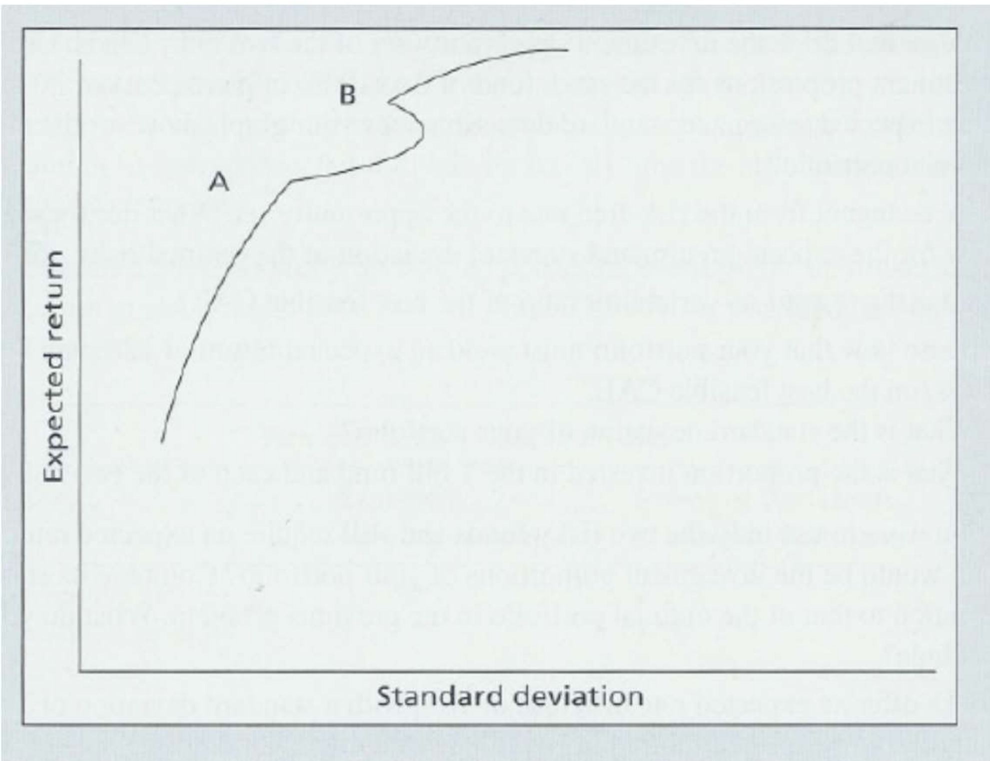
b. How would you answer (*a*) if the correlation coefficient between gold and stocks were 1.0? Draw a graph illustrating why one would or would not hold gold. Could these expected returns, standard deviations, and correlation represent an equilibrium for the security market?



- The graph shows that the stock-only portfolio dominates any portfolio containing gold.
- This cannot be an equilibrium; the price of gold must fall and its expected return must rise.

Problem 5

Your assistant gives you the following diagram, see next page, as the efficient frontier of the group of stocks you asked him to analyze. The diagram looks a bit odd, but your assistant insists he got the diagram from his analysis. Would you trust him? Is it possible to get such a diagram?



- No, it is not possible to get such a diagram.
- Even if the correlation between A and B were 1.0, the frontier would be a straight line connecting A and B.

Problem 6

. Investors expect the market rate of return this year to be 10%. The expected rate of return on a stock with a beta of 1.2 is currently 12%. If the market return this year turns out to be 8%, how would you revise your expectation of the rate of return on the stock?

- **The expected rate of return on the stock will change by beta times the unanticipated change in the market return:**

  $1.2 \times (8\% - 10\%) = -2.4\%$

- **Therefore, the expected rate of return on the stock should be revised to:**

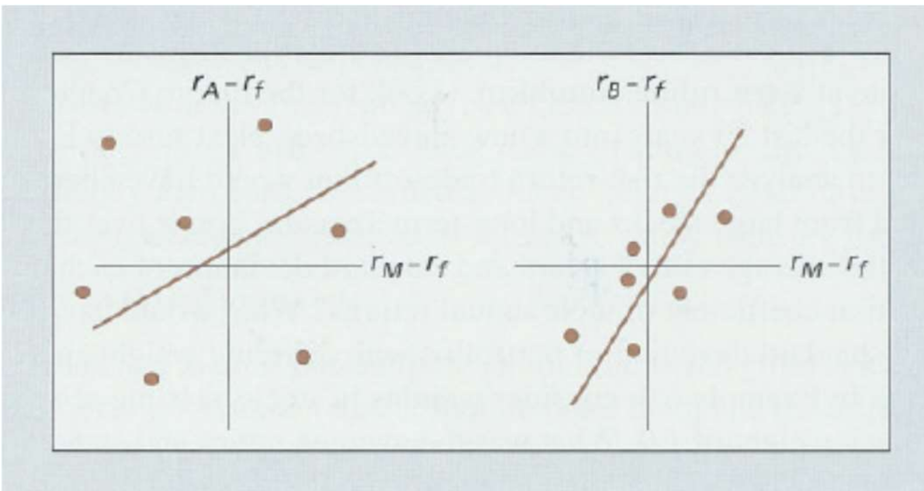
  $12\% - 2.4\% = 9.6\%$

Problem 7

The following figure shows plots of monthly rates of return and the stock market for two stocks.

a. Which stock is riskiest to an investor currently holding her portfolio in a diversified portfolio of common stock?

b. Which stock is riskiest to an undiversified investor who puts all of his funds in only one of these stocks?



b. The undiversified investor is exposed to both firm-specific and systematic risk. Stock A has higher firm-specific risk because the deviations of the observations from the SCL are larger for Stock A than for Stock B.

Stock A may therefore be riskier to the undiversified investor.

a. The risk of the diversified portfolio consists primarily of systematic risk. Beta measures systematic risk, which is the slope of the security characteristic line (SCL). The two figures depict the stocks' SCLs. Stock B's SCL is steeper, and hence Stock B's systematic risk is greater. The slope of the SCL, and hence the systematic risk, of Stock A is lower. Thus, for this investor, stock B is the riskiest.