An alternative technique for the generation of a frequency-modulated signal which permits the use of crystal control is called the Indirect method. In this technique the phase angle is caused to vary while holding the frequency constant. What is really generated by this technique is what is called a *phase-modulated signal.* With some minor doctoring, this phase-modulated signal can be passed off as an FM signal, and it is.

Frequency Multipliers

Most often, FM signals are initially generated in low-power circuits and circuits providing frequency deviations which are too small to meet FCC requirements.

The mathematical description of a frequency-modulated signal is

$$v = A\sin\left[2\pi(f_c + \Delta f\sin(2\pi f_a t))t\right] \tag{4.11}$$

The frequency is

$$f = f_c + \Delta f\sin 2\pi f_a t \tag{4.12}$$

Any means which will multiply the frequency of the FM signal by S will produce a new signal having a frequency deviation of $S\Delta f$.

$$Sf = S(f_c + \Delta f\sin 2\pi f_a t)$$
$$Sf = Sf_c + S\Delta f\sin 2\pi f_a t$$

Thus,

$$\Delta f_{\text{new}} = S\Delta f_{\text{old}} \tag{4.13}$$

with a new center frequency of $Sf_{c\,\text{old}}$.

Frequency multiplication is not difficult to obtain since harmonics which are generated by nonlinear devices such as class C amplifiers and varactor diodes provide outputs rich in harmonics, harmonics being signals having a frequency which is an integer multiple of the fundamental which is the input signal. It then becomes merely a chore of choosing the appropriate harmonic by using a frequency-selective circuit.

Frequency multipliers are generally limited by practical considerations to multiplications by 2, 3, or 4. Larger multiplication factors may be obtained by cascading these smaller multipliers.

Heterodyning

It sometimes becomes necessary to be able to adjust the frequency of the modulated signal without affecting frequency deviation. This can be accomplished by mixing, beating, or heterodyning, all three terms meaning the same thing. This is the same process used in the superheterodyne receiver to generate the intermediate frequency by heterodyning the local oscillator signal with the received signal.

The difference between heterodyning and multiplying is that in heterodyning the sinusoid angle is *added to* or *subtracted from*, while a multiplier *multiplies* the sinusoid angle by some factor. It is not unusual to find frequency multipliers followed by a heterodyner in an FM transmitter.

Solved Problems

4.1 A 107.6-MHz carrier is frequency modulated by a 7-kHz sine wave. The resultant FM signal has a frequency deviation of 50 kHz.

(*a*) Find the carrier swing of the FM signal.

(*b*) Determine the highest and lowest frequencies attained by the modulated signal.

(*c*) What is the modulation index of the FM wave?

SOLUTION

Given: $f_c = 107.6\,\text{MHz}$
$f_a = 7\,\text{kHz}$
$\Delta f = 50\,\text{kHz}$

Find: (*a*) c.s. (*b*) f_H, f_L (*c*) m_f

(*a*) Relating carrier swing to frequency deviation

$$\text{c.s.} = 2\Delta f$$
$$= 2 \times 50 \times 10^3$$
$$\boxed{\text{c.s.} = 100\,\text{kHz}}$$

(*b*) The upper frequency reached is equal to the rest or carrier frequency plus the frequency deviation:

$$f_H = f_c + \Delta f$$
$$= 107.6 \times 10^6 + 50 \times 10^3$$
$$= (107\,600 \times 10^3) + (50 \times 10^3)$$
$$= 107\,650 \times 10^3$$
$$\boxed{f_H = 107.65\,\text{MHz}}$$

The lowest frequency reached by the modulated wave is equal to the rest or carrier frequency minus the frequency deviation.

$$f_L = f_c - \Delta f$$
$$= 107.6 \times 10^6 - 50 \times 10^3$$
$$= 107\,600 \times 10^3 - 50 \times 10^3$$
$$= 107\,550 \times 10^3$$
$$\boxed{f_L = 107.55\,\text{MHz}}$$

(*c*) The modulation index is determined by

$$m_f = \frac{\Delta f}{f_a}$$
$$= \frac{50 \times 10^3}{7 \times 10^3}$$
$$\boxed{m_f = 7.143}$$

4.2 Determine the frequency deviation and carrier swing for a frequency-modulated signal which has a resting frequency of 105.000 MHz and whose upper frequency is 105.007 MHz when modulated by a particular wave. Find the lowest frequency reached by the FM wave.

SOLUTION

Given: $f_0 = 105.000\,\text{MHz}$
$f_{\text{upper}} = 105.007\,\text{MHz}$

Find: Δf, c.s., f_{lower}

Frequency deviation is defined as the maximum change in frequency of the modulated signal away from the rest or carrier frequency.

$$\Delta f = (105.007 - 105.000) \times 10^6$$
$$= 0.007 \times 10^6$$
$$= 7000$$
$$\boxed{\Delta f = 7\,\text{kHz}}$$

Carrier swing can now be determined by

$$\text{c.s.} = 2\Delta f$$
$$= 2(7 \times 10^3)$$
$$= 14 \times 10^3$$
$$\boxed{\text{c.s.} = 14\,\text{kHz}}$$

The lowest frequency reached by the modulated wave can be found by subtracting the frequency deviation from the carrier or rest frequency.

$$f_{\text{lower}} = f_0 - \Delta_f$$
$$= (105.000 - 0.007) \times 10^6$$
$$\boxed{f_{\text{lower}} = 104.993\,\text{MHz}}$$

4.3 What is the modulation index of an FM signal having a carrier swing of 100 kHz when the modulating signal has a frequency of 8 kHz?

SOLUTION

Given: c.s. = 100 kHz
$f_a = 8$ kHz

Find: m_f

From the defining equation,

$$m_f = \frac{\Delta f}{f_a}$$

First determining Δf,

$$\Delta f = \frac{\text{c.s.}}{2}$$
$$= \frac{100 \times 10^3}{2}$$
$$= 50\,\text{kHz}$$

Now substituting into the equation for m_f,

$$m_f = \frac{50 \times 10^3}{8 \times 10^3}$$
$$\boxed{m_f = 6.25}$$

4.4 A frequency-modulated signal which is modulated by a 3-kHz sine wave reaches a maximum frequency of 100.02 MHz and minimum frequency of 99.98 MHz.

(*a*) Determine the carrier swing.
(*b*) Find the carrier frequency.
(*c*) Calculate the frequency deviation of the signal.
(*d*) What is the modulation index of the signal?

SOLUTION

Given:
$$f_{max} = 100.02\ \text{MHz}$$
$$f_{min} = 99.98\ \text{MHz}$$
$$f_a = 3\ \text{kHz}$$

Find: (*a*) c.s. (*b*) f_c (*c*) Δf (*d*) m_f

(*a*) The carrier swing is defined as the total variation in frequency from the highest to lowest reached by the modulated wave.

$$\begin{aligned} \text{c.s.} &= f_{max} - f_{min} \\ &= 100.02 \times 10^6 - 99.98 \times 10^6 \\ &= 0.04 \times 10^6 \\ &= 40 \times 10^3 \end{aligned}$$

$$\boxed{\text{c.s.} = 40\ \text{kHz}}$$

(*b*) The carrier frequency or rest frequency is midway between the maximum frequency and minimum frequency reached by the modulated wave.

$$\begin{aligned} f_c &= \frac{f_{max} + f_{min}}{2} \\ &= \frac{100.02 \times 10^6 + 99.98 \times 10^6}{2} \\ &= 100 \times 10^6 \end{aligned}$$

$$\boxed{f_c = 100.00\ \text{MHz}}$$

(*c*) Since the carrier swing is equal to twice the frequency deviation,

$$\begin{aligned} \Delta f &= \frac{\text{c.s.}}{2} \\ &= \frac{40 \times 10^3}{2} \end{aligned}$$

$$\boxed{\Delta f = 20\ \text{kHz}}$$

(*d*) The modulation index for a frequency modulated wave is defined as

$$\begin{aligned} m_f &= \frac{\Delta f}{f_a} \\ &= \frac{20 \times 10^3}{3 \times 10^3} \end{aligned}$$

$$\boxed{m_f = 6.667}$$

4.5 An FM transmission has a frequency deviation of 20 kHz.

(*a*) Determine the percent modulation of this signal if it is broadcast in the 88–108 MHz band.

(*b*) Calculate the percent modulation if this signal were broadcast as the audio portion of a television broadcast.

SOLUTION

Given: $\Delta f = 20\,\text{kHz}$

Find: (*a*) Percent modulation—FM broadcast band

(*b*) Percent modulation—TV

(*a*) Percent modulation for an FM wave is defined as

$$M = \frac{\Delta f_{\text{actual}}}{\Delta f_{\text{max}}} \times 100$$

The maximum frequency deviation in the FM broadcast band permitted by the FCC is 75 kHz:

$$M = \frac{20 \times 10^3}{75 \times 10^3} \times 100$$

$$\boxed{M = 26.67\%}$$

(*b*)

$$M = \frac{\Delta f_{\text{actual}}}{\Delta f_{\text{max}}} \times 100$$

The maximum frequency deviation for the FM audio portion of a TV broadcast is 25 kHz as set by the FCC.

$$M = \frac{20 \times 10^3}{25 \times 10^3} \times 100$$

$$\boxed{M = 80.0\%}$$

4.6 (*a*) What is the frequency deviation and carrier swing necessary to provide 75% modulation in the FM broadcast band?

(*b*) Repeat for an FM signal serving as the audio portion of a TV broadcast.

SOLUTION

Given: $M = 75\%$

Find: (*a*) Δf_{FM}, c.s.$_{\text{FM}}$ (*b*) Δf_{TV}, c.s.$_{\text{TV}}$

(*a*) Frequency deviation is defined as

$$M = \frac{\Delta f_{\text{actual}}}{\Delta f_{\text{max}}} \times 100$$

The maximum frequency deviation permitted in the FM broadcast band, 88–108 MHz, by the FCC is 75 kHz.

$$75 = \frac{\Delta f_{\text{FM}}}{75 \times 10^3} \times 100$$

$$\Delta f_{\text{FM}} = \frac{75 \times 75 \times 10^3}{100}$$

$$= 56.25 \times 10^3$$

$$\boxed{\Delta f_{\text{FM}} = 56.25\,\text{kHz}}$$

Carrier swing is related to frequency deviation by

$$\text{c.s.}_{\text{FM}} = 2\Delta f_{\text{FM}}$$
$$= 2 \times 56.25 \times 10^3$$
$$\boxed{\text{c.s.}_{\text{FM}} = 112.5\,\text{kHz}}$$

(*b*)
$$M = \frac{\Delta f_{\text{actual}}}{\Delta f_{\text{max}}} \times 100$$

The maximum frequency deviation permitted by the FCC for the audio portion of a TV signal is 25 kHz. Thus,

$$75 = \frac{\Delta f_{\text{TV}}}{25 \times 10^3} \times 100$$
$$\Delta f_{\text{TV}} = \frac{75 \times 25 \times 10^3}{100}$$
$$\boxed{\Delta f_{\text{TV}} = 18.75\,\text{kHz}}$$
$$\text{c.s.}_{\text{TV}} = 2\Delta f_{\text{TV}}$$
$$= 2 \times 18.75 \times 10^3$$
$$\boxed{\text{c.s.}_{\text{TV}} = 37.5\,\text{kHz}}$$

4.7 Determine the percent modulation of an FM signal which is being broadcast in the 88–108 MHz band, having a carrier swing of 125 kHz.

SOLUTION

Given: c.s. = 125 kHz

Find: M

Frequency deviation and carrier swing are related by

$$\Delta f = \frac{\text{c.s.}}{2}$$
$$= \frac{125 \times 10^3}{2}$$
$$= 62.5\,\text{kHz}$$
$$M = \frac{\Delta f_{\text{actual}}}{\Delta f_{\text{max}}} \times 100$$

Maximum frequency deviation for the FM broadcast band permitted by the FCC is 75 kHz:

$$M = \frac{62.5 \times 10^3}{75 \times 10^3} \times 100$$
$$\boxed{M = 83.3\%}$$

4.8 The percent modulation of the sound portion of a TV signal is 80%. Determine the frequency deviation and carrier swing of the signal.

SOLUTION

Given: $M = 80\%$

Find: Δf, c.s.

The percent modulation of an FM signal is

$$M = \frac{\Delta f_{\text{actual}}}{\Delta f_{\max}} \times 100$$

The maximum frequency deviation for the sound portion of a TV signal as specified by the FCC is 25 kHz. Thus,

$$80 = \frac{\Delta f_{\text{actual}}}{25 \times 10^3} \times 100$$

$$\Delta f_{\text{actual}} = \frac{80 \times 25 \times 10^3}{100}$$

$$\boxed{\Delta f_{\text{actual}} = 20\,\text{kHz}}$$

Carrier swing is related to frequency deviation by

$$\text{c.s.} = 2\Delta f_{\text{actual}}$$
$$= 2 \times 20 \times 10^3$$

$$\boxed{\text{c.s.} = 40\,\text{kHz}}$$

4.9 A 5-kHz audio tone is used to modulate a 50-MHz carrier causing a frequency deviation of 20 kHz. Determine (*a*) the modulation index and (*b*) the bandwidth of the FM signal.

SOLUTION

Given: $f_a = 5\,\text{kHz}$
$f_c = 50.0\,\text{MHz}$
$\Delta f = 20\,\text{kHz}$

Find: (*a*) m_f (*b*) BW

(*a*) Modulation index is defined as

$$m_f = \frac{\Delta f}{f_a}$$
$$= \frac{20 \times 10^3}{5 \times 10^3}$$

$$\boxed{m_f = 4}$$

(*b*) Referring to the Schwartz bandwidth curve, Fig. 4-3, and entering on the horizontal axis with $m_f = 4$, it is found that

$$\frac{\text{BW}}{\Delta f} = 3.8$$

This is shown in Fig. 4-11.

Substituting 20×10^3 for Δf as given,

$$\frac{\text{BW}}{20 \times 10^3} = 3.8$$

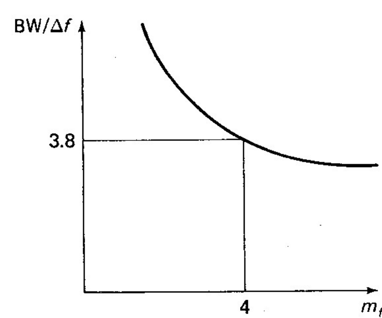

Fig. 4-11

Solving for BW,

$$\mathrm{BW} = 3.8 \times 20 \times 10^3$$
$$= 76 \times 10^3$$

$$\boxed{\mathrm{BW} = 76\,\mathrm{kHz}}$$

4.10 Determine the frequency of the modulating signal which is producing an FM signal having a bandwidth of 50 kHz when the frequency deviation of the FM signal is 10 kHz.

SOLUTION

Given: $\mathrm{BW} = 50\,\mathrm{kHz}$
$\Delta f = 10\,\mathrm{kHz}$

Find: f_a

In order to find f_a, reference must be made to the Schwartz bandwidth curve, Fig. 4-3. In order to enter this curve, determine BW/Δf:

$$\frac{\mathrm{BW}}{\Delta f} = \frac{50 \times 10^3}{10 \times 10^3}$$
$$= 5$$

From Fig. 4-3,

$$m_f = 2$$
$$= \frac{\Delta f}{f_a}$$

So,

$$2 = \frac{10 \times 10^3}{f_a}$$
$$f_a = \frac{10 \times 10^3}{2}$$

$$\boxed{f_a = 5\,\mathrm{kHz}}$$

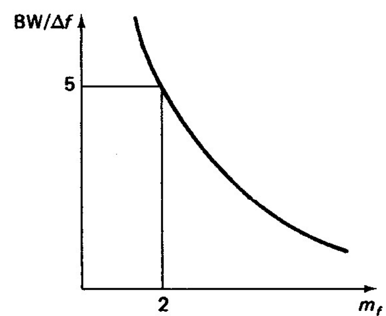

Fig. 4-12

This is shown in Fig. 4-12.

4.11 A 103.0-MHz carrier is frequency modulated by a 10-kHz sine wave. Determine the modulation index of the FM signal. Referring to Schwartz's curve, Fig. 4-3, determine the bandwidth when the carrier swing is 80 kHz.

SOLUTION

Given: $f_c = 103.0\,\mathrm{MHz}$
$f_a = 10\,\mathrm{kHz}$
c.s. $= 80\,\mathrm{kHz}$

Find: m_f, BW

The defining equation for the modulation index is

$$m_f = \frac{\Delta f}{f_a}$$

However, before using this equation, it is necessary to determine the frequency deviation, Δf.

$$\Delta f = \frac{\text{c.s.}}{2}$$
$$= \frac{80\,\text{kHz}}{2}$$
$$= 40\,\text{kHz}$$

Returning to the defining equation for modulation index:

$$m_f = \frac{40 \times 10^3}{10 \times 10^3}$$

$$\boxed{m_f = 4}$$

Entering the Schwartz bandwidth curve of Fig. 4.3 with $m_f = 4$ results in

$$\frac{\text{BW}}{\Delta f} = 3.5$$
$$\text{BW} = 3.5 \times 40 \times 10^3$$
$$= 140 \times 10^3$$

$$\boxed{\text{BW} = 140\,\text{kHz}}$$

4.12 If a 6-MHz band were being considered for use with the same standards that apply to the 88–108 MHz band, how many FM stations could be accommodated?

SOLUTION

Given: BW = 6 MHz

Find: Number of stations

Each station requires a total bandwidth of 400 kHz; 150 kHz for the signal and a 25-kHz guard band above and below with only alternate channels used.

$$\text{Number of stations} = \frac{6 \times 10^6}{400 \times 10^3}$$

$$\boxed{\text{Number of stations} = 15}$$

4.13 Determine the bandwidth of a narrowband FM signal which is generated by a 4-kHz audio signal modulating a 125-MHz carrier.

SOLUTION

Given: Narrowband FM

$f_a = 4\,\text{kHz}$

$f_c = 125\,\text{MHz}$

Find: BW

Since this is a narrowband FM signal, the bandwidth is found merely by doubling the modulating frequency:

$$\text{BW} = 2f_a$$
$$= 2 \times 4 \times 10^3$$

$$\boxed{\text{BW} = 8\,\text{kHz}}$$