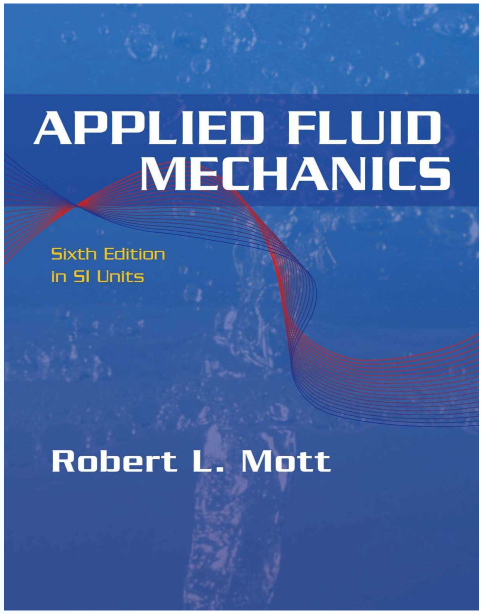
Applied Fluid Mechanics



1. The Nature of Fluid and the Study of Fluid Mechanics
2. Viscosity of Fluid
3. Pressure Measurement
4. Forces Due to Static Fluid
5. Buoyancy and Stability
6. Flow of Fluid and Bernoulli's Equation
7. General Energy Equation
8. Reynolds Number, Laminar Flow, Turbulent Flow and Energy Losses Due to Friction

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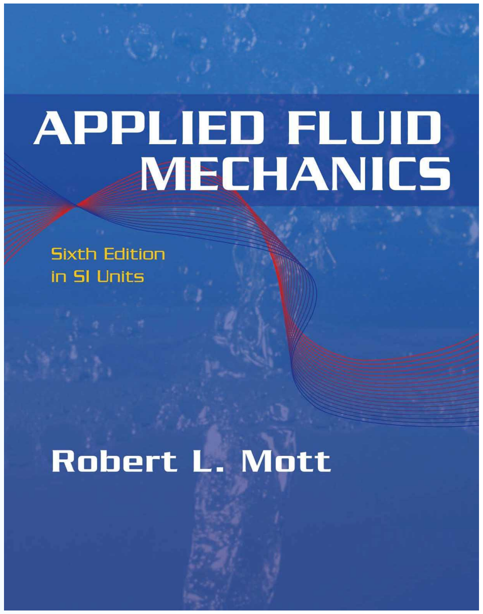

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections
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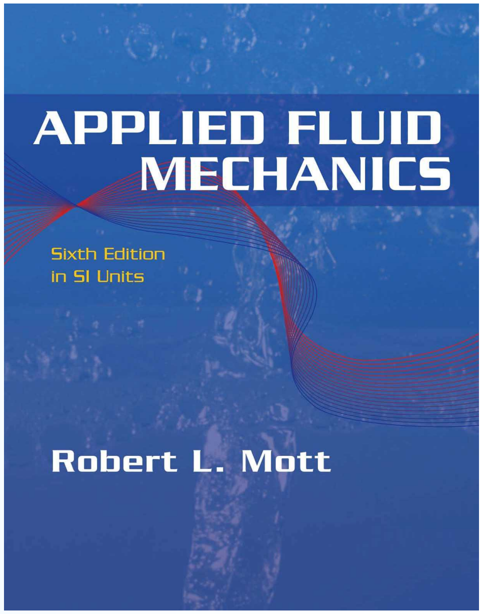


Chapter Objectives

- Describe the appearance of laminar flow and turbulent flow.
- State the relationship used to compute the Reynolds number.
- Identify the limiting values of the Reynolds number by which you can predict whether flow is laminar or turbulent.
- Compute the Reynolds number for the flow of fluids in round pipes and tubes.
- State *Darcy's equation* for computing the energy loss due to friction for either laminar or turbulent flow.

Chapter Objectives

- State the *Hagen–Poiseuille equation* for computing the energy loss due to friction in laminar flow.
- Define the *friction factor* as used in Darcy's equation.
- Determine the friction factor using Moody's diagram for specific values of Reynolds number and the relative roughness of the pipe.

Chapter Outline

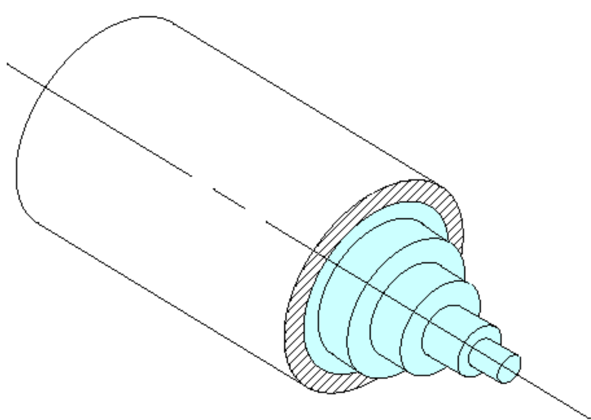
1. Introductory Concepts
2. Reynolds Number
3. Critical Reynolds Numbers
4. Darcy’s Equation
5. Friction Loss in Laminar Flow
6. Friction Loss in Turbulent Flow
7. Equations for the Friction Factor
8. Hazen-Williams Formula for Water Flow
9. Other Forms of the Hazen-Williams Formula
10. Nomograph for Solving the Hazen-Williams Formula

8.1 Introductory Concepts

- As the water flows from a faucet at a very low velocity, the flow appears to be smooth and steady. The stream has a fairly uniform diameter and there is little or no evidence of mixing of the various parts of the stream. This is called *laminar flow*.
- When the faucet is nearly fully open, the water has a rather high velocity. The elements of fluid appear to be mixing chaotically within the stream. This is a general description of *turbulent flow*.

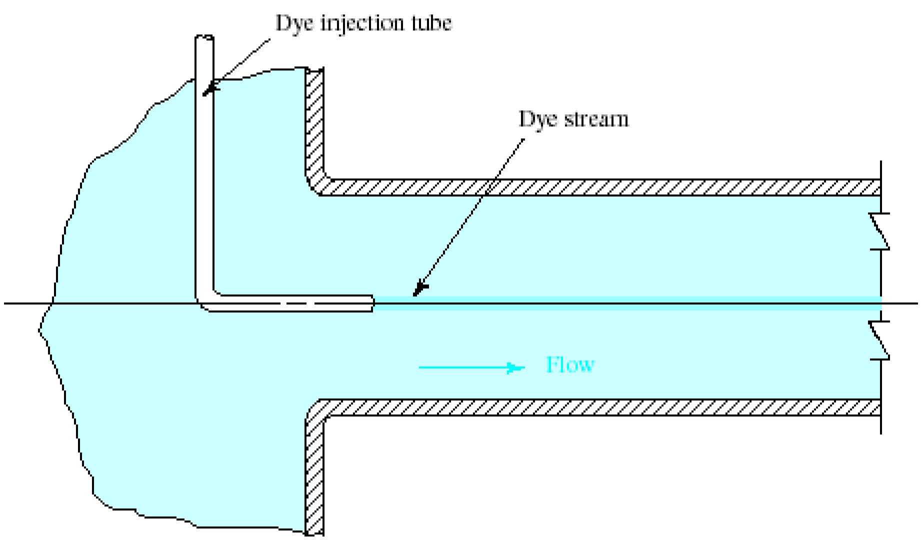
8.1 Introductory Concepts

- Figure 8.1 shows one way of visualizing laminar flow in a circular pipe.
- Concentric rings of fluid are flowing in a straight, smooth path.
- There is little or no mixing of the fluid across the "boundaries" of each layer as the fluid flows along in the pipe.



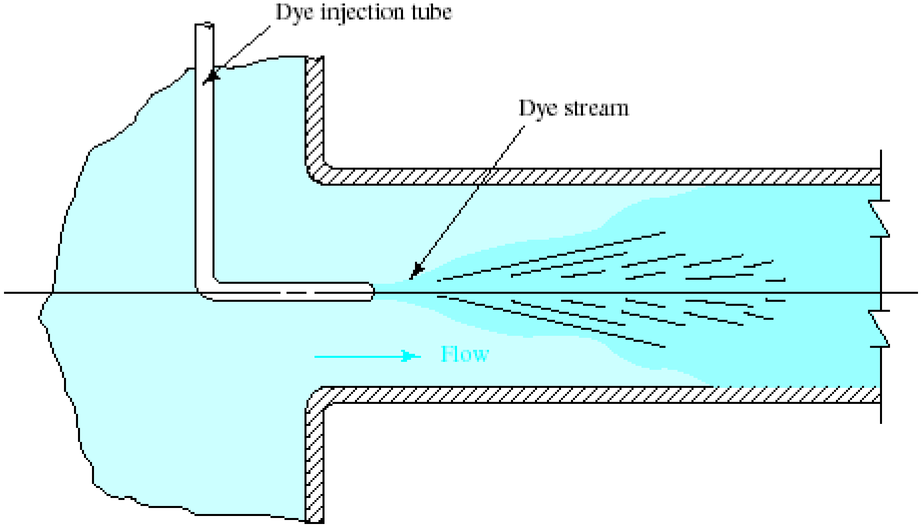
8.1 Introductory Concepts

- Figure 8.2 shows a transparent fluid such as water flowing in a clear glass tube.
- When a stream of a dark fluid such as a dye is injected into the flow, the stream remains intact as long as the flow remains laminar.
- The dye stream will not mix with the bulk of the fluid.



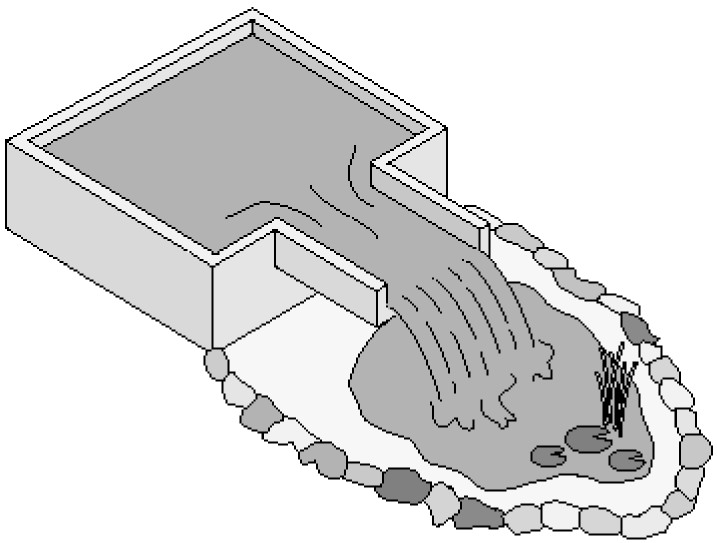
8.1 Introductory Concepts

- Figure 8.3 shows that when a dye stream is introduced into turbulent flow, it immediately dissipates throughout the primary fluid.



8.1 Introductory Concepts

- Figure 8.4 shows a reservoir discharging fluid into an open channel that eventually allows the stream to fall into a lower pool.



8.2 Reynolds Number

- Osborne Reynolds was the first to demonstrate that laminar or turbulent flow can be predicted if the magnitude of a dimensionless number, now called the Reynolds number is known.
- The following equation shows the basic definition of the Reynolds number, N_R:

$$N_R = \frac{vD\rho}{\mu} = \frac{vD}{\nu} \qquad (8\text{–}1)$$

where fluid density ρ, fluid viscosity v, pipe diameter *D*, and average velocity of flow v.

8.2 Reynolds Number

- Table 8.1 lists the required units in both the SI metric unit system and the U.S. Customary unit system.
- Converting to these standard units prior to entering data into the calculation for is recommended.

Quantity	SI Units	U.S. Customary Units
Velocity	m/s	ft/s
Diameter	m	ft
Density	kg/m^3 or $N{\cdot}s^2/m^4$	$slugs/ft^3$ or $lb{\cdot}s^2/ft^4$
Dynamic viscosity	$N{\cdot}s/m^2$ or $Pa{\cdot}s$ or $kg/m{\cdot}s$	$lb{\cdot}s/ft^2$ or $slugs/ft{\cdot}s$
Kinematic viscosity	m^2/s	ft^2/s

8.2 Reynolds Number

- We can demonstrate that the Reynolds number is dimensionless by substituting standard SI units into Eq. (8–1):

$$N_R = \frac{vD\rho}{\mu} = v \times D \times \rho \times \frac{1}{\mu}$$

$$N_R = \frac{\text{m}}{\text{s}} \times \text{m} \times \frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}\cdot\text{s}}{\text{kg}}$$

- Because all units can be cancelled, N_R is dimensionless.

8.2 Reynolds Number

- The Reynolds number is the ratio of the inertia force on an element of fluid to the viscous force.
- The inertia force is developed from Newton's second law of motion, $F = ma$.

8.3 Critical Reynolds Number

- For practical applications in pipe flow we find that if the Reynolds number for the flow is less than 2000, the flow will be laminar.
- If the Reynolds number is greater than 4000, the flow can be assumed to be turbulent.
- In the range of Reynolds numbers between 2000 and 4000, it is impossible to predict which type of flow exists; therefore this range is called the *critical region*.
- We will assume the following:

If $N_R < 2000$, the flow is laminar.

If $N_R > 4000$, the flow is turbulent.

Example 8.1

Determine whether the flow is laminar or turbulent if glycerine at 25°C flows in a pipe with a 150-mm inside diameter. The average velocity of flow is 3.6 m/s.

We must first evaluate the Reynolds number using Eq. (8–1):

$$N_R = vD\rho/\mu$$

$$v = 3.6 \text{ m/s}$$

$$D = 0.15 \text{ m}$$

$$\rho = 1258 \text{ kg/m}^3 \qquad \text{(from Appendix B)}$$

$$\mu = 9.60 \times 10^{-1} \text{ Pa·s} \qquad \text{(from Appendix B)}$$

$$N_R = \frac{(3.6)(0.15)(1258)}{9.60 \times 10^{-1}} = 708$$

Example 8.1

Because N_R = 708, which is less than 2000, the flow is laminar. Notice that each term was expressed in consistent SI units before N_R was evaluated.

Example 8.2

Determine whether the flow is laminar or turbulent if water at 70°C flows in a 1-in Type K copper tube with a flow rate of 285 L/min.

For a 1-in Type K copper tube, *D*=0.02527 m and *A*=5.017 x 10–4 m2 (from Appendix H). Then we have

$$v = \frac{Q}{A} = \frac{285\ \text{L/min}}{5.017 \times 10^{-4}\ \text{m}^2} \times \frac{1\ \text{m}^3/\text{s}}{60\,000\ \text{L/min}} = 9.47\ \text{m/s}$$

$$\nu = 4.11 \times 10^{-7}\ \text{m}^2/\text{s} \qquad \text{(from Appendix A)}$$

$$N_R = \frac{(9.47)(0.02527)}{4.11 \times 10^{-7}} = 5.82 \times 10^5$$

Because the Reynolds number is greater than 4000, the flow is turbulent.

Example 8.3

Determine the range of average velocity of flow for which the flow would be in the critical region if SAE 10 oil at 15°C is flowing in a 2-in Schedule 40 steel pipe. The oil has a specific gravity of 0.89.

The flow would be in the critical region if 2000<N_R<4000. First, we use the Reynolds number and solve for velocity:

$$N_R = \frac{vD\rho}{\mu}$$

$$v = \frac{N_R \mu}{D\rho} \qquad (8\text{–}2)$$

Example 8.3

Then we find the values for *n*, *D*, and ρ

$$D = 52.5 \text{ mm} \qquad \text{(from Appendix F)}$$
$$\mu = 1 \times 10^{-1} \text{ N·s/m}^2 \qquad \text{(from Appendix D)}$$
$$\rho = 0.89(1000 \text{ kg/m}^3) = 890 \text{ kg/m}^3$$

Substituting these values into Eq. (8–2), we get

$$\upsilon = \frac{N_R(1 \times 10^{-1})}{(0.0525)(890)} = (2.14 \times 10^{-3})N_R$$

Example 8.3

For N_R=2000, we have

$$v = (2.14 \times 10^{-3})(2 \times 10^{3}) = 4.3 \text{ m/s}$$

For N_R=4000, we have

$$v = (2.14 \times 10^{-3})(4 \times 10^{3}) = 8.56 \text{ m/s}$$

Therefore, if 4.3<v<8.56 m/s, the flow will be in the critical region.

8.4 Darcy's Equation

- In the general energy equation

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_A - h_R - h_L = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

  the term h_L is defined as the energy loss from the system.

- Darcy's equation shows that

$$h_L = f \times \frac{L}{D} \times \frac{v^2}{2g} \qquad (8\text{–}3)$$

h_L = energy loss due to friction (N·m/N, m, lb-ft/lb, or ft)
L = length of flow stream (m or ft)
D = pipe diameter (m or ft)
v = average velocity of flow (m/s or ft/s)
f = friction factor (dimensionless)

8.5 Friction Loss in Laminar Flow

- Because laminar flow is so regular and orderly, we can derive a relationship between the energy loss and the measurable parameters of the flow system.
- This relationship is known as the *Hagen–Poiseuille equation*:

$$h_L = \frac{32\mu L v}{\gamma D^2} \tag{8–4}$$

- The Hagen–Poiseuille equation is valid only for laminar flow ($N_R < 2000$).

8.5 Friction Loss in Laminar Flow

- If the two relationships for h_L are set equal to each other, we can solve for the value of the friction factor:

$$f \times \frac{L}{D} \times \frac{v^2}{2g} = \frac{32\mu L v}{\gamma D^2}$$

$$f = \frac{32\mu L v}{\gamma D^2} \times \frac{D2g}{Lv^2} = \frac{64\mu g}{vD\gamma}$$

- As $\rho = \gamma/g$,

$$f = \frac{64\mu}{vD\rho}$$

- Since $N_R = vD\rho/\mu$,

$$f = \frac{64}{N_R} \qquad (8\text{–}5)$$

8.5 Friction Loss in Laminar Flow

- In summary, the energy loss due to friction in *laminar flow* can be calculated either from the Hagen–Poiseuille equation,

$$h_L = \frac{32\mu L v}{\gamma D^2}$$

- or from Darcy's equation,

$$h_L = f \times \frac{L}{D} \times \frac{v^2}{2g}$$

where $f = 64/N_R$.

Example 8.4

Determine the energy loss if glycerine at 25°C flows 30 m through a 150-mm-diameter pipe with an average velocity of 4.0 m/s.

First, we must determine whether the flow is laminar or turbulent by evaluating the Reynolds number:

$$N_R = \frac{vD\rho}{\mu}$$

From Appendix B, we find that for glycerin at 25°C

$$\rho = 1258\ \text{kg/m}^3$$

$$\mu = 9.60 \times 10^{-1}\ \text{Pa·s}$$

$$N_R = \frac{(4.0)(0.15)(1258)}{9.60 \times 10^{-1}} = 786$$

Example 8.4

Because N_R < 2000, the flow is laminar. Using Darcy's equation, we get

$$h_L = f \times \frac{L}{D} \times \frac{v^2}{2g}$$

$$f = \frac{64}{N_R} = \frac{64}{786} = 0.081$$

$$h_L = 0.081 \times \frac{30}{0.15} \times \frac{(4.0)^2}{2(9.81)}\,\mathrm{m} = 13.2\,\mathrm{m}$$

Notice that each term in each equation is expressed in the units of the SI unit system. Therefore, the resulting units for h_L are m or Nm/N. This means that 13.2 Nm of energy is lost by each newton of the glycerine as it flows along the 30 m of pipe.

Example 8.4

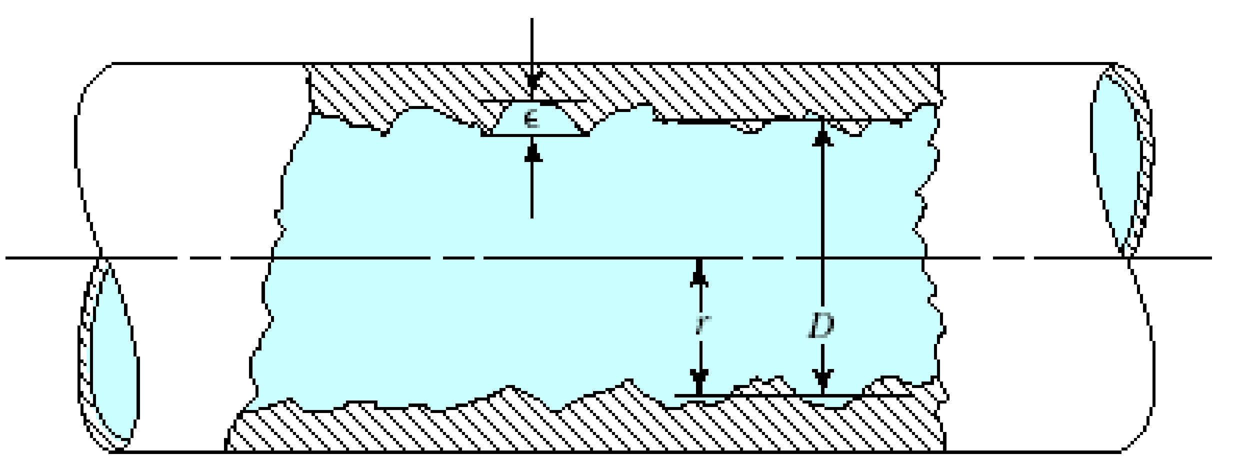
Because $N_R < 2000$, the flow is laminar. Using Darcy's equation, we get

Notice that each term in each equation is expressed in the units of the SI unit system. Therefore, the resulting units for h_L are m or Nm/N. This means that 13.2 Nm of energy is lost by each newton of the glycerine as it flows along the 30 m of pipe.

8.6 Friction Loss in Turbulent Flow

- For turbulent flow of fluids in circular pipes it is most convenient to use Darcy's equation to calculate the energy loss due to friction.
- Turbulent flow is rather chaotic and is constantly varying.
- For these reasons we must rely on experimental data to determine the value of *f*.
- Figure 8.5 illustrates pipe wall roughness (exaggerated) as the height of the peaks of the surface irregularities.
- Because the roughness is somewhat irregular, averaging techniques are used to measure the overall roughness value.

8.6 Friction Loss in Turbulent Flow



8.6 Friction Loss in Turbulent Flow

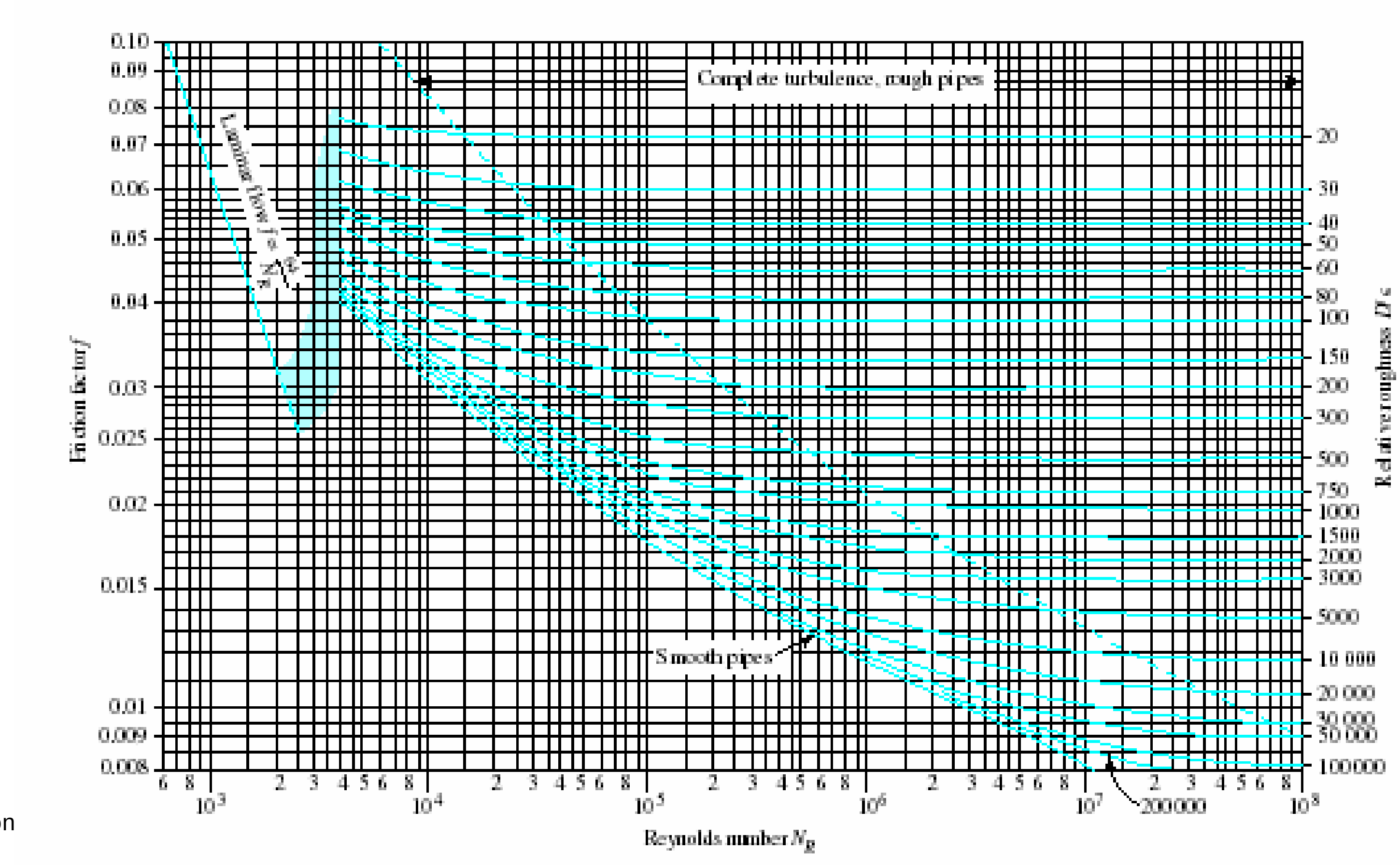
- For commercially available pipe and tubing, the design value of the average wall roughness has been determined as shown in Table 8.2.
- *These are only average values for new, clean pipe. Some variation should be expected. After a pipe has been in service for a time, the roughness could change due to the formation of deposits on the wall or due to corrosion*.

8.6 Friction Loss in Turbulent Flow

Material	Roughness ϵ (m)	Roughness ϵ (ft)
Glass	Smooth	Smooth
Plastic	3.0×10^{-7}	1.0×10^{-6}
Drawn tubing; copper, brass, steel	1.5×10^{-6}	5.0×10^{-6}
Steel, commercial or welded	4.6×10^{-5}	1.5×10^{-4}
Galvanized iron	1.5×10^{-4}	5.0×10^{-4}
Ductile iron—coated	1.2×10^{-4}	4.0×10^{-4}
Ductile iron—uncoated	2.4×10^{-4}	8.0×10^{-4}
Concrete, well made	1.2×10^{-4}	4.0×10^{-4}
Riveted steel	1.8×10^{-3}	6.0×10^{-3}

8.6.1 Moody Diagram

- One of the most widely used methods for evaluating the friction factor employs the Moody diagram shown in Fig. 8.6.



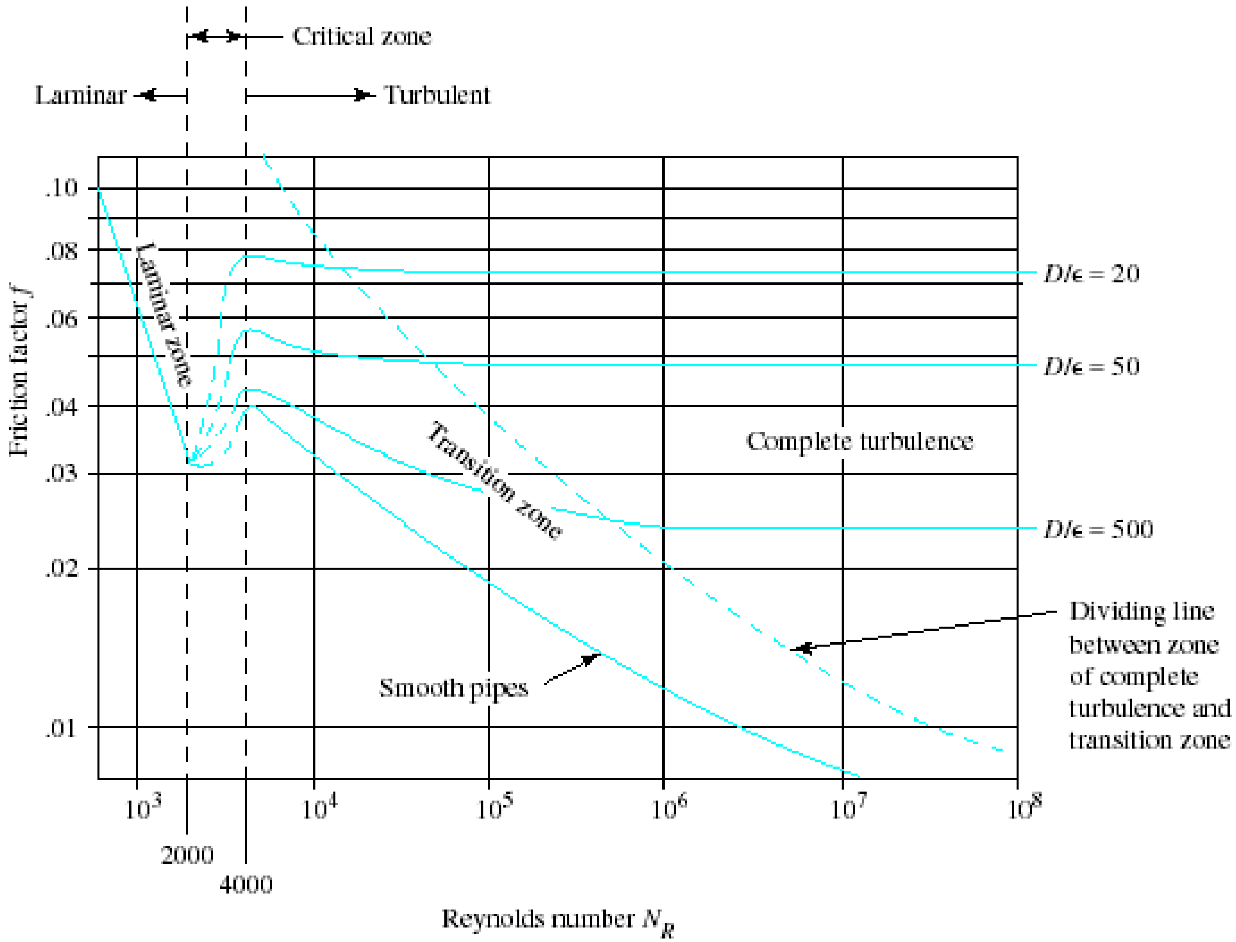
8.6.1 Moody Diagram

- Several important observations can be made from these curves:

1. For a given Reynolds number of flow, as the relative roughness is increased, the friction factor *f* decreases.
2. For a given relative roughness , the friction factor *f* decreases with increasing Reynolds number until the zone of complete turbulence is reached.
3. Within the zone of complete turbulence, the Reynolds number has no effect on the friction factor.
4. As the relative roughness increases, the value of the Reynolds number at which the zone of complete turbulence begins also increases.

8.6.1 Moody Diagram

- Figure 8.7 is a simplified sketch of Moody's diagram in which the various zones are identified.



8.6.1 Moody Diagram

- The *laminar zone* at the left has already been discussed.
- At the right of the dashed line downward across the diagram is the *zone of complete turbulence*.
- Between the smooth pipes line and the line marking the start of the complete turbulence zone is the *transition zone*.

8.6.1 Moody Diagram

- Check your ability to read the Moody diagram correctly by verifying the following values for friction factors for the given values of Reynolds number and relative roughness, using Fig. 8.6:

N_R	D/ϵ	f
6.7×10^3	150	0.0430
1.6×10^4	2000	0.0284
1.6×10^6	2000	0.0171
2.5×10^5	733	0.0223

8.6.1 Use of the Moody Diagram

- The Moody diagram is used to help determine the value of the friction factor *f* for turbulent flow.
- The value of the Reynolds number and the relative roughness must be known.
- Therefore, the basic data required are the pipe inside diameter, the pipe material, the flow velocity, and the kind of fluid and its temperature, from which the viscosity can be found.

Example 8.5

Determine the friction factor *f* if water at 70°C is flowing at 9.14 m/s in an uncoated ductile iron pipe having an inside diameter of 25 mm.

The Reynolds number must first be evaluated to determine whether the flow is laminar or turbulent:

$$N_R = \frac{vD}{\nu}$$

Here *D*=0.025 m and v=4.11x10^{-7} m^2/s. We now have

$$N_R = \frac{(9.14)(0.025)}{4.11 \times 10^{-7}} = 5.6 \times 10^5$$

Example 8.5

Thus, the flow is turbulent. Now the relative roughness must be evaluated. From Table 8.2 we find $\varepsilon = 2.4 \times 10^{-4}$ m. Then, the relative roughness is

$$\frac{D}{\epsilon} = \frac{0.025 \text{ m}}{2.4 \times 10^{-4} \text{ m}} = 104$$

The final steps in the procedure are as follows:

1. Locate the Reynolds number on the abscissa of the Moody diagram:

$$N_R = 5.6 \times 10^5$$

Example 8.5

2. Project vertically until the curve for $D/\varepsilon = 104$ is reached. Because 104 is so close to 100, that curve can be used.

3. Project horizontally to the left, and read $f = 0.038$.

Example 8.6

If the flow velocity of water in Problem 8.5 was 0.14 m/s with all other conditions being the same, determine the friction factor *f*. Write

$$N_R = \frac{vD}{\nu} = \frac{(0.14)(0.025)}{4.11 \times 10^{-7}} = 8.52 \times 10^3$$

$$\frac{D}{\epsilon} = \frac{0.025 \text{ m}}{2.4 \times 10^{-4} \text{ m}} = 104$$

Then, from Fig. 8.6, $f = 0.044$. Notice that this is on the curved portion of the curve D/ε and that there is a significant increase in the friction factor over that in Example Problem 8.5.

Example 8.7

Determine the friction factor *f* if ethyl alcohol at 25°C is flowing at 5.3 m/s in a standard 1.5-in Schedule 80 steel pipe.

Evaluating the Reynolds number, we use the equation

$$N_R = \frac{vD\rho}{\mu}$$

From Appendix B, ρ = 787 kg/m^3 and μ =1.00 x 10^{-3} Pa•s. Also, for a 11.2-in Schedule 80 pipe, *D* = 0.0381 m. Then we have

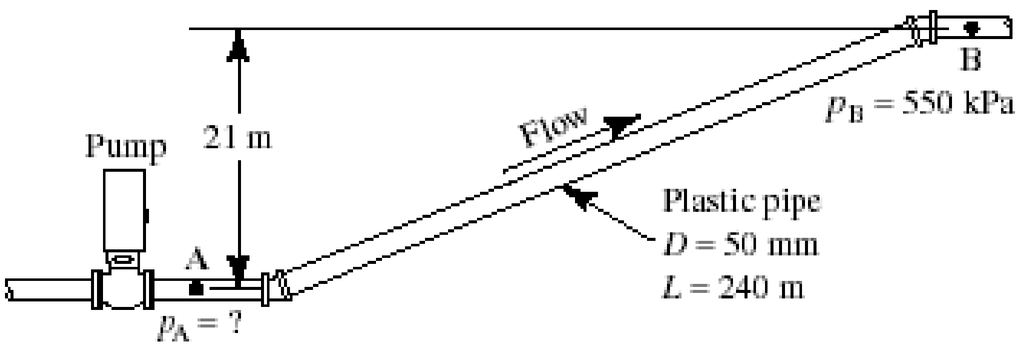
$$\frac{D}{\epsilon} = \frac{0.0381\ \mathrm{m}}{4.6 \times 10^{-5}\ \mathrm{m}} = 828$$

Example 8.7

From Fig. 8.6, f = 0.0225. You must interpolate on both N_R and D/ε to determine this value, and you should expect some variation. However, you should be able to read the value of the friction factor f within 0.0005 in this portion of the graph.

Example 8.8

See Fig. 8.8. In a chemical processing plant, benzene at 50°C (sg = 0.86) must be delivered to point B with a pressure of 550 kPa. A pump is located at point A 21 m below point B, and the two points are connected by 240 m of plastic pipe having an inside diameter of 50 mm. If the volume flow rate is 110 L/min, calculate the required pressure at the outlet of the pump.



Example 8.8

The relation is

$$\frac{p_A}{\gamma} + z_A + \frac{v_A^2}{2g} - h_L = \frac{p_B}{\gamma} + z_B + \frac{v_B^2}{2g}$$

$$\frac{p_A}{\gamma} + z_A + \cancel{\frac{v_A^2}{2g}}^{0} - h_L = \frac{p_B}{\gamma} + z_B + \cancel{\frac{v_B^2}{2g}}^{0}$$

$$p_A = p_B + \gamma[(z_B - z_A) + h_L] \qquad (8\text{–}6)$$

We find that because point B is higher than point A. z_B - z_A = +21 m. The evaluation of the Reynolds number is the first step. The type of flow, laminar or turbulent, must be determined.

Example 8.8

For a 50-mm pipe, D = 0.050 m and A = 1.963 x 10^{-3} m^2. Then, we have

$$Q = (110\ \text{L/min})\left(\frac{1\ \text{m}^3/\text{s}}{60\,000\ \text{L/min}}\right) = 1.83 \times 10^{-3}\ \text{m}^3/\text{s}$$

$$v = \frac{Q}{A} = \frac{1.83 \times 10^{-3}\ \text{m}^3/\text{s}}{1.963 \times 10^{-3}\ \text{m}^2} = 0.932\ \text{m/s}$$

We find that

$$\rho = (0.86)(1000\ \text{kg/m}^3) = 860\ \text{kg/m}^3$$

$$\mu = 4.2 \times 10^{-4}\ \text{Pa}\cdot\text{s} \qquad \text{(from Appendix D)}$$

Thus

$$N_R = \frac{(0.932)(0.050)(860)}{4.2 \times 10^{-4}} = 9.54 \times 10^4$$

Example 8.8

For turbulent flow, Darcy's equation should be used:

$$h_L = f \times \frac{L}{D} \times \frac{v^2}{2g}$$

From Table 8.2, roughness is 3.0 x 10-7 m. Then

$$\frac{D}{\epsilon} = \frac{0.050\ \mathrm{m}}{3.0 \times 10^{-7}\ \mathrm{m}} = 166\ 700 = 1.667 \times 10^5$$

Thus,

$$h_L = f \times \frac{L}{D} \times \frac{v^2}{2g} = 0.018 \times \frac{240}{0.050} \times \frac{(0.932)^2}{2(9.81)}\ \mathrm{m}$$

$$h_L = 3.83\ \mathrm{m}$$

Example 8.8

You should have the pressure as follows:

$$p_A = p_B + \gamma[(z_B - z_A) + h_L]$$

$$p_A = 550\,\text{kPa} + \frac{(0.86)(9.81\,\text{kN})}{\text{m}^3}(21\,\text{m} + 3.83\,\text{m})$$

$$p_A = 550\,\text{kPa} + 209\,\text{kN/m}^2 = 550\,\text{kPa} + 209\,\text{kPa}$$

$$p_A = 759\,\text{kPa}$$

8.7 Equation for the Friction Factor

- In the *laminar flow zone*, for values below 2000, *f* can be found from Eq. (8–5),

$$f = 64/N_R$$

- The following equation, which allows the direct calculation of the value of the friction factor for turbulent flow,

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{N_R^{0.9}}\right)\right]^2} \qquad (8\text{–}7)$$

8.7 Equation for the Friction Factor

- To calculate the value of the friction factor *f* when the Reynolds number and relative roughness are known, use Eq. (8–5) for laminar flow and Eq. (8–7) for turbulent flow.

Example 8.9

Compute the value for the friction factor if the Reynolds number for the flow 1 x 10^5 is and the relative roughness is 2000.

Because this is in the turbulent zone, we use Eq. (8–7),

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(2000)} + \frac{5.74}{(1 \times 10^5)^{0.9}}\right)\right]^2}$$

$$f = 0.0204$$

8.8 Hazen-Williams Formula for Water Flow

- The *Hazen–Williams formula* is one of the most popular formulas for the design and analysis of water systems. Its use is limited to the flow of water in pipes larger than 2.0 in and smaller than 6.0 ft in diameter.
- The Hazen–Williams formula is unit-specific. In the U.S. Customary unit system it takes the form

$$v = 1.32 C_h R^{0.63} s^{0.54} \qquad (8\text{–}8)$$

8.8 Hazen-Williams Formula for Water Flow

where

v = Average velocity of flow (ft/s)

C_h = Hazen–Williams coefficient (dimensionless)

R = Hydraulic radius of flow conduit (ft)

s = Ratio of h_L/L: energy loss/length of conduit (ft/ft)

- The coefficient C_h is dependent only on the condition of the surface of the pipe or conduit. Table 8.3 gives typical values.

	C_h	
Type of Pipe	Average for New, Clean Pipe	Design Value
Steel, ductile iron, or cast iron with centrifugally applied cement or bituminous lining	150	140
Plastic, copper, brass, glass	140	130
Steel, cast iron, uncoated	130	100
Concrete	120	100
Corrugated steel	60	60

8.8 Hazen-Williams Formula for Water Flow

- The Hazen–Williams formula for SI units is

$$v = 0.85 C_h R^{0.63} s^{0.54} \quad (8\text{–}9)$$

v = Average velocity of flow (m/s)
C_h = Hazen–Williams coefficient (dimensionless)
R = Hydraulic radius of flow conduit (m)
s = Ratio h_L/L: energy loss/length of conduit (m/m)

As before, the volume flow rate can be computed from $Q = Av$.

Example 8.10

For what velocity of flow of water in a new, clean, 6-in Schedule 40 steel pipe would an energy loss of 6.1 m of head occur over a length of 304.8 m? Compute the volume flow rate at that velocity. Then refigure the velocity using the design value of C_h for steel pipe.

We can use Equation (8–8). Write

$$
\begin{aligned}
s &= h_L/L = (6.1\ \text{m})/(304.8\ \text{m}) = 0.02 \\
R &= D/4 = (0.154\ \text{m})/4 = 0.0385\ \text{m} \\
C_h &= 130
\end{aligned}
$$

Example 8.10

Then

$$v = 0.85C_h R^{0.63} s^{0.54}$$
$$v = 0.85(130)(0.0385)^{0.63}(0.02)^{0.54} = 1.717 \text{ m/s}$$
$$Q = Av = (0.019 \text{ m}^2 \times 1.717 \text{ m/s}) = 0.033 \text{ m}^3\text{/s}$$

Note that the velocity and volume flow rate are both directly proportional to the value of C_h. If the pipe degrades after use so the value of C_h = 100, the allowable volume flow rate to limit the energy loss to the same value of 6.1 m per 304.8 m of pipe length would be

$$v = (1.717 \text{ m/s})(100/130) = 1.321 \text{ m/s}$$
$$Q = (0.033)(100/130) = 0.025 \text{ m}^3\text{/s}$$

8.9 Other Forms of Hazen-Williams Formula

- Other types of calculations that are often desired are:

1. To determine the required size of pipe to carry a given flow rate while limiting the energy loss to some specified value.
2. To determine the energy loss for a given flow rate through a given type and size of pipe of a known length.

- Table 8.4 shows several forms of the Hazen–Williams formula that facilitate such calculations.

8.9 Other Forms of Hazen-Williams Formula

U.S. Customary Units	SI Units
$v = 1.32 C_h R^{0.63} s^{0.54}$	$v = 0.85 C_h R^{0.63} s^{0.54}$
$Q = 1.32 A C_h R^{0.63} s^{0.54}$	$Q = 0.85 A C_h R^{0.63} s^{0.54}$
$h_L = L\left[\dfrac{Q}{1.32 A C_h R^{0.63}}\right]^{1.852}$	$h_L = L\left[\dfrac{Q}{0.85 A C_h R^{0.63}}\right]^{1.852}$
$D = \left[\dfrac{2.31 Q}{C_h s^{0.54}}\right]^{0.380}$	$D = \left[\dfrac{3.59 Q}{C_h s^{0.54}}\right]^{0.380}$
Note: Units must be consistent:	
v in ft/s	v in m/s
Q in ft^3/s	Q in m^3/s
A in ft^2	A in m^2
h_L, L, R, and D in ft	h_L, L, R, and D in m
s in ft/ft (dimensionless)	s in m/m (dimensionless)

8.10 Nomograph for solving the Hazen-Williams Formula

- The nomograph shown in Fig. 8.9 allows the solution of the Hazen–Williams formula to be done by simply aligning known quantities with a straight edge and reading the desired unknowns at the intersection of the straight edge with the appropriate vertical axis.
- *Note that this nomograph is constructed for the value of the Hazen–Williams coefficient of $C_h = 100$.*

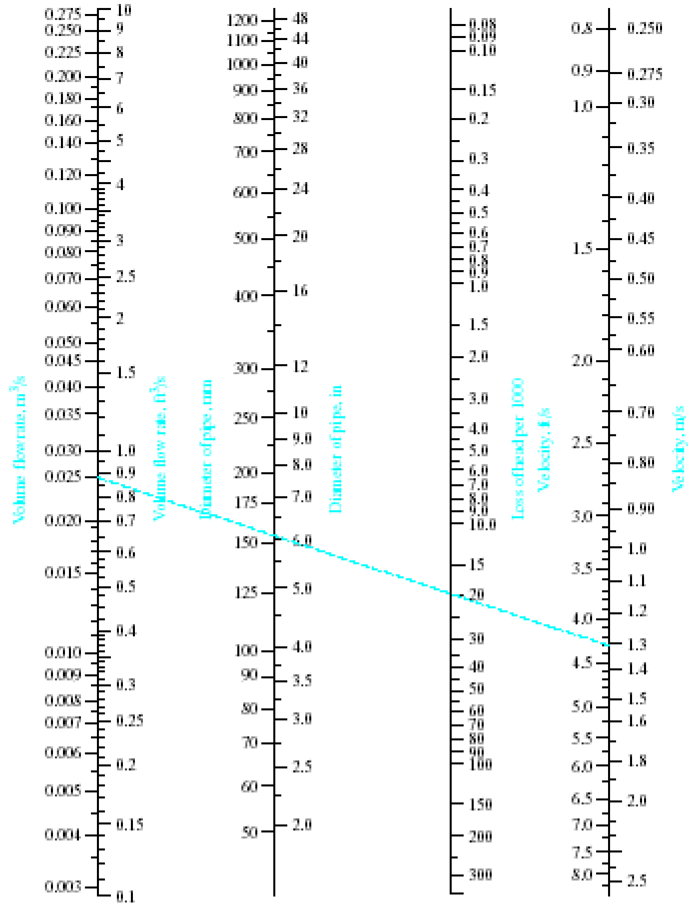
$$v_c = v_{100}(C_h/100) \quad \text{[velocity]} \quad (8\text{–}10)$$

$$Q_c = Q_{100}(C_h/100) \quad \text{[volume flow rate]} \quad (8\text{–}11)$$

$$D_c = D_{100}(100/C_h)^{0.38} \quad \text{[pipe diameter]} \quad (8\text{–}12)$$

$$s_c = s_{100}(100/C_h)^{1.85} \quad \text{[head loss/length]} \quad (8\text{–}13)$$

8.10 Nomograph for solving the Hazen-Williams Formula



Example:
Given: 6-in Schedule 40
steel pipe
$C_h = 100$
$s = h_L/1000 = 20$
Result: Allowable
velocity = 1.31 m/s

Example 8.11

Specify the required size of Schedule 40 steel pipe to carry 0.034 m^3/s of water with no more than 4.0 m of head loss over a 1000 m length of pipe. Use the design value for C_h.

Table 8.3 suggests $C_h = 100$. Now, using Fig. 8.9, we can place a straight edge from Q=0.034 m^3/s on the volume flow rate line to the value of s=(4 m)/(1000 m) on the energy loss line. The straight edge then intersects the pipe size line at approximately 245 mm. The next larger standard pipe size listed in Appendix F is the nominal 10-in pipe with an inside diameter of 254.5 mm.

Example 8.11

Returning to the chart in Fig. 8.9 and slightly realigning Q=0.034 m^3/s with D=255 mm, we can read an average velocity of v=0.65 m/s. This is relatively low for a water distribution system, and the pipe is quite large. If the pipeline is long, the cost for piping would be excessively large. If we allow the velocity of flow to increase to approximately 1.8 m/s for the same volume flow rate, we can use the chart to show that a 6-in pipe could be used with a head loss of approximately 35 m per 1000 m of pipe. The lower cost of the pipe compared with the 10-in pipe would have to be compared with the higher energy cost required to overcome the additional head loss.