Chapter 7
Trip Distribution

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TRANSPORTATION PLANNING

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BASIC CONCEPTS IN TRIP DISTRIBUTION

- **Trip distribution is a process by which the trips generated in one zone are allocated to other zones in the study area.**
- **These trips may be within the study area (internal-internal) or between the study area and areas outside the study area (internal-external).**

BASIC CONCEPTS IN TRIP DISTRIBUTION

- For example, if the trip generation analysis results in an estimate of 200 HBW trips in zone 10,
- Then the trip distribution analysis would determine how many of these trips would be made between zone 10 and all the other internal zones.
- In addition, the trip distribution process considers internal-external trips (or vice versa) where one end of the trip is within the study area and the other end is outside the area.

TRIP DISTRIBUTION MODELS

- **Several basic methods are used for trip distribution. Among these are:**

  - Gravity model

  - Growth factor models

  - Intervening opportunities model

- **The gravity model is preferred because it uses the attributes of the transportation system and land-use characteristics and has been calibrated extensively for many urban areas.**

TRIP DISTRIBUTION MODELS

- The gravity model has achieved virtually universal use because of its simplicity, its accuracy, and its support from the U.S. DOT.
- Growth factor models, which were used more widely in the 1950s and 1960s, require that the origin-destination matrix be known for the base (or current) year, as well as an estimate of the number of future trip ends in each zone.
- The intervening opportunities model and other models are available but not widely used in practice.

GRAVITY MODEL

- **The most widely used and documented trip distribution model is the gravity model.**
- **It states that the number of trips between two zones is directly proportional to the number of trip attractions generated by the zone of destination and inversely proportional to a function of time of travel between the two zones.**
- **Mathematically, the gravity model is expressed as:**

GRAVITY MODEL

$$T_{ij} = P_i \left[\frac{A_j F_{ij} K_{ij}}{\sum_j A_j F_{ij} K_{ij}} \right]$$

where

T_{ij} = number of trips that are produced in zone i and attracted to zone j
P_i = total number of trips produced in zone i
A_j = number of trips attracted to zone j
F_{ij} = a value which is an inverse function of travel time
K_{ij} = socioeconomic adjustment factor for interchange ij

GRAVITY MODEL

- **The values of P_i and A_j have been determined in the trip generation process.**
- **The sum of P_i for all zones must equal the sum of A_j for all zones.**
- **K_{ij} values are used when the estimated trip interchange must be adjusted to ensure that it agrees with the observed trip interchange.**
- **The values for F_{ij} are determined by a calibrating process in which trip generation values as measured in the O-D survey are distributed using the gravity model.**

GRAVITY MODEL

- **After each distribution process is completed, the percentage of trips in each trip length category produced by the gravity model is compared with the percentage of trips recorded in the O-D survey.**
- **If the percentages do not agree, then the F_{ij} factors that were used in the distribution process are adjusted and another gravity model trip distribution is performed.**
- **The calibration process is continued until the trip length percentages are in agreement.**

GRAVITY MODEL

- Figure 12.7 illustrates F values for calibrations of a gravity model. (Normally a semilog plot.)
- F values can also be determined using travel time values and an inverse relationship between F and t.
- For example, the relationship for F might be in the form t^{-1}, t^{-2}, e^{-t}, and so forth, since F values decrease as travel time increases.
- The friction factor can be expressed as:

  $F = ab^{t} e^{-ct}$, where parameters a, b, and c are
- based on national data sources.

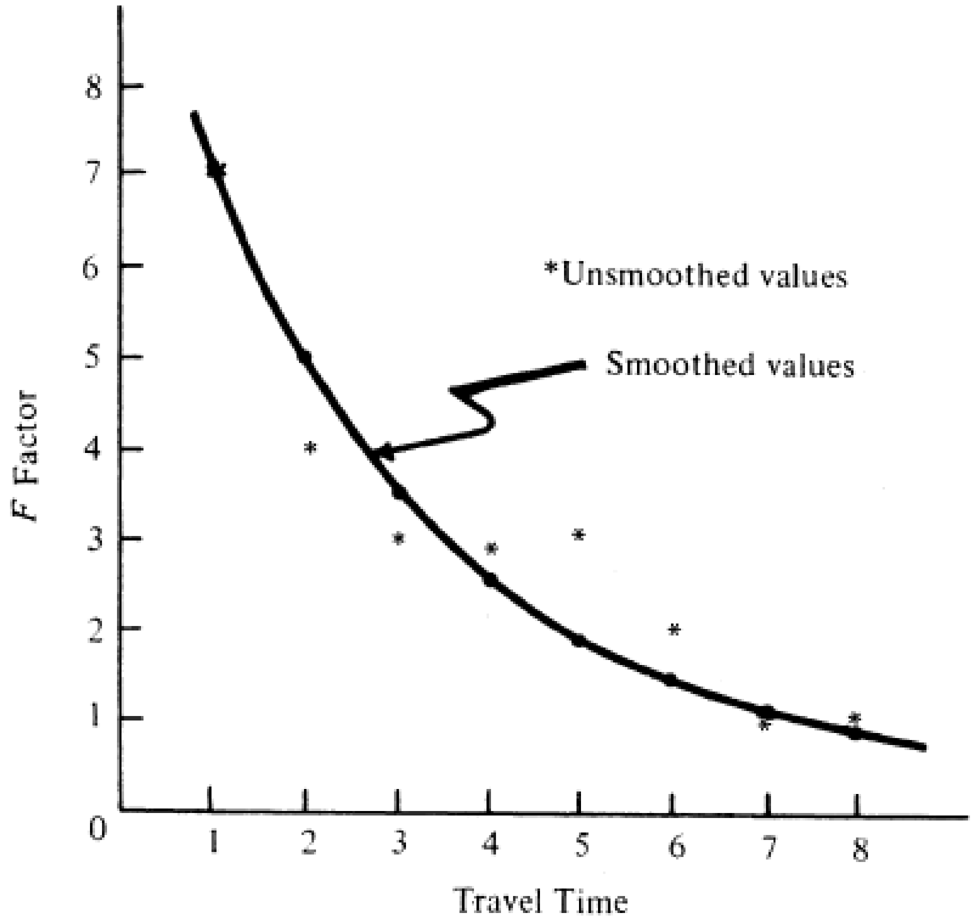

Figure 12.7 Calibration of *F* Factors

GRAVITY MODEL

- The socioeconomic factor is used to make adjustments of trip distribution K_{ij} values between zones where differences between estimated and actual values are significant.
- The K value is referred to as the "socioeconomic factor" since it accounts for variables other than travel time.
- The values for K are determined in the calibration process, but it is used judiciously when a zone is considered to possess unique
- characteristics.

Example 12.4 Use of Calibrated *F* Values and Iteration

To illustrate the application of the gravity model, consider a study area consisting of three zones. The data have been determined as follows: the number of productions and attractions has been computed for each zone by methods described in the section on trip generation, and the average travel times between each zone have been determined. Both are shown in Tables 12.9 and 12.10. Assume K_{ij} is the same unit value for all zones. Finally, the F values have been calibrated as previously described and are shown in Table 12.11 for each travel time increment. Note that the intrazonal travel time for zone 1 is larger than those of most other inter-zone times because of the geographical characteristics of the zone and lack of access within the area. This zone could represent conditions in a congested downtown area.

Determine the number of zone-to-zone trips through two iterations.

Table 12.9 Trip Productions and Attractions for a Three-Zone Study Area

Zone	*1*	*2*	*3*	*Total*
Trip productions	140	330	280	750
Trip attractions	300	270	180	750

Table 12.10 Travel Time between Zones (min)

Zone	*1*	*2*	*3*
1	5	2	3
2	2	6	6
3	3	6	5

Table 12.11 Travel Time versus Friction Factor

Time (min)	*F*
1	82
2	52
3	50
4	41
5	39
6	26
7	20
8	13

Note: *F* values were obtained from the calibration process.

Solution: The number of trips between each zone is computed using the gravity model and the given data. (*Note*: F_{ij} is obtained by using the travel times in Table 12.10 and selecting the correct F value from Table 12.11. For example, travel time is 2 min between zones 1 and 2. The corresponding F value is 52.)

Use Eq. 12.3.

$$T_{ij} = P_i \left[\frac{A_j F_{ij} K_{ij}}{\sum_{j=1}^{n} A_j F_{ij} K_{ij}} \right] \qquad K_{ij} = 1 \text{ for all zones}$$

$$T_{1-1} = 140 \times \frac{300 \times 39}{(300 \times 39) + (270 \times 52) + (180 \times 50)} = 47$$

$$T_{1-2} = 140 \times \frac{270 \times 52}{(300 \times 39) + (270 \times 52) + (180 \times 50)} = 57$$

$$T_{1-3} = 140 \times \frac{180 \times 50}{(300 \times 39) + (270 \times 52) + (180 \times 50)} = 36$$

$$P_1 = 140$$

Make similar calculations for zones 2 and 3.

$$T_{2-1} = 188 \quad T_{2-2} = 85 \quad T_{2-3} = 57 \quad P_2 = 330$$
$$T_{3-1} = 144 \quad T_{3-2} = 68 \quad T_{3-3} = 68 \quad P_3 = 280$$

The results summarized in Table 12.12 represent a *singly constrained* gravity model. This constraint is that the sum of the productions in each zone is equal to the number of productions given in the problem statement. However, the number of attractions estimated in the trip distribution phase differs from the number of attractions given. For zone 1, the correct number is 300, whereas the computed value is 379. Values for zone 2 are 270 versus 210, and for zone 3, they are 180 versus 161.

Table 12.12 Zone-to-Zone Trips: First Iteration, Singly Constrained

Zone	*1*	*2*	*3*	*Computed P*	*Given P*
1	47	57	36	140	140
2	188	85	57	330	330
3	144	68	68	280	280
Computed *A*	379	210	161	750	750
Given *A*	300	270	180	750	

To create a doubly constrained gravity model where the computed attractions equal the given attractions, calculate the adjusted attraction factors according to the formula

$$A_{jk} = \frac{A_j}{C_{j(k-1)}} A_{j(k-1)} \tag{12.4}$$

where

A_{jk} = adjusted attraction factor for attraction zone (column) j, iteration k
$A_{jk} = A_j$ when $k = 1$
C_{jk} = actual attraction (column) total for zone j, iteration k
A_j = desired attraction total for attraction zone (column) j
j = attraction zone number, $j = 1, 2, \ldots, n$
n = number of zones
k = iteration number, $k = 1, 2, \ldots, m$
m = number of iterations

To produce a doubly constrained gravity model, repeat the trip distribution computations using modified attraction values so that the numbers attracted will be increased or reduced as required. For zone 1, for example, the estimated attractions were too great. Therefore, the new attraction factors are adjusted downward by multiplying the original attraction value by the ratio of the original to estimated attraction values.

$$\text{Zone 1:} A_{12} = 300 \times \frac{300}{379} = 237$$

$$\text{Zone 2:} A_{22} = 270 \times \frac{270}{210} = 347$$

$$\text{Zone 3:} A_{32} = 180 \times \frac{180}{161} = 201$$

Table 12.13 Zone-to-Zone Trips: Second Iteration, Doubly Constrained

Zone	*1*	*2*	*3*	*Computed P*	*Given P*
1	34	68	38	140	140
2	153	112	65	330	330
3	116	88	76	280	280
Computed A	303	268	179	750	750
Given A	300	270	180	750	

Apply the gravity model (Eq. 12.3) for all iterations to calculate zonal trip interchanges using the adjusted attraction factors obtained from the preceding iteration. In practice, the gravity model becomes

$$T_{ij} = P_i \left[\frac{A_j F_{ij} K_{ij}}{\sum_j A_j F_{ij} K_{ij}} \right]$$

where T_{ijk} is the trip interchange between i and j for iteration k, and $A_{jk} = A_j$ when $k = 1$. Subscript j goes through one complete cycle every time k changes, and i goes through one complete cycle every time j changes. This formula is enclosed in parentheses and subscripted to indicate that the complete process is performed for each trip purpose.

Perform a second iteration using the adjusted attraction values.

$$T_{1-1} = 140 \times \frac{237 \times 39}{(237 \times 39) + (347 \times 52) + (201 \times 50)} = 34$$

$$T_{1-2} = 140 \times \frac{347 \times 52}{(237 \times 39) + (347 \times 52) + (201 \times 50)} = 68$$

$$T_{1-3} = 140 \times \frac{201 \times 50}{(237 \times 39) + (347 \times 52) + (201 \times 50)} = 37$$

$$P_1 = 140$$

Make similar calculations for zones 2 and 3.

$$T_{2-1} = 153 \qquad T_{2-2} = 112 \qquad T_{2-3} = 65 \qquad P_2 = 330$$

$$T_{3-1} = 116 \qquad T_{3-2} = 88 \qquad T_{3-3} = 76 \qquad P_3 = 280$$

The results are summarized in Table 12.13. Note that, in each case, the sum of the attractions is now much closer to the given value. The process will be continued until there is a reasonable agreement (within 5%) between the A that is estimated using the gravity model and the values that are furnished in the trip generation phase.

GRAVITY MODEL

- When should a singly constrained gravity model or the doubly constrained gravity model be used?
- The singly constrained gravity model may be preferred if the friction factors are more reliable than the attraction values.
- The doubly constrained gravity model is appropriate if the attraction values are more reliable than friction factors.
- To illustrate either choice, consider the following example:

Example 12.5 Selecting Singly or Doubly Constrained Gravity Model Results

A three-zone system with 900 home-based shopping productions is shown in Table 12.14. Zones 1 and 2 each generate 400 productions, while zone 3 generates 100 productions. Each zone contains a shopping mall with 300 attractions. The shopping mall in zone 1 can be easily reached due to the parking availability and transit service. Thus, F_{11}, F_{21}, and $F_{31} = 1.0$. Parking costs at the shopping mall in zone 2 are moderate with some transit service. Thus, F_{12}, F_{22}, and $F_{32} = 0.5$. Parking costs at the mall in zone 3 is high and transit service is unavailable. Thus, F_{13}, F_{23}, and $F_{33} = 0.2$.

Table 12.14 Home-Based Shopping Productions and Attractions

Zone	*Productions*	*Attractions*
1	400	300
2	400	300
3	100	300
Total	900	900

Application of the singly constrained gravity model yields the results shown in Table 12.15 and application of the doubly constrained gravity model yields the results shown in Table 12.16.

Table 12.15 Zone-to-Zone Trips: Singly Constrained Gravity Model

Zone	*1*	*2*	*3*	*Computed P*	*Given P*
1	235	118	47	400	400
2	235	118	47	400	400
3	59	29	12	100	100
Computed *A*	529	265	106	900	900
Given *A*	300	300	300	900	

Table 12.16 Zone-to-Zone Trips: Doubly Constrained Gravity Model

Zone	*1*	*2*	*3*	*Computed P*	*Given P*
1	133	133	133	400	400
2	133	133	133	400	400
3	33	33	33	100	100
Computed *A*	300	300	300	900	900
Given *A*	300	300	300	900	

Which of the results shown for the singly constrained gravity model and for the doubly constrained gravity model are more likely to be the most accurate?

Solution: Table 12.15 is more likely to be accurate if engineering judgment suggests the occurrence of travel impedances and thus the friction factors are more accurate than trip attractions. Table 12.16 is more likely to be accurate if the attractions are more accurate than the friction factors.

In practice, these judgments must be made based on the quality of the data set. For example, if local land-use data had been recently used to develop trip attraction rates whereas friction factors had been borrowed from another area, then the selection of the doubly constrained gravity model results in Table 12.16 is recommended.