Chapter 3: Alternating Current Circuits

Dr. Muna HajjYahya
Dr. Maen Ishtaiwi

Physics Department
An-Najah National University
Electrical Instrumentation

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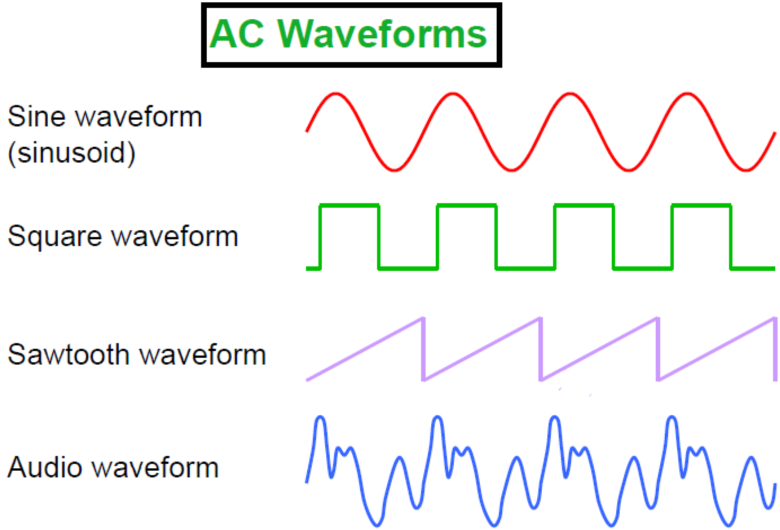
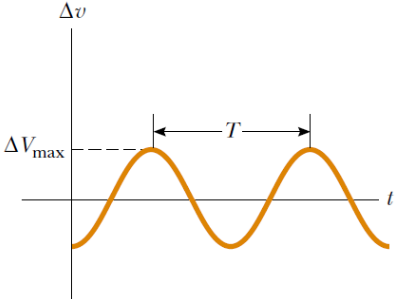

AC Waveforms
Sine waveform
(sinusoid)
Square waveform
Sawtooth waveform
Audio waveform

AC Sources

- **Frequency:** the number of cycles per second of an AC waveform is known as the frequency f, and is expressed in Hertz (H).
- **Period:** the period T of an AC waveform is the time taken for a complete cycle.

$$\text{Period} = \frac{1}{f} \qquad (1)$$



AC Sources

- We will use lowercase symbols Δv and i to indicate the instantaneous values of time-varying voltages and currents. Capital letters represent fixed values of voltage and current, such as ΔV_{max} and I_{max}.
- An AC circuit consists of circuit elements and a power source that provides an alternating voltage Δv. This time-varying voltage is described by

$$\Delta v = \Delta V_{max} sin(\omega t) \tag{2}$$

where ΔV_{max} is the maximum output voltage of the AC source, or the voltage amplitude.
- The angular frequency of the AC voltage is

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{3}$$

where f is the frequency of the source and T is the period.

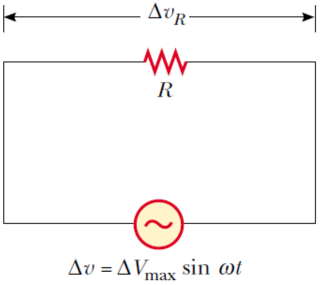
2. Resistors in an AC Circuit

- Consider a simple AC circuit consisting of a resistor and an AC source.
- KVL for loop:

$$\Delta v + \Delta v_R = 0 \tag{4}$$

$$\Delta v = \Delta v_R = \Delta V_{max} sin(\omega t) \tag{5}$$

where Δv_R is the instantaneous voltage across the resistor.



Resistors in an AC Circuit

- The instantaneous current in the resistor is

$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R} sin(\omega t) = I_{max} sin(\omega t) \tag{6}$$

- The maximum current in a resistor.

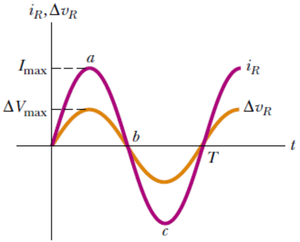
$$I_{max} = \frac{\Delta V_{max}}{R} \tag{7}$$

- The voltage across a resistor is

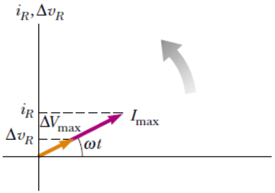
$$\Delta v_R = I_{max} R sin(\omega t) \tag{8}$$

- Because i_R and Δv_R both vary as sin(ωt) and reach their maximum values at the same time, they are said to be in **phase**.

- (a) Plots of the instantaneous current i_R and instantaneous voltage Δv across a resistor as functions of time.
- (b) Phasor diagram for the resistive circuit showing that the current is in phase with the voltage.
- **For a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor.**



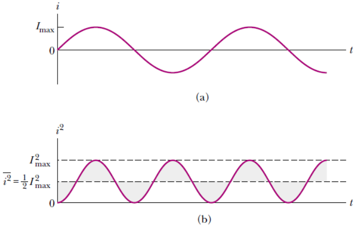
(a)



(b)

Resistors in an AC Circuit

- Phasor diagrams is a vector whose length is proportional to the maximum value of the variable it represents (ΔV_{max} for voltage and I_{max} for current) and which rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable.
- The average value of the current over one cycle is **zero**.
- However, the direction of the current has no effect on the behavior of the resistor.



Resistors in an AC Circuit

- What is of importance in an AC circuit is an average value of current, referred to as the rms (root-mean-square) current.
- RMS current is

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max} \tag{9}$$

- Average power delivered to a resistor

$$P_{avg} = I_{rms}^2 R \tag{10}$$

- RMS voltage is

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707 \Delta V_{max} \tag{11}$$

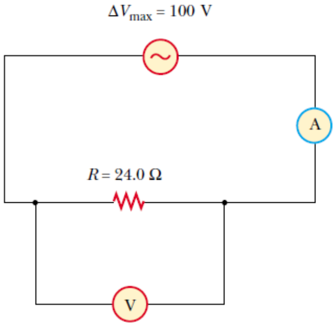
Example 1

The voltage output of an AC source is given by the expression $\Delta v = (200$ V$)\sin(\omega t)$. Find the rms current in the circuit when this source is connected to a 100 Ω resistor.

.

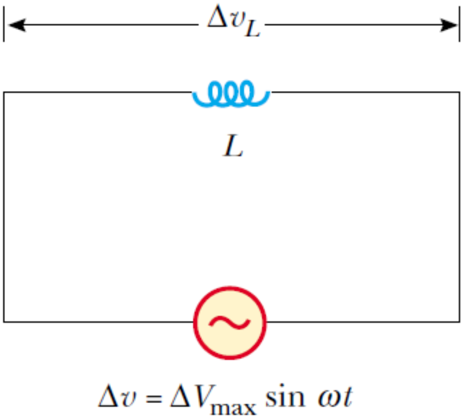
Example 2

An AC power supply produces a maximum voltage $\Delta V_{max} = 100$ V. This power supply is connected to a 24 Ω resistor, and the current and resistor voltage are measured with an ideal AC ammeter and voltmeter, as shown. What does each meter read?



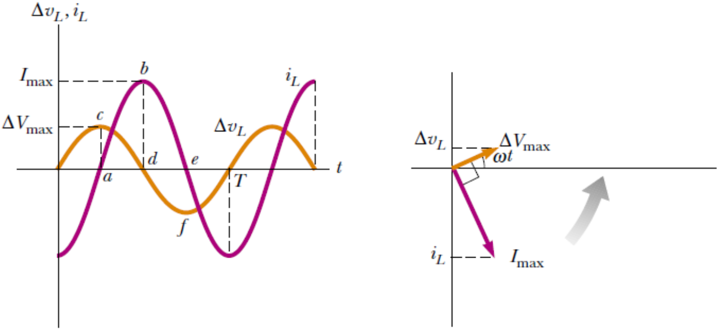
3. Inductors in an AC Circuit

- Now consider an AC circuit consisting only of an inductor connected to the terminals of an AC source, as shown in Figure.



Inductors in an AC Circuit

- (a) Plots of the instantaneous current i_L and instantaneous voltage Δv_L across a inductor as functions of time.
- (b) Phasor diagram for the inductive circuit, showing that the current leads the voltage by 90°.



(a) (b)

Example 2

In a purely inductive AC circuit, L = 25 mH and the rms voltage is 150 V. Calculate the inductive reactance and rms current in the circuit if the frequency is 60 Hz.

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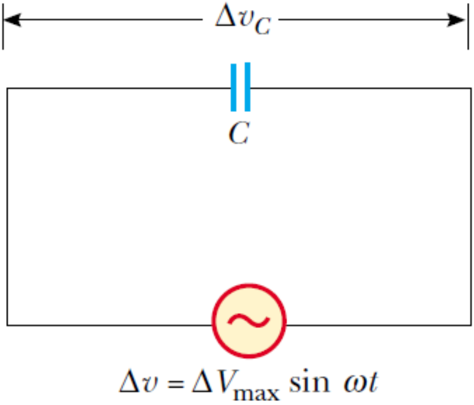
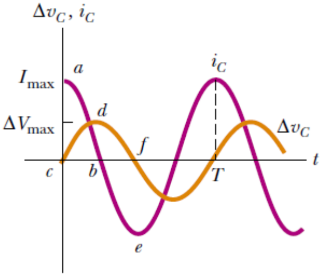
4. Capacitors in an AC Circuit

- Now consider an AC circuit consisting only of an capacitor connected to the terminals of an AC source, as shown in Figure.

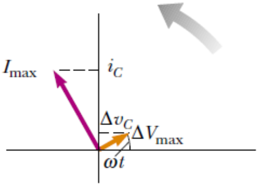


Capacitors in an AC Circuit

- (a) Plots of the instantaneous current i_C and instantaneous voltage Δv_C across a capacitor as functions of time.
- (b) Phasor diagram for the capacitive circuit, showing that the current leads the voltage by 90° .



(a)



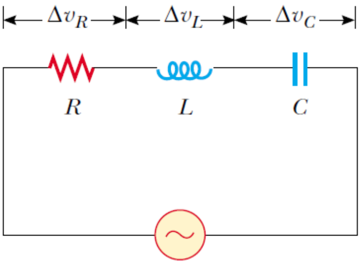
(b)

Example 3

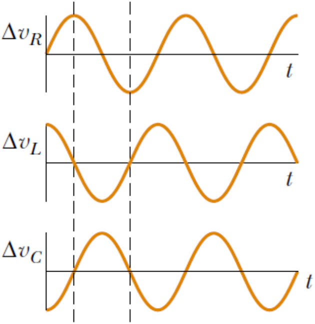
An 8 μF capacitor is connected to the terminals of a 60 Hz AC source whose rms voltage is 150 V. Find the capacitive reactance and the rms current in the circuit.

If the frequency is doubled? What happens to the rms current in the circuit?

- The Figure shows a circuit that contains a resistor, an inductor, and a capacitor connected in series across an alternating voltage source.



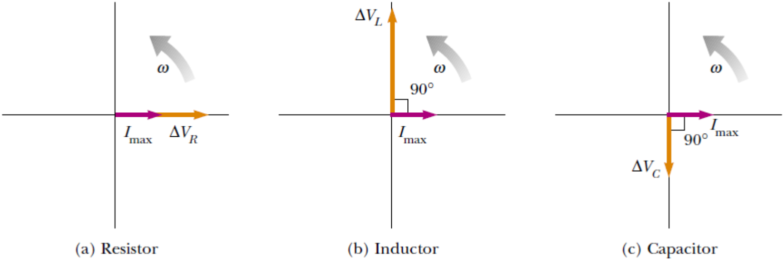
(a)



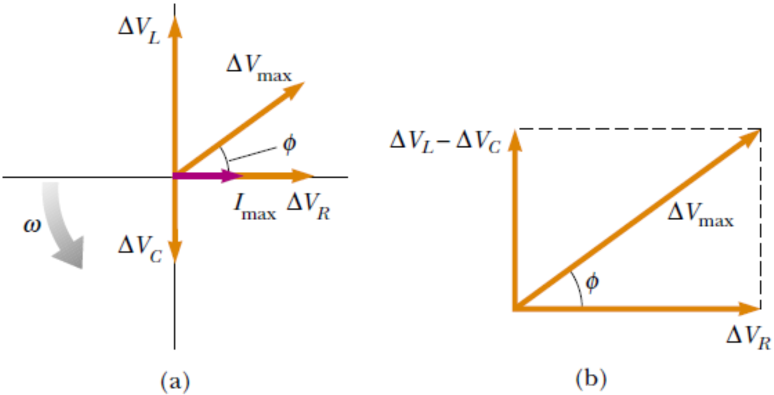
(b)

The RLC Series Circuit

- Phase relationships between the voltage and current phasors for (a) a resistor, (b) an inductor, and (c) a capacitor connected in series.



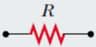
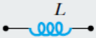
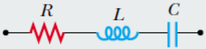
- Phasor diagram for the series RLC circuit.



The RLC Series Circuit

- Impedance Values and Phase Angles for Various Circuit-Element Combinationsa

Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
	R	$0°$
	X_C	$-90°$
	X_L	$+90°$
	$\sqrt{R^2 + X_C{}^2}$	Negative, between $-90°$ and $0°$
	$\sqrt{R^2 + X_L{}^2}$	Positive, between $0°$ and $90°$
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Example 4

A series RLC AC circuit has R $=$ 425 Ω L $=$ 1.25 H, C $=$ 3.5 μF, $\omega = 377$ s^{-1}, and ΔVmax $=$ 150 V.

1. Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.
2. Find the maximum current in the circuit.
3. Find the phase angle between the current and voltage.
4. Find the maximum voltage across each element.
5. Find the instantaneous voltage across each element.