Applied Fluid Mechanics

Chapter 8

Reynolds Number, Laminar Flow, Turbulent Flow, and Energy Losses Due to Friction

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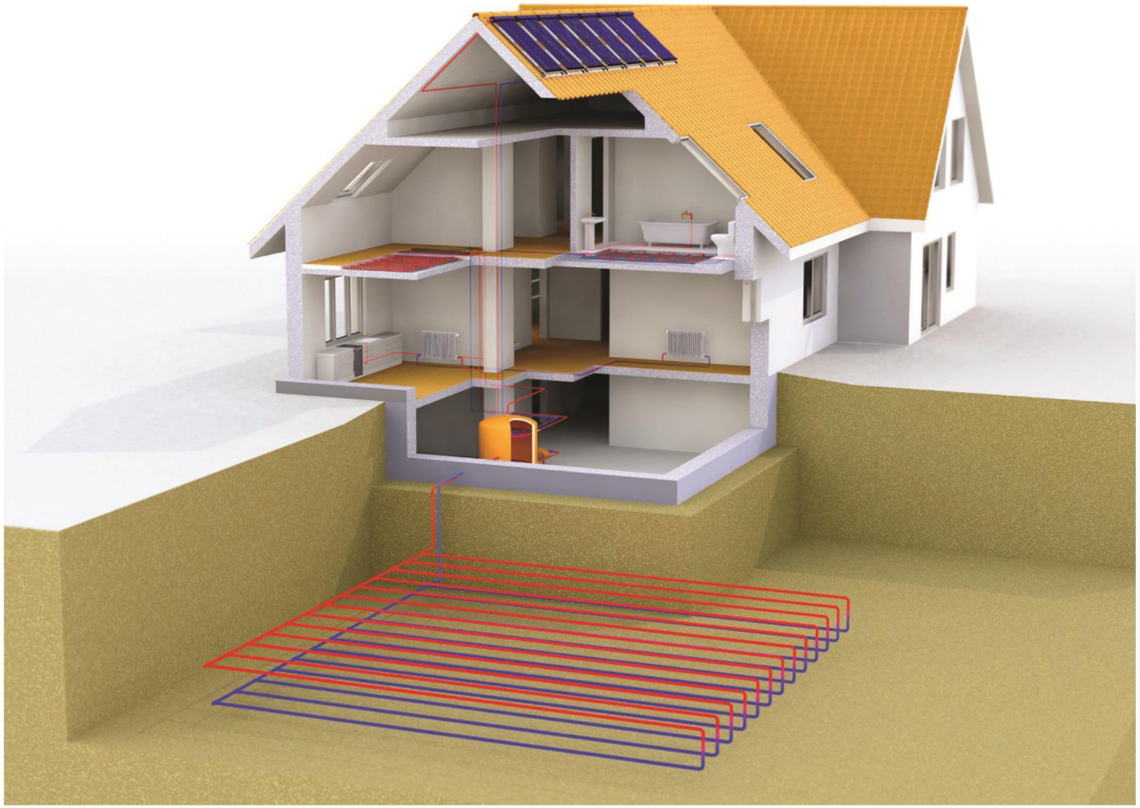
In some applications, losses are negligible. In others, like this residential geothermal system with extremely long tubing, losses must be analyzed.



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Characterizing Flow as Laminar or Turbulent

- Flows with low Reynolds numbers appear slow and smooth and are called *laminar.*

- Flows with high Reynolds numbers appear fast, chaotic, and rough and are called *turbulent.*
- Reynolds number is calculated as

$$N_R = \frac{vD\rho}{\eta} = \frac{vD}{\nu}$$

Reynolds Number is Dimensionless and Characterizes Flow

$$N_R = \frac{vD\rho}{\eta} = \frac{vD}{\nu}$$

If $N_R < 2000$, the flow is laminar.

If $N_R > 4000$, the flow is turbulent.

$$N_R = \frac{vD\rho}{\eta} = v \times D \times \rho \times \frac{1}{\eta}$$

$$N_R = \frac{\text{m}}{\text{s}} \times \text{m} \times \frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}\cdot\text{s}}{\text{kg}}$$

Reynolds Number is Dimensionless and Characterizes Flow

TABLE 8.1 Standard units for quantities used in the calculation of Reynolds number to ensure that it is dimensionless

Quantity	SI Units	U.S. Customary Units
Velocity	m/s	ft/s
Diameter	M	ft
Density	kg/m^3 or $N{\cdot}s^2/m^4$	$slugs/ft^3$ or $lb{\cdot}s^2/ft^4$
Dynamic viscosity	$N{\cdot}s/m^2$ or $Pa{\cdot}s$ or $kg/m{\cdot}s$	$lb{\cdot}s/ft^2$ or $slugs/ft{\cdot}s$
Kinematic viscosity	m^2/s	ft^2/s

Pearson

Example Problem 8.1

Determine whether the flow is laminar or turbulent if glycerin at 25°C flows in a circular passage within a fabricated chemical processing device. The diameter of the passage is 150 mm. The average velocity of flow is 3.6 m/s.

We must first evaluate the Reynolds number using Equation (8–1):

$$N_R = vD\rho/\eta$$

$$v = 3.6 \text{ m/s}$$

$$D = 0.15 \text{ m}$$

$$\rho = 1258 \text{ kg/m}^3 \text{ (from Appendix B)}$$

$$\eta = 9.60 \times 10^{-1} \text{ Pa·s (from Appendix B)}$$

Then we have

$$N_R = \frac{(3.6)(0.15)(1258)}{9.60 \times 10^{-1}} = 708$$

Because $N_R = 708$, which is less than 2000, the flow is laminar. Notice that each term was expressed in consistent SI units before N_R was evaluated.

Example Problem 8.2

Determine whether the flow is laminar or turbulent if water at 70°C flows in a hydraulic copper tube with a 32 mm OD × 2.0 mm wall. The flow rate is 285 L/min.

Evaluate the Reynolds number, using Eq. (8–1):

$$N_R = \frac{vD\rho}{\eta} = \frac{vD}{\nu}$$

For the copper tube, $D = 28$ mm $= 0.028$ m, and $A = 6.158 \times 10^{-4}$ m^2 (from Appendix G.2). Then we have

$$v = \frac{Q}{A} = \frac{285 \text{ L/min}}{6.158 \times 10^{-4} \text{ m}^2} \times \frac{1 \text{ m}^3/\text{s}}{60\,000 \text{ L/min}} = 7.71 \text{ m/s}$$

$$\nu = 4.11 \times 10^{-7} \text{ m}^2/\text{s} \quad \text{(from Appendix A)}$$

$$N_R = \frac{(7.71)(0.028)}{4.11 \times 10^{-7}} = 5.25 \times 10^5$$

Because the Reynolds number is greater than 4000, the flow is turbulent.

Example Problem 8.3

Determine the range of average velocity of flow for which the flow would be in the critical region if SAE 10 oil at 60°F is flowing in a 2-in Schedule 40 steel pipe. The oil has a specific gravity of 0.89.

Solution The flow would be in the critical region if $2000 < N_R < 4000$. First, we use the Reynolds number and solve for velocity:

$$N_R = \frac{vD\rho}{\eta}$$

$$v = \frac{N_R\eta}{D\rho} \qquad \textbf{(8–2)}$$

Then we find the values for η, D, and ρ:

$$D = 0.1723 \text{ ft} \quad \text{(from Appendix F)}$$

$$\eta = 2.10 \times 10^{-3} \text{ lb-s/ft}^2 \quad \text{(from Appendix D)}$$

$$\rho = (\text{sg})(1.94 \text{ slugs/ft}^3) = (0.89)(1.94 \text{ slugs/ft}^3) = 1.73 \text{ slugs/ft}^3$$

Substituting these values into Eq. (8–2), we get

$$v = \frac{N_R(2.10 \times 10^{-3})}{(0.1723)(1.73)} = (7.05 \times 10^{-3})N_R$$

For $N_R = 2000$, we have

$$v = (7.05 \times 10^{-3})(2 \times 10^3) = 14.1 \text{ ft/s}$$

For $N_R = 4000$, we have

$$v = (7.05 \times 10^{-3})(4 \times 10^3) = 28.2 \text{ ft/s}$$

Therefore, if $14.1 < v < 28.2$ ft/s, the flow will be in the critical region.

Darcy's Equation

In the general energy equation

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_A - h_R - h_L = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

the term h_L is defined as the energy loss from the system. One component of the energy loss is due to friction in the flowing fluid. Friction is proportional to the velocity head of the flow and to the ratio of the length to the diameter of the flow stream, for the case of flow in pipes and tubes. This is expressed mathematically as Darcy's equation:

⇨ Darcy's Equation for Energy Loss

$$h_L = f \times \frac{L}{D} \times \frac{v^2}{2g}$$

where

h_L = energy loss due to friction (N·m/N, m, lb-ft/lb, or ft)

L = length of flow stream (m or ft)

D = pipe diameter (m or ft)

v = average velocity of flow (m/s or ft/s)

f = friction factor (dimensionless)

Friction Loss in Laminar Flow

Because laminar flow is so regular and orderly, we can derive a relationship between the energy loss and the measurable parameters of the flow system. This relationship is known as the Hagen–Poiseuille equation:

Hagen–Poiseuille Equation

$$h_L = \frac{32\eta L v}{\gamma D^2}$$

Pearson

Friction Factor for Laminar Flow

For orderly laminar flow, the friction factor has a simple, linear relationship with the Reynolds Number

Friction Factor for Laminar Flow

$$f \times \frac{L}{D} \times \frac{v^2}{2g} = \frac{32\eta L v}{\gamma D^2}$$

$$f = \frac{32\eta L v}{\gamma D^2} \times \frac{D2g}{Lv^2} = \frac{64\eta g}{vD\gamma}$$

Because $\rho = \gamma/g$, we get

$$f = \frac{64\eta}{vD\rho}$$

$$f = \frac{64}{N_R}$$

Example Problem 8.4

Determine the energy loss if glycerin at 25°C flows 30 m through a standard DN 150-mm Schedule 80 pipe with an average velocity of 4.0 m/s.

Solution First, we must determine whether the flow is laminar or turbulent by evaluating the Reynolds number:

$$N_R = \frac{vD\rho}{\eta}$$

From Appendix B, we find that for glycerin at 25°C

$$\rho = 1258 \text{ kg/m}^3$$
$$\eta = 9.60 \times 10^{-1} \text{ Pa·s}$$

Then, we have

$$N_R = \frac{(4.0)(0.1463)(1258)}{9.60 \times 10^{-1}} = 767$$

Because $N_R < 2000$, the flow is laminar.

Using Darcy's equation, we get

$$h_L = f \times \frac{L}{D} \times \frac{v^2}{2g}$$

$$f = \frac{64}{N_R} = \frac{64}{767} = 0.0835$$

$$h_L = 0.0835 \times \frac{30}{0.1463} \times \frac{(4.0)^2}{2(9.81)} \text{m} = 13.96 \text{ m}$$

Notice that each term in each equation is expressed in the units of the SI unit system. Therefore, the resulting units for h_L are m or N·m/N. This means that 13.96 N·m of energy is lost by each newton of the glycerin as it flows along the 30 m of pipe.

Friction Factor for Turbulent Flow

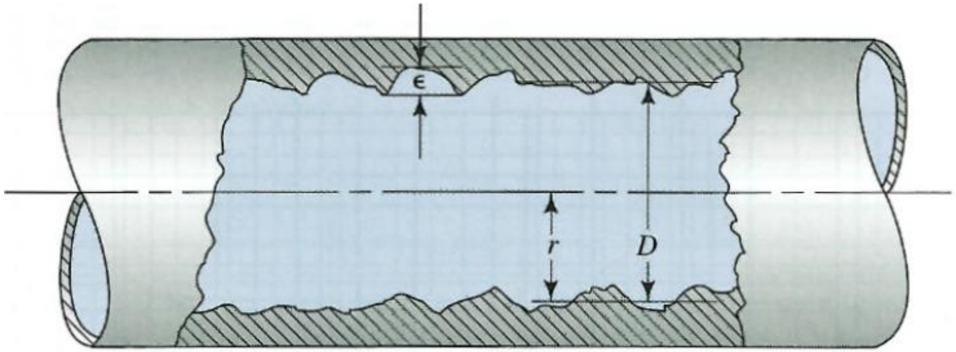


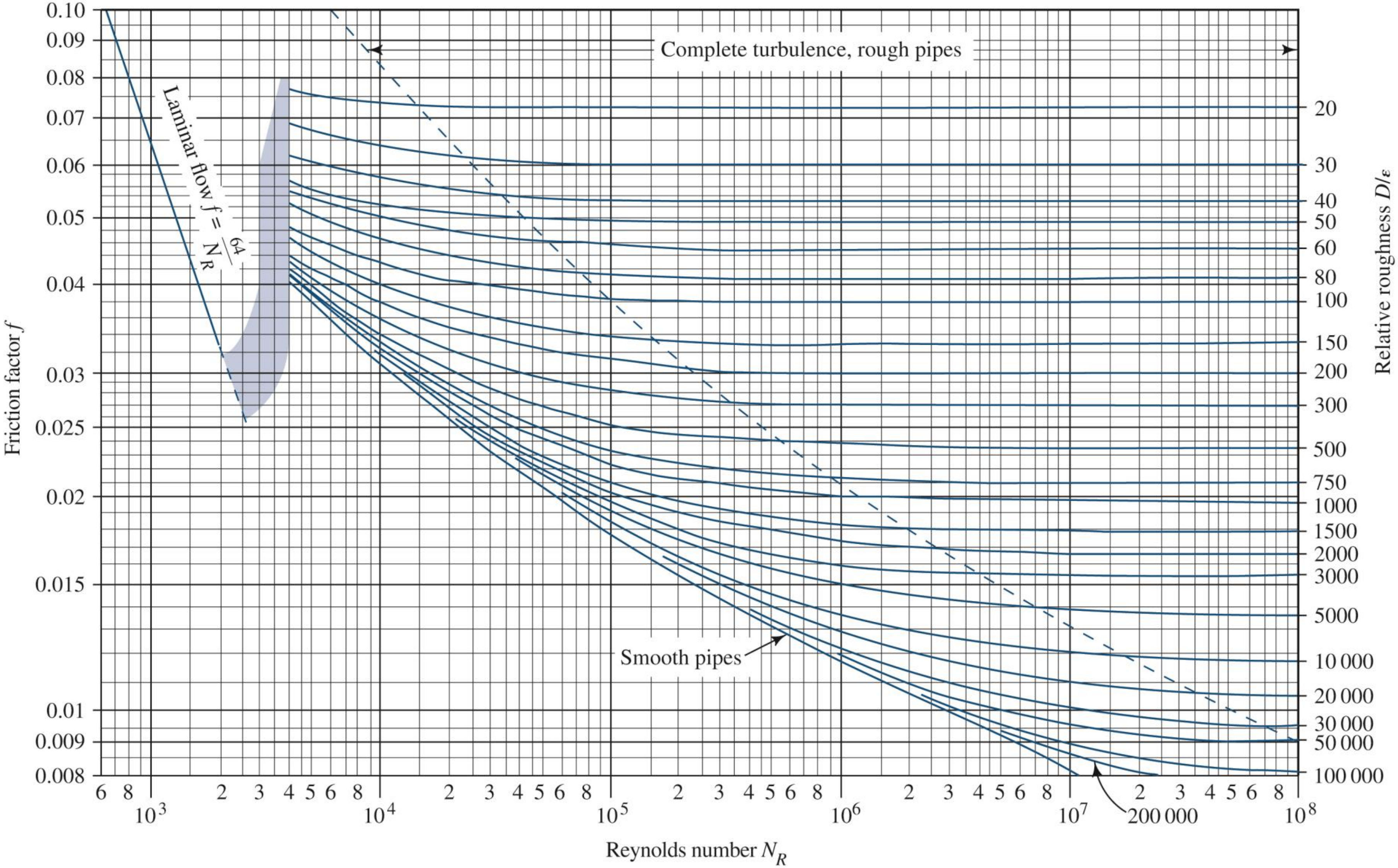
TABLE 8.2 Pipe roughness—design values

Material	Roughness ε (m)	Roughness ε (ft)
Glass	Smooth	Smooth
Plastic	3.0×10^{-7}	1.0×10^{-6}
Drawn tubing; copper, brass, steel	1.5×10^{-6}	5.0×10^{-6}
Steel, commercial or welded	4.6×10^{-5}	1.5×10^{-4}
Galvanized iron	1.5×10^{-4}	5.0×10^{-4}
Ductile iron—coated	1.2×10^{-4}	4.0×10^{-4}
Ductile iron—uncoated	2.4×10^{-4}	8.0×10^{-4}
Concrete, well made	1.2×10^{-4}	4.0×10^{-4}
Riveted steel	1.8×10^{-3}	6.0×10^{-3}

Friction Factor for Turbulent Flow

- Most industrial systems are turbulent, not laminar.
- Determining the friction factor is a more complex process, either relying on careful interpolation from Moody's Diagram or a lengthy equation given in Section 8.8 of the textbook.
- The friction factor depends on the velocity of fluid flow, the diameter of the pipe, the inside surface finish of the pipe, and the viscosity of the fluid.
- It take some practice to become proficient at reading the Moody's Diagram, and practice exercises with solutions are provided in the book.

Moody’s diagram



Example Problem 8.5

Determine the friction factor f if water at 160°F is flowing at 30.0 ft/s in a 1-in Schedule 40 steel pipe.

The Reynolds number must first be evaluated to determine whether the flow is laminar or turbulent:

$$N_R = \frac{vD}{\nu}$$

From Appendix F: $D = 1.049$ in $= 0.0874$ ft. For the water, from Appendix A.2, $\nu = 4.38 \times 10^{-6}$ ft^2/s, then

$$N_R = \frac{(30.0)(0.0874)}{4.38 \times 10^{-6}} = 5.98 \times 10^5$$

Thus, the flow is turbulent. Now the relative roughness must be evaluated. From Table 8.2 we find $\varepsilon = 1.5 \times 10^{-4}$ ft. Then the relative roughness is

$$\frac{D}{\varepsilon} = \frac{0.0874 \text{ ft}}{1.5 \times 10^{-4} \text{ ft}} = 583$$

Notice that for D/ε to be a dimensionless ratio, both D and ε must be in the same units.

The final steps in the procedure are as follows:

1. Locate the Reynolds number on the abscissa of the Moody diagram:

$$N_R = 5.98 \times 10^5$$

2. Project vertically until the curve for $D/\varepsilon = 583$ is reached. You must interpolate between the curve for 500 and the one for 750 on the vertical line for $N_R = 5.98 \times 10^5$.
3. Project horizontally to the left, and read $f = 0.023$.

Example Problem 8.6

If the flow velocity of water in Problem 8.5 was 0.45 ft/s with all other conditions being the same, determine the friction factor f.

$$N_R = \frac{vD}{\nu} = \frac{(0.45)(0.0874)}{4.38 \times 10^{-6}} = 8.98 \times 10^3$$

$$\frac{D}{\varepsilon} = \frac{0.0874}{1.5 \times 10^{-4}} = 583$$

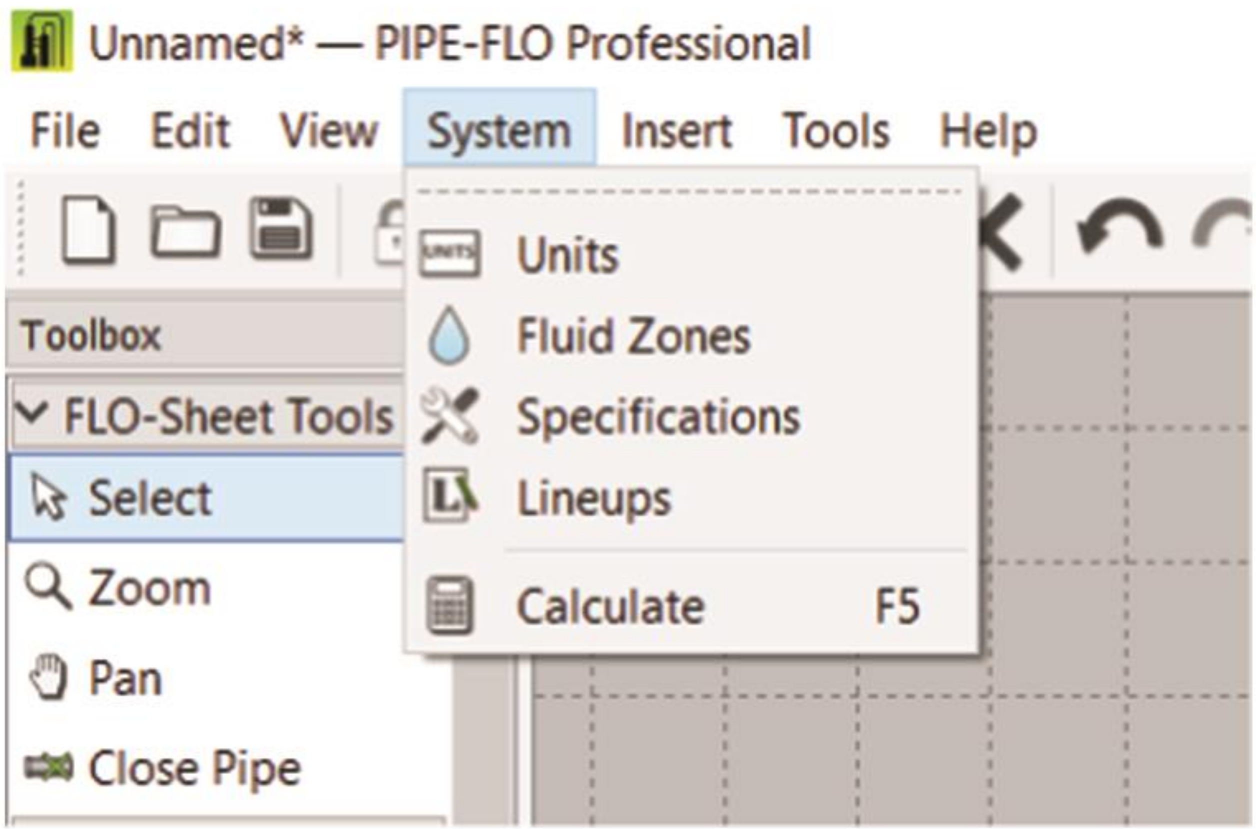
Then, from Fig. 8.7, $f = 0.0343$. Notice that this is on the curved portion of the D/ε curve and that there is a significant increase in the friction factor over that in Example Problem 8.5.

Typically when engineering calculations become tedious, a software solution is available, and this is no exception.

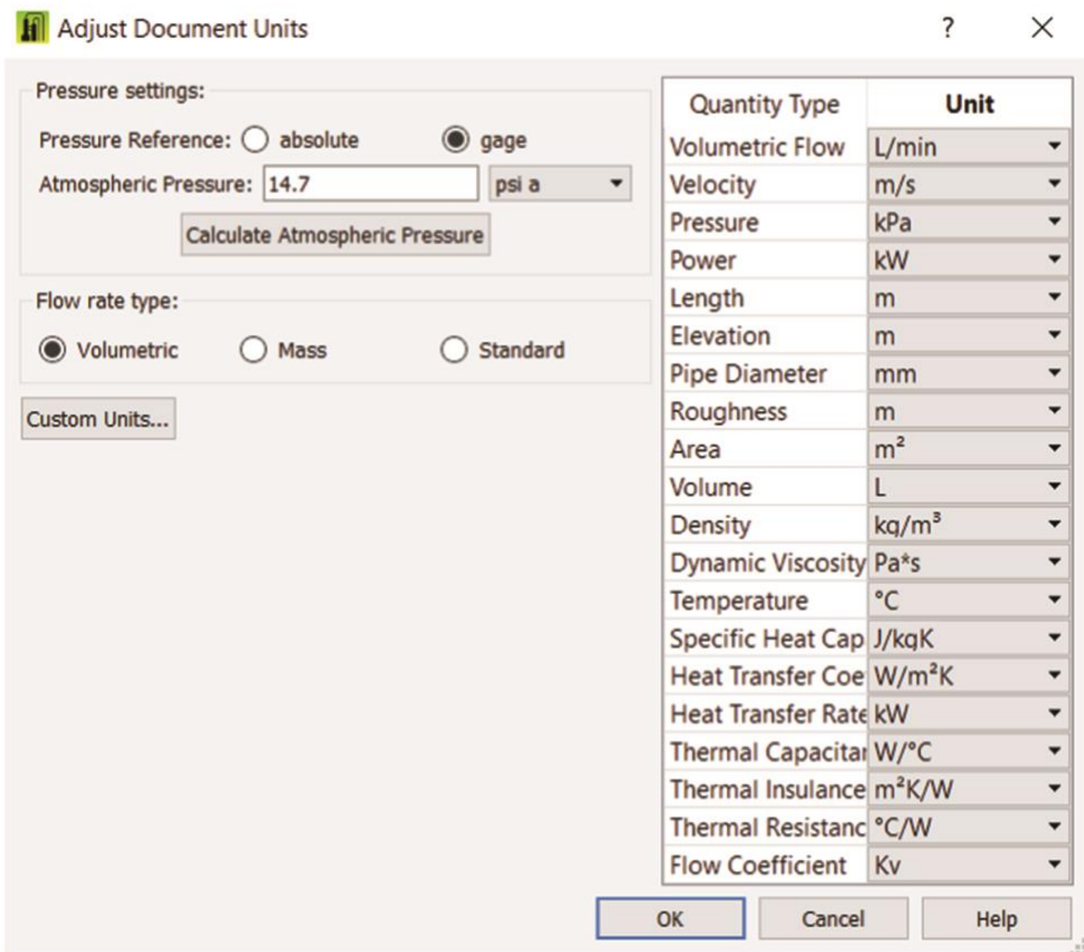
PIPE-FLO is excellent software and is available to all users of this textbook. Tutorials are offered in the textbook, guiding students through responsible use of the program to solve problems like those solved manually.

Contact PIPE-FLO in Lacey, WA. More information is available on PIPE-FLO throughout the textbook.

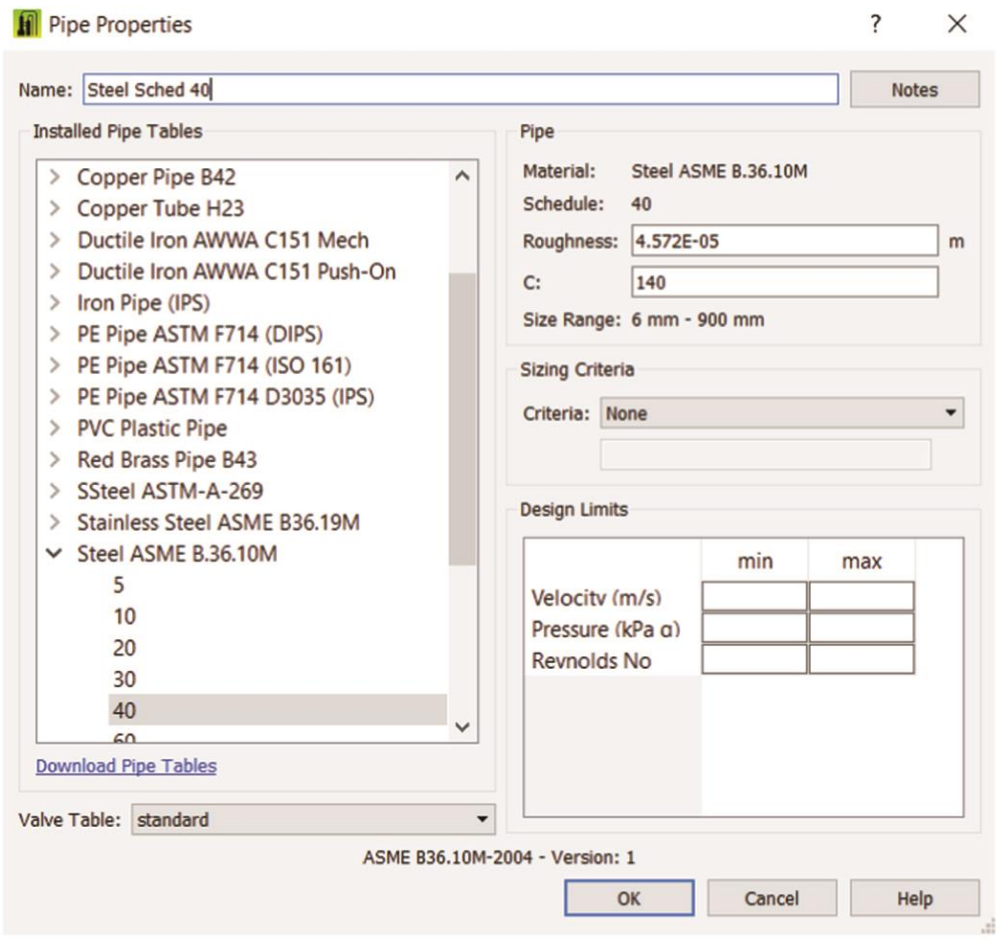
Consider the use of PIPE-FLO software to make these calculations



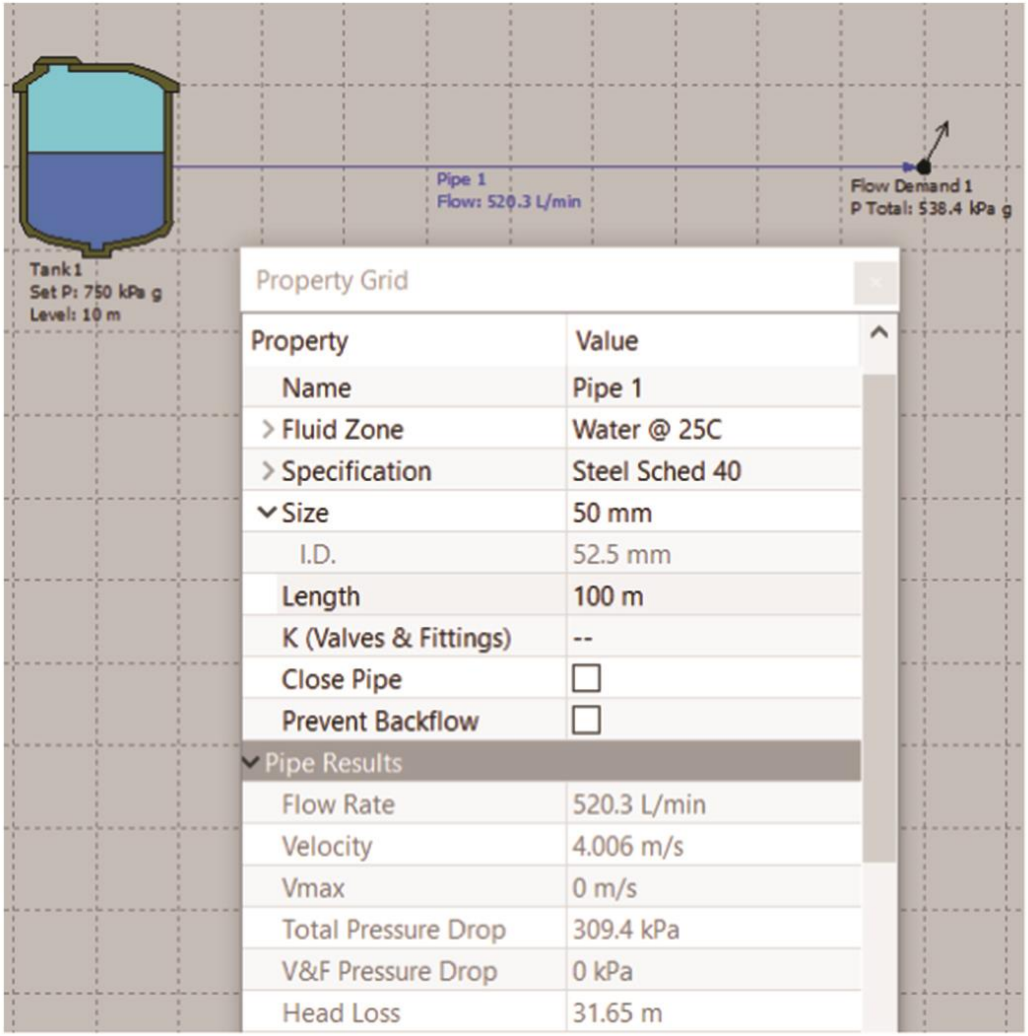
Data entry is clean and complete, and extensive unit systems are available



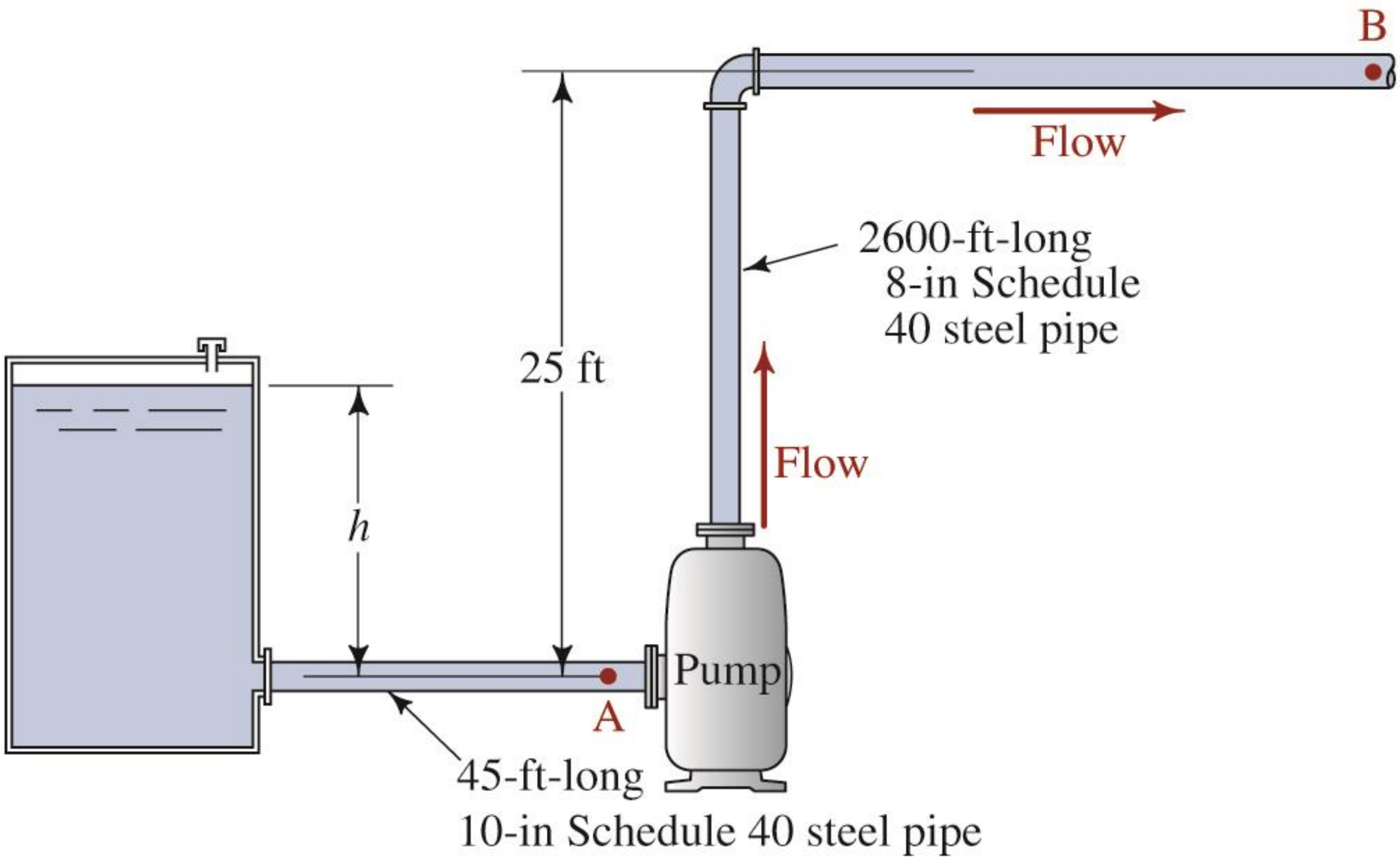
Standard materials are available, saving the time to look up all values



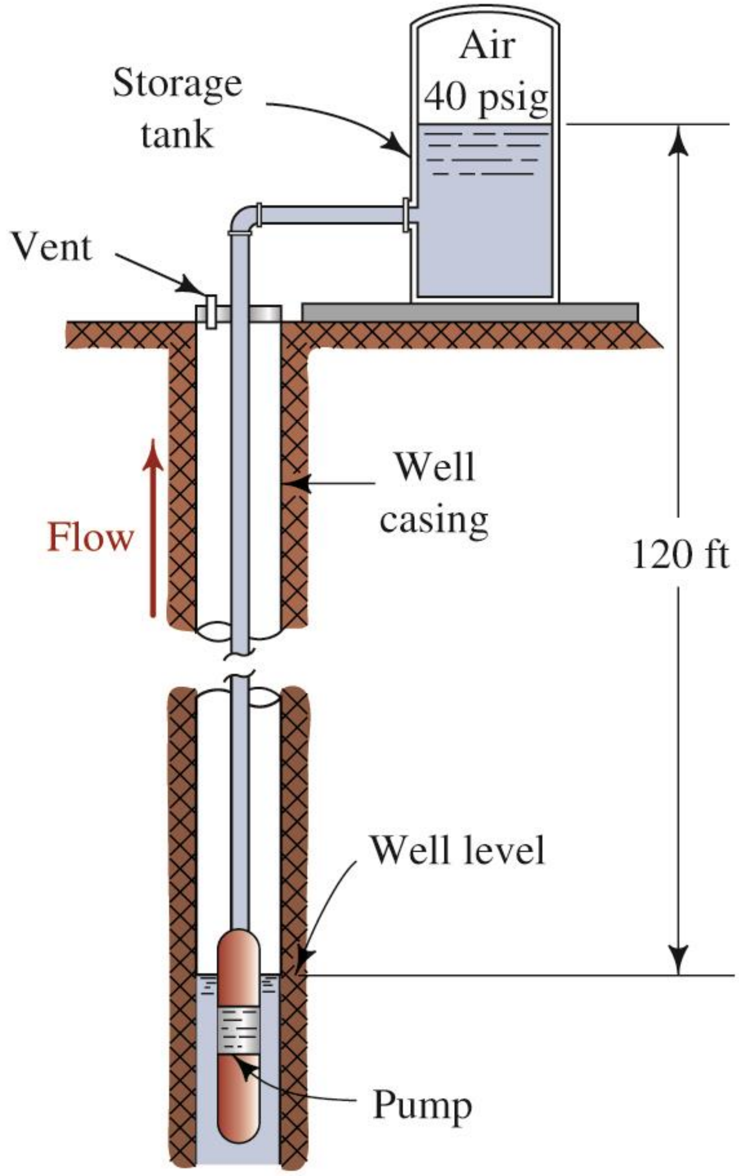
Solutions are graphical, clear, complete, and accurate.



Fire protection system for Problem 8.33



Well Pump for Problem 8.34



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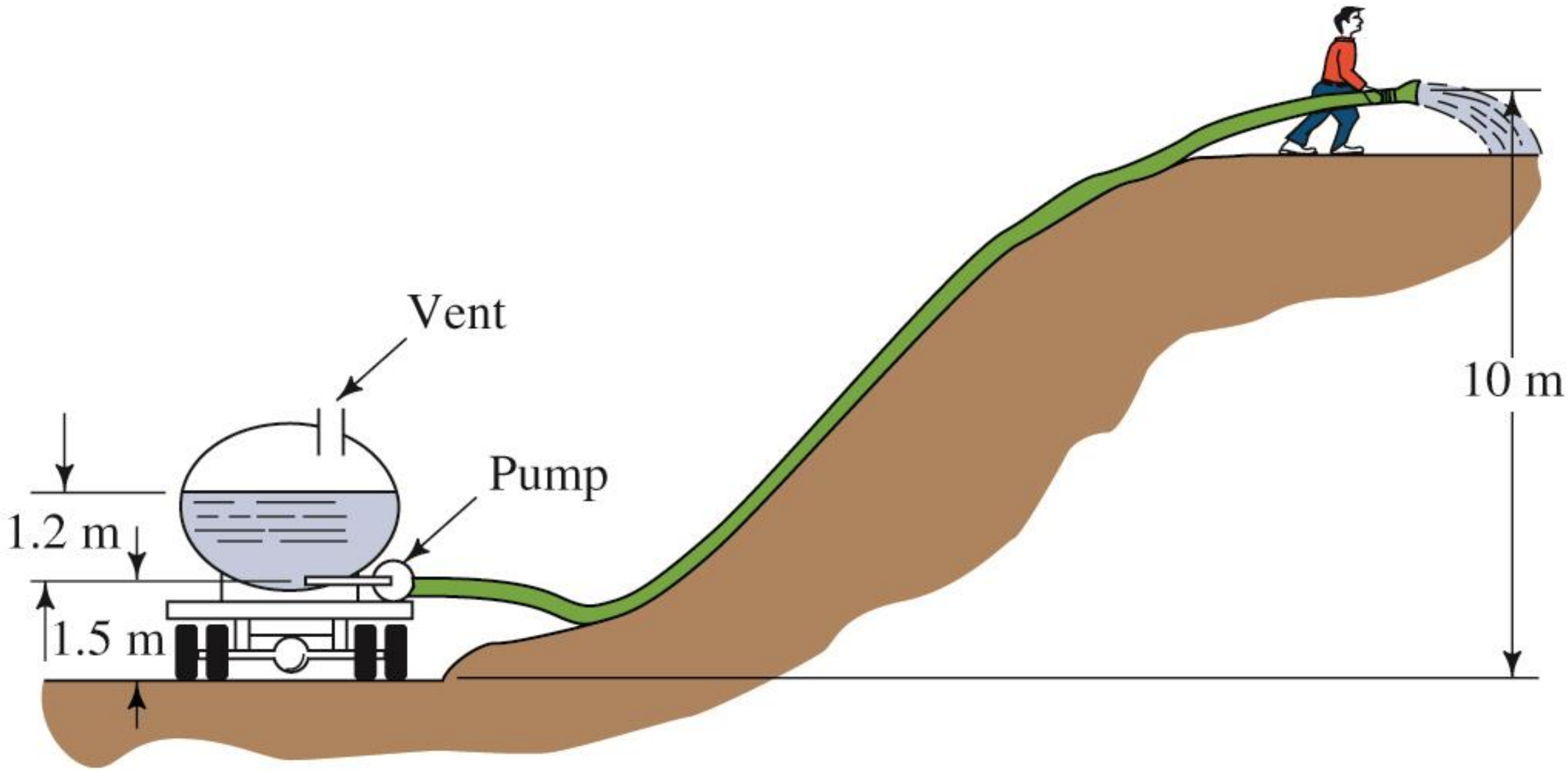
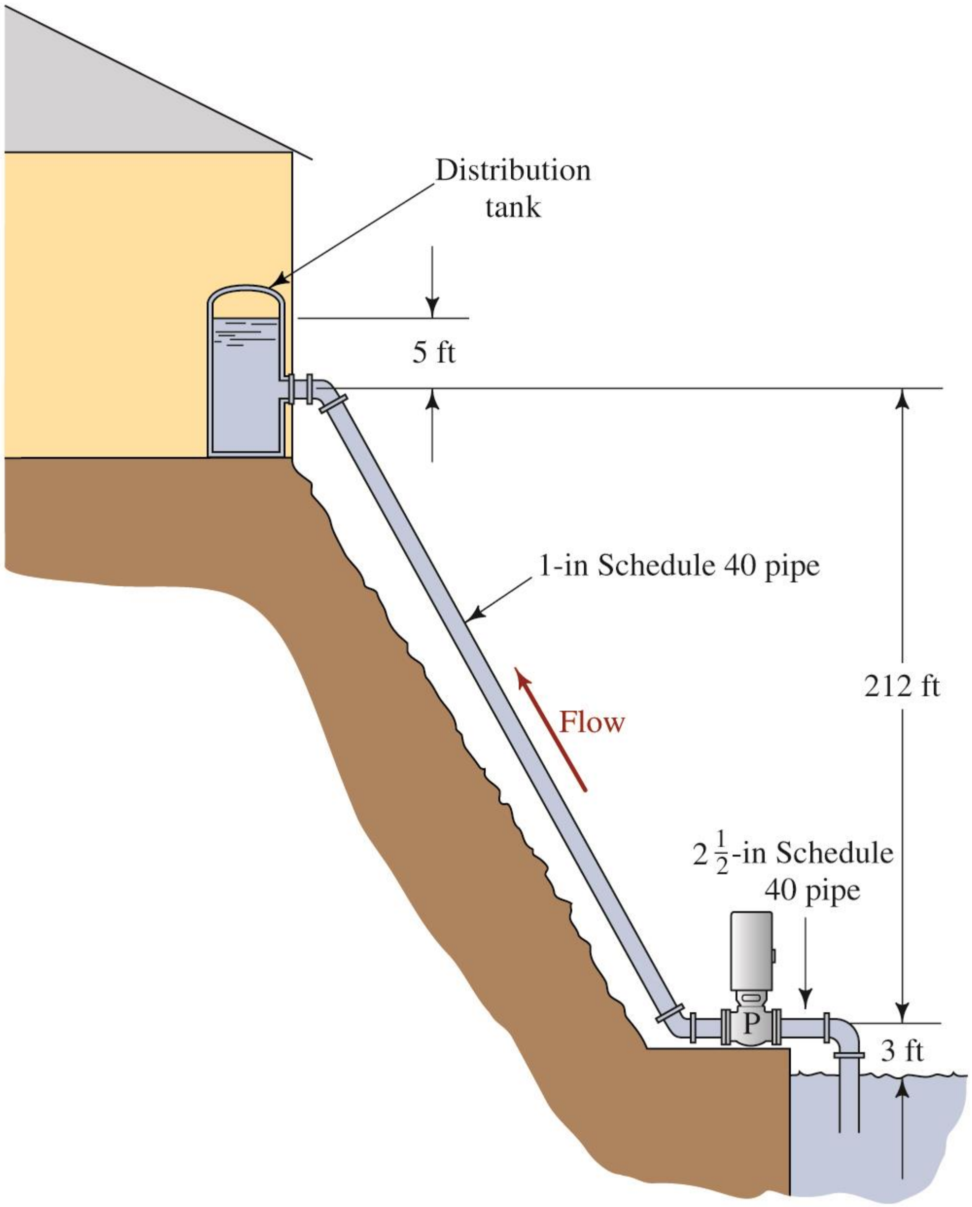
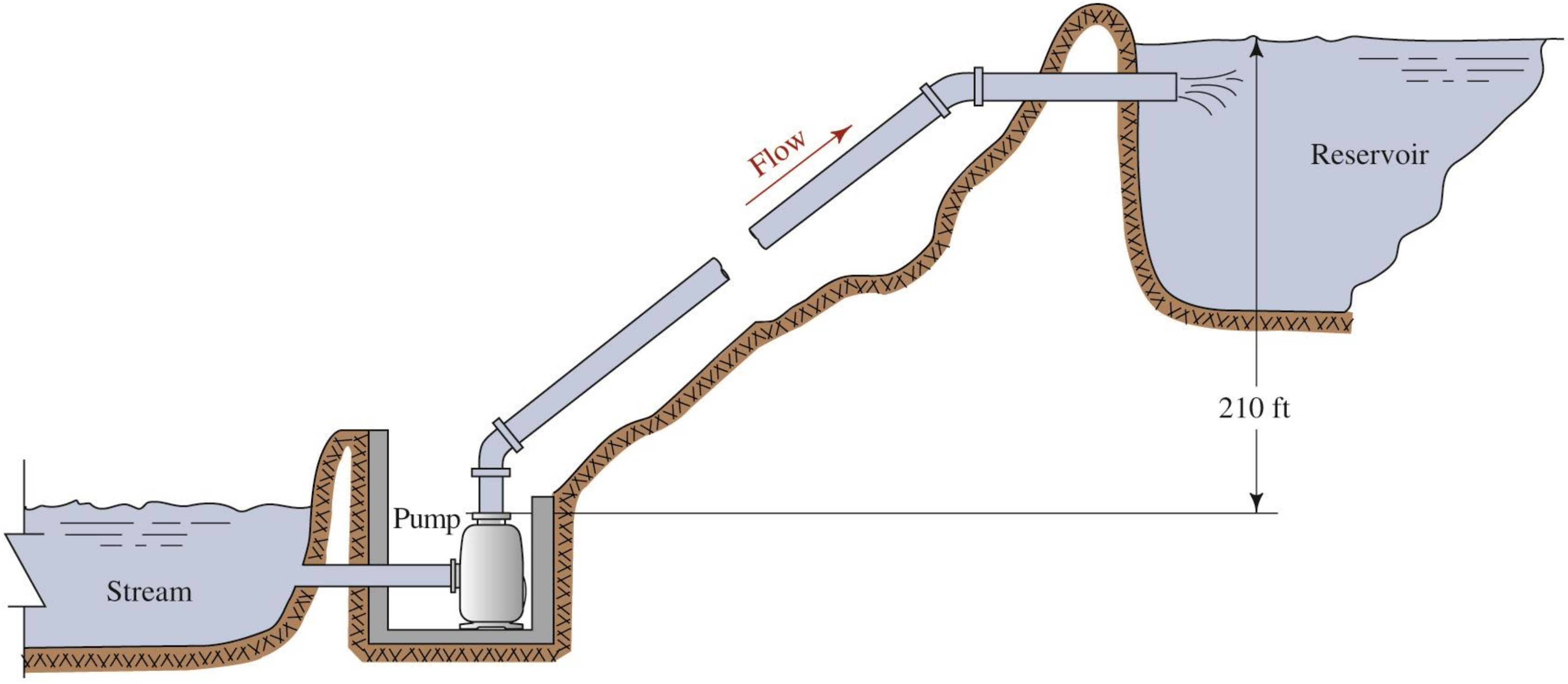
Fertilizer truck for Problem 8.36



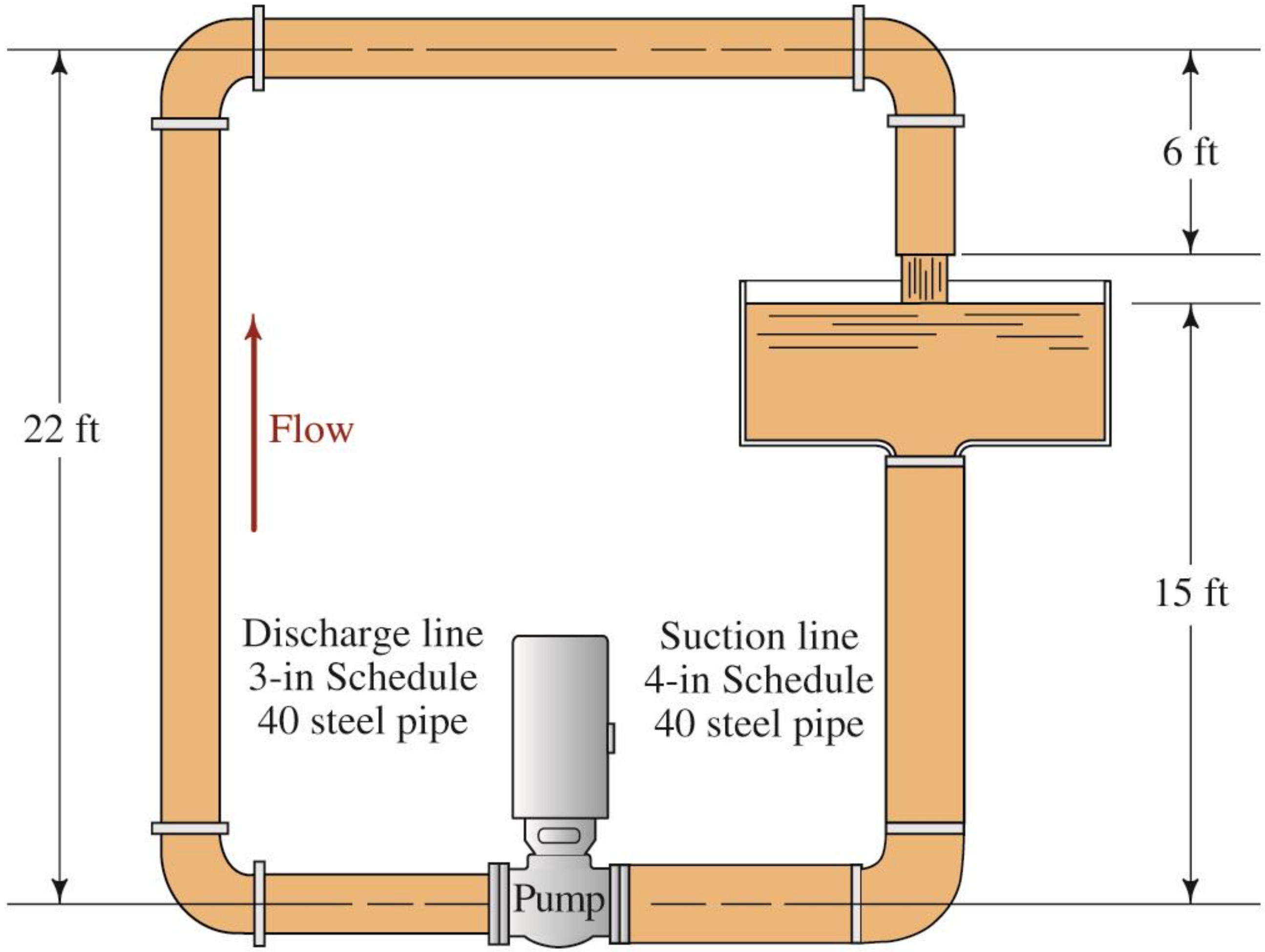


Water supply system for Problem 8.40

Reservoir pump for Problems 8.44 and 8.45



Oil recirculation system for Problem 8.47



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