Solutions to end-of-chapter problems

Engineering Economy, 8th edition

Leland Blank and Anthony Tarquin

Chapter 1
Foundations of Engineering Economy

Basic Concepts

1.1 Financial units for economically best.

1.2 Morale, goodwill, dependability, acceptance, friendship, convenience, aesthetics, etc.

1.3 Measure of worth is a criterion used to select the economically best alternative. Some measures are present worth, rate of return, payback period, benefit/cost ratio.

1.4 The color I like, best fuel rating, roomiest, safest, most stylish, fastest, etc.

1.5 Sustainability: Intangible; installation cost: tangible; transportation cost: *tangible*; simplicity: intangible; taxes: tangible; resale value: tangible; morale: intangible; rate of return: tangible; dependability: intangible; inflation: tangible; acceptance by others: intangible; ethics: intangible.

1.6 Examples are: house purchase; car purchase, credit card (which ones to use); personal loans (and their rate of interest and repayment schedule); investment decisions of all types; when to sell a house or car.

Ethics

1.7 *This problem can be used as a discussion topic for a team-based exercise in class.*

(a) Most obvious are the violations of Canons number 4 and 5. Unfaithfulness to the client and deceptive acts are clearly present.

(b) The Code for Engineer's is only partially useful to the owners in determining sound bases since the contractor is not an engineer. Much of the language of the Code is oriented toward representation, qualifications, etc., not specific acts of deceit and fraudulent behavior. Code sections may be somewhat difficult to interpret in construction of a house.

(c) Probably a better source would be a Code for Contractor's or consulting with a real estate attorney.

1.8 Many sections could be identified. Some are: I.b; II.2.a and b; III.9.a and b.

1.9 Example actions are:

- Try to talk them out of doing it now, explaining it is stealing
- Try to get them to pay for their drinks

- Pay for all the drinks himself
- Walk away and not associate with them again

1.10 *This is structured to be a discussion question; many responses are acceptable.* Responses can vary from the ethical (stating the truth and accepting the consequences) to unethical (continuing to deceive himself and the instructor and devise some on-the-spot excuse).

Lessons can be learned from the experience. A few of them are:

- Think before he cheats again.
- Think about the longer-term consequences of unethical decisions.
- Face ethical-dilemma situations honestly and make better decisions in real time.

Alternatively, Claude may learn nothing from the experience and continue his unethical practices.

Interest Rate and Rate of Return

1.11 Extra amount received = 2865 - 25.80*100 = $285
Rate of return = 285/2580
= 0.110 (11%)

Total invested + fee 2865 + 50 = $2915
Amount required for 11% return = 2915*1.11
= $3235.65

1.12 (a) Payment = 1,600,000(1.10)(1.10) = $1,936,000
(b) Interest = total amount paid – principal
= 1,936,000- 1,600,000
= $336,000

1.13 i = [(5,184,000 – 4,800,000)/4,800,000]*100% = 8% per year

1.14 Interest rate = interest paid/principal
= (312,000/2,600,000)
= 0.12 (12%)

1.15 i = (1125/12,500)*100 = 9%
i = (6160/56,000)*100 = 11%
i = (7600/95,000)*100 = 8%
The $56,000 investment has the highest rate of return

1.16 Interest on loan = 45,800(0.10) = $4,580
Default insurance = $900
Set-up fee = 45,800(0.01) = 458
Total amount paid = 4,580 + 900 + 458 = $5938

Effective interest rate = (5,938/45,800)*100 = 12.97%

Terms and Symbols

1.17 P = ?; F = 8*240,000 = $1,920,000; n = 2; i = 0.10

1.18 P = $20,000,000; A = ?; n = 6; i = 0.10

1.19 P = $2,400,000; A = $760,000: n = 5; i = ?

1.20 P = $1,500,000; F = $3,000,000: n = ?; i = 0.20

1.21 F = $250,000; A = ?: n = 3; i = 0.09

Cash Flows

1.22 Well drilling: *outflow*; maintenance: *outflow*; water sales: *inflow*; accounting: *outflow*; government grants: *inflow*; issuance of bonds: *inflow*; energy cost: *outflow*; pension plan contributions: *outflow*; heavy equipment purchases: *outflow*; used-equipment sales: *inflow*; stormwater fees: *inflow*; discharge permit revenues: *inflow*.

1.23 Let Rev = Revenues; Exp = Expenses

Year	1	2	3	4	5	Total
Rev, $1000	521	685	650	804	929	
Exp, $1000	610	623	599	815	789	
NCF, $1000	-89	62	51	-11	140	153
Exp/Rev, %	117	91	92	101	85	

(a) Total NCF = $153,000
(b) Last row of the table shows the answers

1.24

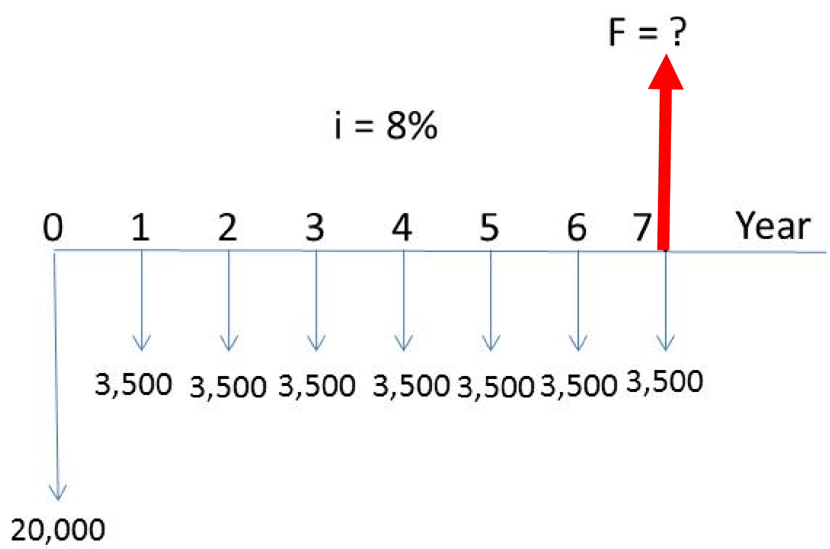
Month	Receipts, $1000	Disbursements, $1000	NCF, $1000
Jan	300	500	-200
Feb	950	500	+450
Mar	200	400	-200
Apr	120	400	-280
May	600	500	+100
June	900	600	+300
July	800	300	+500
Aug	900	300	+600
Sept	900	200	+700
Oct	500	400	+100
Nov	400	400	0
Dec	1800	700	+1100

+3,170

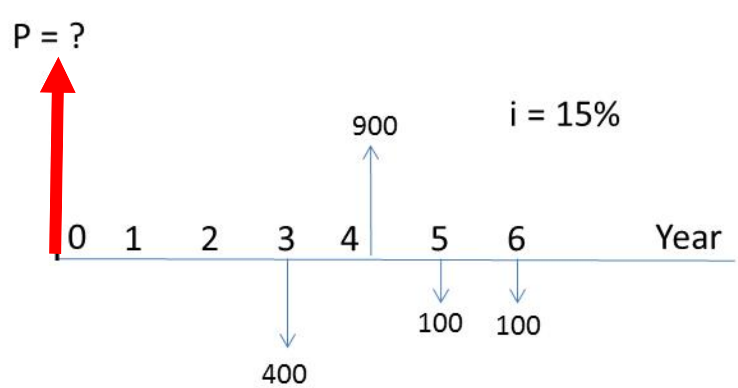
Net cash flow = \$3,170 (\$3,170,000)

1.25 End-of-period amount for March: 50 + 70 = \$120; Interest = 120*0.03 = \$3.60
End-of-period amount for June: 120 + 120 + 20 = \$260; Interest = 260*0.03 = \$7.80
End-of-period amount for September: 260 + 150 + 90 = \$500; Interest = \$15.00
End-of-period amount for Dec: 500 + 40 + 110 = \$650; Interest = \$19.50

1.26



1.27



Equivalence

1.28 (a) i = (5000-4275)/4275 = 0.17 (17%)

(c) Price one year later = 28,000 * 1.04 = \$29,120

(d) Price one year earlier = 28,000/1.04 = $26,923

(e) Jackson: Interest rate = (2750/20,000)*100
= 13.75%

Henri: Interest rate = (2295/15,000)*100
= 15.30%

(f) 81,000 = 75,000 + 75,000(i)
i = 6,000/75,000
= 0.08 (8%)

1.29 (a) Profit = 8,000,000*0.28
= $2,240,000

(b) Investment = 2,240,000/0.15
= $14,933,333

1.30 P + P(0.10) = 1,600,000
1.1P = 1,600,000
P = $1,454,545

1.31 Equivalent present amount = 1,000,000/(1 + 0.10)
= $909,091
Discount = 790,000 – 909,091
= $119,091

1.32 Total bonus next year = (this year's bonus + interest) + next year's bonus
= [4,000 + 4,000(0.10)] + 4,000
= $8,400

1.33 (a) Early-bird: 20,000 – 20,000(0.10) = $18,000

(b) Equivalent future amount = 18,000(1 + 0.06)
= $19,080

Savings = 20,000 - $19,080
= $920

Simple and Compound Interest

1.34 (a) F = P + Pni
1,000,000 = P + P(3)(0.20)
1.60P = 1,000,000
P = $625,000

(b) $P(1+i)(1+i)(1+i) = 1{,}000{,}000$

$P = 1{,}000{,}000/[(1+0.20)(1+0.20)(1+0.20)]$

$= \$578{,}704$

1.35 $F = P + Pni$

$120{,}000 = P + P(3)(0.07)$

$1.21P = 120{,}000$

$P = \$99{,}173.55$

1.36 $F = 240{,}000(1+ 0.12)^3$

$= \$337{,}183$

1.37 (a) $F = P + Pni$

$10{,}000 = 5000 + 5000(n)(0.12)$

$5000 = 600n$

$n = 8.33$ years

(b) $10{,}000 = 5000 + 5000(n)(0.20)$

$n = 5$ years

1.38 (a) Total due; compound interest $= 150{,}000(1.05)(1.05)(1.05)$

$= \$173{,}644$

Total due; simple interest $= P + Pni$

$= 150{,}000 + 150{,}000(3)(0.055)$

$= 150{,}000 + 24{,}750$

$= \$174{,}750$

Select the 5% compound interest rate

(b) Difference $= 174{,}750 - 173{,}644$

$= \$1106$

1.39 $90{,}000 = 60{,}000 + 60{,}000(5)(i)$

$300{,}000\ i = 30{,}000$

$i = 0.10$ (10% per year)

1.40 Simple: $F = 10{,}000 + 10{,}000(3)(0.10)$

$= \$13{,}000$

Compound: $13{,}000 = 10{,}000(1 + i)(1 + i)(1 + i)$

$(1 + i)^3 = 1.3000$

$3\log(1 + i) = \log 1.3$

$$3\log(1 + i) = 0.1139$$
$$\log(1 + i) = 0.03798$$
$$1 + i = 1.091$$
$$i = 9.1\% \text{ per year}$$

Spreadsheet function: = RATE(3,,-10000,13000) displays 9.14%

1.41 Follow plan 4, Example 1.16 as a model

A is 9900 – 2000 = \$7900
B is 7900(0.10) = \$790
C is 7900 + 790 = \$8690
D is 8690 – 2000 = \$6690

1.42 (a) Simple:

$$F = P + Pni$$
$$2{,}800{,}000 = 2{,}000{,}000 + 2{,}000{,}000(4)(i)$$
$$i = 10\% \text{ per year}$$

(b) Compound:

$$F = P(1 + i)(1 + i)(1 + i)(1 + i)$$
$$2{,}800{,}000 = 2{,}000{,}000(1 + i)^4$$
$$(1 + i)^4 = 1.4000$$
$$\log(1 + i)^4 = \log 1.400$$
$$4\log(1 + i) = 0.146$$
$$\log(1 + i) = 0.0365$$
$$(1 + i) = 10^{0.0365}$$
$$(1 + i) = 1.0877$$
$$i = 8.77\%$$

(c) Spreadsheet function: = RATE(4,,-2000000,2800000)

MARR and Opportunity Cost

1.43 Bonds - debt; stock sales – equity; retained earnings – equity; venture capital – debt; short term loan – debt; capital advance from friend – debt; cash on hand – equity; credit card – debt; home equity loan - debt.

1.44 WACC = 0.40(10%) + 0.60(16%) = 13.60%

1.45 WACC = 0.05(10%) + 0.95(19%) = 18.55%

The company should undertake the inventory, technology, warehouse, and maintenance projects.

1.46 Let x = percentage of debt financing; Then, 1- x = percentage of equity financing

$0.13 = x(0.28) + (1-x)(0.06)$
$0.22x = 0.07$
$x = 31.8\%$

Recommendation: debt-equity mix should be 31.8% debt and 68.2% equity financing

1.47 Money: The opportunity cost is the loss of the use of the $5000 plus the $100 interest.

Percentage: The 30% estimated return on the IT stock is the opportunity cost in percentage.

Exercises for Spreadsheets

1.48 (a) PV is P; (b) PMT is A; (c) NPER is n; (d) IRR is i; (e) FV is F; (f) RATE is i

1.49 (a) PV(i%,n,A,F) finds the present value P
(b) FV(i%,n,A,P) finds the future value F
(c) RATE(n,A,P,F) finds the compound interest rate i
(d) IRR(first_cell:last_cell) finds the compound interest rate i
(e) PMT(i%,n,P,F) finds the equal periodic payment A
(f) NPER(i%,A,P,F) finds the number of periods n

1.50 (a) (1) F = ?; i = 8%; n = 10; A = $3000; P = $8000
(2) A = ?; i = 12%; n = 20; P = $-16,000; F = 0
(3) P = ?; i = 9%; n = 15; A = $1000; F = $600
(4) n = ?; i = 10%; A = $-290; P = 0; F = $12,000
(5) F = ?; i = 5%; n = 5; A = $500; P = $-2000

(b) (1) negative
(2) positive
(3) negative
(4) positive (years)
(5) can't determine if 5% per year will cover the 5 withdrawals of $500

1.51 Spreadsheet shows relations only in cell reference format. Cell E10 will indicate $64 more than cell C10.

	A	B	C	D	E
1	Initial amount =	1000		i =	0.1
2					
3		**Simple**		**Compound**	
4	Year	Interest, $	Total, $	Interest, $	Total, $
5	0		= $B$1		= $B$1
6	1	= $B$1*$E$1	= C5 + B6	= $E5 * $E$1	= E5 + D6
7	2	= $B$1*$E$1	= C6 + B7	= $E6 * $E$1	= E6 + D7
8	3	= $B$1*$E$1	= C7 + B8	= $E7 * $E$1	= E7 + D8
9	4	= $B$1*$E$1	= C8 + B9	= $E8 * $E$1	= E8 + D9
10	Total	=SUM(B6:B9)	= C9	=SUM(D6:D9)	= E9

Additional Problems and FE Review Questions

1.52 Answer is (d)

1.53 Answer is (b)

1.54 Answer is (c)

1.55 Answer is (b)

1.56 $F = P + Pni$

$2P = P + P(n)(0.05)$

$n = 20$ years

Answer is (d)

1.57 Amount now $= 10{,}000 + 10{,}000(0.10)$

$= \$11{,}000$

Answer is (c)

1.58 Move both cash flows to year 0 and solve for i

$$1000(1 + i) = 1345.60/(1 + i)$$
$$(1 + i)^2 = 1345.60/1000$$
$$(1 + i) = 1.16$$
$$i = 16\%$$

Answer is (d)

1.59 F in year 2 at 20% compound interest $= P(1.20)(1.20) = 1.44P$

For simple interest, $F = P + Pni = P(1 + ni)$

$$P(1 + 2i) = 1.44P$$
$$(1 + 2i) = 1.44P$$

$i = 22\%$

Answer is (c)

1.60 $\quad \text{WACC} = 0.70(16\%) + 0.30(12\%)$

$= 14.8\%$

Answer is (c)

1.61 Amount available = total principal in year 0 + interest for 2 years + principal added year 1 + interest for 1 year

$$= 850{,}000(1+0.15)^2 + 200{,}000\,(1+0.15)$$
$$= 1{,}124{,}125 + 230{,}000$$
$$= \$1{,}354{,}125$$

Answer is (a)

Solution to Case Study, Chapter 1

There is no definitive answer to case study exercises. The following is only an example.

Renewable Energy Sources for Electricity Generation

3. LCOE approximation uses $1/(1.05)^{11} = 0.5847$ and LCOE last year = 0.1022.
Let $X_{11} = I_{11} + M_{11} + F_{11}$

With the limited data, to estimate the value of X_{11} set the LCOE for year 11 equal to the consumer cost for year 10.

$$0.1027 = 0.1022 + \frac{(0.5847)X_{11}}{(0.5847)(5.052 \text{ billion})}$$

$$0.5847X_{11} = (0.0005)(2.9539 \text{ billion})$$

$$X_{11} = \$2.526 \text{ million}$$

If the sum of investments (I_{11}), M&O (M_{11}) and fuel (F_{11}) is significantly different than \$2.526 million, the breakeven value for year 11 may change. Next step is to find the values of I, M and F for year 11.

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Chapter 2
Factors: How Time and Interest Affect Money

Determination of F, P and A

2.1 (1) (F/P, 10%, 7) = 1.9487
(2) (A/P, 12%,10) = 0.17698
(3) (P/G,15%,20) = 33.5822
(4) (F/A,2%,50) = 84.5794
(5) (A/G,35%,15) = 2.6889

2.2 F = 1,200,000(F/P,7%,4)
= 1,200,000(1.3108)
= $1,572,960

2.3 F = 200,000(F/P,10%,3)
= 200,000(1.3310)
= $266,200

2.4 P = 7(120,000)(P/F,10%,2)
= 840,000(0.8264)
= $694,176

2.5 F = 100,000,000/30(F/A,10%,30)
= 3,333,333(164.4940)
= $548,313,333

2.6 P = 25,000(P/F,10%,8)
= 25,000(0.4665)
= $11,662.50

2.7 P = 8000(P/A,10%,10)
= 8000(6.1446)
= $49,156.80

2.8 P = 100,000((P/A,12%,2)
= 100,000(1.6901)
= $169,010

2.9 F = 12,000(F/A,10%,30)
= 12,000(164.4940)
= $1,973,928

2.10 A = 50,000,000(A/F,20%,3)
= 50,000,000(0.27473)
= $13,736,500

2.11 F = 150,000(F/P,18%,5)
= 150,000(2.2878)
= $343,170

2.12 P = 75(P/F,18%,2)
= 75(0.7182)
= $53.865 million

2.13 A = 450,000(A/P,10%,3)
= 450,000(0.40211)
= $180,950

2.14 P = 30,000,000(P/F,10%,5) – 15,000,000
= 30,000,000(0.6209) – 15,000,000
= $3,627,000

2.15 F = 280,000(F/P,12%,2)
= 280,000(1.2544)
= $351,232

2.16 F = (200 – 90)(F/A,10%,8)
= 110(11.4359)
= $1,257,949

2.17 $F = 125{,}000(F/A,10\%,4)$
$= 125{,}000(4.6410)$
$= \$580{,}125$

2.18 $F = 600{,}000(0.04)(F/A,10\%,3)$
$= 24{,}000(3.3100)$
$= \$79{,}440$

2.19 $P = 90{,}000(P/A,20\%,3)$
$= 90{,}000(2.1065)$
$= \$189{,}585$

2.20 $A = 250{,}000(A/F,9\%,5)$
$= 250{,}000(0.16709)$
$= \$41{,}772.50$

2.21 $A = 1{,}150{,}000(A/P,5\%,20)$
$= 1{,}150{,}000(0.08024)$
$= \$92{,}276$

2.22 $P = (110{,}000 * 0.3)(P/A,12\%,4)$
$= (33{,}000)(3.0373)$
$= \$100{,}231$

2.23 $A = 3{,}000{,}000(10)(A/P,8\%,10)$
$= 30{,}000{,}000(0.14903)$
$= \$4{,}470{,}900$

2.24 $A = 50{,}000(A/F,20\%,3)$
$= 50{,}000(0.27473)$
$= \$13{,}736$

Factor Values

2.25 (a) 1. Interpolate between i = 8% and i = 9% at n = 15:

$0.4/1 = x/(0.3152 – 0.2745)$

$x = 0.0163$

$(P/F,8.4\%,15) = 0.3152 – 0.0163$

$= 0.2989$

2. Interpolate between i = 16% and i = 18% at n = 10:

$1/2 = x/(0.04690 - 0.04251)$

$x = 0.00220$

$(A/F,17\%,10) = 0.04690 – 0.00220$

$= 0.04470$

(b) 1. $(P/F,8.4\%,15) = 1/(1 + 0.084)^{15}$

$= 0.2982$

2. $(A/F,17\%,10) = 0.17/[(1 + 0.17)^{10} - 1]$

$= 0.04466$

(c) 1. = -PV(8.4%,15,,1) displays 0.29824

2. = -PMT(17%,10,,1) displays 0.04466

2.26 (a) 1. Interpolate between i = 18% and i = 20% at n = 20:

$1/2 = x/40.06$

$x = 20.03$

$(F/A,19\%,20) = 146.6280 + 20.03$

$=166.658$

2. Interpolate between i = 25% and i = 30% at n = 15:

$1/5 = x/0.5911$

$x = 0.11822$

$(P/A,26\%,15) = 3.8593 – 0.11822$

$= 3.7411$

(b) 1. $(F/A,19\%,20) = [(1+ 0.19)^{20} – 1]/0.19$

$= 165.418$

2. $(P/A,26\%,15) = [(1 + 0.26)^{15} –1]/[0.26(1 + 0.26)^{15}]$

$= 3.7261$

(c) 1. = -FV(19%,20,1) displays 165.41802
2. = -PV(26%,15,1) displays 3.72607

2.27 (a) 1. Interpolate between n = 32 and n = 34:

$1/2 = x/78.3345$

$x = 39.1673$

$(F/P,18\%,33) = 199.6293 + 39.1673$

$= 238.7966$

2. Interpolate between n = 50 and n = 55:

$4/5 = x/0.0654$

$x = 0.05232$

$(A/G,12\%,54) = 8.1597 + 0.05232$

$= 8.2120$

(b) 1. $(F/P,18\%,33) = (1+0.18)^{33}$

$= 235.5625$

2. $(A/G,12\%,54) = \{(1/0.12) – 54/(1+0.12)^{54} –1\}$

$= 8.2143$

2.28 Interpolated value: Interpolate between n = 40 and n = 45:

$3/5 = x/(72.8905 – 45.2593)$

$x = 16.5787$

$(F/P,10\%,43) = 45.2593 + 16.5787$

$= 61.8380$

Formula value: $(F/P,10\%,43) = (1+ 0.10)^{43}$

$= 60.2401$

% difference = [(61.8380 - 60.2401)/ 60.2401]*100

= 2.65%

Arithmetic Gradient

2.29 (a) G = \$-300 (b) CF_5 = \$2800 (c) n = 9

2.30 $P_0 = 500(P/A,10\%,9) + 100(P/G,10\%,9)$
$= 500(5.7590) + 100(19.4215)$
$= 2879.50 + 1942.15$
$= \$4821.65$

2.31 (a) Revenue = 390,000 + 2(15,000)
= \$420,000

(b) A = 390,000 + 15,000(A/G,10%,5)
= 390,000 + 15,000(1.8101)
= \$417,151.50

2.32 A = 9000 – 560(A/G,10%,5)
= 9000 – 560(1.8101)
= \$7986

2.33 500 = 200 + G(A/G,10%.7)
500 = 200 + G(2.6216)
G = \$114.43

2.34 A = 100,000 + 10,000(A/G,10%,5)
= 100,000 + 10,000(1.8101)
= \$118,101

F = 118,101(F/A,10%,5)
= 118,101(6.1051)
= \$721,018

2.35 3500 = A + 40(A/G,10%,9)
3500 = A + 40(3.3724)
A = \$3365.10

2.36 In \$ billion units,
P = 2.1(P/F,18%,5)
= 2.1(0.4371)
= 0.91791 = \$917,910,000

$917{,}910{,}000 = 100{,}000{,}000(P/A,18\%,5) + G(P/G,18\%,5)$

$917{,}910{,}000 = 100{,}000{,}000(3.1272) + G(5.2312)$

$G = \$115{,}688{,}561$

2.37 $95{,}000 = 55{,}000 + G(A/G,10\%,5)$

$95{,}000 = 55{,}000 + G(1.8101)$

$G = \$22{,}098$

2.38 P in year 0 $= 500{,}000(P/F,10\%,10)$

$= 500{,}000(0.3855)$

$= \$192{,}750$

$192{,}750 = A + 3000(P/G,10\%,10)$

$192{,}750 = A + 3000(22.8913)$

$A = \$124{,}076$

Geometric Gradient

2.39 Find $(P/A,g,i,n)$ using Equation [2.32] and $A_1 = 1$

For n = 1: $P_g = 1*\{1 – [(1 + 0.05)/(1 + 0.10)]^1\}/(0.10 – 0.05)$

$= 0.90909$

For n = 2: $P_g = 1*\{1 – [(1 + 0.05)/(1 + 0.10)]^2\}/(0.10 – 0.05)$

$= 1.77686$

2.40 Decrease deposit in year 4 by 7% per year for three years to get back to year 1.

First deposit $= 5550/(1 + 0.07)^3$

$= \$4530.45$

2.41 $P_g = 35{,}000\{1 – [(1 + 0.05)/(1 + 0.10)]^6\}/(0.10 – 0.05)$

$= \$170{,}486$

2.42 $P_g = 200{,}000\{1 – [(1 + 0.03)/(1 + 0.10)]^5\}/(0.10 – 0.03)$

$= \$800{,}520$

2.43 First find P_g and then convert to F in year 15

$$P_g = (0.10)(160{,}000)\{1 - [(1 + 0.03)/(1 + 0.07)]^{15}/(0.07 - 0.03)\}$$
$$= 16{,}000(10.883) = \$174{,}128.36$$

$$F = 174{,}128.36(F/P,7\%,15)$$
$$= 174{,}128.36\ (2.7590)$$
$$= \$480{,}420.15$$

2.44 (a) $P_g = 260\{1 - [(1 + 0.04)/(1 + 0.06)]^{20}\}/(0.06 - 0.04)$

$$= 260(15.8399)$$
$$= \$4119.37$$

(b) $P_{Total} = (4119.37)(51{,}000)$

$$=\$210{,}087{,}870$$

2.45 Solve for P_g in geometric gradient equation and then convert to A

$$A_1 = 5{,}000{,}000(0.01) = 50{,}000$$

$$P_g = 50{,}000[1 - (1.10/1.08)^5]/(0.08 - 0.10)$$
$$= \$240{,}215$$

$$A = 240{,}215(A/P,8\%,5)$$
$$= 240{,}215(0.25046)$$
$$= \$60{,}164$$

2.46 First find P_g and then convert to F

$$P_g = 5000[1 - (0.95/1.08)^5]/(0.08 + 0.05)$$
$$= \$18{,}207$$

$$F = 18{,}207(F/P,8\%,5)$$
$$= 18{,}207(1.4693)$$
$$= \$26{,}751$$

Interest Rate and Rate of Return

2.47 $1,000,000 = 290,000(P/A,i,5)$

$(P/A,i,5) = 3.44828$

Interpolate between 12% and 14% interest tables or use Excel's RATE function

By RATE, $i = 13.8\%$

2.48 $50,000 = 10,000(F/P,i,17)$

$5.0000 = (F/P,i,17)$

$5.0000 = (1 + i)^{17}$

$i = 9.93\%$

2.49 $F = A(F/A,i\%,5)$

$451,000 = 40,000(F/A,i\%,5)$

$(F/A,i\%,5) = 11.2750$

Interpolate between 40% and 50% interest tables or use Excel's RATE function

By RATE, $i = 41.6\%$

2.50 Bonus/year = $6(3000)/0.05 = \$360,000$

$1,200,000 = 360,000(P/A,i,10)$

$(P/A,i,10) = 3.3333$

$i = 27.3\%$

2.51 Set future values equal to each other

Simple: $F = P + Pni$

$= P(1 + 5*0.15)$

$= 1.75P$

Compound: $F = P(1 + i)^n$

$= P(1 + i)^5$

$1.75P = P(1 + i)^5$

$i = 11.84\%$

2.52 $100,000 = 190,325(P/F,i,30)$

$(P/F,i,30) = 0.52542$

Find i by interpolation between 2% and 3%, or by solving P/F equation, or by Excel

By RATE function, $i = 2.17\%$

2.53 400,000 = 320,000 + 50,000(A/G,i,5)
(A/G,i,5) = 1.6000
Interpolate between i = 22% and i = 24%
i = 22.6%

Number of Years

2.54 160,000 = 30,000(P/A,15%,n)
(P/A,15%,n) = 5.3333
From 15% table, n is between 11 and 12 years; therefore, n = 12 years
By NPER, n = 11.5 years

2.55 (a) 2,000,000 = 100,000(P/A,5%,n)
(P/A,5%,n) = 20.000

From 5% table, n is > 100 years. In fact, at 5% per year, her account earns $100,000 per year. Therefore, she will be able to withdraw $100,000 forever; actually, n is ∞.

(b) 2,000,000 = 150,000(P/A,5%,n)
(P/A,5%,n) = 13.333
By NPER, n = 22.5 years

(c) The reduction is impressive from forever (n is infinity) to n = 22.5 years for a 50% increase in annual withdrawal. It is important to know how much can be withdrawn annually when a fixed amount and a specific rate of return are involved.

2.56 10A = A(F/A,10%,n)
(F/A,10%,n) = 10.000

From 10% factor table, n is between 7 and 8 years; therefore, n = 8 years

2.57 (a) 500,000 = 85,000(P/A,10%,n)
(P/A,10%,n) = 5.8824

From 10% table, n is between 9 and 10 years.

(b) Using the function = NPER(10%,-85000,500000), the displayed n = 9.3 years.

2.58 1,500,000 = 6,000,000(P/F,25%,n)
(P/F,25%,n) = 0.2500

From 25% table, n is between 6 and 7 years; therefore, n = 7 years

2.59 15,000 = 3000 + 2000(A/G,10%,n)
(A/G,10%,n) = 6.0000

From 10% table, n is between 17 and 18 years; therefore, n = 18 years. She is not correct; it takes longer.

2.60 First set up equation to find present worth of $2,000,000 and set that equal to P in the geometric gradient equation. Then, solve for n.

P = 2,000,000(P/F,7%,n)

$2{,}000{,}000(P/F,7\%,n) = 10{,}000\{1 – [(1+0.10)/(1+0.07)]^n\}/(0.07 – 0.10)$

Solve for n using Goal Seek or trial and error.
By trial and error, n = is between 25 and 26; therefore, n = 26 years

Exercises for Spreadsheets

2.61

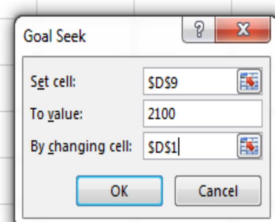
Part	Function	Answer
a	= -FV(10%,30,100000000/30)	**$548,313,409**
b	= -FV(10%,33,100000000/30)	**$740,838,481**
c	= -FV(10%,33,100000000/30) + FV(10%,3,(100000000/30)*2)	**$718,771,814**

2.62

Part		Function	Result	Conclusion
(a) \$12,000 for 30 years		= - FV(10%,30,12000)	\$1,973,928.27	Not quite reached
(a) \$8000 for 15; \$15,000 for 15 years		= - FV(10%,30,8000) - FV(10%,15,7000)	\$ 1,538,359.55	Not reached
(b) \$12,000 for n years		=NPER(10%,-12000,,2000000)	30.13	Years
(c) \$8000 for 15; \$15000 for 15 years				
One solution: Continue the deposits beyond year 30 and determine the future worth year by year.	**Year**	**Function**	**Accumulated**	**Conclusion**
	31	= -FV(10%,\$B10,8000) - FV(10%,\$B10-15,7000)	\$ 1,707,195.51	
	32	= -FV(10%,\$B11,8000) - FV(10%,\$B11-15,7000)	\$ 1,892,915.06	
	33	= -FV(10%,\$B12,8000) - FV(10%,\$B12-15,7000)	\$ 2,097,206.57	33 years
	34	= -FV(10%,\$B13,8000) - FV(10%,\$B13-15,7000)	\$ 2,321,927.22	
	35	= -FV(10%,\$B14,8000) - FV(10%,\$B14-15,7000)	\$ 2,569,119.94	

2.63 Goal Seek template before and result after with solution for G = \$115.69 million

Gradient amount is (\$1000)			\$ 50.00
Year	**Deposit**	**PV in year 0**	**FV in year 5**
0			
1	100.00	\$84.75	
2	150.00	\$192.47	
3	200.00	\$314.20	
4	250.00	\$443.15	
5	300.00	\$574.28	\$1,313.81



	A	B	C	D	E
1	Gradient amount is ($1000)			$ 115.69	
2					
3	Year	Deposit	PV in year 0	FV in year 5	
4	0				
5	1	100.00	$84.75		
6	2	215.69	$239.65		
7	3	331.38	$441.34		
8	4	447.08	$671.94		
9	5	562.77	$917.93	$2,100.00	

2.64 Here is one approach to the solution using NPV and FV functions with results (left) and formulas (right).

Year, n	Deposit	Present worth in year 0	Future worth in year n
0			
1	10,000	9,346	10,000
2	11,000	18,954	21,700
3	12,100	28,831	35,319
4	13,310	38,985	51,101
5	14,641	49,424	69,319
6	16,105	60,155	90,277
7	17,716	71,188	114,312
8	19,487	82,529	141,801
9	21,436	94,189	173,163
10	23,579	106,176	208,864
11	25,937	118,498	249,422
12	28,531	131,167	295,412
13	31,384	144,190	347,475
14	34,523	157,578	406,321
15	37,975	171,342	472,739
16	41,772	185,492	547,603
17	45,950	200,039	631,885
18	50,545	214,993	726,662
19	55,599	230,367	833,127
20	61,159	246,171	952,605
21	67,275	262,419	1,086,563
22	74,002	279,122	1,236,624
23	81,403	296,294	1,404,591
24	89,543	313,947	1,592,455
25	98,497	332,095	1,802,424
26	108,347	350,752	2,036,941
27	119,182	369,932	2,298,709
28	131,100	389,650	2,590,718
29	144,210	409,920	2,916,279
30	158,631	430,759	3,279,049

Year, n	Deposit	Present worth in year 0	Future worth in year n
0			
= $A3+1	10000	=NPV(7%,$B$4:$B4)	= -FV(7%,$A4,,$C4)
= $A4+1	= $B4*1.1	=NPV(7%,$B$4:$B5)	= -FV(7%,$A5,,$C5)
= $A5+1	= $B5*1.1	=NPV(7%,$B$4:$B6)	= -FV(7%,$A6,,$C6)
= $A6+1	= $B6*1.1	=NPV(7%,$B$4:$B7)	= -FV(7%,$A7,,$C7)
= $A7+1	= $B7*1.1	=NPV(7%,$B$4:$B8)	= -FV(7%,$A8,,$C8)
= $A8+1	= $B8*1.1	=NPV(7%,$B$4:$B9)	= -FV(7%,$A9,,$C9)
= $A9+1	= $B9*1.1	=NPV(7%,$B$4:$B10)	= -FV(7%,$A10,,$C10)
= $A10+1	= $B10*1.1	=NPV(7%,$B$4:$B11)	= -FV(7%,$A11,,$C11)
= $A11+1	= $B11*1.1	=NPV(7%,$B$4:$B12)	= -FV(7%,$A12,,$C12)
= $A12+1	= $B12*1.1	=NPV(7%,$B$4:$B13)	= -FV(7%,$A13,,$C13)
= $A13+1	= $B13*1.1	=NPV(7%,$B$4:$B14)	= -FV(7%,$A14,,$C14)
= $A14+1	= $B14*1.1	=NPV(7%,$B$4:$B15)	= -FV(7%,$A15,,$C15)
= $A15+1	= $B15*1.1	=NPV(7%,$B$4:$B16)	= -FV(7%,$A16,,$C16)
= $A16+1	= $B16*1.1	=NPV(7%,$B$4:$B17)	= -FV(7%,$A17,,$C17)
= $A17+1	= $B17*1.1	=NPV(7%,$B$4:$B18)	= -FV(7%,$A18,,$C18)
= $A18+1	= $B18*1.1	=NPV(7%,$B$4:$B19)	= -FV(7%,$A19,,$C19)
= $A19+1	= $B19*1.1	=NPV(7%,$B$4:$B20)	= -FV(7%,$A20,,$C20)
= $A20+1	= $B20*1.1	=NPV(7%,$B$4:$B21)	= -FV(7%,$A21,,$C21)
= $A21+1	= $B21*1.1	=NPV(7%,$B$4:$B22)	= -FV(7%,$A22,,$C22)
= $A22+1	= $B22*1.1	=NPV(7%,$B$4:$B23)	= -FV(7%,$A23,,$C23)
= $A23+1	= $B23*1.1	=NPV(7%,$B$4:$B24)	= -FV(7%,$A24,,$C24)
= $A24+1	= $B24*1.1	=NPV(7%,$B$4:$B25)	= -FV(7%,$A25,,$C25)
= $A25+1	= $B25*1.1	=NPV(7%,$B$4:$B26)	= -FV(7%,$A26,,$C26)
= $A26+1	= $B26*1.1	=NPV(7%,$B$4:$B27)	= -FV(7%,$A27,,$C27)
= $A27+1	= $B27*1.1	=NPV(7%,$B$4:$B28)	= -FV(7%,$A28,,$C28)
= $A28+1	= $B28*1.1	=NPV(7%,$B$4:$B29)	= -FV(7%,$A29,,$C29)
= $A29+1	= $B29*1.1	=NPV(7%,$B$4:$B30)	= -FV(7%,$A30,,$C30)
= $A30+1	= $B30*1.1	=NPV(7%,$B$4:$B31)	= -FV(7%,$A31,,$C31)
= $A31+1	= $B31*1.1	=NPV(7%,$B$4:$B32)	= -FV(7%,$A32,,$C32)
= $A32+1	= $B32*1.1	=NPV(7%,$B$4:$B33)	= -FV(7%,$A33,,$C33)

Answers: (a) 26 years; (b) 30 years, only 4 years more than the $2 million milestone.

2.65 (a) Present worth is the value of the savings for each bid

Bid 1: Savings = $10,000

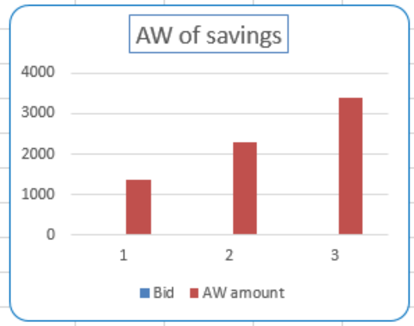
Bid 2: Savings = $17,000

Bid 3: Savings = $25,000

(b) and (c) Spreadsheet for A values and column chart

	Part (b)		
Bid	Savings	AW formula	AW amount
1	10,000	= - PMT(6%,10,$B3)	$1,358.68
2	17,000	= - PMT(6%,10,$B4)	$2,309.76
3	25,000	= - PMT(6%,10,$B5)	$3,396.70

Part (c)



ADDITIONAL PROBLEMS AND FE REVIEW QUESTIONS

2.66 Answer is (a)

2.67 P = 840,000(P/F,10%,2)

= 840,000(0.8264)

= $694,176

Answer is (a)

2.68 P = 81,000(P/F,6%,4)

= 81,000(0.7921)

= $64,160

Answer is (d)

2.69 F = 25,000(F/P,10%,25)

= 25,000(10.8347)

= $270,868

Answer is (c)

2.70 A = 10,000,000(A/F,10%,5)

= 10,000,000(0.16380)

= $1,638,000

Answer is (a)

2.71 $A = 2{,}000{,}000(A/F,8\%,30)$
$= 2{,}000{,}000(0.00883)$
$= \$17{,}660$
Answer is (a)

2.72 $390 = 585(P/F,i,5)$
$(P/F,i,5) = 0.6667$
From tables, i is between 8% and 9%
Answer is (c)

2.73 $AW = 26{,}000 + 1500(A/G,8\%,5)$
$= \$28{,}770$
Answer is (b)

2.74 $30{,}000 = 4202(P/A,8\%,n)$
$(P/A,8\%,5) = 7.1395$
$n = 11$ years
Answer is (d)

2.75 $23{,}632 = 3000\{1- [(1+0.04)^n/(1+0.06)^n]\}/(0.06-0.04)$
$[(23{,}632*0.02)/3000]-1 = (0.98113)^n$
$\log 0.84245 = n\log 0.98113$
$n = 9$
Answer is (b)

2.76 $A = 800 – 100(A/G,8\%,6)$
$= 800 – 100(2.2763)$
$= \$572.37$
Answer is (c)

2.77 $P = 100{,}000(P/A,10\%,5) - 5000(P/G,10\%,5)$
$= 100{,}000(3.7908) - 5000(6.8618)$
$= \$344{,}771$
Answer is (a)

2.78 $109.355 = 7(P/A,i,25)$
$(P/A,i,25) = 15.6221$

From tables, $i = 4\%$

Answer is (a)

2.79 $28,800 = 7000(P/A,10\%,5) + G(P/G,10\%,5)$

$28,800 = 7000(3.7908) + G(6.8618)$

$G = \$330$

Answer is (d)

2.80 $40,000 = 11,096(P/A,i,5)$

$(P/A,i,5) = 3.6049$

$i = 12\ \%$

Answer is (c)

Solution to Case Study, Chapter 2

The Amazing Impact of Compound Interest

1. Ford Model T and a New Car

(a) Inflation rate is substituted for i = 3.10% per year

(b) Model T: Beginning cost in 1909: P = $825
Ending cost: n = 1909 to 2015 + 50 years = 156 years; F = $96,562

$$F = P(1+i)^n = 825(1.031)^{156}$$
$$= 825(117.0447)$$
$$= \$96,562$$

New car: Beginning cost: P = $28,000
Ending cost: n = 50 years; F = $128,853

$$F = P(1+i)^n = 28,000(1.031)^{50}$$
$$= 28,000(4.6019)$$
$$= \$128,853$$

2. Manhattan Island

(a) i = 6.0% per year

(b) Beginning amount in 1626: P = $24
Ending value: n = 391; F = $188.3 billion

$$F = 24(1.06)^{391}$$
$$= 24(7,845,006.7)$$
$$= \$188,280,161 \qquad (\$188.3 \text{ billion})$$

3. Pawn Shop Loan

(a) i per week = (30/200)*100 = 15% per week i per year = $[(1.15)^{52} – 1]*100$ = 143,214% per year

Subtraction of 1 considers repayment of the original loan of \$200 when the interest rate is calculated (see Chapter 4 for details.)

(b) Beginning amount: P = \$200
Ending owed:1 year later, F = \$286,627

F = P(F/P, 15%,52)
= $200(1.15)^{52}$
= 200(1433.1370)
= \$286,627

4. Capital Investment

(a) i = 15^{+}% per year

1,000,000 = 150,000(P/A,i%,60)
(P/A,i%,60) = 6.6667
i = 15^{+}%

(b) Beginning amount: P = \$1,000,000 invested
Ending total amount over 60 years: 150,000(60) = \$9 million

Value: F_{60} = 150,000(F/A,15%,60)
= 150,000(29220.0)
= \$4,383,000,000 (\$4.38 billon)

5. Diamond Ring

(a) i = 4% per year

(b) Beginning price: P = \$50
Ending value after 179 years: F = \$55,968

n = great grandmother + grandmother + mother + girl
= 65 + 60 + 30 + 24

$= 179$ years

$$F = 50(F/P,4\%,179)$$
$$= 50(1119.35)$$
$$= \$55,968$$

Solutions to end-of-chapter problems

Engineering Economy, 8th edition

Leland Blank and Anthony Tarquin

Chapter 3
Combining Factors and Spreadsheet Functions

Present Worth Calculations

3.1 P = 400,000(P/A,10%,15)(P/F,10%,1)
= 400,000(7.6061)(0.9091)
= $2,765,882

3.2 P = 40,000(P/F,12%,4)
= 40,000(0.6355)
= $25,420

3.3 P = 600,000(0.10)(P/F,10%,2) + 1,350,000(0.10)(P/F,10%,5)
= 60,000(0.8264) + 135,000(0.6209)
= $133,406

3.4 P = 12,000(P/A,10%,9)(P/F,10%,1)
= 12,000(5.7590)(0.9091)
= $62,826

3.5 P = 320,000(P/A,10%,4) + 150,000(P/A,10%,2)(P/F,10%,4)
= 320,000(3.1699) + 150,000(1.7355)(0.6830)
= $1,192,170

3.6 P = 100,000(260)(P/A,10%,8)(P/F,10%,2)
= 26,000,000(5.3349)(0.8264)
= $114.628 million

3.7 P = 80(2000)(P/A,18%,3) + 100(2500)(P/A,18%,5)(P/F,18%,3)
= 160,000(2.1743) + 250,000(3.1272)(0.6086)
= $823,691

3.8 P = 15,000(P/A,8%,18)(P/F,8%,2)
= 15,000(9.3719)(0.8573)
= $120,518

3.9 P = 28,000 + 28,000(P/A,8%,3) + 48,000(P/A,8%,7)(P/F,8%,3)
= 28,000 + 28,000(2.5771) + 48,000(5.2064)(0.7938)
= $298,535

3.10 Present worth before = 73,000(P/A,10%,5)
= 73,000(3.7908)
= $276,728

Present worth after = 16,000 + 58,000(P/F,10%,1) + 52,000(P/A,10%,4)(P/F,10%,1)
= 16,000 + 58,000(0.9091) + 52,000(3.1699)(0.9091)
= $218,579

Present worth of savings = 276,728 – 218,579 = $58,149

Annual Worth Calculations

3.11 Find F in year 10 and then convert to A in years 1-10

F = 20,000(F/A,10%,8)
= 20,000(11.4359)
= $228,718

A = 228,718(A/F,10%,10)
= 228,718(0.06275)
= $14,352

Spreadsheet function: = PMT(10%,10,,FV(10%,8,20000))

3.12 Find P in year 0 and then convert to A in years 1-5

P = 7000(P/F,10%,2) + 9000(P/F,10%,4) + 15,000(P/F,10%,5)
= 7000(0.8264) + 9000(0.6830) + 15,000(0.6209)
= $21,245.30

$A = 21{,}245.30(A/P,10\%,5)$
$= 21{,}245.30(0.26380)$
$= \$5605$

Spreadsheet: = PMT(10%,5,PV(10%,2,,7000)+PV(10%,4,,9000)+PV(10%,5,,15000))

3.13 Find F in year 7 and then convert to A in years 1-7

$F = 70{,}000(F/P,10\%,7) + 15{,}000(F/A,10\%,5)$
$= 70{,}000(1.9487) + 15{,}000(6.1051)$
$= 227{,}986$

$A = 227{,}986(A/F,10\%,7)$
$= 227{,}986(0.10541)$
$= \$24{,}032$

3.14 Find P in year -1 and then convert to A over a 6-year period. In \$1000 units,

$P_{-1} = -120(P/F,12\%,2) - 50(P/F,12\%,3) + 90(P/A,12\%,4)(P/F,12\%,3)$
$= -120(0.7972) - 50(0.7118) + 90(3.0373)(0.7118)$
$= \$63.322$

$A = 63.322(A/P,12\%,6)$
$= 63.322(0.24323)$
$= \$15.402$ (\$15,402)

3.15 Find P and the convert to A

$P = 140{,}000(4000) + 140{,}000(6000)(P/F,10\%,1)$
$= 560{,}000{,}000 + 840{,}000{,}000(0.9091)$
$= \$1{,}323{,}644{,}000$

$A = 1{,}323{,}644{,}000(A/P,10\%,4)$
$= 1{,}323{,}644{,}000(0.31547)$
$= \$417{,}569{,}973$

3.16 $A = 900{,}000(F/P,8\%,1)(A/P,8\%,8)$
$= 900{,}000(1.08)(0.17401)$
$= \$169{,}138$

3.17 $A = 80{,}000(A/P,10\%,5) + 80{,}000$
$= 80{,}000(0.26380) + 80{,}000$
$= \$101{,}104$

3.18 $A = 20{,}000(A/P,6\%,5) + 1{,}000{,}000(0.15)(0.75)$
$= 20{,}000(0.2374) + 112{,}500$
$= \$117{,}248$

3.19 Find P in year 0 and then P so that A = \$300
$P_0 = x + 300(P/A,10\%,6) + x(P/F,10\%,7)$

$A = P_0(A/P,10\%,7) = 300$

Substitute equation for P_0

$[x + 300(P/A,10\%,6) + x(P/F,10\%,7)](A/P,10\%,7) = 300$
$[x + 300(4.3553) + x(0.5132)](0.20541) = 300$
$0.31083\, x = 31.61$
$x = \$101.71$

3.20 Plan 1: $A = 700{,}000(A/P,10\%,4)$
$= 700{,}000(0.31547)$
$= \$220{,}829$

Plan 2: Let x = amount of first payment
$700{,}000 = x(P/F,10\%,1) + 2x(P/F,10\%,2) + 4x(P/F,10\%,3)$
$700{,}000 = x(0.9091) + 2x(0.8264) + 4x(0.7513)$
$5.5671x = 700{,}000$
$x = 125{,}739$
$4x = \$502{,}955$

Difference = 502,955 - 220,829 = \$282,126

Future Worth Calculations

3.21 $F = 10{,}000(F/A,10\%,21)$
$= 10{,}000(64.0025)$
$= \$640.025$

3.22 $F = 4000(F/A,10\%,9)(F/P,10\%,1) + 1000(F/A,10\%,4)(F/P,10\%,1)$
$= 4000(13.5795)(1.1000) + 1000(4.6410)(1.1000)$
$= 59{,}750 + 5105$
$= \$64{,}855$

Spreadsheet function: = (-FV(10%,9,4000)-FV(10%,4,1000))*(1.1)

3.23 $F_7 = 15{,}000(F/A,8\%,5)$
$= 15{,}000(5.8666)$
$= \$87{,}999$

$F_{18} = 87{,}999(F/P,8\%,11)$
$= 87{,}999(2.3316)$
$= \$205{,}178$

3.24 $A = 500{,}000(A/F,10\%,11)$
$= 500{,}000(0.05396)$
$= \$26{,}980$

3.25 (a) $F_5 = 3000(F/A,10\%,4)$
$= 3000(4.6410)$
$= \$13{,}923$

(b) $F_4 = 3000[(F/A,10\%,3) + (P/F,10\%,1)]$
$= 3000[3.3100 + 0.9091]$
$= \$12{,}657$

3.26 First find P in year 0 and then move to year 9

$P = 200 + 200(P/A,10\%,3) + 300(P/A,10\%,3)(P/F,10\%,3)$
$= 200 + 200(2.4869) + 300(2.4869)(0.7513)$

$= \$1257.90$

$F = 1257.90(F/P,10\%,9)$
$= 1257.90(2.3579)$
$= \$2966$

3.27 $F = 500(F/A,10\%,3)(F/P,10\%,6) + 800(F/A,10\%,4)(F/P,10\%,1)$
$= 500(3.3100)(1.7716) + 800(4.6410)(1.1000)$
$= \$7016$

3.28 $100{,}000 = x(F/A,10\%,4)(F/P,10\%,4) + 2x(F/A,10\%,4)$
$100{,}000 = x(4.6410)(1.4641) + 2x(4.6410)$
$16.0769x = 100{,}000$
$x = \$6220$
$2x = \$12{,}440$

Random Single Amounts and Uniform Series

3.29 In \$1000 units

$P = 100(P/F,6\%,1) + 200(P/A,6\%,2)(P/F,6\%,2) + 90(P/A,6\%,4)(P/F,6\%,4)$
$= 100(0.9434) + 200(1.8334)(0.8900) + 90(3.4651)(0.7921)$
$= 94.34 + 326.35 + 247.02$
$= \$667.71$ (\$667,710)

3.30 Move \$85,000 to year 1, subtract \$42,000, and find the equivalent over 4 remaining years.

$A = [85{,}000(F/P,10\%,1) - 42{,}000](A/P,10\%,4)$
$= [85{,}000(1.1000) - 42{,}000](0.31547)$
$= \$16{,}247$

3.31 $P = 40 + x(P/F,10\%,1) + 40(P/A,10\%,2)(P/F,10\%,1) + x(P/F,10\%,4)$
$+ 40(P/A,10\%,2)(P/F,10\%,4)$
$300 = 40 + x(0.9091) + 40(1.7355)(0.9091) + x(0.6830) + 40(1.7355)(0.6830)$
$1.5921x = 300 – 40 - 63.110 – 47.414$
$x = \$93.89$

3.32 First find P in year 0, then convert to equivalent A value over 10 years; add the annual maintenance. In $1 million units,

P = 1.5(P/F,6%,1) + 2(P/F,6%,2)
= 1.5(0.9434) + 2(0.8900)
= $3.1591 million

A of investments = 3.1591(A/P,6%,10)
= 3.1591(0.13587)
= $434,118

Total A = 434,118 + 65,000 = $ 499,118

Spreadsheet function: = -PMT(6%,10, -PV(6%,1,,1500000) - PV(6%,2,,2000000)) + 65000

3.33 (a) Using factors, determine the annual cash flow with arithmetic gradient values of G_{tax} = $1 for the tax and G_{stu} = 1000 for students. The resulting cash flows do not form an arithmetic gradient series.

Year	Students	Tax, $	Cash flow, $
0			
1	50,000	56	2,800,000
2	51,000	57	2,907,000
3	52,000	58	3,016,000
4	53,000	59	3,127,000
5	54,000	60	3,240,000

P = 2,800,000(P/F,8%,1) + 2,907,000(P/F,8%,2) + … + 3,240,000(P/F,8%,5)
= 2,800,000(0.9259) + 2,907,000(0.8573) + … + 3,240,000(0.6806)
= $11,982,281

F = 11,982,281(F/P,8%,5)
= 11,982,281(1.4693)
= $17,605,565

(b) Spreadsheet solution

	A	B	C	D	E
1					
2	**Year**	**Students**	**Tax, $**	**Cash flow, $**	**Functions**
3	0				
4	1	50,000	56	2,800,000	= B4*C4
5	2	51,000	57	2,907,000	= B5*C5
6	3	52,000	58	3,016,000	= B6*C6
7	4	53,000	59	3,127,000	= B7*C7
8	5	54,000	60	3,240,000	= B8*C8
9	PW in 0 using NPV function			$11,982,602	=NPV(8%,E4:E8)+E3
10	FW in 5 using FV function			$17,606,374	=-FV(8%,5,,E9)

3.34 A = [70,000(F/P,15%,2) + 50,000(F/A,15%,2)](A/P,15%,8)
= [70,000(1.3225) + 50,000(2.1500)](0.22285)
= $44,587

3.35 F = 6000(F/P,12%,9) + 9000(F/P,12%,7) + 10,000(F/A,12%,5)
= 6000(2.7731) + 9000(2.2107) + 10,000(6.3528)
= $100,063

3.36 (a) F = -2500(F/P,10%,10) + (700 – 200)(F/A,10%,4)(F/P,10%,6)
+ (2000 – 300)(F/A,10%,6)
= -2500(2.5937) + (500)(4.6410)(1.7716) + (1700)(7.7156)
= $10,743.268

Actual worth today is $10,743,268

(b) Yes, because the F value >> 0

Shifted Gradients

3.37 P = 200(P/F,10%,1) + [50(P/A,10%,7) +20(P/G,10%,7)](P/F,10%,1)
= 200(0.9091) + [50(4.8684) +20(12.7631)](0.9091)
= $635.17

3.38 P = 26,000(P/F,10%.1) + [26,000(P/A,10%,4) + 2000(P/G,10%,4)](P/F,10%,1)
= 26,000(0.9091) + [26,000(3.1699) + 2000(4.3781)](0.9091)
= $106,522

3.39 First find P in year 0, then find A over 4 years.

$$P = 250{,}000 + 275{,}000(P/A,10\%,4) + 25{,}000(P/G,10\%,4) + 25{,}000(P/F,10\%,4)$$
$$= 250{,}000 + 275{,}000(3.1699) + 25{,}000(4.3781) + 25{,}000(0.6830)$$
$$= \$1{,}248{,}250$$

$$A = 1{,}248{,}250(A/P,10\%,4)$$
$$= 1{,}248{,}250(0.31547)$$
$$= \$393{,}785$$

3.40 (a) Factors: Find P in year –1 using gradient factor and then move forward 1 year

$$P_{-1} = 2{,}500{,}000(P/A,10\%,21) + 200{,}000(P/G,10\%,21)$$
$$= 2{,}500{,}000(8.6487) + 200{,}000(58.1095)$$
$$= \$33{,}243{,}650$$

$$F = P_0 = 33{,}243{,}650(F/P,10\%,1)$$
$$= 33{,}243{,}650\ (1.1000)$$
$$= \$36{,}568{,}015$$

(b) Spreadsheet: If entries are in cells B2 through B22, the function = NPV(10%,B3:B22)+B2 displays $36,568,004, which is the present worth in year 0.

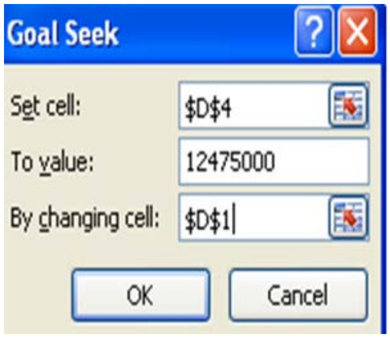
3.41 $$A = 550{,}000(A/P,10\%,12) + 550{,}000 + 40{,}000(A/G,10\%,12)$$
$$= 550{,}000(0.14676) + 550{,}000 + 40{,}000(4.3884)$$
$$= \$806{,}254$$

3.42 (a) Using tabulated factors

$$12{,}475{,}000(F/P,15\%,2) = 250{,}000(P/A,15\%,13) + G(P/G,15\%,13)$$
$$12{,}475{,}000(1.3225) = 250{,}000(5.5831) + G(23.1352)$$
$$G(23.1352) = 15{,}102{,}413$$
$$G = \$652{,}789$$

(b) Spreadsheet solution using Goal Seek

	A	B	C	D
1	Year	Income, $	Gradient, G =	$ 652,788
2	0			
3	1	0		
4	2	0		$12,475,000
5	3	250,000		
6	4	902,788		
7	5	1,555,576		
8	6	2,208,365		
9	7	2,861,153		
10	8	3,513,941		
11	9	4,166,729		
12	10	4,819,517		
13	11	5,472,306		
14	12	6,125,094		
15	13	6,777,882		
16	14	7,430,670		
17	15	8,083,458		



3.43 Development cost, year 0 = 600,000(F/A,15%,3)
= 600,000(3.4725)
= $2,083,500

Present worth of contract, year –1 = 250,000(P/A,15%,6) + G(P/G,15%,6)
= 250,000(3.7845) + G(7.9368)

Move development cost to year –1 and set equal to income

2,083,500(P/F,15%,1) = 250,000(3.7845) + G(7.9368)
2,083,500(0.8696) = 250,000(3.7845) + G(7.9368)
G = $109,072

3.44 Move $20,000 to year 0, add and subtract $1600 in year 4 to use gradient, and solve for x. Use + signs for cash flows for convenience.

20,000(P/F,10%,8) = 1000(P/A,10%,8) + 200(P/G,10%,8) – 1600(P/F,10%,4) + x(P/F,10%,4)

20,000(0.4665) = 1000(5.3349) + 200(16.0287) – 1600(0.6830) + x(0.6830)

$$9330 = 5334.90 + 3205.74 - 1092.80 + 0.683x$$
$$x = \$2755.72$$

3.45 (a) Add and subtract \$2400 and \$2600 in periods 3 and 4, respectively, to use gradient factors. Use + signs for cash flows for convenience.

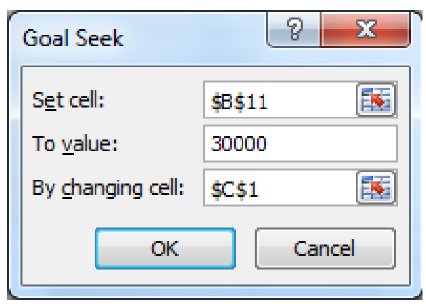
$$30{,}000 = 2000 + 200(A/G,10\%,8) - 2400(P/F,10\%,3)(A/P,10\%,8)$$
$$-2600(P/F,10\%,4)(A/P,10\%,8) + x(P/F,10\%,3)(A/P,10\%,8)$$
$$+ 2x(P/F,10\%,4)(A/P,10\%,8)$$

$$30{,}000 = 2000 + 200(3.0045) - 2400(0.7513)(0.18744)$$
$$-2600(0.6830)(0.18744) + x(0.7513)(0.18744)$$
$$+ 2x(0.6830)(0.18744)$$

$$30{,}000 = 2000 + 600.90 - 337.98 - 332.86 + 0.14082x + 0.25604x$$
$$0.39686x = 28{,}069.94$$
$$x = \$70{,}730$$

(b) Spreadsheet uses Goal Seek to find x = \$70,726

	A	B	C
1	**Year**	**Cash Flow, \$**	**70726.04**
2	0		**Functions**
3	1	2,000	`2000
4	2	2,200	`=B3+200
5	3	70,726	`=C$1
6	4	141,452	`=2*B$5
7	5	2,800	`=B4+600
8	6	3,000	`=B7+200
9	7	3,200	`=B8+200
10	8	3,400	`=B9+200
11		30,000	`=-PMT(10%,8,NPV(10%,B3:B10)+B2)



3.46 First determine the discount cost schedule. Develop the annual cost series for the flat rate of \$15.00 and the corresponding series using the discounted rates. The future worth values using a spreadsheet are:

(a) F_{flat} = \$136,307
(b) F_{disc} = \$132,549

10-kilo packs purchased	Discount, %	Unit cost, $ per10-kilo pack	Cost function
0-100	None	19.95	19.95
101-250	10% off	17.96	=$C$2*(1-1*(0.1))
251-1000	10% more	15.96	=$C$2*(1-2*(0.1))
1001-10,000	10% more	13.97	=$C$2*(1-3*(0.1))
10,001-50,000	20% more	9.98	=$C$2*(1-5*(0.1))
50,001-100,000	20% more	5.99	=$C$2*(1-7*(0.1))
> 100,000	20% more	2.00	=$C$2*(1-9*(0.1))

		Purchases at flat rates		
Years ago	n value	10-kilos packs purchased	Unit cost, $ per 10-kilo pack	Total cost for year, $
8	0	100	15.00	1,500
7	1	150	15.00	2,250
6	2	500	15.00	7,500
5	3	800	15.00	12,000
4	4	1100	15.00	16,500
3	5	1400	15.00	21,000
2	6	1700	15.00	25,500
1	7	2000	15.00	30,000
FW at 8%			=-FV(8%,7,,NPV(8%,J4:J10)+J3)	$ 136,307

	Purchases at discounted rates		
Years ago	10-kilos packs purchased	Unit cost, $ per 10-kilo pack	Total cost for year, $
	100	19.95	1,995
7	150	17.96	2,694
6	500	15.96	7,980
5	800	15.96	12,768
4	1100	13.97	15,367
3	1400	13.97	19,558
2	1700	13.97	23,749
1	2000	13.97	27,940
FW @ 8%		=-FV(8%,7,,NPV(8%,D15:D21)+D14)	$ 132,549

3.47 $P = 29{,}000 + 13{,}000(P/A,10\%,3) + 13{,}000[7/(1 + 0.10)](P/F,10\%,3)$

$= 29{,}000 + 13{,}000(2.4869) + 82{,}727(0.7513)$

$= \$123{,}483$

3.48 (a) Find P in year –1 and then move to year 5

$P_{-1} = 210{,}000[6/(1 + 0.08)]$

$= 210{,}000(0.92593)$

$= \$1{,}166{,}667$

$F = 1{,}166{,}667(F/P,8\%,6)$

$= 1{,}166{,}667(1.5869)$

$= \$1{,}851{,}383$

(b) Spreadsheet function: = -FV(8%,5,,NPV(8%,B3:B7)+B2)

3.49 Find P in year 1 for geometric gradient and add to amount in year 1; move total back to year 0.

$$P_1 = 22{,}000 + 22{,}000[1 - (1.08/1.10)^9]/(0.10 - 0.08)$$
$$= \$189{,}450$$

$$P_0 = 189{,}450(P/F,10\%,1)$$
$$= 189{,}450(0.9091)$$
$$= \$172{,}229$$

3.50 (a) Find P in year 4 for the geometric gradient, then move all cash flows to future

$$P_4 = 500{,}000[1 - (1.15/1.12)^{16}]/(0.12 - 0.15)$$
$$= \$8{,}773{,}844$$

$$F = 500{,}000(F/A,12\%,4)(F/P,12\%,16) + P_4(F/P,12\%,16)$$
$$= 500{,}000(4.7793)(6.1304) + 8{,}773{,}844(6.1304)$$
$$= \$68{,}436{,}684$$

(b) Spreadsheet

	A	B
1	Year	Cash Flow, $
2	0	
3	1	500,000
4	2	500,000
5	3	500,000
6	4	500,000
7	5	500,000
8	6	575,000
9	7	661,250
10	8	760,438
11	9	874,503
12	10	1,005,679
13	11	1,156,530
14	12	1,330,010
15	13	1,529,511
16	14	1,758,938
17	15	2,022,779
18	16	2,326,196
19	17	2,675,125
20	18	3,076,394
21	19	3,537,853
22	20	4,068,531
23		$68,436,701.40
24	`=-FV(12%,20,,NPV(12%,B3:B22)+B2)	

Shifted Decreasing Gradients

3.51 $P = [2000(P/A,10\%,6) - 200(P/G,10\%,6)](F/P,10\%,1)$
$$= [2000(4.3553) - 200(9.6842](1.10)$$
$$= \$7451.14$$

3.52 First find P in year 1, move to year 0, and then convert to A

$$P_1 = 500(P/A,10\%,4) - 50(P/G,10\%,4)$$
$$= 500(3.1699) - 50(4.3781)$$
$$= \$1366.05$$

$P_0 = P_1(P/F,10\%,1)$
$= 1366.05(0.9091)$
$= \$1241.87$

$A = P_0(A/P,10\%,5)$
$= 1241.87(0.26380)$
$= \$327.61$

3.53 $P = 1{,}800{,}000(P/A,12\%,3) + [1{,}800{,}000(P/A,12\%,7) - 30{,}000(P/G,12\%,7)](P/F,12\%,3)$
$= 1{,}800{,}000(2.4018) + [1{,}800{,}000(4.5638) - 30{,}000(11.6443)](0.7118)$
$= \$9{,}921{,}910$

3.54 $20{,}000 = 5000 + 4500(P/A,8\%,n) - 500(P/G,8\%,n)$

Solve for n by trial and error:
Try n = 5: \$19,281 < \$20,000
Try n = 6: \$20,541 > \$20,000

By interpolation, n = 5.6 years

3.55 Find present worth of gradient in year 1, move back to year 0, and then set equal to \$2500

$P_1 = 900(P/A,10\%,4) - G(P/G,10\%,4)$
$= 900(3.1699) - G(4.3781)$
$= 2852.91 - 4.3781G$

$P_0 = (2852.91 - 4.3781G)(P/F,10\%,1)$
$= (2852.91 - 4.3781G)(0.9091)$
$= 2593.58 - 3.9801G$

$2500 = 2593.58 - 3.9801G$
$-93.58 = -3.9801G$
$G = \$23.51$

3.56 $P_3 = 4{,}100{,}000[1 - (0.90/1.06)^{17}]/(0.06 + 0.10)$
$= \$24{,}037{,}964$

$P_0 = 4{,}100{,}000(P/A,6\%,3) + P_3(P/F,6\%,3)$
$= 4{,}100{,}000(2.6730) + 24{,}037{,}964(0.8396)$

= \$31,141,574

3.57 Find P in year 5, then find future worth of all cash flows

$$P_5 = 4000[1 - (0.85/1.10)^9]/(0.10 + 0.15)$$
$$= \$14,428$$

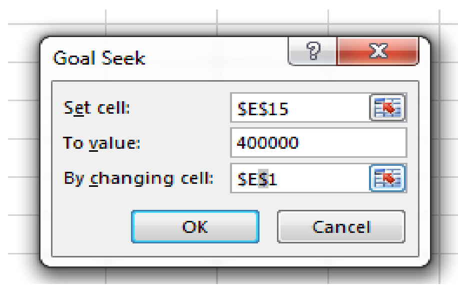
$$F = [4000(F/A,10\%,5) + P_5](F/P,10\%,9)$$
$$= [4000(6.1051) + 14,428](2.3579)$$
$$= [24,420 + 14,428](2.3579)$$
$$= \$91,601$$

Exercises for Spreadsheets

3.58 (a) P = \$298,542 using NPV function shown

(b) Use Goal Seek. With the savings estimated, only a 2.07% return is possible.

(a) Rate	**8.00%**	**(b) Rate**	**2.07%**
Year	**Savings**	**Year**	**Savings**
0	28,000	0	28,000
1	28,000	1	28,000
2	28,000	2	28,000
3	28,000	3	28,000
4	48,000	4	48,000
5	48,000	5	48,000
6	48,000	6	48,000
7	48,000	7	48,000
8	48,000	8	48,000
9	48,000	9	48,000
10	48,000	10	48,000
NPV value	\$ 298,542	NPV value	\$ 400,000
Function	= NPV(\$B\$1, B5:B14) + B5	Function	= NPV(\$E\$1, B5:B14) + B5



3.59 A = $101,104

	A	B
1		
2	**Year**	**Cost, $**
3	0	80,000
4	1	80,000
5	2	80,000
6	3	80,000
7	4	80,000
8	5	80,000
9		
10	Function	= -PMT(10%,5,NPV(10%,B4:B8)+B3)
11	A, $/year	$101,104

3.60 (a) Amount re-paid is $195,000 for Schedule A, and less at $180,000 for Schedule B.

(b) Annual equivalent loss at 5% over 12 years of $685 is less for Schedule A, even though Schedule B requires less total re-payment by $15,000. Placement of payments and the time value of money make the difference.

	A	B	C	D	E	F	G	H
2		Amount	Amount			Amount	Amount	Amount
3	Year	received	to repay		Year	received	to repay	to repay
4	0	50,000	0		0	50,000	0	0
5	1	0	0		1	0	20,000	0
6	2	0	0		2	0	20,000	20,000
7	3	50,000	19,500		3	50,000	20,000	0
8	4	0	19,500		4	0	20,000	0
9	5	0	19,500		5	0	20,000	0
10	6	50,000	19,500		6	50,000	20,000	0
11	7		19,500		7			40,000
12	8		19,500		8			
13	9		19,500		9			
14	10		19,500		10			
15	11		19,500		11			
16	12		19,500		12			
17	**(a) Amount re-paid is $195,000 for Schedule A and less at $180,000 for Schedule B**							
18		**150,000**	**195,000**			**150,000**	**120,000**	**60,000**
19								
20	NPV function	= NPV(5%, B5:B10)+B4	= NPV(5%, C5:C16)+C4			= NPV(5%, B5:B10)+B4	= NPV(5%, G5:G10)+G4	=NPV(5%, H5:H11) + H4
21	NPV result	$ 130,503	$ 136,575			$ 130,503	$ 101,514	$ 46,568
22								
23	PMT function	= -PMT(5%,12, B21-C21)				=-PMT(5%,12, F21-G21-H21)		
24	**PMT result**	$ (685)				($1,983)		
25								
26	**(b) Annual equivalent loss at 5% over 12 years of $685 is less for Schedule A,**							
27	**even though Schedule B requires less total re-payment by $15,000**							
28								

3.61 Value of *x* is $101.74

	A	B	C
1	Amount x, $	300.00	
2		Cash flow, $	
3	Year	Function	Amount
4	0	=$B$1	300.00
5	1	300	300
6	2	300	300
7	3	300	300
8	4	300	300
9	5	300	300
10	6	300	300
11	7	=$B$1	300.00
12			
13	Function		= -PMT(10%,7,$B$1 +NPV(10%,C5:C10) -PV(10%,7,,$B$1))
14	Annual worth, $/year		361.62

Goal Seek
Set cell: $C$14
To value: 300
By changing cell: $B$1
OK Cancel

With Goal Seek template set-up

	A	B	C
1	Amount x, $	101.74	
2		Cash flow, $	
3	Year	Function	Amount
4	0	=$B$1	101.74
5	1	300	300
6	2	300	300
7	3	300	300
8	4	300	300
9	5	300	300
10	6	300	300
11	7	=$B$1	101.74
12			
13	Function		= -PMT(10%,7,$B$1 +NPV(10%,C5:C10) -PV(10%,7,,$B$1))
14	Annual worth, $/year		300.00

Solution for x = $101.74

3.62 (a) F_5 = $13,923 using the function = - FV(10%,4,3000)

(b) F_4 = $12,657 (Note: Function intentionally omitted)

3.63 Two approaches are: (1) use trial and error in the function = NPV(10%,B5:B10)+B4, and (2) use Goal Seek. Either results in $93.88 in years 1 and 4.

	A	B	C	D	E	F	G
1	Value of x	93.88					
2		Cash flow,					
3	Year	1,000					
4	0	40					
5	1	93.88					
6	2	40					
7	3	40					
8	4	93.88					
9	5	40					
10	6	40					
11							
12	NPV function	= NPV(10%,B5:B10)+B4					
13	NPV value	300.00					
14							

Goal Seek Status

Goal Seeking with Cell B13 found a solution.

Target value: 300

Current value: 300.00

Step | Pause | OK | Cancel

3.64 (a) Worth today = $10,732,345

(b) If net worth is $5 million after 10 years, initial investment would have had to increase from $2.5 million to $4.712 million.

	A	B	C	D
1				
2	Investment	-2500.000		
3	Year	CF, x$1000	Function	
4	0	-2500	=$B$2	
5	1	500		
6	2	500		
7	3	500		
8	4	500		
9	5	1700		
10	6	1700		
11	7	1700		
12	8	1700		
13	9	1700		
14	10	1700		
15				
16	(a) FW in 10	$10,732,345	= -FV(10%,10,,NPV(10%,B5:B14)+B4)*1000	

Goal Seek

Set cell: $B$16

To value: 5000

By changing cell: $B$2

OK | Cancel

(a) Worth today

	A	B	C	D
1				
2	Investment	-4712.279		
3	Year	CF, x$1000	Function	
4	0	-4712	=$B$2	
5	1	500		
6	2	500		
7	3	500		
8	4	500		
9	5	1700		
10	6	1700		
11	7	1700		
12	8	1700		
13	9	1700		
14	10	1700		
15				
16	(a) FW in 10	$5,000,000	= -FV(10%,10,,NPV(10%,B5:B14)+B4)*1000	

(b) Initial investment using Goal Seek tool

3.65 Payment in year 4, x = \$2755.89

Extra above arithmetic gradient value of \$-1600 is \$1155.89

	A	B	C	D	E
1	Year	CF, \$	Function	Extra amt, yr 4 =	**-1155.89**
2	0	0	0		
3	1	-1000	-1000		
4	2	-1200	=C3-200		
5	3	-1400	=C4-200		
6	**4**	**-2755.89**	Function omitted		
7	5	-1800	=C6-200		
8	6	-2000	=C7-200		
9	7	-2200	=C8-200		
10	8	-2400	=C9-200		
11					
12	P in year 0	(\$9,330.15)	= NPV(10%,C3:C10)+C2		
13	F in year 8	**\$20,000.00**	= FV(10%,8,,\$B\$12)		

ADDITIONAL PROBLEMS AND FE REVIEW QUESTIONS

3.66 Answer is (b)

3.67 A = 100,000(A/F,10%,4)
= 100,000(0.21547)
= \$21,547
Answer is (b)

3.68 Answer is (d)

3.69 Answer is (a)

3.70 9,500(F/P,20%,5) + x(F/P,20%,3) = 38,000
9500(2.4883) + x(1.7280) = 38,000
x = \$8311
Answer is (c)

3.71 A(F/A,20%,5)(F/P,20%,8) = 50,000
A(7.4416)(4.2998) = 50,000
A = \$1563
Answer is (a)

3.72 125,000 = 190,000 – G(A/G,20%,5)
125,000 = 190,000 – G(1.6405)
G = \$39,622
Answer is (d)

3.73 24,000 = 3695(P/A,10%,n)
(P/A,10%,n) = 6.4952
From 10% tables, n is close to 11
Answer is (c)

3.74 P = 100,000(P/F,10%,2)
= \$100,000(0.8264)
= \$82,640
Answer is (b)

3.75 10,000 = 2x(P/F,10%,2) + x(P/F,10%,4)
10,000 = 2x(0.8264) + x(0.6830)
2.3358x = 10,000
x = \$4281
Answer is (a)

3.76 Answer is (b)

3.77 Answer is (d)

Solution to Case Study, Chapter 3

There are not always definitive answers to case studies. The following are examples only.

Preserving Land for Public Use

Cash flows for purchases at g = –25% start in year 0 at $4 million. Cash flows for parks development at G = $100,000 start in year 4 at $550,000. All cash flow signs are +.

	Cash flow	
Year	Land	Parks
0	$4,000,000	
1	3,000,000	
2	2,250,000	
3	1,678,000	
4	1,265,625	$550,000
5	949,219	650,000
6		750,000

1. Find P. In $1 million units,

   P = 4 + 3(P/F,7%,1) + … + 0.750(P/F,7%,6)

   = $13.1716 ($13,171,600)

   Amount to raise in years 1 and 2:

   A = (13.1716 – 3.0)(A/P,7%,2)

   = (10.1716)(0.55309)

   = $5.6258 ($5,625,800 per year)

2. Find remaining project fund needs in year 3, then find the A for the next 3 years

   F_3 = (13.1716 – 3.0)(F/P,7%,3)

   = $12.46019

   A = 12.46019(A/P,7%,3)

   = $4.748 ($4,748,000 per year)

Solutions to end-of-chapter problems

Engineering Economy, 8th edition

Leland Blank and Anthony Tarquin

Chapter 4
Nominal and Effective Interest Rates

Nominal and Effective Rates

4.1 (a) week (b) quarter (c) six months

4.2 APR is a nominal interest rate while APY is an effective rate. The APY must be used in all interest formulas, factors and functions.

4.3 (a) month (b) month (c) week

4.4 (a) 4 (b) 12 (c) 6

4.5 (a) 1 (b) 2 (c) 4 (d) 12

4.6 (a) r/6-mths: 1*2 = 2% (b) r/year: 1*4 = 4% (c) r/2-years: 1*8 = 8%

4.7 APR = 2%*2
= 4%

4.8 (a) r/8-mths: (0.09/3)*2 = 6%
(b) r/12-mths: (0.09/3)*3 = 9%
(c) r/2-yrs: (0.09/3)*6 = 18%

4.9 (a) 5% (b) 20%

4.10 (a) effective (b) effective (c) nominal (d) effective (e) nominal

4.11 Interest period = year; CP = quarter; m = 4

4.12 (a) Interest period = year; CP = month; m = 12

(b) $i_a = (1 + r/m)^m - 1$
$= (1 + 0.12/12)^{12} - 1$
$= 0.12683 \quad (12.683\%)$

4.13 APY $= (1 + 0.12/12)^{12} - 1$
$= 12.683\%$ per year

Spreadsheet function: = EFFECT(12%,12)

4.14 (a) First find effective monthly rate; then find APR with m = 3 mths/quarter

$0.035 = (1 + r/3)^3 - 1$
$r/3 = (1 + 0.035)^{1/3} - 1$
$= 1.153\%$ per month

APR = 1.153*12
= 13.8% per year

(b) APY $= (1 + 0.138/12)^{12} - 1$
$= 14.71\%$

(c) = EFFECT(13.8%,12) displays the APY of 14.71% per year.
The NOMINAL functions displays only annual nominal or APR rates. Therefore, the quoted effective 3.5% per quarter, compounded monthly can't be entered to get the correct answer of 13.8%. The effective APY of 14.71% can be entered into = NOMINAL(14.71%,12) to display the APR of 13.8%, however.

4.15 (a) Interest rate per month = 4%

$r = (0.04)(12)$
$= 48\%$ per year

(b) $i_a = (1 + 0.48/12)^{12} - 1$
$= 60.1\%$ per year

4.16 (a) $0.1664 = (1 + 0.16/m)^m - 1$
By trial and error, m= 2
Compounding period = 6 months

(b) = EFFECT(16%,n)
By trial and error to display 16.64%
m = 2; CP = 6 months

4.17 $0.75 = (1 + r/12)^{12} – 1$
$r/12 = (1 + 0.75)^{1/12} – 1$
$= 4.77\%$ per month

or using Equation [4.4] with m = 12

$i = (1 + 0.75)^{1/12} – 1$
$= (1.75)^{0.08333} – 1$
$= 0.477$ (4.77% per month)

Equivalence When PP ≥ CP

4.18 (a) Since n is in years, need effective i/year
$i/year = (1 + 0.12/4)^4 - 1$
$= 12.551\%$

(b) Since n is semiannual periods, need effective i/6-mths
$i/6\text{-mths} = (1 + 0.06/2)^2 – 1$
$= 6.09\%$

(c) Since n is quarters, need i/quarter
$i/quarter = 0.12/4$
$= 3\%$

4.19 (a) Find the effective annual rates

Jennifer: $(1 + 0.05/4)^4 – 1 * 100\% = 5.095\%$
Rex: $(1 + 0.0485/12)^{12} – 1 * 100 = 4.959\%$

Rex is <u>not</u> correct, as no frequency of compounding, even continuous, will make the APY go above 5%.

(b) Determine the effective i per for a 6-month payment period.

Jennifer: For a nominal rate of 2.5% semiannually, compounded quarterly, m = 2
Effective i = $(1 + 0.025/2)^2$ - 1 * 100 = 2.516% per 6-month period

Rex: For a nominal; rate of 2.425% semiannually, compounded monthly, m = 6
Effective i = $(1 + 0.02425/6)^6$ – 1 * 100 = 2.450% per 6-month period

4.20 (a) P = 65,000,000(P/F,3%,12)
= 65,000,000(0.7014)
= $45,591,00

(b) Spreadsheet function: = PV(EFFECT(12%,4),3,,65000000) displays $-45,589,692

4.21 Compound: F = 20,000,000(F/P,1.5%,12)
= 20,000,000(1.1956)
= $23,912,000

Simple: F = 20,000,000(1.18)
= $23,600,000

Difference = 23,912,000 – 23,600,000
= $312,000

4.22 (a) P = 10,000(P/F,2%,28)
= 10,000(0.5744)
= $5744

(b) = EFFECT(8%,4) displays 8.243% per year

4.23 P = 15,000,000(P/F,5%,6)
= 15,000,000(0.7462)
= $11,193,000

4.24 F = 2.3(F/P,2%,60) (in $ billion)
= 2.3(3.2810)
= $7.546

4.25 $F = 3.9(F/P,0.5\%,120)$ (in \$ billion)
$= 3.9(1.8194)$
$= \$7,095,660,000$

4.26 $P = 190,000(P/F,4\%,4) + 120,000(P/F,4\%,8)$
$= 190,000(0.8548) + 120,000(0.7307)$
$= \$250,096$

4.27 $F = 18,000(F/P,12\%,4) + 26,000(F/P,12\%,3) + 42,000(F/P,12\%,2)$
$= 18,000(1.5735) + 26,000(1.4049) + 42,000(1.2544)$
$= \$117,535$

4.28 $P = 1,000,000(0.01)(P/F,5\%,20)$
$= 10,000(0.3769)$
$= \$3769$

4.29 $P = 1.3(P/A,1\%,28)(P/F,1\%,2)$ (in \$ million)
$= 1.3(24.3164)(0.9803)$
$= \$30.9886$
Spreadsheet function: = PV(1%,2,,PV(1%,28,1.3)) displays \$30.9885 (in \$ million)

4.30 PP = quarter; CP = quarter; effective i = 4% per quarter
$A = 3.5(A/P,4\%,12)$ (in \$ million)
$= 3.5(0.10655)$
$= \$0.37293$ (\$372,930 per quarter)

4.31 $P = 51(100,000)(0.25)(P/A,0.5\%,60)$
$= (1,275,000)(51.7256)$
$= \$65,950,140$

4.32 PP = year; CP = quarter; effective per PP needed

$i = (1 + 0.15/4)^4 - 1 * 100$
$= 15.865\%$ per year

$A = 4,100,000(A/P,15.865\%,4)$

Find factor value by interpolation, formula, or spreadsheet.

By formula: $A = 4,100,000(0.35641) = \$1,461,281$ per year

By spreadsheet function: = PMT(15.865%,4,4100000)

$A = \$1,461,291$ per year

4.33 (a) Need effective i per PP of 6 months

$i/6 \text{ months} = (1 + 0.03)^2 - 1 = 0.0609$

$$
\begin{aligned}
A &= 30,000(A/P,6.09\%,4) \\
&= 30,000\{[0.0609(1 + 0.0609)^4]/[(1 + 0.0609)^4-1]\} \\
&= 30,000(0.28919) \\
&= \$8675.70 \text{ per 6 months}
\end{aligned}
$$

(b) = PMT(EFFECT(6%,2),4,30000) displays \$8675.59 per 6 months

4.34
$$
\begin{aligned}
F &= 100,000(F/A,0.25\%,8)(F/P,0.25\%,3) \\
&= 100,000(8.0704)(1.0075) \\
&= \$813,093
\end{aligned}
$$

Subsidy = 813,093 – 800,000 = \$13,093

4.35 PP = year; CP = month; need effective annual i

$$
\begin{aligned}
i_a &= (1 + 0.18/12)^{12} - 1 \\
&= 19.562\% \text{ per year}
\end{aligned}
$$

First find F in year 5 and then convert to A over 5 years. By factor,

$$
\begin{aligned}
F_5 &= 350,000(F/A,19.562\%,3) \\
&= 350,000(3.6251) \\
&= \$1,268,785
\end{aligned}
$$

$$
\begin{aligned}
A &= F_5(A/F,19.562\%,5) \\
&= 1,268,785(0.13554) \\
&= \$171,971
\end{aligned}
$$

By spreadsheet, function = -PMT(19.562%,5,,FV(19.562%,3,350000)) displays

A = \$171,975 per year

4.36 i/week = 0.25%

F = 2.99(F/A,0.25%,52)
= 2.99(55.4575)
= \$165.82

4.37 F = (14.99 – 6.99)(F/A,1%,24)
= 8(26.9735)
= \$215.79

4.38 PP = 6 months; CP = quarterly; need effective i per PP

i/6-mths = $(1 + 0.08/2)^2$ -1 * 100
= 8.160%

F = 100,000(F/P,8.160%,4) + 25,000(F/A,8.160%,4)

Find answer by interpolation, formula, or spreadsheet.

Function: = - FV(8.16%,4,,100000) - FV(8.16%,4,25000) displays \$249,776

F = \$249,776

4.39 (a) 3,000,000 = 200,000(P/A,1%,n)
(P/A,1%,n) = 15.000

From 1% table, n is between 16 and 17; therefore, n = 17 months

(b) Spreadsheet and calculator functions are = NPER(1%,200000,-3000000) and NPER(1,200000,-3000000,0), respectively. Display is 16.3 months.

4.40 P = 80(P/A,3%,12) + 2(P/G,3%,12)
= 80(9.9540) + 2(51.2482)
= \$898.82

4.41 PP = quarter; CP = quarter; need i per quarter

$$2{,}000{,}000 = A(P/A,3\%,8) + 50{,}000(P/G,3\%,8)$$
$$2{,}000{,}000 = A(7.0197) + 50{,}000(23.4806)$$
$$A = \$117{,}665$$

4.42 PP = month; CP = month; use i = 1.5% per month; find P for g = 1%

$$P = 140{,}000[1 - (1.01/1.015)^{24}]/(0.015 - 0.01)$$
$$= 140{,}000(22.35297)$$
$$= \$3{,}129{,}416$$

4.43 PP = year; CP = 6 months; use effective annual i

(a) $i_a = (1 + 0.10/2)^2 - 1 = 0.1025$ (10.25% per year)

$$P_g = 125{,}000\{1 - [(1 + 0.03)/(1 + 0.1025)]^5\}/(0.1025 - 0.03)$$
$$= 125{,}000(3.9766)$$
$$= \$497{,}080$$

$$A = 497{,}080(A/P,10.25\%,5)$$
$$= 497{,}080(0.26548)$$
$$= \$131{,}965$$

(b) Use the NPV and PMT functions at 10.25%

	A	B	C
1		Year	Cash flow, $
2		0	0
3		1	125,000
4		2	128,750
5		3	132,613
6		4	136,591
7		5	140,689
8			
9	P at 10.25%		$497,080
10	A for 5 years		$131,967

4.44 PP = month; CP = month; use i = 0.005 per month

(a) A = 80,000(A/P,0.5%,60) = 80,000(0.01933) = $1546.40 per month
(b) Current principal balance is $77,701.47 (see table below)
(c) Sum of interest paid: 400.00 + 394.27 = $794.27
(d) Now i = 0.042/12 = 0.0035 per month
    A = 77,701.47(A/P,0.35%,60) = 77,701.47(0.01851) = $1438.25 per month

Month (1)	i per month (2)	Interest owed (3)=(7 prior)(2)	Total owed (4)=(7 prior)+(3)	Monthly payment (5)	Principal reduction (6)=(5)-(3)	Principal remaining after payment (7)=(7 prior)-(6)
0						80,000.00
1	0.005	400.00	80,400.00	1546.40	1146.40	78,853.60
2	0.005	394.27	79,247.87	1546.40	1152.13	**77,701.47 (b)**
3	0.0035	271.96	77,973.43	**1438.25 (d)**	1166.29	76,535,18

Equivalence When PP < CP

4.45 PP < CP. Move monthly deposits to end of 6-month compounding period; find F

F = 1200(6)(F/A,4%,50)
= 7200(152.6671)
= $1,099,203

4.46 Move deposit to end of annual compounding period; find F

F = 1500(12)(F/A,18%,3)
= 18,000(3.5724)
= $64,303

4.47 Move cash flows to end of each quarter; find F

F = 9000(F/A,2%,12)
= 9000(13.4121)
= $120,709

4.48 First find equivalent amount per 6-months and then divide by 6 to get monthly deposit

A per semiannual period = 80,000(A/F,6%,6)
= 80,000(0.14336)
= $11,468.80

Monthly deposit = 11,468.80/6
= $1911.47

Spreadsheet function: = PMT(6%,6,,80000)/6 displays -$1911.50

4.49 Chemical cost/month = 11(30) = $330

A = 1200(A/P,1%,48) + 330
= 1200(0.02633) + 330
= $361.60 per month

4.50 Move withdrawals to beginning of semiannual periods; find F

F = (10,000 – 1000)(F/P,4%,6) – 1000(F/P,4%,5) – 1000(F/P,4%,3)
= 9000(1.2653) – 1000(1.2167) – 1000(1.1249)
= $9046

Continuous Compounding

4.51 Formula: $i = (e^{0.10} - 1) \times 100 = 10.517\%$

Spreadsheet: = EFFECT(10%,10000)

4.52 r = 2% per month = 12% per 6 months

$i = e^{0.12} - 1$
$= 12.75\%$

4.53 $0.127 = e^r - 1$ is the relation; $r = \ln(1 + i)$ provides the solution

r/year = ln(1.127)
= 0.1196 (11.96%)

r /quarter = 0.1196/4 = 0.0299 (2.99%)

4.54 r = 15% per year = 1.25% per month; need effective i per month

$i = e^{0.0125} - 1 = 0.0126$ (1.26% per month)

$F = 100{,}000(F/A,1.26\%,24)$
$= 100{,}000\{[(1 + 0.0126)^{24} - 1]/0.0126\}$
$= 100{,}000(27.8213)$
$= \$2{,}782{,}130$

Spreadsheet: = -FV(EFFECT(1.25%,10000),24,100000) displays $2,781,414

4.55 18% per year = 18/12 = 1.50% per month

$i = e^{0.015} - 1 = 1.51\%$ per month

$P = 6000(P/A,1.51\%,60)$
$= 6000\{[(1 + 0.0151)^{60} - 1]/[0.0151(1 + 0.0151)^{60}]\}$
$= 6000(39.2792)$
$= \$235{,}675$

Spreadsheet: = = - PV(EFFECT(1.5%,10000),60,6000,) displays $235,596

4.56 (a) $i = e^{0.02} - 1 = 2.02\%$ per month

$A = 50{,}000{,}000(A/P,2.02\%,36)$
$= 50{,}000{,}000\{[0.0202(1 + 0.0202)^{36}]/[(1 + 0.0202)^{36} - 1]\}$
$= 50{,}000{,}000(0.03936)$
$= \$1{,}967{,}941$

(b) = - PMT(EFFECT(2%,10000),36,50000000) displays $1,967,990

4.57 (a) $i = e^{0.015} - 1 = 1.51\%$ per month

$2P = P(1 + 0.0151)^n$
$2.000 = (1.0151)^n$
Take log of both sides and solve for n

$n = 46.2$ months

(b) Spreadsheet: = NPER(EFFECT(1.5%,10000),,-1,2) displays 46.2

4.58 (a) Set up F/P equation in months.

$$3P = P(1 + i)^{60}$$
$$3.000 = (1 + i)^{60}$$
$$1.01848 = 1 + i$$
$$i = 1.848\% \text{ per month (effective)}$$

(b) $(1 + 0.01848)^{12} - 1 = 0.24575$ (24.575%)

Varying Interest Rates

4.59 P = 150,000(P/F,12%,2)(P/F,10%,3)
= 150,000(0.7972)(0.7513)
= $89,840

4.60 Rates are 1%, increasing to 1.25% per month

F = 300,000(F/P,1%,4)(F/P,1.25%,8)
= 300,000(1.0406)(1.1045)
= $344,803

4.61 F value:
F = - 100(F/A,14%,5)(F/P,10%,3) - 160(F/A,10%,3)
= - 100(6.6101)(1.3310) - 160(3.3100)
= $-1409.40

P value:
P = - 100(P/A,14%,5) – 160(P/A,10%,3)(P/F,14%,5)
= - 100(3.4331) – 160(2.4869)(0.5194)
= $-549.98

A value using relation for P:
-549.98 = A(3.4331) + A(2.4869)(0.5194)
= A(4.7248)
A = $-116.40

A value using relation for F:

$$-1409.40 = A(6.6101)(1.3310) + A(3.3100)$$
$$= A(12.1080)$$
$$A = \$-116.40$$

4.62 First move cash flow in years 0-4 to year 4 at i = 12%; move the total to year 5 at i = 20%.

$$F_4 = 5000(F/P,12\%,4) + 6000(F/A,12\%,4)$$
$$= 5000(1.5735) + 6000(4.7793)$$
$$= \$36,543$$

$$F_5 = 36,543(F/P,20\%,1) + 9000$$
$$= 36,543(1.20) + 9000$$
$$= \$52,852$$

4.63 $P = 5000(P/A,10\%,3) + 7000(P/A,12\%,2)(P/F,10\%,3)$
$$= 5000(2.4869) + 7000(1.6901)(0.7513)$$
$$= 12,434.50 + 8888.40$$
$$= \$21,323$$

Now substitute A value for cash flows.

$$21,323 = A(P/A,10\%,3) + A(P/A,12\%,2)(P/F,10\%,3)$$
$$= A(2.4869) + A(1.6901)(0.7513)$$
$$= A(3.7567)$$
$$A = \$5676$$

Exercises for Spreadsheets

4.64 Solution of problem 4.19 using a spreadsheet.

(a) Rex is not correct since his effective rate is less than that for Jennifer

(b) Effective rates per 6-months are shown in column O.

K	L	M	N	O
	APY		i per 6-mths	
	Formula	Rate, %	Formula	Rate, %
Jennifer	= EFFECT(5%,4) * 100	5.095	= EFFECT(2.5%,2) * 100	**2.516**
Rex	= EFFECT(4.85%,12) * 100	4.959	= EFFECT(2.425%,6) * 100	**2.450**

4.65 Solution to Problem 4.44 using a spreadsheet.

	A	B	C	D	E	F	G
1	**Month**	**I per month**	**Interest owed**	**Total owed**	**Monthly payment**	**Principal reduction**	**Principal remaining after payment**
2	0						$ 80,000.00
3	1	0.0050	$ 400.00	$ 80,400.00	$1,546.62	$ 1,146.62	$ 78,853.38
4	2	0.0050	$ 394.27	$ 79,247.64	$1,546.62	$ 1,152.36	**$ 77,701.02**
5	3	0.0035	$ 271.95	$ 77,972.97	**$1,438.01**	$1,166.05	$ 76,534.97
6							
7							
8	(a) A @ 0.5%	= -PMT(0.5%,60,80000)	$1,546.62			=$E5-$C5	= $G3 - $F4
9	(b)	Value in G4	$ 77,701.02				
10	(c)	= SUM(C3:C4)	$ 794.27				
11	(d) A @ 0.35%	= -PMT(0.35%,60,$G4)	$1,438.01				

4.66 (a) F = \$9046; (b) F = \$9139; Difference ≈ \$92

	A	B	C	D	E	F
1	i values =		4.0%			0.656%
2			per 6-mths			per month
3		**Part (a) No interperiod compounding**			**Part (a) Interperiod compounding provided**	
4	Semiannual period	Cash flows, $ (moved)	F value after 3 years	Month	Cash flows, $ (not moved)	F value after 3 years
5	0	-9000	**$9,046.35**	0	-10000	**$9,138.74**
6	1	1000		1	0	
7	2	0		2	1000	
8	3	1000		3	0	
9	4	0		4	0	
10	5	0		5	0	
11	6	0		6	0	
12				7	0	
13				8	0	
14				9	0	
15	**Difference**	**= F5 - C5**	**$92.39**	10	0	
16				11	1000	
17				12	0	
18				13	0	
19				14	0	
20				15	0	
25				20	0	
26				21	0	
27				22	0	
28				23	1000	
29				24	0	
30				25	0	
36				31	0	
37				32	0	
38				33	0	
41				36	0	

4.67 Two spreadsheets follow the answers. You must write the functions.

(a) i/month = 0.06/12 = 0.005 (0.5%)
= -PMT(0.5%,60,20000) displays $386.66. Payment amount is correct

(b) Payment #60 is reduced to $386.38 to accommodate round-off error throughout.
Total paid = $23,199.32

(c) Total interest paid = $3199.32
Percentage of principal = 3199.32/20,000 * 100 = 16%

(d) Remaining principal after #36 is $8723.91. Increase i as of payment #37
New effective i = 0.10/12 = = 0.00833 (0.833%/month)

= -PMT(0.833%,624,8723.91) displays $402.56

(e) With interest due, payment #48 of 4,981.54 will pay off the entire loan balance

Parts (a) through (c)

Month (payment #)	i per month	Interest owed	Total owed	Monthly payment	Principal reduction	Principal remaining after payment
-						20,000.00
1.00	0.01	100.00	20,100.00	386.66	286.66	19,713.34
2.00	0.01	98.57	19,811.91	386.66	288.09	19,425.25
3.00	0.01	97.13	19,522.37	386.66	289.53	19,135.71
4.00	0.01	95.68	19,231.39	386.66	290.98	18,844.73
5.00	0.01	94.22	18,938.96	386.66	292.44	18,552.30
6.00	0.01	92.76	18,645.06	386.66	293.90	18,258.40
7.00	0.01	91.29	18,349.69	386.66	295.37	17,963.03
8.00	0.01	89.82	18,052.84	386.66	296.84	17,666.18
9.00	0.01	88.33	17,754.51	386.66	298.33	17,367.85
10.00	0.01	86.84	17,454.69	386.66	299.82	17,068.03
11.00	0.01	85.34	17,153.37	386.66	301.32	16,766.71
12.00	0.01	83.83	16,850.55	386.66	302.83	16,463.89
13.00	0.01	82.32	16,546.21	386.66	304.34	16,159.55
14.00	0.01	80.80	16,240.34	386.66	305.86	15,853.68
15.00	0.01	79.27	15,932.95	386.66	307.39	15,546.29
16.00	0.01	77.73	15,624.02	386.66	308.93	15,237.36
17.00	0.01	76.19	15,313.55	386.66	310.47	14,926.89
18.00	0.01	74.63	15,001.53	386.66	312.03	14,614.87
19.00	0.01	73.07	14,687.94	386.66	313.59	14,301.28
20.00	0.01	71.51	14,372.79	386.66	315.15	13,986.13
21.00	0.01	69.93	14,056.06	386.66	316.73	13,669.40
22.00	0.01	68.35	13,737.74	386.66	318.31	13,351.08
23.00	0.01	66.76	13,417.84	386.66	319.90	13,031.18
24.00	0.01	65.16	13,096.34	386.66	321.50	12,709.68
25.00	0.01	63.55	12,773.22	386.66	323.11	12,386.56
26.00	0.01	61.93	12,448.50	386.66	324.73	12,061.84
27.00	0.01	60.31	12,122.15	386.66	326.35	11,735.49
28.00	0.01	58.68	11,794.16	386.66	327.98	11,407.50
29.00	0.01	57.04	11,464.54	386.66	329.62	11,077.88
30.00	0.01	55.39	11,133.27	386.66	331.27	10,746.61
31.00	0.01	53.73	10,800.34	386.66	332.93	10,413.68
32.00	0.01	52.07	10,465.75	386.66	334.59	10,079.09
33.00	0.01	50.40	10,129.49	386.66	336.26	9,742.83
34.00	0.01	48.71	9,791.54	386.66	337.95	9,404.88
35.00	0.01	47.02	9,451.91	386.66	339.64	9,065.25
36.00	0.01	45.33	9,110.57	386.66	341.33	8,723.91
37.00	0.01	43.62	8,767.53	386.66	343.04	8,380.87
38.00	0.01	41.90	8,422.78	386.66	344.76	8,036.12
39.00	0.01	40.18	8,076.30	386.66	346.48	7,689.64
40.00	0.01	38.45	7,728.08	386.66	348.21	7,341.42
41.00	0.01	36.71	7,378.13	386.66	349.95	6,991.47
42.00	0.01	34.96	7,026.43	386.66	351.70	6,639.77
43.00	0.01	33.20	6,672.97	386.66	353.46	6,286.31
44.00	0.01	31.43	6,317.74	386.66	355.23	5,931.08
45.00	0.01	29.66	5,960.74	386.66	357.00	5,574.08
46.00	0.01	27.87	5,601.95	386.66	358.79	5,215.29
47.00	0.01	26.08	5,241.36	386.66	360.58	4,854.70
48.00	0.01	24.27	4,878.98	386.66	362.39	4,492.32
49.00	0.01	22.46	4,514.78	386.66	364.20	4,128.12
50.00	0.01	20.64	4,148.76	386.66	366.02	3,762.10
51.00	0.01	18.81	3,780.91	386.66	367.85	3,394.25
52.00	0.01	16.97	3,411.22	386.66	369.69	3,024.56
53.00	0.01	15.12	3,039.68	386.66	371.54	2,653.02
54.00	0.01	13.27	2,666.29	386.66	373.39	2,279.63
55.00	0.01	11.40	2,291.03	386.66	375.26	1,904.37
56.00	0.01	9.52	1,913.89	386.66	377.14	1,527.23
57.00	0.01	7.64	1,534.86	386.66	379.02	1,148.20
58.00	0.01	5.74	1,153.94	386.66	380.92	767.28
59.00	0.01	3.84	771.12	386.66	382.82	384.46
60.00	0.01	1.92	386.38	386.38	384.46	0.00
Totals		3,199.32		23,199.32		

Parts (d) and (e)

Month (payment #)	i per month	Interest owed	Total owed	Monthly payment	Principal reduction	Principal remaining after payment	
-						20,000.00	
1	0.005	100.00	20,100.00	386.66	286.66	19,713.34	
2	0.005	98.57	19,811.91	386.66	288.09	19,425.25	
3	0.005	97.13	19,522.37	386.66	289.53	19,135.71	
4	0.005	95.68	19,231.39	386.66	290.98	18,844.73	
5	0.005	94.22	18,938.96	386.66	292.44	18,552.30	
6	0.005	92.76	18,645.06	386.66	293.90	18,258.40	
7	0.005	91.29	18,349.69	386.66	295.37	17,963.03	
8	0.005	89.82	18,052.84	386.66	296.84	17,666.18	
9	0.005	88.33	17,754.51	386.66	298.33	17,367.85	
10	0.005	86.84	17,454.69	386.66	299.82	17,068.03	
11	0.005	85.34	17,153.37	386.66	301.32	16,766.71	
12	0.005	83.83	16,850.55	386.66	302.83	16,463.89	
13	0.005	82.32	16,546.21	386.66	304.34	16,159.55	
14	0.005	80.80	16,240.34	386.66	305.86	15,853.68	
15	0.005	79.27	15,932.95	386.66	307.39	15,546.29	
16	0.005	77.73	15,624.02	386.66	308.93	15,237.36	
17	0.005	76.19	15,313.55	386.66	310.47	14,926.89	
18	0.005	74.63	15,001.53	386.66	312.03	14,614.87	
19	0.005	73.07	14,687.94	386.66	313.59	14,301.28	
20	0.005	71.51	14,372.79	386.66	315.15	13,986.13	
21	0.005	69.93	14,056.06	386.66	316.73	13,669.40	
22	0.005	68.35	13,737.74	386.66	318.31	13,351.08	
23	0.005	66.76	13,417.84	386.66	319.90	13,031.18	
24	0.005	65.16	13,096.34	386.66	321.50	12,709.68	
25	0.005	63.55	12,773.22	386.66	323.11	12,386.56	
26	0.005	61.93	12,448.50	386.66	324.73	12,061.84	
27	0.005	60.31	12,122.15	386.66	326.35	11,735.49	
28	0.005	58.68	11,794.16	386.66	327.98	11,407.50	
29	0.005	57.04	11,464.54	386.66	329.62	11,077.88	
30	0.005	55.39	11,133.27	386.66	331.27	10,746.61	
31	0.005	53.73	10,800.34	386.66	332.93	10,413.68	
32	0.005	52.07	10,465.75	386.66	334.59	10,079.09	
33	0.005	50.40	10,129.49	386.66	336.26	9,742.83	
34	0.005	48.71	9,791.54	386.66	337.95	9,404.88	
35	0.005	47.02	9,451.91	386.66	339.64	9,065.25	
36	0.005	45.33	9,110.57	386.66	341.33	8,723.91	Late
37	0.00833	72.70	8,796.61	402.56	329.86	8,394.05	i=0.833%
38	0.00833	69.95	8,464.00	402.56	332.61	8,061.43	A = $402.56
39	0.00833	67.18	8,128.61	402.56	335.39	7,726.05	
40	0.00833	64.38	7,790.43	402.56	338.18	7,387.87	
41	0.00833	61.57	7,449.43	402.56	341.00	7,046.87	
42	0.00833	58.72	7,105.59	402.56	343.84	6,703.03	
43	0.00833	55.86	6,758.89	402.56	346.71	6,356.32	
44	0.00833	52.97	6,409.29	402.56	349.59	6,006.73	
45	0.00833	50.06	6,056.79	402.56	352.51	5,654.22	
46	0.00833	47.12	5,701.34	402.56	355.45	5,298.78	
47	0.00833	44.16	5,342.93	402.56	358.41	4,940.37	
48	0.00833	41.17	4,981.54	402.56	361.39	4,578.97	
49	0.00833	38.16	4,617.13	402.56	364.41	4,214.57	
50	0.00833	35.12	4,249.69	402.56	367.44	3,847.13	
51	0.00833	32.06	3,879.19	402.56	370.50	3,476.62	
52	0.00833	28.97	3,505.59	402.56	373.59	3,103.03	
53	0.00833	25.86	3,128.89	402.56	376.71	2,726.32	
54	0.00833	22.72	2,749.04	402.56	379.84	2,346.48	
55	0.00833	19.55	2,366.03	402.56	383.01	1,963.47	

Additional Problems and FE Exam Review Questions

4.68 i/year = $(1 + 0.01)^{12} - 1 = 0.1268$ (12.68%)
Answer is (d)

4.69 Answer is (d)

4.70 Calculate quarterly and semiannual rates to determine correct answer
i/quarter = $e^{0.045} - 1 = 0.0460$ (4.60%)
i/6-mths = $e^{0.09} - 1 = 0.0942$ (9.42%)
Answer is (c)

4.71 Answer is (c)

4.72 Answer is (c)

4.73 PP of month < CP of 6 months. Assume no interperiod compounding
i/6-mths = 3%
Answer is (b)

4.74 r = 2% per quarter = 4% per 6-mth period

i /6-mths = $e^{0.04} - 1$ = 4.08% per semi
Answer is (d)

4.75 Answer is (a)

4.76 A = 500,000(A/F,7%,13)
= 500,000(0.04965)
= $24,825
Answer is (c)

4.77 x(F/P,7%,20) + x(F/P,7%,14) = 300,000
x(3.8697) + x(2.5785) = 300,000
6.4482x = 300,000
x = $46,525
Answer is (b)

4.78 P_1 = 470(P/A,7%,6) – 50(P/G,7%,6)
= 470(4.7665) – 50(10.9784)
= $1691.34

$F_4 = 1691.34(F/P,7\%,7) + 200$
$= 1691.34(1.6058) + 200$
$= 2715.95 + 200$
$= \$2915.95$
Answer is (b)

4.79 Answer is (a)

4.80 Answer is (b)

4.81 i/6-mths $= (1 + 0.04/2)^2 - 1 = 0.0404$ (4.04%)
Answer is (d)

4.82 Answer is (c)

Solutions to end-of-chapter problems

Engineering Economy, 8^{th} edition

Leland Blank and Anthony Tarquin

Chapter 5
Present Worth Analysis

Types of Projects

5.1 (a) The DN alternative (b) Each other

5.2 (a) Service alternatives all have the same revenues; revenue alternatives each have different amounts of income estimated. (b) Cost alternative.

5.3 Capitalized cost (CC) represents the evaluation of an alternative with a very long or (essentially) an infinite time (life). Real world examples that might be analyzed using CC evaluation are: Yellowstone National Park, Golden Gate Bridge, Hoover Dam, railroads, etc.

5.4 (a) Total possible = $2^5 = 32$

(b) Because of restrictions to combinations of 3, 4, or 5, only 12 remain acceptable: DN, 1, 2, 3, 4, 5, 1&3, 1&4, 1&5, 2&3, 2&4, and 2&5.

Alternative Comparison - Equal Lives

5.5 (a) For independent projects, select all that have PW $\geq$ 0.
(b) For mutually exclusive alternatives, select the one that has the largest numerical value of PW.

5.6 Equal service means that the alternatives have to provide service over the same time period (life).

5.7 PW_X = -45,000 – 8000(P/A,12%,5) + 2000(P/F,12%,5)
= -45,000 – 8000(3.6048) + 2000(0.5674)
= $-72,704

PW_Y = -58,000 – 4000(P/A,12%,5) + 12,000(P/F,12%,5)
= -58,000 – 4000(3.6048) + 12,000(0.5674)
= $-65,615

Select Alternative Y, because it has the larger PW value, that is, the lower PW of costs.

5.8 PW_A = -80,000 – 30,000(P/A,12%,3) + 15,000(P/F,12%,3)
= -80,000 – 30,000(2.4018) + 15,000(0.7118)
= $-141,377

PW_B = -120,000 – 8,000(P/A,12%,3) + 40,000(P/F,12%,3)
= -120,000 – 8,000(2.4018) + 40,000(0.7118)
= $-110,742

Select Method B

Spreadsheet functions: For PW_A: = -PV(12%,3,-30000,15000) - 80000
For PW_B: = -PV(12%,3,-8000,40000) - 120000

5.9 (a) PW_{Full} = $-122,000,000

PW_{Small} = -80,000,000 -100,000,000(P/F,6%,20) - 25,000(P/A,6%,20)
= -80,000,000 -100,000,000(0.3118) - 25,000(11.4699)
= $-111,466,748

Smaller pipeline is more economical

(b) Spreadsheet function for PW_{Small}

= -PV(6%,20,-25000) - 80000000 - PV(6%,20,,-100000000) displays $-111,467,221

5.10 PW_{old} = -1200(3.50)(P/A,15%,5)
= -4200(3.3522)
= $-14,079

PW_{new} = -14,000 – 1200(1.20)(P/A,15%,5)
= -14,000 – 1440(3.3522)
= $-18,827

Keep old brackets

5.11 PW_1 = -900,000 – 560,000(P/F,20%,2) – 79,000(P/A,20%,10)
= -900,000 – 560,000(0.6944) – 79,000(4.1925)
= $-1,620,072

PW_2 = -280,000 – 280,000(P/A,20%,10)
= -280,000 – 280,000(4.1925)
= $-1,453,900

Select option 2 - subcontracting

5.12 PW_{single} = -4000 - 4000(P/A,12%,4)
= -4000 - 4000(3.0373)

$= \$-16{,}149$

$PW_{site} = \$-15{,}000$

Buy the site license

5.13 Units are $ million

$PW_{ponds} = -13 – 2.1(P/A,10\%,5)$
$= -13 – 2.1(3.7908)$
$= \$-20.961$ ($-20,960,680)

$PW_{tubes} = -18 – 0.41(P/A,10\%,5)$
$= -18 – 0.41(3.7908)$
$= \$-19.554$ ($-19,554,228)

Use plastic tubes; PW of cost is lower

5.14 Compare each alternative against DN and select all with PW ≥ 0. Monetary units are $1000.

$PW_A = -1200 + 200(P/A,15\%,10) + 5(P/F,15\%,10)$
$= -1200 + 200(5.0188) + 5(0.2472)$
$= \$-195.004$ (Reject)

$PW_B = -2000 + 400(P/A,15\%,10) + 6(P/F,15\%,10)$
$= -2000 + 400(5.0188) + 6(0.2472)$
$= \$9.003$ (Accept)

$PW_C = -5000 + 1100(P/A,15\%,10) + 8(P/F,15\%,10)$
$= -5000 + 1100(5.0188) + 8(0.2472)$
$= \$522.658$ (Accept)

$PW_D = -7000 + 1300(P/A,15\%,10) + 7(P/F,15\%,10)$
$= -7000 + 1300(5.0188) + 7(0.2472)$
$= \$-473.830$ (Reject)

5.15 $PW_A = -80{,}000 – [30{,}000(P/A,12\%,3) + 4000(P/G,12\%,3)] + 15{,}000(P/F,12\%,3)$
$= -80{,}000 – [30{,}000(2.4018) + 4000(2.2208)] + 15{,}000(0.7118)$
$= \$-150{,}260$

$PW_B = -120{,}000 – [8{,}000(P/A,12\%,3) + 6500(P/G,12\%,3)] + 40{,}000(P/F,12\%,3)$
$= -120{,}000 – [8{,}000(2.4018) + 6500(2.2208)] + 40{,}000(0.7118)$
$= \$-125{,}178$

Select Method B

5.16 Municipal water: Cost/mth = -5(30)(2.90)/1000 = \$0.435

PW = -0.435(P/A,0.5%,12)
= -0.435(11.6189)
= \$-5.05

Evian bottled water: Cost/mth = (2)(0.60)(30) = \$36.00
PW = -36.00(P/A,0.5%,12)
= -36.00(11.6189)
= \$-418.28

Local bottled water: Cost/mth = (2)(0.25)(30) = \$15.00
PW = -15.00(P/A,0.5%,12)
= -15.00(11.6189)
= \$-174.28

Drinking bottled water varies from 34 to 83 times more expensive than drinking tap water.

Alternative Comparison - Different Lives

5.17 PW_{DDM} = -164,000 – 55,000(P/A,20%,4) – 164,000(P/F,20%,2)
= -164,000 – 55,000(2.5887) – 164,000(0.6944)
= \$-420,260

PW_{LS} = -370,000 – 21,000(P/A,20%,4) + 30,000(P/F,20%,4)
= -370,000 – 21,000(2.5887) + 30,000(0.4823)
= \$-409,894

Select method LS

5.18 PW_{M} = -205,000 – 29,000(P/A,10%,4) – 203,000(P/F,10%,2)
+ 2000(P/F,10%,4)
= -205,000 – 29,000(3.1699) – 203,000(0.8264) + 2000(0.6830)
= \$-463,320

PW_{FF} = -235,000 – 27,000(P/A,10%,4) + 20,000(P/F,10%,4)
= -235,000 – 27,000(3.1699) + 20,000(0.6830)
= \$-306,927

Select material FF

5.19 (a) PW_1 = -400,000 – 140,000(P/A,15%,6)-360,000(P/F,15%,3)+ 40,000(P/F,15%,6)
= -400,000 – 140,000(3.7845) – 360,000(0.6575) + 40,000(0.4323)
= \$-1,149,238

PW_2 = -600,000 – 100,000(P/A,15%,6) + 60,000(P/F,15%,6)
= -600,000 – 100,000(3.7845) + 60,000(0.4323)
= \$-952,512

Select method 2

(b) Incorrect PW_1 over 3 years, PW_2 is correct in (a)

Incorrect PW_1 = -400,000 – 140,000(P/A,15%,3) + 40,000(P/F,15%,3)
= -400,000 – 140,000(2.2832) + 40,000(0.6575)
= \$-693,348

Correct PW_2 = -600,000 – 100,000(P/A,15%,6) + 60,000(P/F,15%,6)
= -600,000 – 100,000(3.7845) + 60,000(0.4323)
= \$-952,512

Select method 1, which is an economically incorrect decision for Lego. The analysis does not meet the equal-service requirement of PW analysis.

5.20 Study period = LCM = 8 years

PW_A = -15,000 – 6000(P/A,10%,8) – 12,000(P/F10%,4) + 3000(P/F,10%,8)
= -15,000 – 6000(5.3349) – 12,000(0.6830) + 3000(0.4665)
= \$-53,806

PW_B = -28,000 – 9000(P/A,10%,8) – 2000(P/F,10%,4) + 5000(P/F,10%,8)
= -28,000 – 9000(5.3349) – 2000(0.6380) + 5000(0.4665)
= \$-75,048

Select Alternative A

5.21(a) PW_{BFP} = -203,000 – 90,500(P/A,6%,10) – 182,700(P/F,6%,5) + 20,300(P/F,6%,10)
= -203,000 – 90,500(7.3601) – 182,700(0.7473) + 20,300(0.5584)
= \$-994,285

PW_{Cent} = -396,000 – (119,000 - 37,000)(P/A,6%,10) + 39,600(P/F,6%,10)
= -396,000 – 82,000(7.3601) + 39,600(0.5584)

$= \$-977,416$

Centrifuges have a slightly lower PW

(b) $PW_{BFP} = -203,000 – 90,500(P/A,6\%,8) – 182,700(P/F,6\%,5) + 20,300(P/F,6\%,8)$
$= -203,000 – 90,500(6.2098) – 182,700(0.7473) + 20,300(0.6274)$
$= \$-888,782$

$PW_{Cent} = -396,000 – (119,000 - 37,000)(P/A,6\%,8) + 39,600(P/F,6\%,8)$
$= -396,000 – 82,000(6.2098) + 39,600(0.6274)$
$= \$-880,359$

Centrifuges still have a slightly lower PW, but by a smaller amount

5.22 $PW_A = -400 + (360 – 100)(P/A,20\%,3)$
$= -400 + 260(2.1065)$
$= \$147.69$ (\$147,690) **Accept**

$PW_B = -510 + (235 – 140)(P/A,20\%,10) + 22(P/F,20\%,10)$
$= -510 + 95(4.1925) + 22(0.1615)$
$= \$-108.160$ (\$-108,060) **Reject**

$PW_C = -660 + (400 – 280)(P/A,20\%,5)$
$= -660 + 120(2.9906)$
$= \$-301.128$ (\$-301,128) **Reject**

$PW_D = -820 + (605 – 315)(P/A,20\%,8) + 80(P/F,20\%,8)$
$= -820 + 290(3.8372) + 80(0.2326)$
$= \$311.396$ (\$311,396) **Accept**

$PW_E = -900 + (790 – 450)(P/A,20\%,4) + 95(P/F,20\%,4)$
$= -900 + 340(2.5887) + 95(0.4823)$
$= \$25.977$ (\$25,977) **Accept**

Projects A, D and E are acceptable with PW > 0 at 20%

5.23 $PW_C = -40,000 – [7000(P/A,10\%,10) + 1000(P/G,10\%,10)] + 9000(P/F,10\%,10)$
$= -40,000 – [7000(6.1446) + 1000(22.8913)] + 9000(0.3855)$
$= \$-102,434$

$PW_D = -32,000 – 3000(P/A,10\%,10) - 31,500(P/F10\%,5) + 500(P/F,10\%,10)$
$= -32,000 – 3000(6.1446) - 31,500(0.6209) + 500(0.3855)$
$= \$-69,799$

Select alternative D

5.24 PW_K = -160,000 – 7000(P/A,2%,16) –120,000(P/F,2%,8) + 40,000(P/F,2%,16)
= -160,000 – 7000(13.5777) –120,000(0.8535) + 40,000(0.7284)
= $-328,328

PW_L = -210,000 – 5000(P/A,2%,16) + 26,000(P/F,2%,16)
= -210,000 – 5000(13.5777) + 26,000(0.7284)
= $-258,950

Purchase package L

5.25 (a) Factor solution. Must use a 6-year evaluation period for equal-service analysis.

Plan A: effective i/year = $(1 + 0.03)^2 – 1$ = 6.09%
PW_A = -1,000,000 - 1,000,000(P/A,6.09%,5)
= -1,000,000 - 1,000,000(4.2021)
= $-5,202,100

PW_B = -600,000[1 + (P/A,3%,2) + (P/A,3%,3)(P/F,3%,3) + (P/A,3%,3)(P/F,3%,7)]
= -600,000[1 + 1.9135 + (2.8286)(0.9151) + (2.8286)(0.8131)]
= -600,000(7.8019)
= $-4,681,132

PW_C = -1,500,000 – 500,000(P/F,3%,4) – 1,500,000(P/F,3%,6)
- 500,000(P/F,3%,10)
= -1,500,000 – 500,000(0.8885) – 1,500,000(0.8375) – 500,000(0.7441)
= $-3,572,550

Select plan C

(b) Spreadsheet: Cash flow details not included. PW functions and displays are:
Plan A: = NPV($B2,B6:B10)+B5 displays $-5,202,070
Plan B: = NPV($D2,D6:D15)+D5 displays $-4,681,182
Plan C: = NPV($D2,E6:E15)+E5 displays $-3,572,517

Select plan C

5.26 Set the PW_S relation equal to $-33.16, and solve for the first cost X_S (a positive number) with repurchase in year 5. In $1 million units,

PE

$$-33.16 = -X_S[1 + (P/F,12\%,5)] - 1.94(P/A,12\%,10) + 0.05X_S[(P/F,12\%,5) + (P/F,12\%,10)]$$
$$= -1.5674X_S - 1.94(5.6502) + 0.0445X_S$$
$$1.5229X_S = -10.9614 + 33.16$$

$X_S = \$14.576 \qquad (\$14,576,000)$

Select seawater option for any first cost ≤ \$14.576 million

Future Worth Comparison

5.27 $FW_{solar} = -12,600(F/P,10\%,4) – 1400(F/A,10\%,4)$
$= -12,600(1.4641) – 1400(4.6410)$
$= \$-24,945$

$FW_{line} = -11,000(F/P,10\%,4) – 800(F/A,10\%,4)$
$= -11,000(1.4641) – 800(4.6410)$
$= \$-19,818$

Install power line

5.28 Calculate FW at LCM of 6 years

$FW_D = -62,000[(F/P,15\%,6) + (F/P,15\%,3)] – 15,000(F/A,15\%,6) + 8,000[(F/P,15\%,3) + 1]$
$= -62,000[(2.3131) + (1.5209)] – 15,000(8.7537) + 8,000[(1.5209) + 1]$
$= \$-348,846$

$FW_E = -77,000(F/P,15\%,6) – 21,000(F/A,15\%,6) + 10,000$
$= -77,000(2.3131) – 21,000(8.7537) + 10,000$
$= \$-351,936$

Select option D

Spreadsheet functions:
Option D: = - FV(15%,6,-15000,-62000) - FV(15%,3,,-54000) + 8000
Option E: = - FV(15%,6,-21000,-77000) + 10000

5.29 $FW_{20\%} = -100(F/P,10\%,15) – 80(F/A,10\%,15)$
$= -100(4.1772) – 80(31.7725)$
$= \$-2959.52$

$FW_{35\%} = -240(F/P,10\%,15) – 65(F/A,10\%,15)$
$= -240(4.1772) – 65(31.7725)$
$= \$-3067.74$

20% standard is slightly more economical

5.30 (a) $FW_A = -40,000[(F/P,10\%,8) + (F/P,10\%,6) + (F/P,10\%,4) + (F/P,10\%,2)]$

$- 9000(F/A,10\%,8)$
$= -40{,}000[(2.1436) + (1.7716) + (1.4641) + (1.2100)] – 9000(11.4359)$
$= \$-366{,}495$

$FW_B = -80{,}000[(F/P,10\%,8) + (F/P,10\%,4)] - 6000(F/A,10\%,8)$
$= -80{,}000[(2.1436) + (1.4641)] - 6000(11.4359)$
$= \$-357{,}231$

$FW_C = -130{,}000(F/P,10\%,8) - 4000(F/A,10\%,8) + 13{,}000$
$= -130{,}000(2.1436) - 4000(11.4359) + 13{,}000$
$= \$-311{,}412$

Select method C

(b) Find the PW values from the FW values with n = 8 years. Select method C

$PW_A = FW_A(P/F,10\%,8) = -366{,}495(0.4665) = \$-170{,}970$

$PW_B = FW_B(P/F,10\%,8) = \$-166{,}648$

$PW_C = FW_C(P/F,10\%,8) = \$-145{,}274$

5.31 $FW_{purchase} = -150{,}000(F/P,15\%,6) + 12{,}000(F/A,15\%,6) + 65{,}000$
$= -150{,}000(2.3131) + 12{,}000(8.7537) + 65{,}000$
$= \$-176{,}921$

$FW_{lease} = -20{,}000(F/A,15\%,6)(F/P,15\%,1)$
$= -20{,}000(8.7537)(1.15)$
$= \$-201{,}335$

Purchase the clamshell

Capitalized Cost

5.32 $CC = -78{,}000 - 3500(A/F,8\%,5)/0.08$
$= -78{,}000 - 43{,}750(0.17046)$
$= \$-85{,}458$

5.33 $CC = - [38(120{,}000) + 17(150{,}000)]/0.08$
$= \$-88{,}875{,}000$

5.34 $PW_{50} = 10{,}000(P/A,10\%,50)$
$= 10{,}000(9.9148)$ (by factor)
$= \$99{,}148$

$CC = AW/i = 10{,}000/0.10$
$= \$100{,}000$

Difference $= 100{,}000 – 99{,}148$
$= \$852$

5.35 $CC = -1{,}700{,}000 – 350{,}000(A/F,6\%,3)/0.06$
$= - 1{,}700{,}000 – 350{,}000(0.31411)/0.06$
$= \$-3{,}532{,}308$

5.36 $P_{-1} = 10{,}000/0.10 = 100{,}000$
$P_{-35} = 100{,}000(P/F,10\%,34)$
$= 100{,}000(0.0391)$
$= \$3910$

5.37 (a) $CC = -200{,}000 – 25{,}000(P/A,12\%,4)(P/F,12\%,1) – (40{,}000/0.12)(P/F,12\%,5)$
$= -200{,}000 – 25{,}000(3.0373)(0.8929) – (40{,}000/0.12)(0.5674)$
$= \$-456{,}933$

(b) Find future value in year 25 and multiply by i

$A = F_{25} \times i$
$= [200{,}000(F/P,12\%,25) + 25{,}000(F/A,12\%,4)(F/P,12\%,20) + 40{,}000(F/A,12\%,19)](0.12)$
$= [200{,}000(17.0001) + 25{,}000(4.7793)(9.6463) + 40{,}000(63.4397)](0.12)$
$= 7{,}090{,}172(0.12)$
$= \$850{,}821$ per year (forever)

5.38 (a) Find future value in year 19, then multiply by i

$F_{19} = 10{,}000(F/P,12\%,19) + 30{,}000(F/P,12\%,16) + 8000(F/A,12\%,5)(F/P,12\%,11)$
$= 10{,}000(8.6128) + 30{,}000(6.1304) + 8000(6.3528)(3.4785)$
$= \$446{,}826$

$A = 446{,}826(0.12)$
$= \$53{,}619$ per year (forever)

(b) $CC = -10{,}000 – 30{,}000(P/F,12\%,3) – (8000/0.12)(P/F,12\%,3)$
$= -10{,}000 – 30{,}000(0.7118) – 66{,}667(0.7118)$
$= \$-78{,}808$

5.39 $CC = -250{,}000{,}000 – 800{,}000/0.08 – [950{,}000(A/F,8\%,10)]/0.08$
$- 75{,}000(A/F,8\%,5)/0.08$
$= -250{,}000{,}000 – 800{,}000/0.08 – [950{,}000(0.06903)]/0.08$

$-[75{,}000(0.17046)]/0.08$
$= -250{,}000{,}000 – 10{,}000{,}000 – 819{,}731 – 159{,}806$
$= \$-260{,}979{,}538$

5.40 For alternatives E and F, find AW, then divide by i using CC = AW/i

$$\begin{aligned}AW_E &= -50{,}000(A/P,10\%,2) – 30{,}000 + 5000(A/F,10\%,2)\\ &= -50{,}000(0.57619) – 30{,}000 + 5000(0.47619)\\ &= \$-56{,}428.60\end{aligned}$$

$$\begin{aligned}CC_E &= 56{,}428.60/0.10\\ &= \$-564{,}286\end{aligned}$$

$$\begin{aligned}AW_F &= -300{,}000(A/P,10\%,4) – 10{,}000 + 70{,}000(A/F,10\%,4)\\ &= -300{,}000(0.31547) – 10{,}000 + 70{,}000(0.21547)\\ &= \$-89{,}558.10\end{aligned}$$

$$\begin{aligned}CC_F &= 89{,}558.10/0.10\\ &= \$-895{,}581\\ AW_G &= -900{,}000 - 3000/0.10\\ &= \$-930{,}000\end{aligned}$$

Select alternative E

5.41 Use C to identify the contractor option. PE

(a) $CC_C = -5 \text{ million}/0.12 = \$-41.67 \text{ million}$

Between the three options, select the contractor

(b) Find P_g and A of the geometric gradient (g = 2%), then CC.

$$\begin{aligned}P_g &= -5{,}000{,}000[1 - (1.02/1.12)^{50}]/(0.12 – 0.02)\\ &= -5{,}000{,}000[9.9069]\\ &= \$-49.53 \text{ million}\end{aligned}$$

$$\begin{aligned}A &= P_g(A/P,12\%,50)\\ &= -49.53 \text{ million}(0.12042)\\ &= \$-5.96 \text{ million per year}\end{aligned}$$

$$\begin{aligned}CC_C &= A/i = -5.96 \text{ million}/0.12\\ &= \$-49.70 \text{ million}\end{aligned}$$

Now, select groundwater ($CC_G = \$-48.91$) source by a relatively small margin.

Exercises for Spreadsheets

5.42 Effective $i = i_a = (1 + 0.15/4)^4 - 1 = 0.15865$

Solution shows functions. Select option 3.

	A	B	C	D
1	MARR	15% nominal	15.865%	Effective
2				
3	Year	Option 1	Option 2	Option 3
4	0	-900,000	-280,000	0
5	1	-79,000	-280,000	-400,000
6	2	-639,000	-280,000	-420,000
7	3	-79,000	-280,000	-441,000
8	4	-79,000	-280,000	-463,050
9	5	-79,000	-280,000	-486,203
10	6	-79,000	-280,000	50,000
11	7	-79,000	-280,000	50,000
12	8	-79,000	-280,000	50,000
13	9	-79,000	-280,000	50,000
14	10	-79,000	-280,000	150,000
15				
16	PW value	-$1,700,894	-$1,640,133	-$1,329,792
17	NPV function	= NPV($C$1,B5:B14) + B4	= NPV($C$1,C5:C14) + C4	= NPV($C$1,D5:D14) + D4
18				

5.43 (a) Use PV and logical IF functions, as in top half of the spreadsheet.

(b) Simply update the parameter estimates for A and B. New decisions are displayed.

	A	B	C	D	E
1	Part (a) PW analysis using preliminary estimates and logical IF functions				
2		Present worth analysis		Decision analysis	
3	Project	Function	PW, $1000	Function	Decision
4	A	= -PV(15%,10,200,5) - 1200	-$195.010	= IF($C4>=0, "Accept", "Reject")	Reject
5	B	= -PV(15%,10,400,6) - 2000	$8.991	= IF($C5>=0, "Accept", "Reject")	Accept
6	C	= -PV(15%,10,1100,8) - 5000	$522.623	= IF($C6>=0, "Accept", "Reject")	Accept
7	D	= -PV(15%,10,1300,7) - 7000	-$473.870	= IF($C7>=0, "Accept", "Reject")	Reject
8					
9	Part (b) Reevaluation of projects A and B				
10		Present worth analysis		Decision analysis	
11	Project	Function	PW, $1000	Function	Decision
12	A	= -PV(15%,10,300,8) - 1000	$507.608	= IF($C12>=0, "Accept", "Reject")	Accept
13	B	= -PV(15%,10,440,0) - 2200	$8.258	= IF($C13>=0, "Accept", "Reject")	Accept
14	C	= -PV(15%,10,1100,8) - 5000	$522.623	= IF($C14>=0, "Accept", "Reject")	Accept
15	D	= -PV(15%,10,1300,7) - 7000	-$473.870	= IF($C15>=0, "Accept", "Reject")	Reject

5.44 Select alternative A

	A	B	C
1		Cash flow, $	
2	Year	A	B
3	0	-15,000	-28,000
4	1	-6,000	-9,000
5	2	-6,000	-9,000
6	3	-6,000	-9,000
7	4	-18,000	-11,000
8	5	-6,000	-9,000
9	6	-6,000	-9,000
10	7	-6,000	-9,000
11	8	-3,000	-4,000
12			
13	PW @ 10%	-$53,806	-$75,048

5.45 Selection changes from Ferguson (5 years) to Halgrove (8 or 10 years)

	A	B	C	D	E	F	G
1	Study period	5 years		8 years		10 years	
2	Year	Ferguson	Halgrove	Ferguson	Halgrove	Ferguson	Halgrove
3	0	-203,000	-396,000	-203,000	-396,000	-203,000	-396,000
4	1	-90,000	-82,000	-90,000	-82,000	-90,000	-82,000
5	2	-90,000	-82,000	-90,000	-82,000	-90,000	-82,000
6	3	-90,000	-82,000	-90,000	-82,000	-90,000	-82,000
7	4	-90,000	-82,000	-90,000	-82,000	-90,000	-82,000
8	5	-69,700	-42,400	-272,700	-82,000	-272,700	-82,000
9	6			-90,000	-82,000	-90,000	-82,000
10	7			-90,000	-82,000	-90,000	-82,000
11	8			-69,700	-42,400	-90,000	-82,000
12	9					-90,000	-82,000
13	10					-69,700	-42,400
14	PW @ 6%	-$566,943	-$711,822	-$885,669	-$880,358	-$990,596	-$977,415
15	Selection	Ferguson			Halgrove		Halgrove

5.46 Selection is D in both cases, but by a larger margin for the maximum life plan.

	A	B	C	D	E	F
1		Option	LCM, years	Function	FW value	Selection
2	(a) Usage	D	6	= - FV(15%,6,-15000,-62000) - FV(15%,3,,-54000) + 8000	-$348,843	D
3		E		= - FV(15%,6,-21000,-77000) + 10000	-$351,934	
4						
5	(b) Life	D	8	= - FV(15%,8,-15000,-62000) - FV(15%,4,,-58000) + 4000	-$493,003	D
6		E		= - FV(15%,8,-21000,-77000) + 5000	-$518,807	

5.47 (a) Develop the spreadsheet with 2% increases in M&O for purchase and a constant $10,000 for contract (C5 is the anchor cell). Decision: Purchase the equipment.

(b) Use Goal Seek to change cell C5. Manal would have to decrease its M&O cost to a much smaller $902 per year from the estimated $10,000.

Part (a) Part (b)

	A	B	C
1	M&O cost factor	102.00%	100%
2			
3	Year	Purchase, $	Manal and Assoc, $
4	0	-300,000	-850,000
5	1	-10,000	-10,000
6	2	-10,200	-10,000
7	3	-10,404	-10,000
8	4	-10,612	-10,000
9	5	-10,824	-10,000
10	6	-11,041	-10,000
11	7	-11,262	-10,000
12	8	58,513	-10,000
13	9+		continues at -10,000
14	PW	-$322,303	
15	AW	-$51,902	
16	CC	-$865,039	-$1,016,667

	A	B	C	D	E	F
1	M&O cost factor	102.00%	100%			
2						
3	Year	Purchase, $	Manal and Assoc, $			
4	0	-300,000	-850,000			
5	1	-10,000	-902			
6	2	-10,200	-902			
7	3	-10,404	-902			
8	4	-10,612	-902			
9	5	-10,824	-902			
10	6	-11,041	-902			
11	7	-11,262	-902			
12	8	58,513	-902			
13	9+		continues at -10,000			
14	PW	-$322,303				
15	AW	-$51,902				
16	CC	-$865,039	-$865,039			
17						

Goal Seek
Set cell: C16
To value: -865039
By changing cell: $C$5
OK Cancel

Additional Problems and FE Exam Review Questions

5.48 Answer is (c)

5.49 Answer is (b)

5.50 CC = -200,000 - 30,000/0.10
= $-500,000
Answer is (c)

5.51 AW = -200,000(A/P,10%,8) – 60,000 + 50,000(A/F,10%,8)
= -200,000(0.18744) – 60,000 + 50,000(0.08744)
= $-93,116

CC = -93,116/0.10
= $-931,160
Answer is (c)

5.52 CC = -70,000 – 70,000(A/F,10%,10)/0.10
= -70,000[1 + (0.06275)/0.10]
= $-113,925
Answer is (d)

Problems 5.53 through 5.55 are based on the following cash flows for alternatives X and Y at an interest rate of 10% per year.

	Machine X	Machine Y
Initial cost, $	-146,000	-220,000
AOC, $/year	-15,000	-10,000

Annual revenue, $/year	80,000	75,000
Salvage value, $	10,000	25,000
Life, years	3	6

5.53 $PW_X = -146{,}000 + (80{,}000-15{,}000)(P/A,10\%,6) - 136{,}000(P/F,10\%,3) + 10{,}000(P/F,10\%,6)$
$= -146{,}000 + 65{,}000(4.3553) - 136{,}000(0.7513) + 10{,}000(0.5645)$
$= \$40{,}563$
Answer is (b)

5.54 $FW_X = -146{,}000(F/P,10\%,6) + (80{,}000-15{,}000)(F/A,10\%,6) - 136{,}000(F/P,10\%,3) + 10{,}000$
$= -146{,}000(1.7716) + (65{,}000)(7.7156) - 136{,}000(1.3310) + 10{,}000$
$= \$71{,}861$
Answer is (d)

5.55 First find AW over n = 6 years and then divide by i
$AW_Y = -220{,}000(A/P,10\%,6) + 65{,}000 + 25{,}000(A/F,10\%,6)$
$= -220{,}000(0.22961) + 65{,}000 + 25{,}000(0.12961)$
$= \$17{,}726$

$CC_Y = 17{,}726/0.10$
$= \$177{,}260$
Answer is (c)

5.56 Answer is (d)

Problems 5.57 through 5.61 are based on the following cash flows for alternatives A and B at an interest rate of 10% per year:

Alternative	A	B
First cost, $	-90,000	-750,000
Annual operating cost, $/year	-50,000	-10,000
Salvage value, $	8,000	2,000,000
Life, years	5	∞

5.57 LCM is ∞; first find AW and then divide by i
$AW_A = -90{,}000(A/P,10\%,5) -50{,}000 + 8000(A/F,10\%,5)$
$= -90{,}000(0.26380) -50{,}000 + 8000(0.16380)$
$= -\$72{,}432$

$CC_A = -72{,}432/0.10$
$= \$-724{,}320$
Answer is (a)

5.58 $CC_B = -750{,}000 - 10{,}000/0.10$
$= \$-850{,}000$
Answer is (c)

5.59 $CC_B = -750{,}000 - 10{,}000/0.10$
$= \$-850{,}000$
Answer is (d)

5.60 Answer is (b)

5.61 Answer is (d)

Solution to Case Study, Chapter 5

There is not always a definitive answer to case study exercises. Here are example responses

COMPARING SOCIAL SECURITY BENEFITS

1. Total payments are shown in row 30 of the spreadsheet.

2. Future worth values at 6% per year are shown in row 29.

	A	B	C	D	E	F	G	H	I	J
1	Rate =	6.00%	**Plan A**		**Plan B**		**Plan C**		**Plan D**	
2		Remaining		Future worth		Future worth		Future worth		Future worth
3	Age	Years	Reduced	Reduced	Full	Full	Self-Delayed	Self- Delayed	Spouse-Delayed	Spouse-Delayed
4	61	25								
5	62	24	16,800	16,800					0	
6	63	23	16,800	34,608					0	
7	64	22	16,800	53,484					0	
8	65	21	16,800	73,494					0	
9	66	20	16,800	94,703					0	
10	67	19	16,800	117,185	24,000	24,000			12,000	12,000
11	68	18	16,800	141,016	24,000	49,440			12,000	24,720
12	69	17	16,800	166,277	24,000	76,406			12,000	38,203
13	70	16	16,800	193,054	24,000	104,991	29,760	29,760	29,760	70,255
14	71	15	16,800	221,437	24,000	135,290	29,760	61,306	29,760	104,231
15	72	14	16,800	251,524	24,000	167,408	29,760	94,744	29,760	140,245
16	73	13	16,800	283,415	24,000	201,452	29,760	130,189	29,760	178,419
17	74	12	16,800	317,220	24,000	237,539	29,760	167,760	29,760	218,884
18	75	11	16,800	353,053	24,000	275,792	29,760	207,585	29,760	261,777
19	76	10	16,800	391,036	24,000	316,339	29,760	249,801	29,760	307,244
20	77	9	16,800	431,298	24,000	359,319	29,760	294,549	29,760	355,439
21	78	8	16,800	473,976	24,000	404,879	29,760	341,982	29,760	406,525
22	79	7	16,800	519,215	24,000	453,171	29,760	392,260	29,760	460,677
23	80	6	16,800	567,168	24,000	504,362	29,760	445,556	29,760	518,077
24	81	5	16,800	617,998	24,000	558,623	29,760	502,049	29,760	578,922
25	82	4	16,800	671,878	24,000	616,141	29,760	561,932	29,760	643,417
26	83	3	16,800	728,990	24,000	677,109	29,760	625,408	29,760	711,782
27	84	2	16,800	789,530	24,000	741,736	29,760	692,693	29,760	784,249
28	85	1	16,800	853,702	24,000	810,240	29,760	764,014	29,760	861,064
29	Total FW		$ 853,702		$810,240		$ 764,014		$ 861,064	
30	Sum		$ 403,200		$456,000		$ 476,160		$ 512,160	
31										
32	Answers to #1		Answers to #2							
33										

3. Plots of FW values by year are shown in the (x-y scatter) graph below.

4. Develop all feasible plans for the couple and use the summed FW values to determine which is largest.

Spouse #1	Spouse #2	FW, $
A	A	1,707,404
A	B	1,663,942
A	C	1,617,716
B	B	1,620,480
B	C	1,574,254
B	D	1,671,304
C	C	1,528,028

Solutions to end-of-chapter problems

Engineering Economy, 8th edition
Leland Blank and Anthony Tarquin

Chapter 6
Annual Worth Analysis

Annual Worth and Capital Recovery Calculations

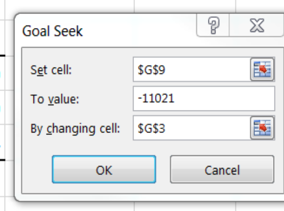
6.1 Multiply the FW values by (A/F,i%,n), where n is equal to the LCM or stated study period.

6.2 The estimate obtained from the three-year AW would *not* be valid, because the AW calculated over one life cycle is valid only for the *entire cycle*, not part of the cycle. Here the asset would be used for only a part of its second 3-year life cycle.

6.3 Factors: -10,000(A/P,10%,3) – 7000 = -10,000(A/P,10%,2) – 7000 + S(A/F,10%,2)
-10,000(0.40211) – 7000 = -10,000(0.57619) – 7000 + S(0.47619)
-11,021 = -12,762 + 0.47619S
S = $3656

Spreadsheet: The answer is the G3 display of $3656

	B	C	D	E	F	G
1						
2	Goal Seek (S = 0)			After Goal Seek to find S		
3	S =	0			S =	$ 3,656
4	n = 3years	n = 2 years		Year	n = 3years	n = 2 years
5	-10,000	-10,000		0	-10,000	-10,000
6	-7,000	-7,000		1	-7,000	-7,000
7	-7,000	-7,000		2	-7,000	-3,344
8	-7,000			3	-7,000	
9	-$11,021	-$12,762		AW @ 10	-$11,021	-$11,021



6.4 (a) AW_4 = -30,000(A/P,10%,6) – 12,000 + 4000(A/F,10%,6)
= -30,000(0.22961) – 12,000 + 4000(0.12961)
= $-18,370

-18,370 = -30,000(A/P,10%,2) – 12,000 + S(A/F,10%,2)
-18,370 = -30,000(0.57619) – 12,000 + S(0.47619)
0.47619S = 10,916
S = $22,923

(b) S = \$22,923 is very high for a used delivery car; this market value is over 5.7 times the estimated salvage of \$4000.

6.5 CR_1 = -50,000(A/P,10%,5) + 5,000(A/F,10%,5)
= -50,000(0.26380) + 5,000(0.16380)
= \$-12,371

CR_2 = -[(50,000 – 5,000)(A/P,10%,5) + 5,000(0.10)]
= -[(45,000)(0.26380) + 5,000(0.10)]
= \$-12,371

6.6 (a) Capital recovery = P/n = 6000/3
= \$2000 per year

Interest, year 1 = 6000(0.10)
= \$600

Payment, year 1 = 2000 + 600
= \$2600

Interest, year 2 = (unrecovered balance)(0.10)
= 4000(0.10)
= \$400

Payment, year 2 = 2000 + 400
= \$2400

Interest, year 3 = (unrecovered balance)(0.10)
= 2000(0.1)
= \$200

Payment, year 3 = 2000 + 200
= \$2200

(b) A = 6,000(A/P,10%,3)
= 6,000(0.40211)
= \$2,412.66

6.7 (a) CR = -750,000(A/P,25%,5) + 125,000(A/F,25%,5)
= -750,000(0.37185) + 125,000(0.12185)
= \$-263,656

The company has to have revenue of at least to \$263,656 per year to recover its capital investment and make a return of 25% per year.

(b) AW = CR + AOC = -263,656 – 80,000
= \$-343,656

(c) CR function: = - PMT(25%,5,-750000,125000) displays \$-263,654
AW function: = - PMT(25%,5,-750000,125000) – 80000 displays \$-343,654

6.8 AW = -638,000(A/P,25%,9) – 240,000 + 184,000(A/F,25%,9)
= -638,000(0.28876) – 240,000 + 184,000(0.03876)
= \$-417,097

6.9 In \$ billion units

AW = -1.16(A/P,15%,60) – 0.0026(P/F,15%,4)(A/P,15%,60) - 0.0063
= -1.16(0.15003) – 0.0026(0.5718)(0.15003) - 0.0063
= \$ - 0.180,557 (\$-180,557,847 per year)

6.10 Find the CR; in \$ million units

CR = [-13 - 10(P/F,15%,1)](A/P,15%,10)
= [-13 - 10(0.8696)](0.19925)
= \$-4.3229

Revenue required is \$4,322,900 per year

6.11 Factors: In \$ million units

AW = - (1.1 + 0.275)(A/P,10%,5) – 13(0.1) + (0.25)(1.1 + 0.275)(A/F,10%,5)
= - 1.375(0.26380) – 1.3 + (0.25)(1.375)(0.16380)
= \$-1.606 (\$-1,606,420)

Function: = - PMT(10%,5,-1375000,343750) – 1300000 displays \$-1,606,416

6.12 AW = -800,000(A/P,10%,4) – 300,000(P/F,10%,2)(A/P/10%,4) - 950,000
+ 250,000(A/F,10%,4)
= -800,000(0.31547) – 300,000(0.8264)(0.31547) - 950,000
+ 250,000(0.21547)
= \$-1,226,720 per year

Alternative Comparison

6.13 (a) In \$1000 units
AW_{Pull} = -1500(A/P,10%,8) -700 + 100(A/F,10%,8)
= -1500(0.18744) -700 + 100(0.08744)
= \$-972.416 (\$-972,416)

AW_{Push} = -2250(A/P,10%,8) - 600 + 50(A/F,10%,8)
= -2250(0.18744) - 600 + 50(0.08744)
= \$-1017.368 (\$-1,017,368)

Select the Pull System

(b) Set AW_{Push} equal to \$-972,416, and solve for S_{Push}. In \$1000 units

-972.416 = -2250(A/P,10%,8) - 600 + S_{Push} (A/F,10%,8)
= -2250(0.18744) - 600 + S_{Push} (0.08744)
0.08744 S_{Push} = 49.324
S_{Push} = \$564,090

6.14 AW_A = -2,000,000(A/P,12%,3) – 60,000 + 2,000,000(0.10)(A/F,12%,3)
= -2,000,000(0.41635) – 60,000 + 200,000(0.29635)
= \$-833,430

AW_B = -795,000(A/P,12%,3) – [85,000(P/F,12%,1)
+ 46,000(P/A,12%,2)(P/F,12%,1)](A/P,12%,3)
= -795,000(0.41635) – [85,000(0.8929) + 46,000(1.6901)(0.8929)](0.41635)
= \$-391,500

Select Plan B

6.15 AW_1 = -550,000(A/P,10%,3) – 160,000 + 125,000(A/F,10%,3)
= -550,000(0.40211) – 160,000 + 125,000(0.30211)
= \$-343,397

AW_2 = -830,000(A/P,10%,3) – 120,000 + 240,000(1.35)(A/F,10%,3)
= -830,000(0.40211) – 120,000 + 240,000(1.35)(0.30211)
= \$-355,868

Select method 1

6.16 Factors:
$AW_{Resturant}$ = -26,000(A/P,1%,60) -2200 -3700 + 14,100 + 26,000(0.10)(A/F,1%,60)
= -26,000(0.02224) - 2200 - 3700 + 14,100 + 2600(0.01224)
= \$7653.58 per month

AW_{Truck} = -17,900(A/P,1%,60) - 900 + 6200 + 17,900(0.35)(A/F,1%,60)
= -17,900(0.02224) - 900 + 6200 + 6265(0.01224)
= \$4978.59 per month

Open the restaurant

Spreadsheet: Restaurant: = - PMT(1%,60,-26000,2600) + 8200 displays \$+7653.48

Truck: = - PMT(1%,60,-17900,6265) + 5300 displays $+4978.54

6.17 (a) $AW_{GM} = -36{,}000(A/P,15\%,3) – 4000 + 15{,}000(A/F,15\%,3)$
$= -36{,}000(0.43798) – 4000 + 15{,}000(0.28798)$
$= \$-15{,}448$

$AW_{Ford} = -32{,}000(A/P,15\%,4) – 3100 + 15{,}000(A/F,15\%,4)$
$= -32{,}000(0.35027) – 3100 + 15{,}000(0.20027)$
$= \$-11{,}305$

Purchase the Ford SUV

(b) $PW_{GM} = -15{,}448(P/A,15\%,12)$
$= -15{,}448(5.4206)$
$= \$-83{,}737$

$PW_{Ford} = -11{,}305(P/A,15\%,12)$
$= -11{,}305(5.4206)$
$= \$-61{,}280$

6.18 (a) Factors

$AW_{Round} = -250{,}000(A/P,10\%,6) – 31{,}000 + 40{,}000(A/F,10\%,6)$
$= -250{,}000(0.22961) – 31{,}000 + 40{,}000(0.12961)$
$= \$-83{,}218$

$AW_{Straight} = -170{,}000(A/P,10\%,4) – 35{,}000 – 26{,}000(P/F,10\%,2)(A/P,10\%,4) + 10{,}000(A/F,10\%,4)$
$= -170{,}000(0.31547) – 35{,}000 – 26{,}000(0.8264)(0.31547) + 10{,}000(0.21547)$
$= \$-93{,}254$

Select Round Knife

(b) Round knife: = - PMT(10%,6,-250000,40000) – 31000 displays $-83,218
Straight knife: = - PMT(10%,4,-170000,10000) - 35000 - PMT(10%,4,-PV(10%,2,, -26000)) displays $-93,254

Select Round knife

6.19 $AW_{Cart} = -300{,}000(A/P,10\%,4) - 60{,}000 + 70{,}000(A/F,10\%,4)$
$= -300{,}000(0.31547) - 60{,}000 + 70{,}000(0.21547)$
$= \$-139{,}558$

$AW_{Art} = -430{,}000(A/P,10\%,6) - 40{,}000 + 95{,}000(A/F,10\%,6)$
$= -430{,}000(0.22961) - 40{,}000 + 95{,}000(0.12961)$

$= \$-126,420$

Select articulated robot

6.20 (a) $AW_C = -40,000(A/P,15\%,3) - 10,000 + 12,000(A/F,15\%,3)$
$= -40,000(0.43798) - 10,000 + 12,000(0.28798)$
$= \$-24,063$

$AW_D = -65,000(A/P,15\%,6) - 12,000 + 25,000(A/F,15\%,6)$
$= -65,000(0.26424) - 12,000 + 25,000(0.11424)$
$= \$-26,320$

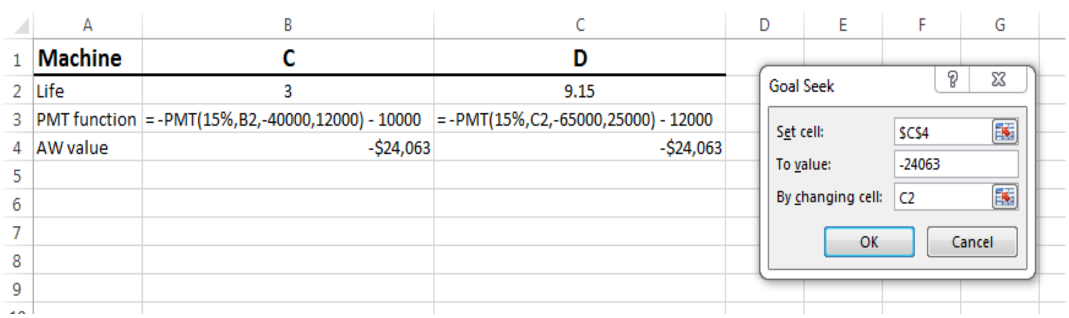
Select machine C

(b) Factors: Set $AW_D = AW_C$ with n_D as an unknown.

$-24,063 = -65,000(A/P,15\%,n_D) - 12,000 + 25,000(A/F,15\%,n_D)$

Solve by trial and error; n_D is between 9 and 10 years. An expected life of 10 years will indicate D as selected.

Spreadsheet: Use Goal Seek tool to display 9.15 years



	A	B	C
1	**Machine**	**C**	**D**
2	Life	3	9.15
3	PMT function	=-PMT(15%,B2,-40000,12000) - 10000	=-PMT(15%,C2,-65000,25000) - 12000
4	AW value	-$24,063	-$24,063

6.21 (a) $AW_A = -25,000(A/P,12\%,2) - 4000$
$= -25,000(0.59170) - 4,000$
$= \$-18,793$

$AW_B = -88,000(A/P,12\%,6) - 1400$
$= -88,000(0.24323) - 1400$
$= \$-22,804$

Select plan A

(b) Use the LCM of 6 years

$FW_A = AW_A(F/A,12\%,6) = -18,793(8.1152) = \$-152,509$

$FW_B = AW_B(F/A,12\%,6) = -22,804(8.1152) = \$-185060$

Select plan A

6.22 (a) $AW_{Land} = -150,000(A/P,10\%,4) – 95,000 + 25,000(A/F,10\%,4)$
$= -150,000(0.31547) – 95,000 + 25,000(0.21547)$
$= \$-136,934$

$AW_{Incin} = -900,000(A/P,10\%,6) – 60,000 + 300,000(A/F,10\%,6)$
$= -900,000(0.22961) – 60,000 + 300,000(0.12961)$
$= \$-227,766$

$AW_{Cont} = \$-140,000$

Select land application

(b) Use the LCM of 12 years

$PW_{Land} = -136,934(P/A,10\%,12) = -136,934(6.8137) = \$-933,027$

$PW_{Incin} = -227,766(P/A,10\%,12) = -227,766(6.8137) = \$-1,551,929$

$PW_{Cont} = -140,000(P/A,10\%,12) = -140,000(6.8137) = \$-953,918$

Select land application

6.23 (a) $AW_{Forklift}$ = CR – AOC – salary – AW of pallets
$= -30,000(A/P,8\%,12) + 8000(A/F,8\%,12) – 1000 - 32,000$
$-500(10)[1+(P/F,8\%,2)+(P/F,8\%,4)+(P/F,8\%,6)+(P/F,8\%,8)$
$+(P/F,8\%,10)](A/P,8\%,12)$
$= -30,000(0.13270) + 8000(0.05270) – 33,000$
$-5000[1+(0.8573)+(0.7350)+(0.6302)+(0.5403)+(0.4632)](0.13270)$
$= \$-39,363$

$AW_{Walkies} = -(2)2,000(A/P,8\%,4) – 2(150) – 55,000$
$- 800(10)[1+(P/F,8\%,2)](A/P,8\%,4)$
$= -4000(0.30192) – 55,300 – 8000[1+(0.8573)](0.30192)$
$= \$-60,994$

Select forklift

(b) Functions shown. Select forklift.

	A	B	C
1	**Year**	**Forklift**	**Walkies**
2	0	= -30000-5000	=-4000 -8000
3	1	= -32000-1000	=-55000-300
4	2	= -32000-1000-5000	=-55000-300-8000
5	3	= -32000-1000	=-55000-300
6	4	= -32000-1000-5000	=-55000-300
7	5	= -32000-1000	
8	6	= -32000-1000-5000	
9	7	= -32000-1000	
10	8	= -32000-1000-5000	
11	9	= -32000-1000	
12	10	= -32000-1000-5000	
13	11	= -32000-1000	
14	12	= -32000-1000+ 8000	
15	AW - Function	=-PMT(8%,12,NPV(8%,B3:B14)+B2)	=-PMT(8%,4,NPV(8%,C3:C6)+C2)
16	**AW - Value, $**	**-39363**	**-60994**

6.24 $AW_Q = -42{,}000(A/P,10\%,2) – 6000$
$= -42{,}000(0.57619) – 6000$
$= \$-30{,}200$

$AW_R = -80{,}000(A/P,10\%,4) – [7000 + 1000(A/G,10\%,4)] + 4000(A/F,10\%,4)$
$= -80{,}000(0.31547) – [7000 + 1000(1.3812)] + 4000(0.21547)$
$= \$-32{,}757$

Select project Q

6.25 $AW_{small} = -1{,}700{,}000(A/P,1\%,120) – 12{,}000 + 170{,}000(A/F,1\%,120)$
$= -1{,}700{,}000(0.01435) – 12{,}000 + 170{,}000(0.00435)$
$= \$-35{,}656$

$AW_{large} = -2{,}100{,}000(A/P,1\%,120) – 8{,}000 + 210{,}000(A/F,1\%,120)$
$= -2{,}100{,}000(0.01435) – 8{,}000 + 210{,}000(0.00435)$
$= \$-37{,}222$

Select small pipeline

Permanent Investments

6.26 AW = P(i) = CC(i)

AW = [1,000,000 + 1,000,000(P/F,10%,3)](0.10)
= [1,000,000 + 1,000,000(0.7513)](0.10)
= $175,130

6.27 Find P in year 29 using (P/F,i%,20) to move back to year 9, and then use (A/F,i%,10) factor to find A

(a) A = [80,000/0.10](P/F,10%,20)(A/F,10%,10)
= [80,000/0.10](0.1486)(0.06275)
= $7459.72 per year

(b) A = [80,000/0.04](P/F,4%,20)(A/F,4%,10)
= [80,000/0.04](0.4564)(0.08329)
= $76,027.11 per year

It takes approximately 10 times more per year at 4% than at 10% per year.

6.28 AW = -300,000(0.10) – 100,000(A/F,10%,5)
= -30,000 – 100,000(0.16380)
= $-46,380

6.29 Find PW in year -1; multiply by i

PW_{-1} = 5,000,000(P/F,10%,1) + 2,000,000(P/F,10%,11) + (100,000/0.10)(P/F,10%,11]
= 5,000,000(0.9091) + 2,000,000(0.3505) + 1,000,000(0.3505)
= $5,597,000

AW = 5,597,000(0.10)
= $559,700 per year

6.30 (a) AW_{50} = 100,000(A/P,5%,50)
= 100,000(0.05478)
= $5478

AW_{∞} = Pi = 100,000(0.05)
= $5,000

Difference is $478

(b) AW_{50} = 100,000(A/P,10%,50)
= 100,000(0.10086)

$= \$10,086$

$AW_{\infty} = Pi = 100,000(0.10)$
$= \$10,000$

Difference is \$86, a lot smaller than at 5% per year

6.31 $AW_{contract} = -1 + 2.5$
$= \$1.5$ (\$1,500,000)

$AW_{license} = -2(A/P,10\%,10) - 0.2 + 1.3$
$= -2(0.16275) + 1.1$
$= \$0.7745$ (\$774,500)

$AW_{in\text{-}house} = -20(0.10) – 4 + 8$
$= \$2.0$ (\$2,000,000)

Select in-house alternative

6.32 (a) Perpetual AW is equal to AW over one life cycle

$AW = -[6000(P/A,8\%,28) + 1000(P/G,8\%,28)](P/F,8\%,2)(A/P,8\%,30)$
$= -[6000(11.0511) + 1000(97.5687)](0.8573)(0.08883)$
$= \$-12,480$

(b) With no function to directly accommodate gradients, list cash flows in cells B2 through B32 and use PMT function to verify.

	A	B	C	D
1	**Year**	**Cost, $**		**AW @ 8%**
2	0	0	**Function**	= -PMT(8%,30,NPV(8%,B3:B32)+B2)
3	1	0	**Value**	**-$12,480**
4	2	0		
5	3	-6000		
6	4	-7000		
7	5	-8000		
8	6	-9000		
9	7	-10000		
10	8	-11000		
11	9	-12000		

6.33 $AW_{brush} = -400,000(A/P,8\%,10) + 50,000(A/F,8\%,10)$
$- [60,000 - 5000(A/G,8\%,10)]$
$= -400,000(0.14903) + 50,000(0.06903) – 60,000 + 5000(3.8713)$

$= \$-96,804$

$AW_{blast} = -400,000(0.08) – 70,000$
$= \$-102,000$

Select brush alternative

Life Cycle Cost

6.34 PW of LCC = -6.6 – 3.5(P/F,7%,1) – 2.5(P/F,7%,2) – 9.1(P/F,7%,3) – 18.6(P/F,7%,4)
- 21.6(P/F,7%,5) - 17(P/A,7%,5)(P/F,7%,5) – 14.2(P/A,7%,10)(P/F,7%,10)
- 2.7(P/A,7%,3)(P/F,7%,17)
= -6.6 – 3.5(0.9346) – 2.5(0.8734) – 9.1(0.8163) – 18.6(0.7629) - 21.6(0.7130)
- 17(4.1002)(0.7130) – 14.2(7.0236)(0.5083) - 2.7(2.6243)(0.3166)
= \$-151,710,860

AW of LCC = -151,710,860(A/P,7%,20)
= -151,710,860(0.09439)
= \$-14,319,988

6.35 PW of LCC = – 2.6(P/F,6%,1) – 2.0(P/F,6%,2) – 7.5(P/F,6%,3) – 10.0(P/F,6%,4)
-6.3(P/F,6%,5) – 1.36(P/A,6%,15)(P/F,6%,5) -3.0(P/F,6%,10)
- 3.7(P/F,6%,18)
= – 2.6(0.9434) – 2.0(0.8900) – 7.5(0.8396) – 10.0(0.7921) -6.3(0.7473)
– 1.36(9.7122)(0.7473) -3.0(0.5584) - 3.7(0.3503)
= \$-36,000,921

AW of LCC = -36,000,921(A/P,6%,20)
= -36,000,921(0.08718)
= \$-3,138,560

6.36 PW of LCC_A = -250,000 – 150,000(P/A,8%,4) – 45,000 – 35,000(P/A,8%,2)
-50,000(P/A,8%,10) – 30,000(P/A,8%,5)
= -250,000 – 150,000(3.3121) – 45,000 – 35,000(1.7833)
-50,000(6.7101) – 30,000(3.9927)
= \$-1,309,517

AW of LCC_A = -1,309,517(A/P,8%,10)
= -1,309,517(0.14903)
= \$-195,157

PW of LCC_B = -10,000 – 45,000 - 30,000(P/A,8%,3) – 80,000(P/A,8%,10)
- 40,000(P/A,8%,10)
= -10,000 – 45,000 - 30,000(2.5771) – 80,000(6.7101) - 40,000(6.7101)
= \$-937,525

AW of LCC_B = -937,525(A/P,8%,10)
= -937,525(0.14903)
= $-139,719

PW of LCC_C = -175,000(P/A,8%,10)
= -175,000(6.7101)
= $-1,174,268

AW of LCC_C = -1,174,268(A/P,8%,10)
= -1,174,268(0.14903)
= $-175,001

Select Alternative B (adapted system)

Spreadsheet Exercises

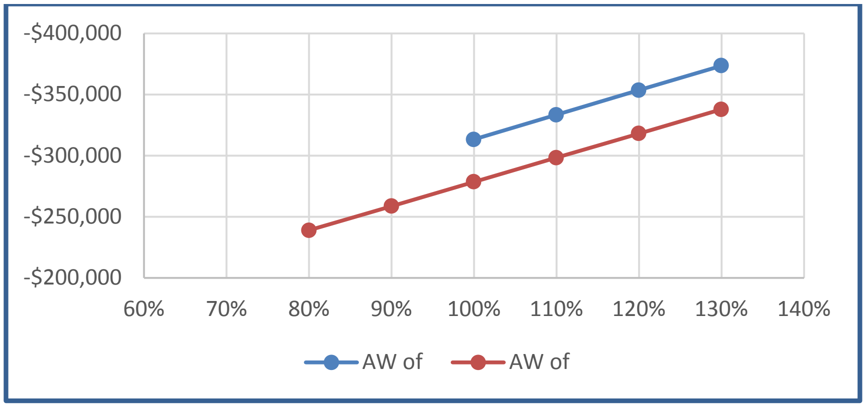
6.37 (a) From the spreadsheet, the difference between the 60-year PW value and the CC value is $-274,578 with CC being the larger amount, as expected.

(b) Google must obtain revenues of the CR amounts each year (column B) plus the annual M&O costs. The CR goes down as *n* increases; but, the M&O is constant, as estimated at $6.3 million per year.

	A	B	C	D
1	**Part (a)**	Function	Amount	
2	AW		-$180,562,733	
3	PW	= - PV(15%,60,C2)	-$1,203,476,978	
4	CC	= C2/0.15	-$1,203,751,557	
5	Difference	= C4-C3	-$274,578	
6				
7	**Part (b)** Capital recovery analysis			
8	Principal is rental plus P of refurbishment			-$1,161,486,558
9	Years, n	CR value	Function for CR	
10	20	-$185,560,800	= -PMT(15%,$A$10,$D$8)	
11	30	-$176,894,633	= -PMT(15%,$A$11,$D$8)	
12	40	-$174,875,838	= -PMT(15%,$A$12,$D$8)	
13	50	-$174,383,905	= -PMT(15%,$A$13,$D$8)	
14	60	-$174,262,733	= -PMT(15%,$A$14,$D$8)	
15	∞	-$174,222,984	= -PMT(15%,10000,$D$8)	

6.38 (a) Spreadsheet details omitted intentionally; resulting chart shown.

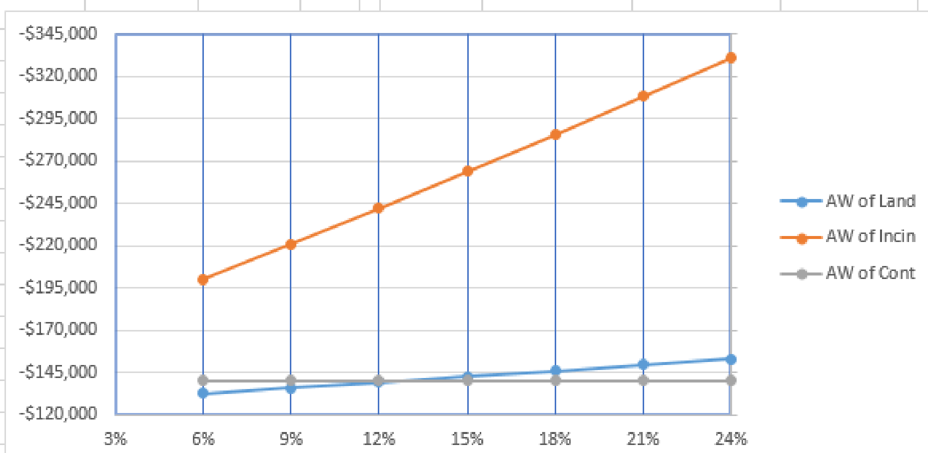
(b) 1. Select Extra-S   2. Select Hi Tone   3. Select Extra-S   4. Select Extra-S



6.39 Spreadsheet functions omitted

Selections: Land application for i = 6%, 9% and 12%
Private disposal contract for i = 15% to 24%

	A	B	C	D	E	F	G	H	I	J
1	**Plan**	**First cost**	**AOC**	**Salvage**	**n**		**Rate, i%**	**AW of Land**	**AW of Incin**	**AW of Cont**
2	**Land**	-150000	-95000	25000	4		6%	-132,574	-200,018	-140,000
3	**Incineration**	-900000	-60000	300000	6		9%	-135,834	-220,752	-140,000
4	**Contract**	0	-140000	0	2		12%	-139,154	-241,935	-140,000
5							15%	-142,533	-263,542	-140,000
6							18%	-145,967	-285,546	-140,000
7							21%	-149,454	-307,922	-140,000
8							24%	-152,991	-330,644	-140,000



6.40 Testing stage: Equipment first cost increases to $-30,000,000; S remains at $600,000
Manufacturing stage: Unit cost to manufacture increases to 5*2.75 = $13.75; now a loss per
unit of $1.00.

Conclusion: AW goes strongly negative to $-5.027 M per year when the manufacturing stage is entered.

E12 = -300000 + (12.75 -2.75*5)*500000 - 0.1*$B$2

	A	B	C	D	E
1	Year	Conceptual stage, $	Production planning stage, $	Testing stage, $	Manufacturing stage, $
2	0	-6,000,000	-6,000,000	-30,000,000	-30,000,000
3	1	4,700,000	4,700,000	4,700,000	-800,000
4	2	4,700,000	4,700,000	4,700,000	-800,000
5	3	4,700,000	4,700,000	4,700,000	-800,000
6	4	4,700,000	4,700,000	4,700,000	-800,000
7	5	4,700,000	4,700,000	4,700,000	-800,000
8	6	4,700,000	4,700,000	4,700,000	-800,000
9	7	4,700,000	4,700,000	4,700,000	-800,000
10	8	4,700,000	4,700,000	4,700,000	-800,000
11	9	4,700,000	4,700,000	4,700,000	-800,000
12	10	5,300,000	5,300,000	5,300,000	-200,000
13					
14	PMT function	= -PMT(7%,10, NPV(7%,B3:B12)+ B2)	= -PMT(7%,10, NPV(7%,C3:C12)+ C2)	= -PMT(7%,10, NPV(7%,D3:D12)+ D2)	= -PMT(7%,10, NPV(7%,E3:E12)+ E2)
15	AW, $/yr	3,889,161	3,889,161	472,101	-5,027,899

Additional Problems and FE Exam Review Questions

6.41 Answer is (b)

6.42 Answer is (a)

6.43 Answer is (b)

6.44 AW = -84,000(A/P,8%,10) – 13,000 + 9000(A/F,8%,10)
= -84,000(0.14903) – 13,000 + 9000(0.06903)
= $-24,897

Answer is (c)

6.45 Answer is (d)

6.46 Answer is (a)

6.47 Find PW in year 0 and then multiply by i

PW_0 = 50,000 + 10,000(P/A,10%,15) + (20,000/0.10)(P/F,10%,15)
= 50,000 + 10,000(7.6061) + (20,000/0.10)(0.2394)
= $173,941

AW = 173,941(0.10)
= $17,394

Answer is (c)

6.48 A = [40,000/0.08](P/F,8%,2)(A/F,8%,3)
= [40,000/0.08](0.8573)(0.30803)
= $132,037
Answer is (d)

6.49 A = [50,000/0.10](P/F,10%,20)(A/F,10%,10)
= [50,000/0.10](0.1486)(0.06275)
= $4662
Answer is (b)

6.50 Answer is (a)

6.51 Answer is (d)

6.52 Answer is (b)

6.53 Answer is (b)

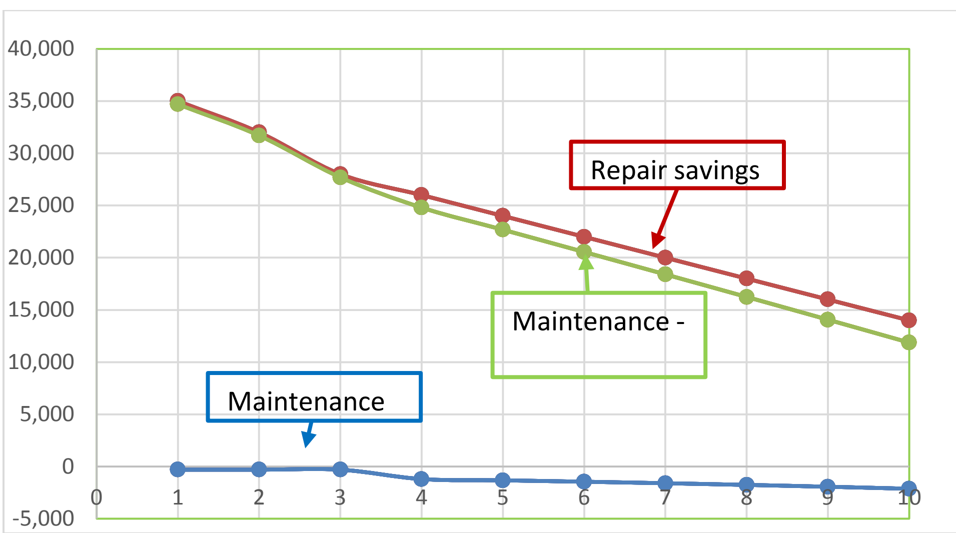
6.54 AW = -800,000(0.10) – 10,000
= $-90,000
Answer is (c)

Solution to Case Study, Chapter 6

ANNUAL WORTH ANALYSIS – THEN AND NOW

1. Spreadsheet and graph are below. Revised costs and savings are in columns F-H. Maintenance costs increase; repair savings decrease.

	A	B	C	D	E	F	G	H
1	MARR =	15%						
2								
3			PowrUp			Lloyd's with changes		
4		Investment	Annual	Repair	Investment	Annual	Repair	Maintenance -
5	Year	and salvage	maintenance	savings	and salvage	maintenance	savings	repair savings
6	0	-26,000	0	0	-36,000	0	0	
7	1	0	-800	25,000	0	-300	35,000	34,700
8	2	0	-800	25,000	0	-300	32,000	31,700
9	3	0	-800	25,000	0	-300	28,000	27,700
10	4	0	-800	25,000	0	-1,200	26,000	24,800
11	5	0	-800	25,000	0	-1,320	24,000	22,680
12	6	2,000	-800	25,000	0	-1,452	22,000	20,548
13	7				0	-1,597	20,000	18,403
14	8				0	-1,757	18,000	16,243
15	9				0	-1,933	16,000	14,067
16	10				0	-2,126	14,000	11,874
17	AW element	-6,642	-800	25,000	-7,173	-977	26,055	
18	**Total AW**			**$17,558**			**$17,904**	



2. In cell G18, the new AW = $17,904. This is only slightly larger than the PowrUp AW = $17,558. Lloyd's would have been selected, but only by a small margin.

3. New CR is $7173 per year (cell E17), an increase from $7025 previously determined when the salvage estimate was $3000 after 10 years.

Solutions to end-of-chapter problems
Engineering Economy, 8th edition
Leland Blank and Anthony Tarquin

Chapter 7
Rate of Return Analysis: One Project

Understanding ROR

7.1 (a) Highest possible is infinity
(b) Lowest possible is -100%

7.2 Total amount owed = principal x interest of 10% per year on principal for 5 years
= P + Pni
= 10,000(1 + 5*0.1)
= 10,000(1.50)
= $15,000

Loan balance = Total amount owed – total amount paid
= 15,000 – 5(2638)
= $1810

7.3 (a) Annual payment = principal/# of periods + interest per year
= 10,000/4 + 10,000(0.10)
= $3500

(b) A = 10,000(A/P,10%,4)
= 10,000(0.31547)
= $3154.70

(c) Difference = 3500 – 3154.70
= $345.30

$345.30 more is required to repay the loan based on the original principal.

7.4 (a) Unrecovered balance of *principal before* payment, year 1 = $60,000,000

Interest on unrecovered balance, year 1 = 60,000,000(0.08) = $4,800,000

(b) Annual payment = 60,000,000(A/P,8%,5)
= 60,000,000(0.25046)
= $15,027,600

Recovered amount, year 1 = payment – interest due
= 15,027,600 – 4,800,000
= $10,227,600

Unrecovered balance of *principal after* payment, year 1 = 60,000,000 – 10,227,600
= $49,772,400

7.5 Monthly payment = 100,000(A/P,0.5%,360)
= 100,000(0.00600)
= $600

Balloon payment = 100,000(F/P,0.5%,60) – 600(F/A,0.5%,60)
= 100,000(1.3489) – 600(69.7700)
= $93,028

7.6 Factors: Annual payment = 6,000,000(A/P,10%,10)
= 6,000,000(0.16275)
= $976,500

Principal remaining after year 1 = Principal(1 + interest rate) – annual payment
= 6,000,000(1.10) – 976,500
= $5,623,500

Interest, year 2 = 5,623,500(0.10)
= $562,350

Spreadsheet function: = - FV(10%,1,-976500,6000000)* 0.1 displays $562,350

Determination of ROR

7.7 (a) Factors and interpolation:

0 = -650,000 + 225,000(P/A,i*,10) + 70,000(P/F,i*,10)

Try 30%: - 650,000 + 225,000(3.0915) + 70,000(0.0725) = $50,663 > 0 too low
Try 35%: - 650,000 + 225,000(2.7150) + 70,000(0.0497) = $-35,646 < 0 too high

Interpolation yields i* = 32.9% per year

(b) Spreadsheet: If cash flows are entered into cells B1:B11
= IRR(B1:B11) displays 32.8%

If RATE is used, a 'guess' is necessary. For example

= RATE(10,225000,-650000,70000,,20%) displays 32.8%

7.8 Interpolation:

Try 5%: - 40,000 + 8000(4.3295) + 8000(0.6768) = \$50.40 > 0 too low

Try 6%: - 40,000 + 8000(4.2124) + 8000(0.6274) = \$-1281.60 < 0 too high

Interpolation yields i* = 5.04% per period

Spreadsheet: Must use IRR function; enter cash flows into cells B1:B9.
Function: = IRR(B1:B9) displays i* = 5.04%

7.9 Factors: Move all cash flows to year 1

0 = -80,000 + 9000(P/F,i*,1) + 70,000(P/F,i*,2) + 30,000(P/F,*i,3)

By trial and error, i* = 15.32%

Spreadsheet: Enter amounts for years 0 to 4 in cells B2:B6 or years 1 to 4 in cells B3:B6
= IRR(B2:B6) displays i* = 15.32%
or = IRR(B3:B6) displays i* = 15.32%

7.10 0 = -50,000(8) + [(10(2500) + 25(650) +70(1200)](P/A,i*%,4)
125,250(P/A,i*%,4) = 400,000
(P/A,i*%,4) = 3.1936

Solve for i* by interpolation in interest tables or spreadsheet

i* = 9.6% per year (spreadsheet)

7.11 0 = -4000 – 300(P/A,i*,4)(P/F,i*,1) - 100(P/A,i*,3)(P/F,i*,5) + 12,000(P/F,i*,8)

Solve for i* by trial and error or spreadsheet

i* = 11.7% (spreadsheet)

7.12 (a) 0 = -650,000 + 105,000(P/A,i*,5) + 50,000(P/F,i*,5)

Since 5(105,000) + 50,000 = 575,000 < 650,000, i* < 0

Use the RATE spreadsheet function to find i*

= RATE(5,105000,-650000,50000) displays i* = -3.74% per year

(b) Let A = annual savings. Solve by factor or Goal Seek

$0 = -650{,}000 + A(P/A,15\%,5) + 50{,}000(P/F,15\%,5)$

$A = \$186{,}490$ (Goal Seek)

7.13 There are no tables for negative interest rates. Write the equation for i < 0% values

$$F = P(1 - i)^n$$
$$10 = 33(1 - i)^7$$
$$0.303030 = (1 - i)^7$$
$$0.303030^{1/7} = (1 - i)$$

$$i = -15.7\% \text{ per year}$$

7.14 $0 = -150{,}000 + (33{,}000 - 27{,}000)(P/A,i^*,30)$

$(P/A,i^*,30) = 25.0000$

$i^* = 1.2\%$ per month (interpolation or spreadsheet)

7.15 $0 = -30{,}000 + (27{,}000 - 18{,}000)(P/A,i^*,5) + 4000(P/F,i^*,5)$

Solve by trial and error or spreadsheet

$i^* = 17.9\ \%$

7.16 $0 = 2{,}000{,}000 - 200{,}000(P/A,i^*,2) - 2{,}200{,}000(P/F,i^*,3)$

Solve for $i^* = 10\%$ per year (spreadsheet)

7.17 Factors: $0 = -130{,}000 - 49{,}000(P/A,i^*,8) + 78{,}000(P/A,i^*,8) + 1000(P/G,i^*,8) + 23{,}000(P/F,i\%,8)$

Solve by trial and error

$i^* = 19.2\%$

Spreadsheet: Entries in cells B2:B10 and the function = IRR(B2:B10) display i* = 19.2%

Year	Cash flow, $
0	-130,000
1	29,000
2	30,000
3	31,000
4	32,000
5	33,000
6	34,000
7	35,000
8	59,000
i* via IRR	19.2%

7.18 (a) In $ million units

Factors: $0 = -4.97 + 1.3(P/A,i^*,10)$

$(P/A,i^*,10) = 3.8231$

Interpolation between 22% and 24% yields i* = 22.83%

Spreadsheet: = RATE(10,1.3,-4.97) displays i* = 22.80%

(b) Cost of guardrail = 72,000(113) = $8.136 million

Factors: $0 = -8.136 + 1.1(P/A,i^*,10)$

$(P/A,i^*,10) = 7.39636$

From interest tables, i* is between 5% and 6%

Spreadsheet: = RATE(10,1100000,-72000*113) displays i* = 5.9%

7.19 $Pi^* = A$

$(100{,}000 - 10{,}000)i^* = 10{,}000$

$i^* = 11.1\%$

7.20 $0 = -110{,}000 + 4800(P/A,i\%,60)$

$(P/A,i^*\%,60) = 22.9167$

Use interpolation or spreadsheet

i* = 3.93% per month

7.21 $0 = -210 - 150(P/F,i^*,1) + [100(P/A,i^*,4) + 60(P/G,i^*,4)](P/F,i^*,1)$

Solve by trial and error or spreadsheet

i* = 24.7% per year

Spreadsheet sample on right

Year	Cash flow, $1000
0	-210
1	-150
2	100
3	160
4	220
5	280
i* via IRR	24.74%

7.22 $0 = -950{,}000 + [450{,}000(P/A,i^*,5) + 50{,}000(P/G,i^*,5)](P/F,i^*,10)$

Solve for i* = 8.45% per year (spreadsheet)

Multiple ROR Values

7.23 Reinvestment rate assumes that any net positive cash inflows to a project make a return of a stated percentage during the next period of time. (1) The PW method assumes the stated percentage is the MARR. (2) The IROR assumes the return is always at the calculated i* rate, no matter what the MARR might be.

7.24 Descartes' rule uses *net cash flows* while Norstrom's criterion is based on *cumulative cash flows*.

7.25 (a) conventional (b) non-conventional (c) conventional (d) non-conventional (e) conventional

7.26 (a) conventional (b) conventional (c) conventional (d) non-conventional (e) non-conventional

7.27 (a) Net cash flow: $-1500, $260, $250, $375, $90, and $230
Cum. cash flow: $-1500, $-1240, $-990, $-615, $-525, and $-295
Conventional; Descartes rule: 1; Norstrom's criterion: inconclusive

(b) Net cash flow: $-1500, $-100, $-10, $60, $25, and $-80
Cum cash flow: $-1500, $-1600, $-1610, $-1550, $-1525, and $-1605
Non-conventional; Descartes rule: 2; Norstrom's criterion: inconclusive

(c) Net cash flow: $1500, $-100, $-10, $-40, $-50, and $-80
Cum cash flow: $1500, $-1400, $1390, $1350, $1300, and $1220
Conventional; Descartes rule: 1; Norstrom's criterion: $S_0 > 0$, does not apply

(d) Net cash flow: $-1500, $-100, $-10, $60, $25, and $10
Cum cash flow: $-1500, $-1600, $-1610, $-1550, $-1525, and $-1515
Conventional; Descartes rule: 1; Norstrom's criterion: inconclusive

7.28 (a) four; (b) one; (c) five; (d) two

7.29 Entering net cash flows into cells B2:B5 results in = IRR(B2:B5), which displays i* = 166.0%

Year	Net cash flow, $	Cumulative cash flow, $
1	-5000	-5000
2	+6000	+1000
3	-2000	-1000
4	+58,000	+57,000

7.30 Tabulate net cash flows and cumulative cash flows.

Quarter	Expenses	Revenue	Net Cash Flow	Cumulative
0	-20	0	-20	-20
1	-20	5	-15	-35
2	-10	10	0	-35
3	-10	25	15	-20
4	-10	26	16	-4
5	-10	20	10	+6
6	-15	17	2	+8
7	-12	15	3	+11
8	-15	2	-13	-2

(a) From net cash flow column, there are two possible i* values
(b) Cumulative cash flow sign starts negative ($S_0 < 0$), and signs change twice. Thus, there is no assurance of a single, positive i* value.
(c) Since the sum of the cumulative cash flows is negative, there is no positive rate of return value.

7.31 (a) Two sign changes; maximum number of i* values is two.

(b) Cumulative cash flow series changes sign once. There is one, nonnegative i* value.

Year	Net Cash Flow, $	Cumulative Cash Flow, $
0	-40,000	-40,000
1	32,000	-8000
2	18,000	+10,000
3	-2000	+8000
4	-1000	+7000

(c) 0 = -40,000 + 32,000(P/F,i*,1) + 18,000(P/F,i*,2) – 2000(P/F,i*,3) – 1000(P/F,i*,4)

= IRR(A1:A5) displays 13.95% for entries in cells A1:A5

7.32 Rule of signs: three possible i* values.
Cumulative cash flow signs: one, positive i* value.

Write PW rate of return equation (in $1000) to find i*

$$0 = -50 + 22(P/F,i^*,1) + 38(P/F,i^*,2) - 2(P/F,i^*,3) - 1(P/F,i^*,4) + 5(P/F,i^*,5)$$

Solve for i* by trial and error or spreadsheet
i* = 12.93% (spreadsheet)

7.33 (a) Two sign changes in the NCF series; maximum of two real-number i* values. Norstrom's criterion indicates one positive value.

(b) 0 = -30,000 + 20,000(P/F,i*,1) + 15,000(P/F,i*,2) - 2000(P/F,i*,3)

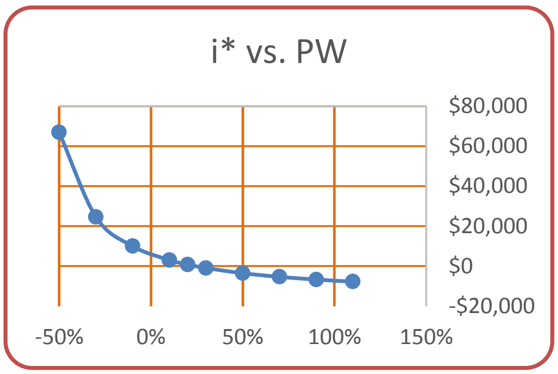
i* = 7.43% per year (spreadsheet)

7.34 (a) Three sign changes in NCF series; three possible i* values. Norstrom's criterion also indicates that there may be more than one i* value.

(b) IRR equation is: 0 = -17,000 + 20,000(P/F,i*,1) - 5000(P/F,i*,2) + 8000(P/F,i*,3)

i* = 24.4% per year

Plot of PW versus i* crosses x-axis only once at i* = 24.4%



7.35 Calculate net cash flows and cumulative cash flows.

Year	Expenses, $	Savings, $	Net Cash Flow, $	Cumulative CF, $
0	-33,000	0	-33,000	-33,000
1	-15,000	18,000	+3,000	-30,000
2	-40,000	38,000	-2000	-32,000
3	-20,000	55,000	+35,000	+3000
4	-13,000	12,000	-1000	+2000

(a) Four sign changes in net cash flow; up to four i* values.
Cumulative cash flow starts negative and changes only once; one positive i*

(b) 0 = -33,000 + 3000(P/F,i*,1) - 2000(P/F,i*,2) + 35,000(P/F,i*,3) -1000(P/F,i*,4)

i* = 2.1% per year (spreadsheet)

7.36 (a) One sign change in NCF and cumulative CF indicates only one i*.

(b) 0 = -5000 - 10,100(P/F,i*,1) + [4500(P/A,i*,5) + 2000(P/G,i*,5)](P/F,i*,1)

i* = 33.7% per year (spreadsheet)

(c) Use ROR equation with i* = 15% and solve for G. Alternatively, enter NCF values onto a spreadsheet and use Goal Seek to find the lowest G value to make i* = 15%

0 = -5000 -10,100(P/F,15%,1) + [4500(P/A,15%,5) + G(P/G,15%,5)](P/F,15%,1)
= -5000 -10,100(0.8696) + [4500(3.3522) + G(5.7751)](0.8696)
5.022G = 665.131
G = $132.44

Spreadsheet: Goal Seek results in G = $132.72

It is possible to have a gradient as low as $133 and still realize a 15% return.

Removing Multiple i* Values

7.37 (a) Rule of signs test: up to 4 values
Cumulative CF sign test: inconclusive since $S_0 > 0$

(b) PW_0 = -2000(P/F,10%,2) - 7000(P/F,10%,3) - 700(P/F,10%,5)
= -2000(0.8264) - 7000(0.7513) - 700(0.6209)
= $-7347

FW_6 = 4100(F/P,20%,5) + 12,000(F/P,20%,2) + 800
= 4100(2.4883) + 12,000(1.4400) + 800
= $28,282

-7347(F/P,i',6) + 28,282 = 0
$-7347(1 + i')^6 + 28,282 = 0$

$i' = (28,282/7347)^{1/6} – 1$
= 0.252 (25.2%)

(c) Enter the NCF values in cells B1 (as a 0) through B7. The function = MIRR(B1:B7,10%,20%) displays i'= 25.2%

7.38 First find net cash flow (NCF)

Year	0	1	2	3	4
Revenue, $	0	25,000	19,000	4,000	28,000
Costs, $	-6,000	-30,000	-7,000	-6,000	-12,000
NCF, $	-6000	-5000	12,000	-2000	16,000

(a) Rule of signs test: up to 3 values
Cumulative CF sign test: 3 changes; no unique, positive i*

(b) Function: = IRR(B1:B5) displays i* = 39.9%

(c) $PW_0 = -6000 - 5000(P/F,10\%,1) - 2000(P/F,10\%,3)$
$= -6000 - 5000(0.9091) - 2000(0.7513)$
$= \$-12,048$

$FW_4 = 12,000(F/P,18\%,2) + 16,000$
$= 12,000(1.3924) + 16,000$
$= \$32,709$

$-12,048(F/P,i',4) + 32,709 = 0$
$-12,048(1 + i')^4 + 32,709 = 0$

$i' = (32,709/12,048)^{1/4} – 1$
$= 0.284 \quad (28.4\%)$

7.39 (a) Cash flow rule of signs: up to three rate of return values.
Cumulative CF test: inconclusive; $S_0 > 0$; series changes signs multiple times

(b) Calculate i" with $i_i = 15\%$ per year

$F_0 = 48,000$	$F_0 > 0$, use i_i
$F_1 = 48,000(1.15) + 20,000 = 75,200$	$F_1 > 0$, use i_i
$F_2 = 75,200(1.15) - 90,000 = -3520$	$F_2 < 0$, use i''
$F_3 = -3520(1 + i'') + 50,000 = 46,480 -3520i''$	$F_3 > 0$ for $i'' < 1320\%$; use i_i
$F_4 = (46,480 – 3520i'')(1.15) -10,000$	
$= 43,452 – 4048i''$	

Set $F_4 = 0$ and solve for i''

$0 = 43,452 – 4048i''$
$i'' = 10.73 \qquad (1073\%)$

(c) With NCF values in cells B2 through B6, both functions = IRR(B2:B6) and = IRR(B2:B6,1073%) display i* = -26.8%

7.40 (a) ROIC method with $i_i = 30\%$ per year

$F_0 = 2000$ $\quad F_0 > 0$; use i_i
$F_1 = 2000(1.30) + 1200$
$= 3800$ $\quad F_1 > 0$; use i_i
$F_2 = 3800(1.30) - 4000$
$= 940$ $\quad F_2 > 0$; use i_i
$F_3 = 940(1.30) - 3000$
$= -1778$ $\quad F_3 < 0$; use i''
$F_4 = -1778(1 + i'') + 2000$

Set $F_4 = 0$ and solve for i''

$0 = -1778(1 + i'') + 2000$
$i'' = 222/1778$
$= 0.1249$ (12.49%)

(b) MIRR method with $i_i = 30\%$ per year and $i_b = 10\%$ per year

$PW_0 = -4000(P/F,i_b,2) – 3000(P/F,i_b,3)$
$= -4000(P/F,10\%,2) – 3000(P/F,10\%,3)$
$= -4000(0.8264) – 3000(0.7513)$
$= \$-5560$

$FW_4 = 2000(F/P,i_i,4) + 1200(F/P,i_i,3) + 2000$
$= 2000(F/P,30\%,4) + 1200(F/P,30\%,3) + 2000$
$= 2000(2.8561) + 1200(2.1970) + 2000$
$= \$10,349$

Find EROR i' at which PW_0 is equivalent to FW_4

$PW_0(F/P,i',4) + FW_4 = 0$
$-5560(1 + i')^4 + 10,349 = 0$
$(1 + i')^4 = 1.8613$
$i' = 0.168$ (16.8%)

7.41 Use MIRR function or hand solution.

(a) By hand, with $i_b = 10\%$ and $i_i = 15\%$

$PW_0 = -7000 – 5000((P/F,10\%,3)$
$= -7000 – 5000((0.7513)$
$= \$-10,757$

$FW_3 = 3000(F/P,15\%,2) + 15{,}000(F/P,15\%,1)$
$= 3000(1.3225) + 15{,}000(1.1500)$
$= \$21{,}218$

Find i′ at which PW_0 is equivalent to FW_3

$-10{,}757(F/P,i',3) = 21{,}218$

$i' = (21{,}218/10{,}757)^{1/3} - 1$
$= 0.254$ (25.4% per year)

(b) By MIRR with $i_b = 10\%$ and $i_i = 30\%$

$i' = 31.7\%$ per year

7.42 $PW_0 = -50{,}000 - 8000(P/F,12\%,7)$
$= -50{,}000 - 8000(0.4523)$
$= \$-53{,}618$

$FW_7 = 15{,}000(F/A,25\%,6)(F/P,25\%,1)$
$= 15{,}000(11.2588)(1.2500)$
$= \$211{,}103$

Find EROR i′ at which PW_0 is equivalent to FW_7

$-53{,}618(F/P,i',7) = 211{,}103$
$i' = (211{,}103/53{,}618)^{1/7} - 1$
$= (3.93717)^{1/7} - 1$
$= 0.216$ (21.6%)

Function: = MIRR(B1:B8,12%,25%) displays 21.6% with B1:B8 containing NCF

7.43 (a) Descartes' rule of signs: 2 sign changes; up to two i* values
Norstrom's criterion: series starts negative; 1 sign change; one positive root

(b) $0 = -65 + 30(P/F,i^*,1) + 84(P/F,i^*,2) - 10(P/F,i^*,3) - 12(P/F,i^*,4)$

Solve for i* by trial and error or spreadsheet

$i^* = 28.6\%$ per year (spreadsheet)

A negative root of -56.0% is discarded. It can be displayed using = IRR(B1:B5,-50%)

(c) Apply net-investment procedure steps because the investment rate $i_i = 15\%$ is not equal to i* rate of 28.6% per year.

Hand solution:

Step 1: $F_0 = -65$ $\quad$ $F_0 < 0$; use i″
$F_1 = -65(1 + i'') + 30$ $\quad$ $F_1 < 0$; use i″
$F_2 = F_1(1 + i'') + 84$ $\quad$ $F_2 > 0$; use i_i (F_2 must be > 0, because last two terms are negative)
$F_3 = F_2(1 + 0.15) - 10$ $\quad$ $F_3 > 0$; use i_i (F_3 must be > 0, because last term is negative)
$F_4 = F_3(1 + 0.15) - 12$

Step 2: Set $F_4 = 0$ and solve for i″ by trial and error.

$$F_1 = -65 - 65i'' + 30$$

$$\begin{aligned} F_2 &= (-65 - 65i'' + 30)(1 + i'') + 84 \\ &= -65 - 65i'' + 30 - 65i'' - 65i''^2 + 30i'' + 84 \\ &= -65i''^2 - 100i'' + 49 \end{aligned}$$

$$\begin{aligned} F_3 &= (-65i''^2 - 100i'' + 49)(1.15) - 10 \\ &= -74.8\, i''^2 - 115i'' + 56.4 - 10 \\ &= -74.8\, i''^2 - 115i'' + 46.4 \end{aligned}$$

$$\begin{aligned} F_4 &= (-74.8\, i''^2 - 115i'' + 46.4)(1.15) - 12 \\ &= -86\, i''^2 - 132.3i'' + 53.3 - 12 \\ &= -86\, i''^2 - 132.3i'' + 41.3 \end{aligned}$$

Solve by quadratic equation, trial and error, or spreadsheet

i″ = 26.6% per year $\quad$ (spreadsheet)

Spreadsheet solution: Using the format and functions of Figure 7-13, i″ = 26.62%.

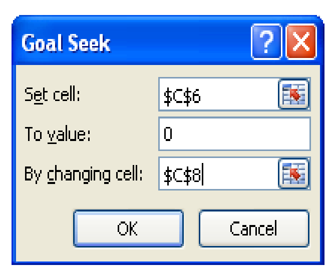
Before Goal Seek

	A	B	C
1	Year	NCF, $	Future worth value, F, $
2	0	-65	-65
3	1	30	-35
4	2	84	49
5	3	-10	46
6	4	-12	41
7	Investment rate		15.00%
8	Result, ROIC		**0.00%**

After Goal Seek

	A	B	C
1	Year	NCF, $	Future worth value, F, $
2	0	-65	-65
3	1	30	-52
4	2	84	18
5	3	-10	10
6	4	-12	0
7	Investment rate		15.00%
8	Result, ROIC		**26.62%**
9			

Goal Seek template



Bonds

7.44 $I = 10{,}000(0.08)/2$
$= \$400$ every 6 months

7.45 $75 = 5000(b)/4$
$b = 6\%$ per year, payable quarterly

7.46 $0 = -8200 + 10{,}000(0.08)(P/A,i^*,5) + 10{,}000(P/F,i^*,5)$

Solve by trial and error or IRR function

$i^* = 13.1\%$ per year (spreadsheet)

7.47 $0 = -9250 + 50{,}000(P/F,i^*,18)$
$(P/F,i^*,18) = 0.1850$

Solve directly or use spreadsheet

$i^* = 9.83\%$ per year (spreadsheet)

7.48 (a) Dividend $= 1000(0.05)/2 = \$25$ per 6 months

$0 = -925 + 25(P/A,i^*,16) + 800(P/F,i^*,16)$

Solve by trial and error or IRR function

$i^* = 1.98\%$ per 6 months (spreadsheet)

(b) Nominal rate $= 1.98*2 = 3.96\%$ per year

7.49 $i^* = 5000(0.10)/2$
$= \$250$ per six months

$0 = -5000 + 250(P/A,i^*,8) + 5{,}500(P/F,i^*,8)$

Solve for i* by trial and error or spreadsheet

$i^* = 6.0\%$ per six months (spreadsheet)

7.50 $0 = -60{,}000 + 50{,}000(0.14)(P/A,i^*,5) + 50{,}000(P/F,i^*,5)$

Solve by trial and error or IRR function

$i^* = 8.9\%$ per year (spreadsheet)

Function: = RATE(5,7000,-60000,50000) displays i* = 8.9% per year

7.51 I = 10,000(0.08)/4 = $200 per quarter

0 = -9200 + 200(P/A,i*,28) + 10,000(P/F,i*,28)

Solve for i* by trial and error or spreadsheet RATE function

i* = 2.4% per quarter (RATE)

Nominal i* per year = 2.4(4) = 9.6% per year

7.52 I = 10,000(0.08)/4 = $200 per quarter

(a) 0 = -6000 + 200(P/A,i*,12) + 11,500(P/F,i*,12)

Solve for i* by trial and error or spreadsheet

i* = 8.13% per quarter (RATE function)

(b) Nominal i*/year = 8.13(4) = 32.5% per year

7.53 The utility would pay a penalty of $2,000,000 in return for saving 4% per year, payable Semiannually on $20 million for 15 years.

0 = -2,000,000 + [20,000,000(0.04)/2](P/A,i*,30)

Solve for i by trial and error or spreadsheet

i* = 19.9% per six months (spreadsheet)

Spreadsheet Exercises

7.54 (a) Balance on loan after payment 4 = $5000
Payment, A = -5000(A/P,3%,5) = -5000(0.21835) = $-1091.75 per month

(b) Total paid = 20% down + no-interest payments + interest-bearing payments
= 2000 + 3000 + 5(1091.75)
= $10,458.86

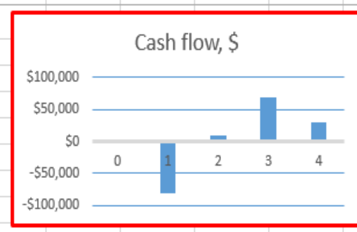
	A	B	C	D	E	F	G	H
1	APR rate		36%		eff i/month	3%		
2								
3	Month	Expected payment	Payment made	New month count	Loan balance	New payment	Total amount to pay	
4	0				8000			
5	1	-800	-800		7200			
6	2	-800	-800		6400			
7	3	-800	-800		5600			
8	4	-800	-600	0	5000	0.00	= PMT(3%,5,$E$8)	
9	5	-800		1		-1,091.77		
10	6	-800		2		-1,091.77		
11	7	-800		3		-1,091.77		
12	8	-800		4		-1,091.77		
13	9	-800		5		-1,091.77		
14	10	-800						
15								
16								
17	Totals	-8000	-3000			-5,458.86	-$8,458.86	

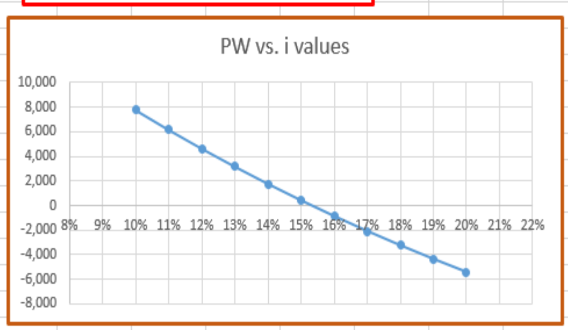
7.55 (a) Enter cash flows for years 0 to 4 in cells B2:B6

Function = IRR(B2:B6) displays i* = 15.32%

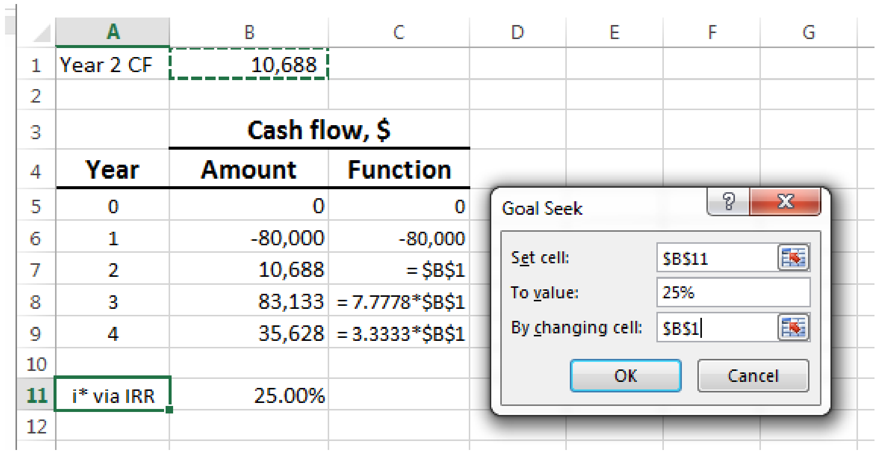
(b) Two charts are included in spreadsheet with i* indicated between 15% and 16% on the second (scatter) chart

	A	B	C	D	E
1	Year	Cash flow, $		Trial i, %	PW, $
2	0	0		10%	7,793
3	1	-80,000		11%	6,178
4	2	9000		12%	4,636
5	3	70000		13%	3,165
6	4	30000		14%	1,760
7				15%	419
8	i* via IRR	15.32%		16%	-862
9				17%	-2,086
10				18%	-3,255
11				19%	-4,372
12				20%	-5,440





(c) Use Goal Seek tool to change \$9000 estimate in year 1 to \$10,688 with multiples shown for years 3 and 4 estimates. Amounts needed are in cells B7 to B9.

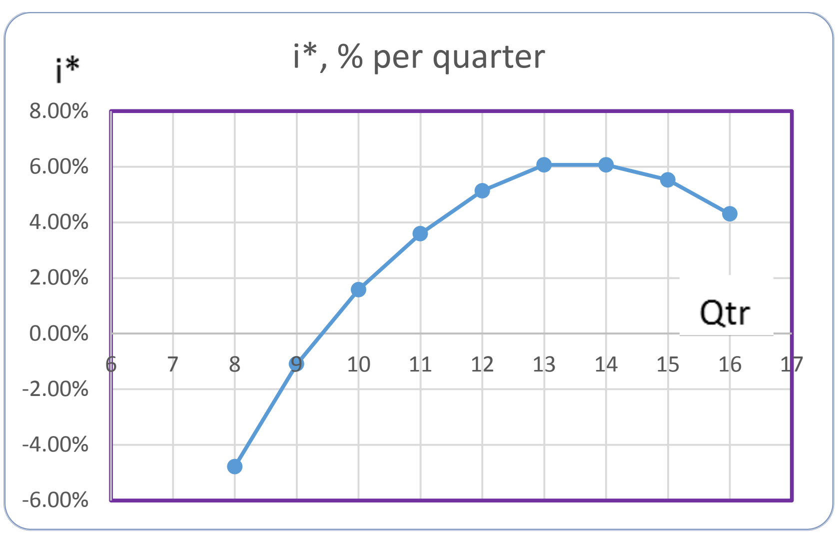


	A	B	C
1	Year 2 CF	10,688	
2			
3		Cash flow, $	
4	Year	Amount	Function
5	0	0	0
6	1	-80,000	-80,000
7	2	10,688	= $B$1
8	3	83,133	= 7.7778*$B$1
9	4	35,628	= 3.3333*$B$1
10			
11	i* via IRR	25.00%	
12			

7.56 (a) Spreadsheet table not shown intentionally; i* per quarter series is:

Quarter	8	9	10	11	12	13	14	15	16
i* value, %/qtr	-4.79%	-1.10%	1.58%	3.59%	5.13%	6.06%	6.06%	5.52%	4.30%

(b) Plot of i* per quarter vs. quarters 8 through 16



(c) A required 24% per year or 6% per quarter (nominal) indicates the equipment should be used for a period of 13 to 15 quarters (over 3 years; less than 4 years)

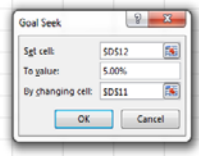
7.57 (a) i* = -2.71% per quarter (see spreadsheet)

(b) Does not meet MARR of 5% per quarter

(c) In round dollar amounts, NCF = $-5 is required for i* = 5%. Revenues must be $10 in quarter 8 to meet MARR.

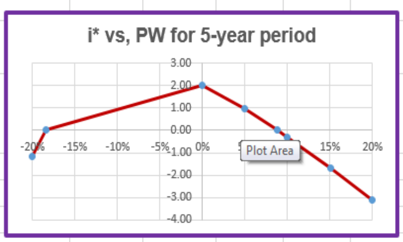
Quarter	Expenses, $	Revenues, $	Net cash flows, $
0	-20	0	-20
1	-20	5	-15
2	-10	10	0
3	-10	25	15
4	-10	26	16
5	-10	20	10
6	-15	17	2
7	-12	15	3
8	-15	2	-13
i* via IRR			-2.71%

Quarter	Expenses, $	Revenues, $	Net cash flows, $
0	-20	0	-20
1	-20	5	-15
2	-10	10	0
3	-10	25	15
4	-10	26	16
5	-10	20	10
6	-15	17	2
7	-12	15	3
8	-15	2	-5
i* via IRR			5.00%



7.58 (a) $i_1^* = -18.38\%$ and $i_2^* = 8.79\%$
(b) i* = 15.03%
(c) Yes for 0 to 5-year period; no for 0 to 10-year period
(d) Plots are shown

Year	NCF, $ M	CNCF, $ M							Rate, i*%	PW, $ M Years 0-5	PW, $ M Years 0-10
0	-40	-40	(a) i* values, years 0-5						-20.00%	-1.17	185.61
1	32	-8	Guess	-99%	-50%	0%	50%	100%	-18.38%	0.00	151.18
2	18	10	i*	-18.38%	-18.38%	8.79%	8.79%	8.79%	0.00%	2.00	17.00
3	10	20							5.00%	0.95	8.79
4	-10	10	(b) i* values, years 0-10						8.79%	0.00	4.65
5	-8	2	Guess	-100%	-50%	0%	50%	100%	10.00%	-0.32	3.57
6	-15	-13	i*	15.03%	15.03%	15.03%	15.03%	15.03%	15.03%	-1.69	0.00
7	0	-13							20.00%	-3.08	-2.58
8	5	-8								Red	Green
9	10	2	(d) Plots of i* vs. PW, years 0-5 and years 0-10								
10	15	17									
Changes, Years 0-5	2	1									
Changes, Years 0-10	3	3									



7.59 ROIC analysis results in i" = 11.26%, same as the IRR result. The MARR of 10% is being met and the company is effectively using the funds invested in it.

	A	B	C	D	E	F	G
1	Year, t	NCF, $1000	CCF, $1000		F_t, $1000		
2	0	-12,000	-12,000		-12,000		
3	1	4,000	-8,000		-9,351		
4	2	-3,000	-11,000		-13,404		
5	3	-7,000	-18,000		-21,913		
6	4	15,000	-3,000		-9,380		
7	5	1,000	-2,000		-9,436		
8	6	4,000	2,000		-6,498		
9	7	-2,000	0		-9,230		
10	8	-5,000	-5,000		-15,269		
11	9	8,000	3,000		-8,988		
12	10	10,000	13,000		0	ROIC analysis	
13					Investment rate, i_i		4.00%
14	Signs	5	3		EROR, i"		11.26%
15	IRR result						
16	IRR	11.26%					

7.60 (a) Sign changes: Descartes: 2; up to two i* values

Norstrom's: 1; one positive i*

(a) Use 'guess' option in IRR function (see spreadsheet)

$i_1^* = -67.2$ and $i_2^* = +52.6\%$

(b) Plot of i% vs. PW crosses at the i* values (see chart)

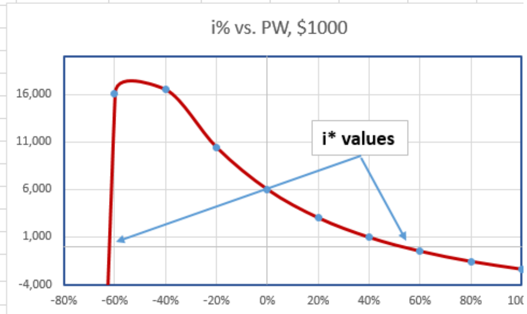
(d, e) EROR values resulting from the MIRR and ROIC analyses are shown on the spreadsheet

(f) Summary intentionally omitted. It's for the learner to write.

Year	NCF, $1000	CCF, $1000	i%	PW, $1000
0	-7,000	-7,000	-90%	-3,477,000
1	3,000	-4,000	-80%	-242,000
2	15,000	11,000	-60%	16,125
3	-5,000	6,000	-40%	16,519
Signs			-20%	10,422
Descartes	2		0%	6,000
Norstrom's		1	20%	3,023
			40%	974
			60%	-486
			80%	-1,561
			100%	-2,375

IRR analysis

Guess	-50%	10%	50%
i* via IRR	-67.2%	52.6%	52.6%

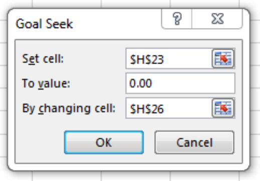


MIRR method (invest rate = 10%)

Borowing rate	Function	EROR, i'
4%	=MIRR($B$2:$B$5,$A20,10%)	**20.71%**
6%	=MIRR($B$2:$B$5,$A21,10%)	**21.59%**
8%	=MIRR($B$2:$B$5,$A22,10%)	**22.43%**
10%	=MIRR($B$2:$B$5,$A23,10%)	**23.23%**
12%	=MIRR($B$2:$B$5,$A24,10%)	**24.00%**
14%	=MIRR($B$2:$B$5,$A25,10%)	**24.73%**

ROIC Method

Year	NCF, $1000	F, $1000
0	-7,000	-7,000
1	3,000	-7,185
2	15,000	4,545
3	-5,000	0
	Investment rate	10.00%
	Resulting EROR, i"	**45.50%**



Additional Problems and FE Exam Review Problems

7.61 Answer is (c)

7.62 Answer is (b)

7.63 Answer is (d)

7.64 700 = V(0.07)/2
V = $20,000
Answer is (b)

7.65 Answer is (d)

7.66 I/quarter = 20,000(0.07)/4
= $350
Answer is (d)

7.67 250 = (10,000)(b)/2
b = 5% per year, payable semiannually
Answer is (c)

7.68 9000 + x = 1000
x = -8000
Answer is (a)

7.69 $0 = 1,000,000 – 20,000(P/A,i^*,24) – 1,000,000(P/F,i^*,24)$
Solve for i* by trial and error or Excel
$i^* = 2\%$ per month (Excel)
Answer is (b)

7.70 $0 = -60,000 + 10,000(P/A,i^*,10)$
$(P/A,i^*,10) = 6.0000$

From tables, i* is between 10% and 11%
Answer is (a)

7.71 $0 = -50,000 + (7500 – 5000)(P/A,i^*,24) + 11,000(P/F,i^*,24)$

$i^* = 2.6\%$ per month
Answer is (a)

7.72 $i^* = 4500/50,000 = 0.09$ (9% per year)
Answer is (c)

7.73 Answer is (c)

7.74 $PW_0 = -40,000 – 29,000(P/F,8\%,2)$
$= -40,000 – 29,000(0.8573)$
$= \$-64,862$

$FW_4 = 13,000(F/P,20\%,3) + 25,000(F/P,20\%,1) + 50,000$
$= 13,000(1.7280) + 25,000(1.2000) + 50,000$
$= \$102,464$

$-64,862(F/P,i',4) + 102,464 = 0$
$-64,862(1 + i')^4 + 102,464 = 0$

$i' = (102,464/64,862)^{1/4} – 1$
$i' = 0.121$ (12.1% per year)
Answer is (b)

Solution to Case Study, Chapter 7

There is not always a definitive answer to case study exercises. Here are example responses

DEVELOPING AND SELLING AN INNOVATIVE IDEA

	A	B	C	D	E	F
1	Year	NCF, $	NCF with sale in year 4 for $500,000	NCF with sale in year 8 for $100,000	NCF with new capital in year 8	Cum NCF, $
2	0	-200,000	-200,000	-200,000	-200,000	-200,000
3	1	55,000	55,000	55,000	55,000	-145,000
4	2	57,750	57,750	57,750	57,750	-87,250
5	3	60,638	60,638	60,638	60,638	-26,613
6	4	63,669	563,669	63,669	63,669	37,057
7	5	40,000		40,000	40,000	77,057
8	6	35,000		35,000	35,000	112,057
9	7	30,000		30,000	30,000	142,057
10	8	25,000		125,000	-175,000	-32,943
11	9	5,000			5,000	-27,943
12	10	10,000			10,000	-17,943
13	11	15,000			15,000	-2,943
14	12	20,000			20,000	17,057
15			47.9%	22.7%	4.7%	
16	ROR after 4 years	7.0%				
17	ROR after 8 years	18.8%				

1. (a) 47.9%; (b) 7.0%

2. (a) 22.7%
   (b) 18.8%

3. 4.7%

4. Descartes' rule of signs: 3 sign changes
   Norstrom's criterion; series starts negative; 3 sign changes

   Could be up to 3 roots in the range ±100%.

5. Continue the NCF series starting in year 13. Next 12 years of NCF at 12% has PW = $284,621. This is the offer based on these estimates.

   Discuss why this is the correct amount to offer.

Solutions to end-of-chapter problems

Engineering Economy, 8th edition

Leland Blank and Anthony Tarquin

Chapter 8
Rate of Return Analysis: Multiple Alternatives

Understanding Incremental ROR

8.1 Alternative B is preferred if the rate of return on the increment of investment between A and B is ≥ MARR.

8.2 (a) The rate of return on the increment is less than 22% per year.

(b) Overall ROR = 150,000(0.25) + 50,000(i^*_{X-Y}) = 200,000(0.22)

$i^*_{X-Y} = 13\%$

8.3 The rate of return on the increment is less than 0%.

8.4 By switching the position of the two cash flows, the interpretation changes completely. The situation would be similar to receiving a loan in the amount of the difference between the two alternatives *if the lower cost alternative* is selected. The rate of return would represent the interest *paid* on the loan. Since it is higher than what the company would consider attractive (i.e., 15% *or less*), the loan should not be accepted. Therefore, select the alternative with the higher initial investment, that is, A.

8.5 The company should select the lower cost infrared model because the rate of return on the increment of investment for the microwave model has to be lower than 19%

8.6 (a) The rate of return on the increment has to be larger than 18%.
(b) The rate of return on the increment has to be smaller than 10%.

8.7 There is *no income* associated with a cost alternative. Therefore, the only way to obtain a rate of return is based on the increment of investment and annual cost estimates.

8.8 Overall ROR = [30,000(0.20) + 70,000(0.14)]/100,000
= 0.158 (15.8%)

8.9 Overall ROR: 100,000(i*) = 30,000(0.30) + 20,000(0.25) + 50,000(0.20)

i* = 0.24 (24% per year)

8.10 (a) $40{,}000(0.14) + (200{,}000 – 40{,}000)(i^*_{Z1}) = 200{,}000(0.26)$

$$i^*_{Z1} = 0.29 \qquad (29\% \text{ per year})$$

(b) Goal Seek results are shown. Thresehold investment in Z2 is \$15,385. Any more than \$15,385, and the overall ROR falls below 26%.

	A	B	C
1	Stock	Invested, $	ROR
2	Z2	**15,385**	14%
3	Z1	184,615	27%
4	Total	200,000	
5			
6			
7	Calculated ROR		26.00%

8.11 $200{,}000(0.28) + 100{,}000(0.42) + 400{,}000(0.19) = 700{,}000(x)$
$x = 0.249 \qquad (24.9\%)$

8.12 (a) Size of investment in Y = 50,000 – 20,000 = \$30,000

(b) $30{,}000(i^*) + 20{,}000(0.15) = 50{,}000(0.40)$
$i^* = 0.567 \qquad (56.7\%)$

8.13 (a) Incremental investment analysis is *not* required. Alternative X should be selected because the rate of return on the increment is known to be lower than 20%
(b) Incremental investment analysis is *not* required because only Y has ROR greater than the MARR
(c) Incremental investment analysis is *not* required. Neither alternative should be selected because neither one has a ROR greater than the MARR.
(d) The ROR on the increment is less than 25%, but an incremental investment analysis *is* required to determine if the rate of return on the increment equals or exceeds MARR = 20%
(e) Incremental investment analysis is *not* required because it is known that the ROR on the increment is greater than 22%. Select Y

8.14 (a) Incremental CF, year 0: -25,000 – (-15,000) = \$-10,000
(b) Incremental CF, year 3: -400 - (-1600 – 15,000 + 3000) = \$+13,200
(c) Incremental CF, year 6: (-400 + 6000) – (-1600 + 3000) = \$+4200

8.15

Year	System X, $	System Z, $	(Z – X), $
0	-40,000	-95,000	-55,000
1	-12,000	-5,000	+7,000
2	-12,000	-5,000	+ 7,000
3	-40,000 + 6,000 -12,000	-5,000	+41,000
4	-12,000	-5,000	+7,000
5	-12,000	-5,000	+7,000
6	-12,000 + 6,000	-5,000 + 14,000	+15,000
	-140,000	-111,000	29,000

Sum of CF for (Z – X) = -111,000 - (-140,000)
= $29,000

8.16 (a) - First cost_P - (-30,000) = -53,000
First cost_P = $-83,000

(b) -11,000 – (-$\text{M\&O}_A$) = 21,000
$\text{M\&O}_A$ = $-32,000

(c) Resale_P – 4000 = 8000
Resale_P = $12,000

8.17 The incremental cash flow equation is 0 = -65,000 + x(P/A,25%,4), where x is the difference in the AOC.

0 = -65,000 + x(2.3616)
x = 65,000/2.3616
= $27,524

AOC_B = 60,000 – 27,524
= $32,476

Incremental ROR Comparison (Two Alternatives)

8.18 Solve for Δi* by trial and error or spreadsheet

Δi* = 18.0% per year

Any MARR value greater than 18% favors X

Function: = RATE(10,9000,-40000,-2000) displays 18.05%

8.19 (a) Find rate of return on incremental cash flow

0 = -5500 + 600(P/A,Δi*,3) + 6390(P/F,Δi*,3)
Δi* = 15.5% (spreadsheet)

(b) Incremental ROR is less than MARR; select Ford Explorer

8.20 $0 = -(2{,}300{,}000 – 1{,}200{,}000) + \{[(360{,}000 + 56{,}000 - 125{,}000] – [(270{,}000 – 105{,}000)]\}(P/A,\Delta i^*,10)$

$= -1{,}100{,}000 + \{[(291{,}000] – [(165{,}000)]\}(P/A,\Delta i^*,10)$

$= -1{,}100{,}000 + 126{,}000(P/A,\Delta i^*,10)$

$(P/A,\Delta i^*,10) = 8.7302$

From interest tables, Δi^* is between 2 and 3%

$\Delta i^* = 2.55\%$ per year (spreadsheet)

$\Delta i^* = 2.55\% < \text{MARR of } 5\%$,

Select Alternative B

8.21 Write ROR equation for increment between B and A

$$0 = -50{,}000 + 20{,}000(P/A,\Delta i^*,5)$$

$(P/A,\Delta i^*,5) = 2.5000$

Solve for Δi^* by interpolation or spreadsheet

$\Delta i^* = 28.6\% > \text{MARR} = 20\%$ Select B

Function: = RATE(5,20000,-50000) displays 28.65%

8.22 Write PW-based incremental ROR equation using $CF_{VS} – CF_{DS}$

$$0 = -25{,}000 + 4000(P/A,\Delta i^*,6) + 26{,}000(P/F,\Delta i^*,3) – 39{,}000(P/F,\Delta i^*,4) + 40{,}000(P/F,\Delta i^*,6)$$

$\Delta i^* = 17.4\%$ (spreadsheet)

$\Delta i^* >$ MARR of 15%; select Variable Speed (VS)

8.23 <u>By hand</u>, in \$1000 units

(a) X vs. DN: i^*_X: $0 = -84 + (96 - 31)(P/A,i^*_X,3) + 40(P/F, i^*_X,3)$

$i^*_X = 67.9\%$

Y vs DN: i^*_Y: $0 = -146 + (119 - 28)(P/A,i^*_Y,3) + 47(P/F, i^*_Y,3)$

$i^*_Y = 47.8\%$

Both X and Y have i* values > MARR = 15%; select robot X

(b) Incremental CF amounts for (Y-X)
Incremental first cost = \$-62,000
Incremental M&O = \$3000
Incremental revenue = \$23,000
Incremental salvage = \$7,000

$$0 = -62{,}000 + 3000(P/A,\Delta i^*,3) + 23{,}000(P/A\ \Delta i^*,3) + 7000(P/F,\Delta i^*,3)$$

Solve for Δi^* by trial and error
$\Delta i^* = 16.8\% > MARR = 15\%$
Select robot Y. Different selection than that based on ROR values

(c) Incremental ROR is the correct basis; selecting robot X in part (a) is incorrect

By spreadsheet

	A	B	C	D	E	F	G	H
1	Year	Robot X, \$1000			Robot Y, \$1000			Incremental cash flow
2		Savings	Costs	Cash flow	Savings	Costs	Cash flow	
3	0	0	-84	-84	0	-146	-146	-62
4	1	96	-31	65	119	-28	91	26
5	2	96	-31	65	119	-28	91	26
6	3	136	-31	105	166	-28	138	33
7	**i* value**			**67.85%**			**47.78%**	
8	**Δi* value**							**16.83%**

(a) Select robot X since i^*_x is larger and both exceed MARR

(b) Select robot Y since $\Delta i^* = 16.83\% > MARR = 15\%$

(c) Incremental ROR is the correct basis; selecting robot X in part (a) is incorrect

8.24 (a) $0 = -10{,}000 + 1200(P/A,\Delta i^*,4) + 12{,}000(P/F,\Delta i^*,2) + 1000(P/F,\Delta i^*,4)$

Solve for Δi^* by trial and error or spreadsheet
$\Delta i^* = 30.31\% > MARR = 20\%$ (spreadsheet)
Select Model 200

(b) If $n_{105} = 4$ years, the incremental ROR equation changes to

$$0 = -10{,}000 + 1200(P/A,\Delta i^*,4) + 1000(P/F,\Delta i^*,4)$$

$\Delta i^* = -17.22\%$
Select Model 105; the incremental investment is definitely not economical

8.25 (a) $0 = -17{,}000 + 400(P/A,\Delta i^*,6) + 17{,}000(P/F,\Delta i^*,3) + 1700(P/F,\Delta i^*,6)$
Solve for Δi^* by trial and error or spreadsheet

$\Delta i^* = 6.84\% < \text{MARR} = 10\%$
Select alternative P

(b) $AW_P = -18{,}000(A/P,10\%,3) - 4000 + 1000(A/F,10\%,3)$
$= -18{,}000(0.40211) - 4000 + 1000(0.30211)$
$= \$-10{,}936$

$AW_Q = -35000(A/P,10\%,6) – 3600 + 2700(A/F,10\%,6)$
$= -35{,}000(0.22961) – 3600 + 2700(0.12961)$
$= \$-11{,}286$

Select P, though neither option makes MARR = 10%

8.26 Revenue projects; determine i* first. Monetary units are in $1000

Treated vs. DN: $0 = -5000 + (2500 - 1000)(P/A,i^*,5) + 100(P/F,i^*,5)$
$= -5000 + 1500(P/A,i^*,5) + 100(P/F,i^*,5)$

$i^*_{Trt} = 15.7\%$ (spreadsheet)

Impregnated vs. DN: $0 = -6500 + (2500 - 650)(P/A,i^*,5) + 200(P/F,i^*,5)$
$= -6500 + 1850(P/A,i^*,5) + 200(P/F,i^*,5)$

$i^*_{Imp} = 13.7\%$

Reject treated and impregnated since i* < MARR = 25%

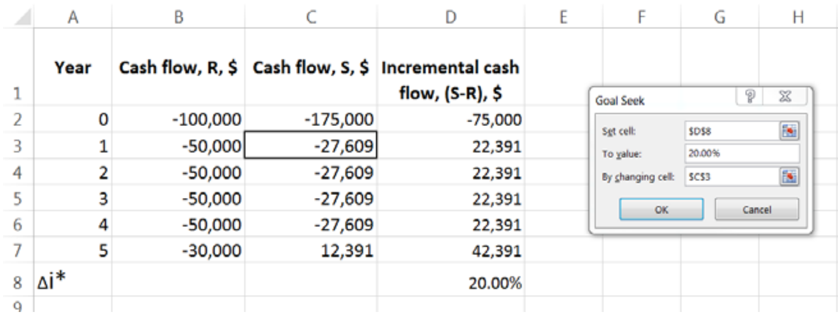
Select DN

8.27 By hand: Let x = M & O costs. Perform an incremental cash flow analysis.

$0 = -75{,}000 + (-x + 50{,}000)(P/A,20\%,5) + 20{,}000(P/F,20\%,5)$
$0 = -75{,}000 + (-x + 50{,}000)(2.9906) + 20{,}000(0.4019)$
$x = \$27{,}609$

M & O cost for S = $27,609

By spreadsheet: Enter any number for M&O in cell C3 and use Goal Seek to display 20.00% in cell D8. If RATE is used, different cash flow values must be entered repeatedly into cell C3.



	A	B	C	D
1	Year	Cash flow, R, $	Cash flow, S, $	Incremental cash flow, (S-R), $
2	0	-100,000	-175,000	-75,000
3	1	-50,000	-27,609	22,391
4	2	-50,000	-27,609	22,391
5	3	-50,000	-27,609	22,391
6	4	-50,000	-27,609	22,391
7	5	-30,000	12,391	42,391
8	Δi*			20.00%

8.28 (a) Breakeven ROR is the Δi* for a perpetual investment

$$0 = -500{,}000(\Delta i^*) + 60{,}000$$

$$\Delta i^* = 60{,}000/500{,}000$$
$$= 0.12 \qquad (12\% \text{ per year})$$

(b) Δi* = 12% > MARR = 10%; select design 1B

8.29 Sample further information items to request before saying 'yes":

- What about multiple Δi* values?
- Basis of negative CF in years 5 and 10
- Basis and percentage of increased savings from year 7 forward
- Required extra savings of FS to reach MARR of 20%

Multiple Alternative (> 2) Comparison

8.30 Select the one with the lowest initial investment cost because none of the increments were justified.

8.31 (a) A vs DN: 0 = -30,000(A/P,i*,8) + 4000 + 1000(A/F,i*,8)
Solve for i* by trial and error or spreadsheet
i* = 2.1% (spreadsheet)
Method A is *not* acceptable

B vs DN: 0 = - 36,000(A/P,i*,8) + 5000 + 2000(A/F,i*,8)
i* = 3.4%
Method B is *not* acceptable

C vs DN: 0 = - 41,000(A/P,i*,8) + 8000 + 500(A/F,i*,8)
i* = 11.3%
Method C *is* acceptable

D vs DN: 0 = - 53,000(A/P,i*,8) + 10,500 - 2000(A/F,i*,8)
i* = 11.1%
Method D *is* acceptable

(b) Revenue alternatives; compare to DN initially

A vs DN: $0 = -30{,}000(A/P,i^*,8) + 4000 + 1000(A/F,i^*,8)$

Solve for i* by trial and error or spreadsheet

$i^* = 2.1\%$ (spreadsheet)

Eliminate A; retain DN

B vs DN: $0 = -36{,}000(A/P,i^*,8) + 5000 + 2000(A/F,i^*,8)$

$i^* = 3.4\%$

Eliminate B; retain DN

C vs DN: $0 = -41{,}000(A/P,i^*,8) + 8000 + 500(A/F,i^*,8)$

$i^* = 11.3\%$

Eliminate DN; retain C

D vs DN: $0 = -53{,}000(A/P,i^*,8) + 10{,}500 - 2000(A/F,i^*,8)$

$i^* = 11.1\%$

Eliminate DN; retain D

D vs C: $0 = -12{,}000(A/P,\Delta i^*,8) + 2{,}500 - 2500(A/F,\Delta i^*,8)$

$\Delta i^* = 10.4\%$

Eliminate D

Select method C

8.32 Rank cost alternatives by increasing initial investment: 1, 3, 4, 2

3 to 1: $0 = -4000 + 1000(P/A,\Delta i^*,5)$

$\Delta i^* = 7.93\% <$ MARR of 25% Eliminate 3

4 to 1: $0 = -5000 + 2000(P/A,\Delta i^*,5)$

$\Delta i^* = 28.65\% >$ MARR of 25% Eliminate 1

2 to 4: $0 = -18{,}000 + 6000(P/A,\Delta i^*,5)$

$\Delta i^* = 19.86\% <$ MARR of 25% Eliminate 2

Select machine 4

8.33 These are revenue alternatives; add DN

(a) 8 vs. DN: $0 = -30{,}000(A/P,i^*,5) + (26{,}500 - 14{,}000) + 2000(A/F,i^*,5)$

Solve for i* by trial and error or spreadsheet

$i^* = 31.7\%$ (spreadsheet)

Eliminate DN

10 vs. 8: $0 = -4000(A/P,\Delta i^*,5) + (14{,}500 – 12{,}500) + 500(A/F,\Delta i^*,5)$
$\Delta i^* = 42.4\%$
Eliminate 8

15 vs. 10: $0 = -4000(A/P,\Delta i^*,5) + (15{,}500 – 14{,}500) + 500(A/F,\Delta i^*,5)$
$\Delta i^* = 10.9\%$
Eliminate 15

20 vs. 10: $0 = -14{,}000(A/P,\Delta i^*,5) + (19{,}500 – 14{,}500) + 1000(A/F,\Delta i^*,5)$
$\Delta i^* = 24.2\%$
Eliminate 10

25 vs. 20: $0 = -9000(A/P,\Delta i^*,5) + (23{,}000 – 19{,}500) + 1100(A/F,\Delta i^*,5)$
$\Delta i^* = 29.0\%$
Eliminate 20

Purchase 25 m^3 truck

(b) For second truck, purchase truck that was eliminated next to last: 20 m^3

8.34 (a) Select all proposals with overall ROR $\geq$ 17%

Select B and C

(b) Compare alternatives incrementally after ranking: DN, A, B, C, D

A to DN:	$\Delta i^* = 11.7\% < 14.5\%$	Eliminate A
B to DN:	$\Delta i^* = 22.2\% > 14.5\%$	Eliminate DN
C to DN:	$\Delta i^* = 17.9\% > 14.5\%$	Eliminate DN
D to DN:	$\Delta i^* = 15.8\% > 14.5\%$	Eliminate DN

Retain B, C, D

C vs. B:	$\Delta i^* = 10.0\% < 14.5\%$	Eliminate C
D vs. B:	$\Delta i^* = 10.0\% < 14.5\%$	Eliminate D

Select alternative B

(c) Compare alternatives incrementally after ranking: DN, A, B, C, D

A to DN:	$\Delta i^* = 11.7\% > 10.0\%$	eliminate DN
B to A:	$\Delta i^* = 43.3\% > 10.0\%$	eliminate A
C to B:	$\Delta i^* = 10.0\% = 10.0\%$	eliminate B
D to C:	$\Delta i^* = 10.0\% = 10.0\%$	eliminate C

Select alternative D

8.35 (a) Select all projects whose ROR ≥ MARR of 15%. Select A, B, and C

(b) Eliminate all alternatives with ROR < MARR; compare others incrementally:
Eliminate D and E
Rank survivors according to increasing first cost: B, C, A

C vs B: $\Delta i^* = 800/5000$
$= 0.16$ (16%) > MARR Eliminate B

A vs C: $\Delta i^* = 200/5000$
$= 0.04$ (4%) < MARR Eliminate A

Select alternative C

8.36 Proposals are independent; compare each against DN only

Product 1: $0 = -340,000 + (180,000 – 70,000)(P/A,i^*,5)$
$i^* = 18.52\% > MARR = 15\%$ Accept

Product 2: $0 = -500,000 + (190,000 - 64,000)(P/A,i^*,5)$
$i^* = 8.23\% < MARR = 15\%$ Reject

Product 3: $0 = -570,000 + (220,000 - 48,000)(P/A,i^*,5)$
$i^* = 15.49\% > MARR = 15\%$ Accept

Product 4: $0 = -620,000 + (205,000 - 40,000)(P/A,i^*,5)$
$i^* = 10.35\% < MARR = 15\%$ Reject

Company should introduce products 1 and 3

8.37 (a) Proposals are independent; Select A and C

(b) Alternative A is justified. B vs. A yields 1%, eliminate B; C vs. A yields 7%, eliminate C; D vs A yields 10%, eliminate A. Select D

(c) Alternative A is justified. B vs A yields 1%, eliminate B; C vs A yields 7%, eliminate C; D vs A yields 10%, eliminate D. Select A

8.38 (a) Initial cost, Machine 1: - 60,000 - (-16,000) = $-44,000
Overall ROR, Machine 2: $0 = -60,000 + 16,000(P/A,i^*,10)$; $i^* = 23.41\%$
Incremental investment, 3 vs. 2: -72,000 - (-60,000) = $-12,000
Incremental cash flow: 4 vs. 3: 24,000 – 19,000 = $5000
Incremental ROR 3 vs. 2: $0 = -12,000 + 3000(P/A,\Delta i^*,10)$; $\Delta i^* = 21.41\%$

Incremental ROR 4 vs. 3: $0 = -26{,}000 + 5000(P/A,\Delta i^*,10)$; $\Delta i^* = 14.08\%$

(b) Machines are ranked according to initial investment: 1, 2, 3, 4; MARR = 18%
Compare 1 to DN: $i^* = 18.6\% >$ MARR eliminate DN
Compare 2 to 1: $\Delta i^* = 35.7\% >$ MARR eliminate 1
Compare 3 to 2: $\Delta i^* = 21.41\% >$ MARR eliminate 2
Compare 4 to 3: $\Delta i^* = 14.08\% <$ MARR eliminate 4

Select Machine 3

8.39 (a) Find ROR for each increment of investment using the general relation

$$AW_1 + AW_{II} = AW_2$$

where II = incremental investment

E vs F: $20{,}000(0.20) + 10{,}000(\Delta i^*) = 30{,}000(0.35)$
$\Delta i^* = 65\%$

E vs G: $20{,}000(0.20) + 30{,}000(\Delta i^*) = 50{,}000(0.25)$
$\Delta i^* = 28.3\%$

E vs H: $20{,}000(0.20) + 60{,}000(\Delta i^*) = 80{,}000(0.20)$
$\Delta i^* = 20\%$

F vs G: $30{,}000(0.35) + 20{,}000(\Delta i^*) = 50{,}000(0.25)$
$\Delta i^* = 10\%$

F vs H: $30{,}000(0.35) + 50{,}000(\Delta i^*) = 80{,}000(0.20)$
$\Delta i^* = 11\%$

G vs H: $50{,}000(0.25) + 30{,}000(\Delta i^*) = 80{,}000(0.20)$
$\Delta i^* = 11.7\%$

(b) Revenue = A = Pi
E: A = 20,000(0.20) = \$4000
F: A = 30,000(0.35) = \$10,500
G: A = 50,000(0.25) = \$12,500
H: A = 80,000(0.20) = \$16,000

(c) Conduct incremental analysis using results from part (a) with MARR = 16%
E vs DN: $i^* = 20\%$ > MARR eliminate DN
E vs F: $\Delta i^* = 65\%$ > MARR eliminate E
F vs G: $\Delta i^* = 10\%$ < MARR eliminate G
F vs H: $\Delta i^* = 11\%$ < MARR eliminate H

Therefore, select Alternative F

(d) Conduct incremental analysis using results from part (a) with MARR = 11%

E vs DN: i* = 20% > MARR, eliminate DN
E vs F: Δi* = 65% > MARR, eliminate E
F vs G: Δi* = 10% < MARR, eliminate G
F vs H: Δi* = 11% = MARR, eliminate F

Select alternative H

(e) Conduct incremental analysis using results from part (a) with MARR = 19%

E vs DN: i* = 20% > MARR, eliminate DN
E vs F: Δi* = 65% > MARR, eliminate E
F vs G: Δi* = 10% < MARR, eliminate G
F vs H: Δi* = 11% < MARR, eliminate H

Select F as first alternative; compare remaining alternatives incrementally.

E vs DN: i* = 20% > MARR, eliminate DN
E vs G: Δi* = 28.3% > MARR, eliminate E
G vs H: Δi* = 11.7% < MARR, eliminate H

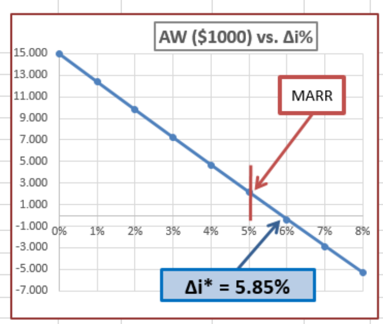
Therefore, select alternatives F and G

Spreadsheet Exercises

8.40 (a) The i* values show that both alternatives are justified for MARR = 5%. Incremental ROR analysis results in Δi* = 5.85 %, which exceeds MARR = 5%. Select Model 1000. Develop plot of AW vs. Δi values to show Δi* = 5.85% per year.

(b) Ranking inconsistency is present. Based on AW analysis over 6 years, select Model 400 (at the higher i* of 7.61%), which is different than incremental analysis result.

	Model 400 Sorter			Model 1000 Sorter					
Year	Savings and SV, \$1000	Expenses, \$1000	Cash flow, \$1000 (B+C)	Savings and SV, \$1000	Expenses, \$1000	Cash flow, \$1000 (E+F)	Incremental cash flow, \$ (G-D)	Rate, Δi%	AW, \$1000
0	0	-700	-700	0	-1,000	-1,000	**-300**	0%	15.000
1	180	-40	140	410	-60	350	**210**	1%	12.386
2	180	-40	140	410	-60	350	**210**	2%	9.792
3	180	-40	140	510	-1,060	-550	**-690**	3%	7.219
4	180	-40	140	410	-60	350	**210**	4%	4.666
5	180	-40	140	410	-60	350	**210**	5%	2.133
6	250	-40	210	510	-60	450	**240**	6%	-0.380
								7%	-2.875
								8%	-5.351
i* for 400 over 6 years			7.61%	i* for 1000 over 6 years		7.01%	Δi* is 5.85%		



8.41 (a) Determine if methods are economically justified. From row 14,

$i^*_B = 6.25\% > MARR = 5\%$; justified
$i^*_I = 4.52\% < MARR = 5\%$, not justified

Incremental ROR is not needed; select method B

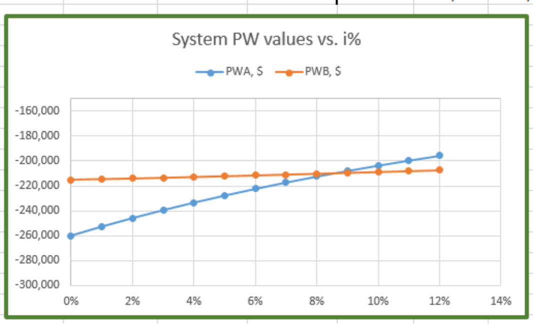
	Method B, \$1000			Method I, \$1000			Incremental
Year	Savings	Costs	Cash flow	Savings	Costs	Cash flow	cash flow
0		-1200	-1200		-2300	-2300	-1100
1	270	-105	165	416	-125	291	126
2	270	-105	165	416	-125	291	126
3	270	-105	165	416	-125	291	126
4	270	-105	165	416	-125	291	126
5	270	-105	165	416	-125	291	126
6	270	-105	165	416	-125	291	126
7	270	-105	165	416	-125	291	126
8	270	-105	165	416	-125	291	126
9	270	-105	165	416	-125	291	126
10	270	-105	165	416	-125	291	126
i* value			**6.25%**			**4.52%**	
Δi* value							**2.55%**

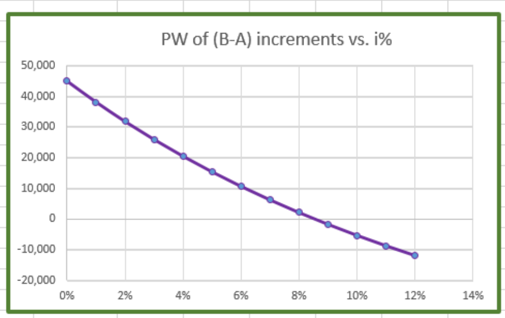
(b) Neither method is justified at MARR = 8%. Since one of the methods must be installed, reduce the MARR expectation; select B with the larger i* and small Δi* = 2.55% per year.

8.42 (a) Graph of breakeven ROR is approximately 8.5% per year in both renditions.

(b) If MARR > approximately 8.5%, the lower-investment system A is selected.

Year	System A Cash flow, \$	System B Cash flow, \$	Incremental cash flow, (B-A), \$	Rate, i%	PW_A, \$	PW_B, \$	PW_{B-A}, \$
				0%	-260,000	-215,000	45,000
0	-100,000	-175,000	-75,000	1%	-252,725	-214,559	38,166
1	-10,000	-8,000	2,000	2%	-245,877	-214,047	31,831
2	-10,000	-8,000	2,000	3%	-239,429	-213,478	25,951
3	-10,000	-8,000	2,000	4%	-233,352	-212,865	20,487
4	-10,000	-8,000	2,000	5%	-227,621	-212,217	15,404
5	-90,000	-8,000	82,000	6%	-222,214	-211,545	10,669
6	-10,000	-8,000	2,000	7%	-217,108	-210,855	6,253
7	-10,000	-8,000	2,000	8%	-212,284	-210,153	2,131
8	-10,000	-8,000	2,000	9%	-207,723	-209,445	-1,722
9	-10,000	-8,000	2,000	10%	-203,409	-208,735	-5,326
10	10,000	32,000	22,000	11%	-199,325	-208,026	-8,702
Δi* value			8.54%	12%	-195,457	-207,323	-11,866





8.43 (a) Select C and D with i* values exceeding MARR (rows 21 and 22).

(b, c) Select C. It is the alternative with the largest investment that has the extra investment justified over another acceptable alternative, DN, in this case.

Alternative		A	B	C	D	D	E
Initial cost, $		-20,000	-10,000	-15,000	-60,000	-60,000	-80,000
Annual cash flow, $ per year		5,500	2,000	3,800	11,000	11,000	9,000
Salvage value, $		0	0	0	0	0	0
Life, years	Year	4	6	6	12	12	12
Incr. ROR analysis		**A to DN**	**B to DN**	**C to DN**	**D to DN**	**D vs. C**	**E to DN**
Incremental investment, $	0	-20,000	-10,000	-15,000	-60,000	-45,000	-80,000
Incremental cash flow	1	5,500	2,000	3,800	11,000	7,200	9000
over the LCM, $ per year	2	5,500	2,000	3,800	11,000	7,200	9000
	3	5,500	2,000	3,800	11,000	7,200	9000
	4	5,500	2,000	3,800	11,000	7,200	9000
	5		2,000	3,800	11,000	7,200	9000
	6		2,000	3,800	11,000	-7,800	9000
	7				11,000	7,200	9000
	8				11,000	7,200	9000
	9				11,000	7,200	9000
	10				11,000	7,200	9000
	11				11,000	7,200	9000
	12				11,000	7,200	9000
Overall i*		3.92%	5.47%	13.46%	14.85%		4.95%
Retain or eliminate?		Eliminate A	Eliminate B	Retain C	Retain D		Eliminate E
Incremental i* (Δi*)						7.92%	
Increment justified?					Must repurchase C in year 6	No	
Alternative selected						**C**	

8.44 (a) Spreadsheet analysis results in selection of 25 m^3 truck bed size.

(b) Purchase truck that was eliminated next to last: 20 m^3

MARR =	18%									
Truck bed size, m^3		**8**	**10**		**15**		**20**		**25**	
Initial cost, $		-30,000	-34,000		-38,000		-48,000		-57,000	
Net cash flow, $ per year		12,500	14,500		15,500		19,500		23,000	
Salvage value, $		2,000	2,500		3,000		3,500		4,600	
Life, years	Year	5	5		5		5		5	
ROR analysis		**8 to DN**	**10 to DN**	**10 vs. 8**	**15 to DN**	**15 vs. 10**	**20 to DN**	**20 vs. 10**	**25 to DN**	**25 vs. 20**
Incremental investment, $	0	-30,000	-34,000	-4,000	-38,000	-4,000	-48,000	-14,000	-57,000	-9,000
Incremental cash flow, $/year	1	12,500	14,500	2,000	15,500	1,000	19,500	5,000	23,000	3,500
	2	12,500	14,500	2,000	15,500	1,000	19,500	5,000	23,000	3,500
	3	12,500	14,500	2,000	15,500	1,000	19,500	5,000	23,000	3,500
	4	12,500	14,500	2,000	15,500	1,000	19,500	5,000	23,000	3,500
	5	14,500	17,000	2,500	18,500	1,500	23,000	6,000	27,600	4,600
Overall i*		31.67%	32.97%		30.75%		30.46%		30.23%	
Retain or eliminate?		Retain 8	Retain 10		Retain 15		Retain 20		Retain 25	
Incremental i* (Δi*)				42.35%		10.93%		24.23%		28.99%
Increment justified?				Eliminate 8		Eliminate 15		Eliminate 10		Eliminate 20
Selection										**25 m^3**

Additional Problems and FE Exam Review Questions

8.45 Answer is (b)

8.46 Answer is (c)

8.47 20,000(0.40) + 80,000(x) = 100,000(0.25)
x = 21.3%
Answer is (c)

8.48 Answer is (c)

8.49 Answer is (a)

8.50 Answer is (b)

8.51 Answer is (d)

8.52 Diesel - Gasoline = -40,000
Diesel - (-150,000) = -40,000
Diesel = -190,000
Answer is (a)

8.53 i* for B, D, and E > MARR
Answer is (c)

8.54 Rank alternatives in terms of increasing first cost: DN, A, B, C, D, E
Eliminate alternatives A and C because i* < MARR =15%

B to DN: i* = 15.1%	i* > MARR	eliminate DN
D to B: Δi* = 38.5%	Δi* > MARR	eliminate B
E to D: Δi* = 26.8%	Δi* > MARR	eliminate D; select E

Answer is (d)

8.55 Rank alternatives in terms of increasing first cost: DN, A, B, C, D, E
Eliminate alternatives A, B, C and E because i* < MARR = 25%
Only D remains; select D
Answer is (b)

8.56 Rank alternatives in terms of increasing first cost: DN, A, B, C
Eliminate alternative B because i* < MARR =16%

A to DN: i* = 23.4%	i* > MARR	eliminate DN
C vs. A: Δi* = 12.0%	Δi* < MARR	eliminate C; select A

Answer is (a)

8.57 Rank Alternatives: C, A, B. D, E; perform incremental ROR analysis for $n = \infty$

A vs. C: $-2000(\Delta i^*) + 400 = 0$; $\Delta i^* = 400/2000 = 20\%$ > MARR; eliminate C
B vs. A: $-1000(\Delta i^*) + 100 = 0$; $\Delta i^* = 100/1000 = 10\%$ < MARR; eliminate B
D vs. A: $-3000(\Delta i^*) + 300 = 0$; $\Delta i^* = 300/3000 = 10\%$ < MARR; eliminate D
E vs. A: $-4{,}000(\Delta i^*) + 700 = 0$; $\Delta i^* = 700/4000 = 17.5\%$ > MARR; eliminate A
Select E
Answer is (d)

Solution to Case Study 1, Chapter 8

Sometimes, there is not a definitive answer to a case study exercise. Here are example responses.

PEFORMING ROR ANALYSIS FOR 3D PRINTER AND IIoT TECHNOLOGY

1. PW at 12% is shown in row 29. Select server #2 (n = 8) with the largest PW value.

2. #1 (n = 3) is eliminated. It has i* < MARR = 12%. Perform an incremental analysis of #1 (n = 4) and #2 (n = 5). Column H shows Δi* = 19.5%. Now perform an incremental comparison of #2 for n = 5 and n = 8. This is not necessary since no extra investment is necessary to expand cash flow by three years. The Δi* is infinity. It is obvious: select #2 (n = 8).

3. PW at 2000% > \$0.05. Δi* is infinity, as shown in cell K45, where an error for IRR(K4:K44) is indicated.

	A	B	C	D	E	F	G	H	I	J	K
1	MARR =	12%						#2(n=5)-to-#1(n=4)			#2(8)-to-#2(5)
2		#1 (n = 3)	#1 (n = 4)	#2 (n = 5)	#2 (n = 8)	#1(n=4)	#2 (n=5)	Incremental	#2 (n=5)	#2 (n = 8)	Incremental
3	Year	Cash flow	Cash flow	Cash flow	Cash flow	20 yr. CF	20 yr. CF	cash flow	40 yr. CF	40 yr. CF	cash flow
4	0	-100,000	-100,000	-200,000	-200,000	-100,000	-200,000	-100,000	-200,000	-200,000	0
5	1	35,000	35,000	50,000	50,000	35,000	50,000	15,000	50,000	50,000	0
6	2	35,000	35,000	55,000	55,000	35,000	55,000	20,000	55,000	55,000	0
7	3	35,000	35,000	60,000	60,000	35,000	60,000	25,000	60,000	60,000	0
8	4		35,000	65,000	65,000	-65,000	65,000	130,000	65,000	65,000	0
9	5			70,000	70,000	35,000	-130,000	-165,000	-130,000	70,000	200,000
10	6				70,000	35,000	70,000	35,000	70,000	70,000	0
11	7				70,000	35,000	70,000	35,000	70,000	70,000	0
12	8				70,000	-65,000	70,000	135,000	70,000	-130,000	-200,000
13	9					35,000	70,000	35,000	70,000	70,000	0
14	10					35,000	-130,000	-165,000	-130,000	70,000	200,000
15	11					35,000	70,000	35,000	70,000	70,000	0
16	12					-65,000	70,000	135,000	70,000	70,000	0
17	13					35,000	70,000	35,000	70,000	70,000	0
18	14					35,000	70,000	35,000	70,000	70,000	0
19	15					35,000	-130,000	-165,000	-130,000	70,000	200,000
20	16					-65,000	70,000	135,000	70,000	-130,000	-200,000
21	17					35,000	70,000	35,000	70,000	70,000	0
22	18					35,000	70,000	35,000	70,000	70,000	0
23	19					35,000	70,000	35,000	70,000	70,000	0
24	20					35,000	70,000	35,000	-130,000	70,000	200,000
25	Overall i*	2.5%	15.0%	14.3%	25.0%	Δi*		19.5%	70,000	70,000	0
26	Retain or		Retain	Retain	Retain		Retain		70,000	70,000	0
27	Eliminate?	Eliminate				Eliminate			70,000	70,000	0
28									70,000	-130,000	-200,000
29	PW @12%	-15,936	6,307	12,224	107,624				-130,000	70,000	200,000
30	26								70,000	70,000	0
31	27								70,000	70,000	0
32	28								70,000	70,000	0
42	38								70,000	70,000	0
43	39								70,000	70,000	0
44	40								70,000	70,000	0
45										Δi*	#DIV/0!
46										PW at 3000%	0.01

LCM = 20 years

LCM = 40 years

Some rows hidden

Solution to Case Study 2, Chapter 8

Sometimes, there is not a definitive answer to a case study exercise. Here are example responses.

HOW A NEW ENGINEERING GRADUATE CAN HELP HIS FATHER

1. Cash flows for each option are summarized at top of the spreadsheet. Rows 9-19 show annual estimates for options in increasing order of initial investment: 3, 2, 1, 4, 5.

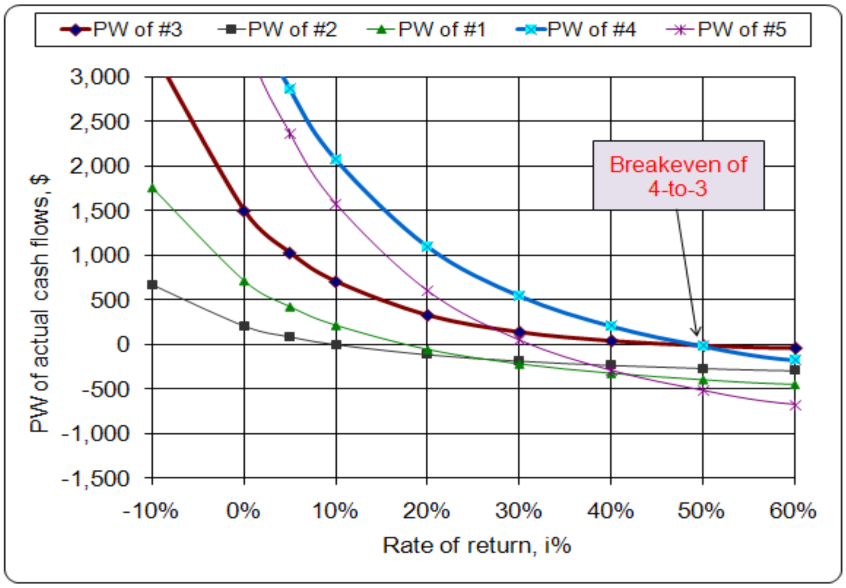
	A	B	C	D	E	F	G	H	I
1	MARR =	25%	ROR, PW, AW analysis		(Cash flows, $1000 units)				
2	Alternative		#3	#2	#1	#4		#5	
3	Initial cost		0	-400	-750	-1,000		-1,500	
4	Est. annual expenses		$-1250,yrs 1-5	$-1400(1-5);-2000(6-10)	$-800+6%/yr	-3,000		-500	
5	Est. annual revenues		$1150 (1-5)	$1400+5%/yr	$1000+4%/yr	3,500		1,000	
6	Sale of business revenue		$500 (5-8)						
7	Life	Year	10	10	10	10		10	
8	Incr. ROR comparison		Actual CF	Actual CF	Actual CF	Actual CF	4-to-3	Actual CF	5-to-4
9	Incremental investment	0	0	-400	-750	-1,000	-1,000	-1,500	-500
10	Incremental cash flow	1	-100	0	200	500	600	500	0
11		2	-100	70	192	500	600	500	0
12		3	-100	144	183	500	600	500	0
13		4	-100	221	172	500	600	500	0
14		5	400	302	160	500	100	500	0
15		6	500	-213	146	500	0	500	0
16		7	500	-124	131	500	0	500	0
17		8	500	-30	113	500	0	500	0
18		9	0	68	93	500	500	500	0
19		10	0	172	72	500	500	500	0
20	Overall i*		46.4%	10.1%	17.4%	49.1%		31.1%	
21	Retain or eliminate?		Retain	Eliminate	Eliminate	Retain		Retain	
22	Incremental i*						49.9%		#NUM!
23	Increment justified?						Yes		No
24	Alternative selected						4		4
25	PW at MARR		215	-152	-146	785		285	-500
26	AW at MARR		60			220		80	
27	Alternative acceptable?		Yes			Yes		Yes	
28	Alternative selected					4			

2. Multiple i* values: Only for option #2; there are 3 sign changes in cash flow and cumulative cash flow series. No values other than 10.1% are found in the 0 to 100% range.

3. Do incremental ROR analysis after removing #1 and #2. See row 22. 4-to-3 comparison yields 49.9%, 5-to-4 has no return because all incremental cash flows are 0 or negative. PW at 25% is $785 for #4, which is the largest PW. Aw is also the largest for #4.

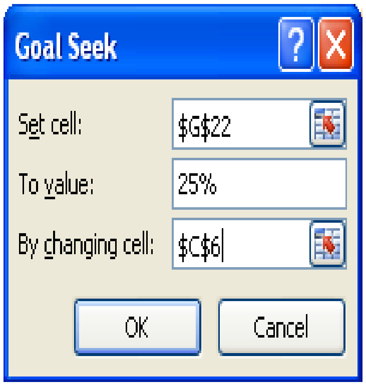
   Conclusion: Select option #4 – trade-out with friend.

4. PW vs. i charts for all 5 options are on the spreadsheet.



Options compared	Approximate breakeven
1 and 2	26%
3 and 5	27
2 and 5	38
1 and 5	42
3 and 4	50

5. Force the breakeven rate of return between options #4 and #3 to be equal to MARR = 25%. Use trial and error or Goal Seek with a target cell of G22 to equal 25% and changing cell of C6 (template at right). Make the values in years 5 through 8 of option #3 equal to the value in cell C6, so they reflect the changes. The answer obtained should be about $1090, which is actually $1,090,000 for each of 4 years.



Required minimum selling price is 4(1090,000) = $4.36 million compared to the current appraised value of $2 million.

Engineering Economy, 8th edition
Leland Blank and Anthony Tarquin

Chapter 9
Benefit/Cost Analysis and Public Sector Economics

Understanding B/C Concepts

9.1 The primary purpose of public sector projects is to provide services for the public good at no profit.

9.2 (a) It is best to take a specific viewpoint in determining costs, benefits and disbenefits because, in the broadest sense, benefits and disbenefits will usually exactly off-set each other.

(b) It depends upon the situation. If it is a personal situation, viewpoints are mine and the other person's. If it is a corporate situation, it may be from my company's and the customer's perspective.

9.3
(a) Bridge across Ohio River – *public*
(b) Coal mine expansion – *private*
(c) Baja 1000 race team – *private*
(d) Consulting engineering firm – *private*
(e) New county courthouse building– *public*
(f) Flood control project – *public*
(g) Endangered species designation – *public*
(h) Freeway lighting (lumen increase) – *public*
(i) Antarctic cruise for you and your spouse – *private*
(j) Crop dusting airplane purchase - *private*

9.4
(a) Municipal bonds – *public*
(b) Retained earnings – *private*
(c) Sales taxes – *public*
(d) Automobile license fees - *public*
(e) Bank loans – *private*
(f) Savings accounts - *private*
(g) Engineer's IRA (Individual Retirement Account) - *private*
(h) State fishing license revenues – *public*
(i) Entrance fees to Tokyo Disneyland – *private*
(j) State park entrance fees – *public*

9.5 A disbenefit commonly has an indirect impact on a person, business, etc., while a cost is a direct impact. For example, economic development may be a benefit from a city viewpoint, but a cost to the shop owner who had his shop and land condemned by the city to do the development.

9.6 (a) $600,000 annual income to area businesses from tourism created by new freshwater reservoir/recreation area – *citizen* or *business owner; benefit*

(b) $450,000 per year for repainting of bridge across the Mississippi River – *budget; cost*

(c) $800,000 per year maintenance by container-ship port authority – *government; cost*

(d) Loss of $1.6 million in salaries for border residents because of strict enforcement of immigration laws – *government; disbenefit* or *citizen; cost*

(e) Reduction of $600,000 per year in car repairs because of improved roadways – *citizen; benefit* or *business owner; cost*

(f) Expenditure of $350,000 for guardrail replacement on freeway – *government; cost*

(g) $1.8 million loss of revenue by farmers because of highway right-of-way purchases - *government; disbenefit* or *farmer; cost*

9.7 (a) Public
(b) Public
(c) Public
(d) Private
(e) Public
(f) Private
(g) Public

9.8 Two advantages of a PPP are: (1) greater efficiency in the private sector, and (2) could be an additional source of funding

Project B/C Value

9.9 The salvage value is placed in the denominator because it is a recovery of cost, which is a consequence to the government. The salvage value is subtracted from costs.

9.10 All values are AW estimates

B/C = (900,000 – 225,000)/750,000 = 0.90 Not justified

9.11 C = 30,000,000(A/P,8%,20)
= $30,000,000(0.10185)
= $3,055,500 per year

B/C = 6,200,000/(3,055,500 + 340,000)
= 1.83

9.12 C = 45,000(0.025)
= $1125

1.5 = B/1125
B = $1687.50

9.13 C = 190,000(A/P,6%,20) + 21,000
= 190,000(0.08718) + 21,000
= $37,564

B = 20,000(2)(1.00)
= $40,000

B/C = 40,000/37,564
= 1.06

9.14 C = P(A/P,7%,50) + 200,000
= P(0.07246) + 200,000

1.3 = 500,000/[P(0.07246) + 200,000]
1.3[P(0.07246) + 200,000] = 500,000
P = $2,547,825

9.15 C = 4000(300)(A/P,6%,10) + 3,200,000
= 4000(300)(0.13587) + 3,200,000
= $3,363,044 per year

B/C = 5,100,000/3,363,044
= 1.52

9.16 (a) AW of B-D = 820,000 – 135,000
= $685,000

AW of C = 9,000,000(A/P,6%,20)
= 9,000,000(0.08718)
= $784,620

B/C = 685,000/784,620 = 0.87 Not justified

(b) Disbenefits not considered: B/C = 820,000/784,620 = 1.05 Marginally justified

(c) With disbenefits: = 685000 /PMT(6%,20,9000000) displays 0.873
Without disbenefits: =820000/PMT(6%,20,9000000) displays 1.045

9.17 (a) Determine AW of benefits and costs

B/C = 4,800,000(A/F,8%,3)/(150)(17,000)
= 4,800,000(0.30803)/2,550,000
= 0.58

(b) Not justified, since B/C < 1.0; but it is a *required project* based on noneconomic criteria --- health of the citizenry

9.18 AW of C = 6,500,000(0.08) + 130,000
= $650,000

AW of B = $820,000

B/C = 820,000/650,000
= 1.26

Economically justified, since B/C > 1.0

9.19 (a) In $1000 units

AW of C = 13,000(A/P,10%,20) + 400
= 13,000(0.11746) + 400
= $1927

AW of B – D = 3800 – 6750(A/F,10%,20)
= 3800 – 6750(0.01746)
= $3682

B/C = 3682/1927
= 1.91 Well justified, since 1.91 > 1.0

(b) Let P = minimum first cost allowed

AW of C = P(A/P,10%,20) + 400
AW of B – D = 3682 from part (a)

1.00 = 3682/[P(A/P,10%,20) + 400]
0.11746P = 3682 - 400
P = $27,941 ($27,941,000)

The first cost must > $27,941,000 to force B/C < 1.0

9.20 AW of C = 1,000,000(A/P,6%,30)
= 1,000,000(0.07265)
= $72,650

Modified B/C = (B - M&O costs)/initial investment
1.7 = (150,000 – M&O)/72,650
1.7(72,650) = 150,000 – M&O
M&O = $26,495

9.21 $C = P(A/P,8\%,10) + 150,000$
$= P(0.14903) + 150,000$

$B - D = 400,000 – 25,000$

$B/C = (400,000 – 25,000)/[P(0.14903) + 150,000]$
$2.1 = 375,000/[P(0.14903) + 150,000]$
$2.1[P(0.14903) + 150,000] = 375,000$
$P = \$191,716$

9.22 Put all cash flows in same units of $/year
AW of benefits $= 3,800,000(A/P,10\%,20)$
$= 3,800,000(0.11746)$
$= \$446,348$

AW of first cost $= 1,200,000(A/P,10\%,20)$
$= 1,200,000(0.11746)$
$= \$140,952$

$(B - D)/C = (446,348 – 45,000)/(140,952 + 300,000)$
$= 0.91$

Project not justified

9.23 Determine annual worth values

$B = 100,000(0.06) + 100,000$
$= 6000 + 100,000$
$= \$106,000$

$D = \$60,000$

$C = 1,800,000(0.06) + 200,000(A/F,6\%,3)$
$= 108,000 + 200,000(0.31411)$
$= \$170,822$

$S = 90,000$

(a) Conventional B/C ratio

$B/C = (106,000 – 60,000)/(170,822 – 90,000)$
$= 0.57$

(b) Modified B/C ratio = (B – D + S)/C

$$= (106{,}000 - 60{,}000 + 90{,}000)/170{,}822$$
$$= 0.80$$

9.24 Convert annual benefits in years 6 through infinity to an A value in years 1 through 5.

B = (A/0.08)(A/F,8%,5)
D = [40,000(66,000) + 1,000,000,000]/5 = \$0.728 billion per year for 5 years
C = 16/5 = \$3.2 billion per year for 5 years

1.0 = (B – D)/C
1.0 = [(A/0.08)(A/F,8%,5) – 0.728]/3.2
1.0 = [(A/0.08)(0.17046) – 0.728]/3.2
3.2 = (0.17046)A/0.08 – 0.728
A = \$1.8435 billion per year

9.25 B = 40(4,000,000) = \$160 million per year
C = 20,000(55,000)(A/P,8%,15)
= 1,100,000,000(0.11683)
= \$128.513 million per year

B/C = 160/128.513
= 1.25

9.26 B = 41,000(33 –18) = \$615,000

D = 1100(85) = \$93,500

C = 750,000(A/P,0.5%,36)
= 750,000(0.03042)
= \$22,815

(a) (B-D)/C = (615,000 – 93,500)/22,815
= 22.86

(b) Disregard disbenefits (tickets)

AW of NCF = 0.5B
= 0.5(41,000)(33-18)
= \$307,500

PI = 307,500/22,815
= 13.48

9.27 In \$10,000 units

$$\begin{aligned}\text{PW of }\Delta\text{NCF} &= 5(P/A,10\%,6) + 2(P/G,10\%,6) + 5(P/F,10\%,6)\\ &= 5(4.3553) + 2(9.6842) + 5(0.5645)\\ &= \$43.97\end{aligned}$$

$$\begin{aligned}\text{PW of investments} &= 15 + 8(P/F,10\%,1) + 10(P/F,10\%,2) + 5(P/F,10\%,5)\\ &\quad + 10(P/F,10\%,6)\\ &= 15 + 8(0.9091) + 10(0.8264) + 5(0.6209) + 10(0.5645)\\ &= \$39.29\end{aligned}$$

$$\begin{aligned}\text{PI} &= 43.97/39.29\\ &= 1.12\end{aligned}$$

The project was economically worthwhile since PI > 1.0

Two Alternative Comparison

9.28 Alternative X should be selected because the B/C ratio on the incremental cash flows between the two alternatives has to be less than 1.0

9.29 $$\begin{aligned}\text{PW of cost of Retention} &= 880,000 + 92,000(P/A,8\%,20)\\ &= 880,000 + 92,000(9.8181)\\ &= \$1,783,265\end{aligned}$$

$$\begin{aligned}\text{PW of cost of Channel} &= 2,900,000 + 30,000(P/A,8\%,20)\\ &= 2,900,000 + 30,000(9.8181)\\ &= \$3,194,543\end{aligned}$$

Channel has higher equivalent total cost

$$\begin{aligned}\text{PW of }\Delta C &= 3,194,543 - 1,783,265\\ &= \$1,411,278\end{aligned}$$

$$\begin{aligned}\text{PW of }\Delta B &= (600,000 - 200,000)[(P/F,8\%,3) + (P/F,8\%,9) + (P/F,8\%,18)]\\ &= 400,000[0.7938 + 0.5002 + 0.2502]\\ &= \$\ 617,680\end{aligned}$$

$$\begin{aligned}\Delta B/C &= 617,680/1,411,278\\ &= 0.44\end{aligned}$$

Build Retention Pond; Channel is not justified since $\Delta B/C < 1.0$

9.30 DT will have the larger equivalent total costs

PW of ΔC for DT = 1 + 10
= \$11 million

PW of ΔB for DT = (700,000 + 400,000)(P/A,6%,30)
= (700,000 + 400,000)(13.7648)
= \$15,141,280

ΔB/C = 15,141,280/11,000,000
= 1.38 > 1.0

Select the DT site

9.31 PW_1 of costs = 600,000 + 50,000(P/A,8%,20)
= 600,000 + 50,000(9.8181)
= \$1,090,905

PW_2 of costs = 1,100,000 + 70,000(P/A,8%,20)
= 1,100,000 + 70,000(9.8181)
= \$1,787,267

Alternative 2 has the larger total cost

ΔC = $PW_2 - PW_1$ = 1,787,267 - 1,090,905
= \$696,362

PW of incremental benefits for alternative 2:
ΔB = 950,000 – 250,000
= \$700,000

ΔB/C = 700,000/696,362
= 1.01

Select alternative 2

9.32 DN is an option. Rank alternatives by increasing PW of total costs: DN, 1, 2

1 vs. DN: B/C = 1,020,000/840,900
= 1.21 eliminate DN

2 vs. 1: ΔB = 1,850,000 – 1,020,000
= \$830,000

$\Delta C = 1{,}780{,}000 – 840{,}900$
$= \$939{,}100$

$\Delta B/C = 830{,}000/939{,}100$
$= 0.88$ eliminate 2

Select Alternative 1

9.33 Location 1 vs DN: $B = \$520{,}000$
$D = \$90{,}000$

$C = 1{,}200{,}000(A/P,8\%,10) + 80{,}000$
$= 1{,}200{,}000(0.14903) + 80{,}000$
$= \$258{,}836$

$B/C = (520{,}000 – 90{,}000)/258{,}836$
$= 1.66$ eliminate DN

Location 2 vs 1: $\Delta B = 580{,}000 - 520{,}000$
$= \$60{,}000$
$\Delta D = 140{,}000 - 90{,}000$
$= \$50{,}000$
$\Delta C = [2{,}000{,}000(A/P,8\%,20) + 75{,}000] – 258{,}836$
$= [2{,}000{,}000(0.10185) + 75{,}000] - 258{,}836$
$= \$19{,}864$

$\Delta B/C = (60{,}000 –50{,}000)/19{,}864$
$= 0.50$ eliminate 2

Select Site 1

9.34 $C_N = 1{,}100{,}000(0.06) + 480{,}000$
$= \$546{,}000$

$C_S = 2{,}900{,}000(0.06) + 390{,}000$
$= \$564{,}000$

South (S) has the larger total cost

ΔB are the difference in disbenefits; S has lower disbenefits

$\Delta B = 70{,}000 – 40{,}000 = \$30{,}000$

$\Delta B/C = 30{,}000/(564{,}000 – 546{,}000)$
$= 30{,}000/18{,}000$

$= 1.67$

Select location S

9.35 Use PW values for B and C

$C_{Conv} = \$200,000$

$C_{Solar} = 1,300,000 – 150,000(P/F,7\%,5)$
$= 1,300,000 – 150,000(0.7130)$
$= \$1,193,050$

Solar has the larger total cost

$\Delta C = 1,193,050 – 200,000$
$= \$993,050$

$\Delta B = (80,000 – 9000)(P/A,7\%,5)$
$= 71,000(4.1002)$
$= \$291,114$

.

$\Delta B/C = 291,114/993,050$
$= 0.29$

Select conventional; solar is not justified

9.36 Compare on a per household basis

(a) Program 1 vs. DN

$B = \$1.25$ per month
$C = 60(A/P,0.5\%,60)$
$= 60(0.01933)$
$= \$1.16$

$B/C = 1.25/1.16$
$= 1.08$ eliminate DN

Program 2 vs. 1

$\Delta B = 8.00 – 1.25 = \$6.75$
$\Delta C = (500 – 60)(A/P,0.5\%,60)$
$= 440(0.01933)$
$= \$8.51$

$\Delta B/C = 6.75/8.51$
$= 0.79$

Select program 1

(b) Program 1: B/C = 1.08 (from above)   Acceptable

Program 2:
B = 8.00 per month
C = 500(A/P,0.5%,60)
= 500(0.01933)
= $9.67

B/C = 8.00/9.67
= 0.83   Not acceptable

9.37 DN is not an option; compare DA vs. CS; use PW values

By hand: (Note: Omit equal salvage values from both computations)

$$PW_{DA} = 200{,}000{,}000 + 360{,}000(P/A,8\%,50) + 10{,}000(P/G,8\%,50) + 4{,}800{,}000(P/F,8\%,25)$$
$$= 200{,}000{,}000 + 360{,}000(12.2335) + 10{,}000(139.5928) + 4{,}800{,}000(0.1460)$$
$$= \$206{,}500{,}788$$

$$PW_{CS} = 50{,}000{,}000 + 175{,}000(P/A,8\%,50) + 8000(P/G,8\%,50) + 100{,}000[(P/F,8\%,10) + (P/F,8\%,20) + (P/F,8\%,30) + (P/F,8\%,40)]$$
$$= 50{,}000{,}000 + 175{,}000(12.2335) + 8000(139.5928) + 100{,}000[(0.4632) + (0.2145) + (0.0994) + (0.0460)]$$
$$= \$53{,}339{,}915$$

Domed arena (DA) has a larger total cost; compare DA vs. CS

$\Delta C = 206{,}500{,}788 - 53{,}339{,}915 = \$153{,}160{,}873$

$$\text{PW of } \Delta B = 10{,}900{,}000(P/A,8\%,15) + 200{,}000(P/G,8\%,15) + 13{,}700{,}000(P/A,8\%,35)(P/F,8\%,15)$$
$$= 10{,}900{,}000(8.5595) + 200{,}000(47.8857) + 13{,}700{,}000(11.6546)(0.3152)$$
$$= \$153{,}203{,}050$$

$$\Delta B/\Delta C = 153{,}203{,}050/153{,}160{,}873 = 1.00$$

Select Domed Arena (very marginal decision)

By spreadsheet: Select domed arena by a very slim margin with ΔB/C = 1.0003. (Note: rows are truncated in image of spreadsheet.)

	A	B	C	D	E	F	G	H
1	**Year**	**Conventional Stadium (CS)**			**Domed Arena (DA)**			**DA extra**
2		**Build/ M&O**	**Periodic**	**Total**	**Build/ M&O**	**Periodic**	**Total**	**revenue**
3	0	50000000		50000000	200000000		200000000	0
4	1	175000		175000	360000		360000	10900000
5	2	183000		183000	370000		370000	11100000
6	3	191000		191000	380000		380000	11300000
7	4	199000		199000	390000		390000	11500000
8	5	207000		207000	400000		400000	11700000
9	6	215000		215000	410000		410000	11900000
10	7	223000		223000	420000		420000	12100000
23	**20**	327000	**100000**	427000	550000		550000	13700000
24	21	335000		335000	560000		560000	13700000
25	22	343000		343000	570000		570000	13700000
26	23	351000		351000	580000		580000	13700000
27	24	359000		359000	590000		590000	13700000
28	**25**	367000		367000	600000	**4800000**	5400000	13700000
29	26	375000		375000	610000		610000	13700000
30	27	383000		383000	620000		620000	13700000
52	49	559000		559000	840000		840000	13700000
53	**50**	567000	**0**	567000	850000		850000	13700000
54	PW @ 8%			$53,339,917			$ 206,500,868	$ 153,209,332
55	**DA vs. CS**							
56	ΔC						$ 153,160,951	
57	ΔB							
58	**ΔB/C**							**1.0003**

Multiple (> 2) Alternatives

9.38 (a) Incremental analysis is needed between alternatives Z (challenger) and Y
(b) Incremental analysis is not needed for independent project; select Y and Z

9.39 Calculate total costs and rank sites; benefits are direct

Cost A = 50(A/P,10%,5) + 3
= 50(0.26380) + 3
= 16.190

Cost B = 90(A/P,10%,5) + 4
= 90(0.26380) + 4
= 27.742

Cost C = 200(A/P,10%,5) + 6
= 200(0.26380) + 6
= 58.760

Ranking: DN, A, B, C

Determine B/C for all sites initially

Site A: $B/C = (B-D)/C$
$= (20 - 0.5)/16.190$
$= 1.20$ Acceptable

Site B: $B/C = (29 - 2.0)/27.742$
$= 0.97$ Not acceptable

Site C: $B/C = (61 - 2.1)/58.760$
$= 1.00$ Acceptable

Compare A vs. DN; then C vs. A or DN; B was eliminated

A vs. DN: $B/C = 1.20$ eliminate DN

C vs. A: $\Delta B = 61 - 20 = 41$
$\Delta D = 2.1 - 0.5 = 1.6$
$\Delta C = 58.76 - 16.19 = 42.57$

$\Delta B/C = (41 - 1.6)/42.57$
$= 0.93$ eliminate C

Select site A

9.40 Eliminate K and M because $B/C < 1.0$; compare L vs. J incrementally

$\Delta B/C$ for L vs. J is 1.42; eliminate J

Select alternative L

9.41 (a) First calculate PW of benefits for each alternative from the PW of cost and B/C ratio values, then calculate $\Delta B/C$ ratios.

PW benefits for P: $B_p/10{,}000{,}000 = 1.1$
$B_p = \$11{,}000{,}000$

PW benefits for Q: $B_Q/40{,}000{,}000 = 2.4$
$B_Q = \$96{,}000{,}000$

PW benefits for R: $B_R/50{,}000{,}000 = 1.4$
$B_R = \$70{,}000{,}000$

PW benefits for S: $B_S/80{,}000{,}000 = 1.8$

$B_s = \$144{,}000{,}000$

Q vs. P: $\Delta B/C = (96 - 11)/(40 - 10) = 2.83$

R vs. P: $\Delta B/C = (70 - 11)/(50 - 10) = 1.48$

S vs. P: $\Delta B/C = (144 - 11)/(80 - 10) = 1.90$

R vs. Q: $\Delta B/C = (70 - 96)/(50 - 40) = -2.60$

S vs. Q: $\Delta B/C = (144 - 96)/(80 - 40) = 1.20$

S vs. R: $\Delta B/C = (144 - 70)/(80 - 50) = 2.47$

b) Alternatives are already ranked according to increasing cost, except add DN

P vs. DN: B/C = 1.1 eliminate DN
Q vs. P: ΔB/C = 2.83 eliminate P
R vs. Q: ΔB/C = -2.60 eliminate R
S vs. Q: ΔB/C = 1.20 eliminate Q

Select alternative S

9.42 Rank alternatives by increasing total cost: DN, G, J, H, I, L, K

Eliminate H and K based on B/C < 1.0

G vs. DN: B/C = 1.15 eliminate DN
J vs. G: ΔB/C = 1.07 eliminate G
I vs. J: ΔB/C = 1.07 eliminate J
L vs. I: ΔB/C = ?

ΔB/C for L vs. I comparison is not shown. Must compare L to I incrementally; survivor is the selected alternative.

9.43 Rank alternatives by total cost: DN, X, Y, Z, Q; eliminate X based on B/C < 1.0

Y vs. DN: B/C = 1.07 eliminate DN
Z vs. Y: ΔB/C = 1.40 eliminate Y
Q vs. Z: ΔB/C = 1.00 eliminate Z

Select Q, marginally, since ΔB/C = 1.00

9.44 Calculate AW of total cost (in $ millions), then rank according to increasing AW value

$$\begin{aligned} AW_{Pond} &= 58(A/P,6\%,40) + 5.5 \\ &= 58(0.06646) + 5.5 \\ &= \$9.35 \end{aligned}$$

$$\begin{aligned} AW_{Expand} &= 76(A/P,6\%,40) + 5.3 \\ &= 76(0.06646) + 5.3 \\ &= \$10.35 \end{aligned}$$

$$\begin{aligned} AW_{Advanced} &= 2(A/P,6\%,40) + 2.1 \\ &= 2(0.06646) + 2.1 \\ &= \$2.23 \end{aligned}$$

$$\begin{aligned} AW_{Partial} &= 48(A/P,6\%,40) + 4.4 \\ &= 48(0.06646) + 4.4 \\ &= \$7.59 \end{aligned}$$

Benefits are directly estimated; ranking is: DN, Advanced, Partial, Pond, Expand

Advanced vs. DN: $\Delta B/C = 2.7/2.23$
$= 1.21$ eliminate DN

Partial vs. Advanced: $\Delta B/C = (8.3 - 2.7)/(7.59 - 2.23)$
$= 1.04$ eliminate Advanced

Pond to Partial: $\Delta B/C = (11.1 - 8.3)/(9.35 - 7.59)$
$= 1.59$ eliminate Partial

Expand to Pond: $\Delta B/C = (12.0 - 11.1)/(10.35 - 9.35)$
$= 0.90$ eliminate Expand

Select the Pond System

9.45 Compare all alternatives against DN using AW values

EC vs. DN: B = $110,000 per year
D = $26,000 per year
$$\begin{aligned} C &= 38{,}000(A/P,7\%,10) + 49{,}000 \\ &= 38{,}000(0.14238) + 49{,}000 \\ &= \$54{,}410 \text{ per year} \end{aligned}$$

$$\begin{aligned} (B-D)/C &= (110{,}000 – 26{,}000)/54{,}410 \\ &= 1.54 \end{aligned}$$ EC is acceptable

NS vs. DN: B = \$160,000 per year
D = \$21,000 per year
C = 87,000(A/P,7%,10) + 64,000
= 87,000(0.14238) + 64,000
= \$76,387 per year

(B-D)/C = (160,000 – 21,000)/76,387
= 1.82 NS is acceptable

ST vs. DN: B = \$74,000 per year
D = \$32,000 per year
C = 99,000(A/P,7%,10) + 42,000
= 99,000(0.14238) + 42,000
= \$56,095 per year

(B-D)/C = (74,000 – 32,000)/56,095
= 0.75 ST is not acceptable

AC vs. DN: B = \$52,000 per year
D = \$14,000 per year
C = 61,000(A/P,7%,10) + 36,000
= 61,000(0.14238) + 38,000
= \$46,685 per year

(B-D)/C = (52,000 – 14,000)/46,685
= 0.81 AC is not acceptable

Select EC and NS

Cost-Effectiveness

9.46 Methods are independent. Calculate CER values, rank in increasing order, select lowest CER until budget is exceeded.

CER = cost/score

$CER_{Aerostats}$ = 3.8/ 8 = 0.48
CER_{Boots} = 31.4/52 = 0.60
CER_{Fence} = 18.7/12 = 1.56
CER_{Motion} = 9.8/7 = 1.40
$CER_{Seismic}$ = 8.3/5 = 1.66
CER_{Drones} = 12.1/26 = 0.47

Ordering: drones, aerostats, boots, motion, seismic, fence

Total cost = 12.1 + 3.8 + 31.4 + 9.8 = \$57.1 million

9.47 Strategies are independent; calculate CER values, rank in increasing order and select those to not exceed \$60/employee.

$CER_A = 5.20/50 = 0.10$
$CER_B = 23.40/182 = 0.13$
$CER_C = 3.75/40 = 0.09$
$CER_D = 10.80/75 = 0.14$
$CER_E = 8.65/53 = 0.16$ (0.163)
$CER_F = 15.10/96 = 0.16$ (0.157)

	A	B	C	D	E
1					Cumulative
2	Strategy	C, \$	E	CER	cost, \$
3	C	3.75	40	0.09	3.75
4	A	5.20	50	0.10	8.95
5	B	23.40	182	0.13	32.35
6	D	10.80	75	0.14	43.15
7	F	15.10	96	0.16	58.25
8	E	8.65	53	0.16	66.90
9					

Select strategies C, A, B, D and F to not exceed \$60 per employee. Parts of E may be a possibility to use the remainder of the \$60.

9.48 (a) Methods are independent. Calculate CER values, rank in increasing order, select lowest CER, determine total cost.

$CER_{Acupuncture} = 700/9 = 78$
$CER_{Subliminal} = 150/1 = 150$
$CER_{Aversion} = 1700/10 = 170$
$CER_{Out\text{-}patient} = 2500/39 = 64$
$CER_{In\text{-}patient} = 2800/41 = 68$
$CER_{NRT} = 1300/20 = 65$

Lowest CER is 64 for out-patient. Annual program cost is

2500(400) = \$1,000,000

(b) Rank by CER and select techniques to treat up to 1100 people. Select out-patient, NRT, and in-patient techniques, yielding a total request of \$2,120,000 to treat 1050 patients per year.

9.49 (a) $CER_W = 355/20 = 17.8$
$CER_X = 208/17 = 12.2$
$CER_Y = 660/41 = 16.1$

$CER_Z = 102/7 \quad = 14.6$

(b) Order alternatives according to E: Z, X, W, Y; perform incremental comparison.

Z to DN: 14.6 Basis for comparison

X to Z: $\Delta C/E = (208 - 102)/(17 - 7) = 10.6 < 14.6$

Z is dominated; eliminate Z

W to X: $\Delta C/E = (355 - 208)/(20 - 17) = 49 > 12.2$

Keep W and X; W is new defender

Y to W: $\Delta C/E = (660 - 355)/41 - 20) = 14.5 < 17.8$

W is dominated; eliminate W

Only X and Y remain.

Y to X: $\Delta C/E = (660 - 208)/(41 - 17) = 18.8 > 12.2$

No dominance; both X and Y are acceptable; final decision made on other criteria.

9.50 Minutes are the cost, C, and points gained are the effectiveness measure, E. Order on basis of E and calculate CER values, then perform $\Delta C/E$ analysis.

E = 5; Friend: C/E = 10/5 = 2
E = 10; Slides: C/E = 20/10 = 2
E = 15; Instructor: C/E = 20/15 = 1.33
E = 20; TA: C/E = 15/20 = 0.75

Friend vs. DN: C/E = 2 Basis for comparison

Slides vs. friend: $\Delta C/E = (20-10)/(10-5) = 2$

No dominance; keep both; slides is new defender

Instructor vs. slides: $\Delta C/E = (20-20)/(15-10) = 0$

$0 < 2$; slides are dominated, eliminate slides; instructor is new defender

TA vs. instructor: $\Delta C/E = (15-20)/(20-15) = -1$

$-1 < 0.75$; professor is dominated; o*nly TA and friend remain*

TA vs. friend: $\Delta C/E = (15-10)/(20-5) = 0.33$

$0.33 < 2$; friend is dominated; go to TA for assistance

Public Sector Ethics

9.51 Use Table 9.5, left column titles to develop the other two columns.

9.52 A discussion question open for different responses.

9.53 Some example projects to be described might be:

- Change of ingress and egress ramps for all major thoroughfares
- Signage changes coordinated to make the switch at the correct time
- Training programs to help drivers understand how to drive this different way
- Notification programs and progress reports to the public

Spreadsheet Exercises

9.54 All monetary units are $10,000

	A	B	C	D	E	F	G
1	Year	ΔNCF, $10000	Investment, $10,000		Year	ΔNCF, $10000	Investment, $10,000
2	0	0	15		0	0	15
3	1	5	8		1	5	8
4	2	7	10		2	7	10
5	3	9	0		3	9	0
6	4	11	0		4	11	0
7	5	13	5		5	13	5
8	6	20	10		6	20	10
9	PW @ 10%	43.97	39.29		7	24.19	15
10					PW @ 10%	56.38	46.98
11	PI =	1.12					
12					PI =	1.20	

Goal Seek

Set cell: $F$12

To value: 1.20

By changing cell: $F$9

OK Cancel

(a) PI computation, 6 years

(b) Goal Seek finds year 7 ΔNCF = $24.19 for PI = 1.20

9.55 (a) Row 9 verifies the overall B/C values (screen image #1, row 9)

(b) No, the increment in total costs is not justified since $\Delta B/C = 0.5 < 1.0$

(c) Goal seek finds $\Delta B/C = 1.00$ at \$1,903,102 (screen image #2, cell C2)

(d) Goal Seek finds $\Delta B/C = 1.0$ at \$589,869 (screen image #3, cell C3)

(e) Columns E and F (screen image #4) indicate that location 2 is economically justified with $\Delta B/C = 3.02$ when disbenefits are neglected.

Screen image #1

	A	B	C
1	Estimated values	Location 1	Location 2
2	Initial cost, $	1,200,000	2,000,000
3	Benefits, $/year	520,000	580,000
4			
5	**Order of analysis**	**Location 1**	**Location 2**
6	AW of cost, $/year	258,835	278,704
7	Annual benefits, $/year	520,000	580,000
8	Annual disbenefits, $/year	90,000	140,000
9	**(a) Overall B/C**	**1.66**	**1.58**
10	Overall B/C w/o disbenefits		
11	Acceptable	**Yes**	**Yes**
12	Comparison		2 vs. 1
13	ΔC $/year		19,869
14	ΔB, $/year		60,000
15	ΔD, $/year		50,000
16	**ΔB/C**		**0.50**
17	ΔB/C w/o disbenefits		
18	Increment justified?		**No**
19	Selection		**1**

Screen image #2

	A	B	C
1	Estimated values	Location 1	Location 2
2	Initial cost, $	1,200,000	**1,903,102**
3	Benefits, $/year	520,000	580,000
4			
5	**Order of analysis**	**Location 1**	**Location 2**
6	AW of cost, $/year	258,835	268,835
7	Annual benefits, $/year	520,000	580,000
8	Annual disbenefits, $/year	90,000	140,000
9	**(a) Overall B/C**	**1.66**	**1.64**
10	Overall B/C w/o disbenefits		
11	Acceptable	**Yes**	**Yes**
12	Comparison		2 vs. 1
13	ΔC $/year		10,000
14	ΔB, $/year		60,000
15	ΔD, $/year		50,000
16	**ΔB/C**		**1.00**
17	ΔB/C w/o disbenefits		
18	Increment justified?		**Yes**
19	Selection		**1**

Screen image #3

	A	B	C
1	Estimated values	Location 1	Location 2
2	Initial cost, $	1,200,000	2,000,000
3	Benefits, $/year	520,000	**589,869**
4			
5	**Order of analysis**	**Location 1**	**Location 2**
6	AW of cost, $/year	258,835	278,704
7	Annual benefits, $/year	520,000	589,869
8	Annual disbenefits, $/year	90,000	140,000
9	**(a) Overall B/C**	**1.66**	**1.61**
10	Overall B/C w/o disbenefits		
11	Acceptable	**Yes**	**Yes**
12	Comparison		2 vs. 1
13	ΔC $/year		19,869
14	ΔB, $/year		69,869
15	ΔD, $/year		50,000
16	**ΔB/C**		**1.00**
17	ΔB/C w/o disbenefits		
18	Increment justified?		**Yes**
19	Selection		**1**

Screen image #4

	A	D	E	F
1				
2				
3				
4			**(e) Without disbenefits**	
5	**Order of analysis**		**Location 1**	**Location 2**
6	AW of cost, $/year		258,835	278,704
7	Annual benefits, $/year		520,000	580,000
8	Annual disbenefits, $/year			
9	**(a) Overall B/C**			
10	Overall B/C w/o disbenefits		2.01	2.08
11	Acceptable		**Yes**	**Yes**
12	Comparison			2 vs. 1
13	ΔC $/year			19,869
14	ΔB, $/year			60,000
15	ΔD, $/year			
16	**ΔB/C**			
17	ΔB/C w/o disbenefits			3.02
18	Increment justified?			**Yes**
19	Selection			**2**

9.56 (a) Site A is selected (Screen image #1)

(b) Screen image #2. Work backwards in a new B vs. A comparison to obtain ΔB/C = 1.01 (cell C16). This requires ΔB = 33.17, which in turn requires B = 33.17 for site B. Perform C vs. B comparison to eliminate C, thus leaving only site B.

(c) Site C is acceptable (marginally) at B/C = 1.00

Screen image #1

	A	B	C	D
1	**Estimated values**	**Site A**	**Site B**	**Site C**
2	Initial cost, $	50	90	200
3	Cost, $/year	3	4	6
4	Benefits, $/year	20	29	61
5	Disbenefits, $/year	0.5	2.0	2.1
6	**Order of analysis**	**Site A**	**Site B**	**Site C**
7	AW of cost, $/year	16.19	27.74	58.76
8	Annual benefits, $/year	20	29	61
9	Annual disbenefits, $/year	0.5	2.0	2.1
10	Overall B/C	1.20	0.97	1.00
11	Acceptable	**Yes**	**No**	**Yes**
12	Comparison			C vs. A
13	ΔC $/year			42.57
14	ΔB, $/year			41.00
15	ΔD, $/year			1.60
16	**ΔB/C**			**0.93**
17	Increment justified?			**No**
18	**Selection**		Eliminated	**A**

Screen image #2

	A	B	C	D
1	**Estimated values**	**Site A**	**Site B**	**Site C**
2	Initial cost, $	50	90	200
3	Cost, $/year	3	4	6
4	Benefits, $/year	20	**33.17**	61
5	Disbenefits, $/year	0.5	2.0	2.1
6	**Order of analysis**	**Site A**	**Site B**	**Site C**
7	AW of cost, $/year	16.19	27.74	58.76
8	Annual benefits, $/year	20	**33.17**	61
9	Annual disbenefits, $/year	0.5	2.0	2.1
10	Overall B/C	1.20	1.12	1.00
11	Acceptable	**Yes**	**Yes**	**Yes**
12	Comparison		B vs. A	C vs. B
13	ΔC $/year		11.55	31.02
14	ΔB, $/year		**13.17**	27.83
15	ΔD, $/year		1.50	0.10
16	**ΔB/C**		**1.01**	**0.89**
17	Increment justified?		**Yes**	**No**
18	**Selection**		**B**	

9.57 (a) Screen image #1: projects M and N are acceptable; O and P are not.

(b) Use Goal Seek to determine maximum M&O costs.
Screen images #2 and #3: O is acceptable if M&O is below $32,465 per year
P is acceptable if M&O is below $32,124 per year

Screen image #1

	A	B	C	D	E
1	Estimated values	M	N	O	P
2	First cost, $	38,000	87,000	99,000	61,000
3	M&O cost, $/year	49,000	64,000	42,000	38,000
4					
5					
6	**Project**	**M**	**N**	**O**	**P**
7	AW of total costs, $/year	52,661	72,382	51,538	43,877
8	Annual benefits, $/year	110,000	160,000	74,000	52,000
9	Annual disbenefits, $/year	26,000	21,000	32,000	14,000
10	**Overall B/C**	**1.60**	**1.92**	**0.81**	**0.87**
11	Acceptable	**Yes**	**Yes**	**No**	**No**
12	Logical IF function				=IF(E10<1,"No","Yes")

Screen image #2

	A	B	C	D	E
1	Estimated values	M	N	O	P
2	First cost, $	38,000	87,000	99,000	61,000
3	M&O costs, $/year	49,000	64,000	**32,465**	38,000
4					
5					
6	**Project**	**M**	**N**	**O**	**P**
7	AW of total costs, $/year	52,661	72,382	42,003	43,877
8	Annual benefits, $/year	110,000	160,000	74000.0	52,000
9	Annual disbenefits, $/year	26,000	21,000	32000.0	14,000
10	Overall B/C	1.60	1.92	**0.99992**	0.87
11	Acceptable	Yes	Yes	**No**	No

Screen image #3

	A	B	C	D	E
1	Estimated values	M	N	O	P
2	First cost, $	38,000	87,000	99,000	61,000
3	M&O costs, $/year	49,000	64,000	42,000	**32,124**
4					
5					
6	**Project**	**M**	**N**	**O**	**P**
7	AW of total costs, $/year	52,661	72,382	51,538	38,001
8	Annual benefits, $/year	110,000	160,000	74000.0	52,000
9	Annual disbenefits, $/year	26,000	21,000	32000.0	14,000
10	Overall B/C	1.60	1.92	0.81	**0.99998**
11	Acceptable	Yes	Yes	No	**No**

Additional Problems and FE Exam Review Questions

9.58 Answer is (c)

9.59 Answer is (d)

9.60 Answer is (c)

9.61 Answer is (b)

9.62 Answer is (c)

9.63 B/C = (50,000 – 27,000)/25,000 = 0.92
Answer is (a)

9.64 Reject B because B/C ratio < 1.0. Since overall B/C ratio was higher for C than for A and it
has a larger cost, the incremental B/C ratio between the two has to be greater than 1.3.
Therefore, select alternative C.
Answer is (b)

9.65 Answer is (d)

9.66 Answer is (d)

9.67 Can use either PW, AW, or FW values; For PW,

B/C = (245,784 – 30,723)/(100,000 + 68,798) = 1.27
Answer is (a)

9.68 B has a larger total cost than A, and $B/C_B < B/C_A$, then $\Delta B/C < B/C_B$.
Answer is (c)

9.69 Answer is (a)

9.70 Use PW values based on capitalized costs and the equation P = A/i

B/C = [10,000/0.10]/[50,000 + 50,000(P/F,10%,2)]
= 1.095
Answer is (b)

9.71 B/C = (600,000 – 400,000)/[700,000(A/P,6%,10) + 25,000]
= 200,000/[700,000(0.13587) + 25,000]
= 1.67
Answer is (b)

9.72 B/C = 310,000/[2,800,000(0.06) + 20,000 + 200,000(A/F,6%,5)]
= 310,000/[168,000 + 20,000 + 200,000(0.17740)]
= 310,000/223,480
= 1.39
Answer is (c)

9.73 B/C_X = (110,000 – 20,000)/(60,000 + 45,000)
= 0.86 Reject X

B/C_Y = (150,000 – 45,000)/(90,000 + 35,000)
= 0.84 Reject Y

Answer is (a)

9.74 B/C = (360,000 – 42,000)/[2,000,000(0.06)]
= 2.65
Answer is (d)

9.75 Answer is (b)

9.76 Answer is (d)

9.77 Answer is (c)

Solution to Case Study, Chapter 9

Sometimes, there is not a definitive answer to a case study exercise. Here are example responses.

HIGHWAY LIGHTING OPTIONS TO REDUCE TRAFFIC ACCIDENTS

Computations similar to those for benefits (B), costs (C) and effectiveness measure (E) of accidents prevented in the case study for each alternative results in the following estimates.

	Benefits	Effectiveness	Cost, $ per year		
Alternative	B, $/year	Measure, C	Poles	Power	Total
W	1,482,000	247	1,088,479	459,024	1,547,503
X	889,200	148	544,240	229,512	773,752
Y	1,111,500	185	777,485	401,646	1,179,131
Z	744,000	124	388,743	200,823	589,566

1. B/C analysis order based on total costs: Z, X, Y, W. Challenger is placed first below.

Z vs. DN: B/C = 744,000/589,566 = 1.26 — eliminate DN
X vs. Z: ΔB/C = (889,200-744,000)/(773,752-589,566) = 0.79 — eliminate X
Y vs. Z: ΔB/C = (1,111,500-744,000)/(1,179,131-589,566) = 0.62 — eliminate Y
W vs. Z: ΔB/C = (1,482,000-744,000)/(1,547,503-589,566) = 0.77 — eliminate W

Select alternative Z -- wider pole spacing, cheaper poles and lower lumens

2. C/E analysis order based on effectiveness measure, E: Z, X, Y, W. Challenger listed first.

Calculate C/E for each alternative.

C/E_W = 1,547,503/247 = 6265
C/E_X = 773,752/148 = 5228
C/E_Y = 1,179,131/185 = 6374
C/E_Z = 589,566/124 = 4755

Z vs. DN: C/E = 4755 basis for comparison
X vs. Z: ΔC/E = (773,752-589,566)/(148-124) = 7674 > 4755 no dominance, keep both
Y vs. X: ΔC/E = (1,179,131-773,752)/(185-148) = 10,956 > 5228 no dominance, keep both
W vs. Y: ΔC/E = (1,547,503-1,179,131)/(247-185) = 5941 < 6374 dominance, eliminate Y

Remaining alternatives in order are: Z, X, W

X vs. Z: ΔC/E = 7674 (calculated above) no dominance, keep both
W vs. X: ΔC/E = (1,547,503-773,752)/(247-148) = 7816 > 5228 no dominance, keep both

Three alternatives -- Z, X and W -- are indicated as a possible choice. The decision for one must be made on a basis other than C/E, probably the amount of budget available.

3. Ratio of night/day accidents, lighted = 839/2069 = 0.406

   If the same ratio is applied to unlighted sections, number of accidents prevented is calculated as follows:

$$0.406 = \frac{\text{no. of accidents}}{379}$$

Number of accidents = 154

Number prevented = 199 – 154 = 45

4. For Z to be justified, the incremental comparison of W vs. Z would have to be ≥ 1.0. The benefits would have to increase. Find B_w in the incremental comparison.

   W vs. Z: ΔB/C = (B_w-744,000)/(1,547,503-589,566)

$$1.0 = (B_w - 744{,}000)/(957{,}937)$$
$$B_w = 1{,}701{,}937$$

   The difference in the number of accidents would have to increase from 247 to:

1,701,937 = (difference)(6000)
Difference = 284

From the day estimate in the case study of 1086 accidents without lights, now

Number of accidents would have to be = 1086 – 284 = 802

New night/day ratio = 802/2069 = 0.387

Chapter 10
Project Financing and Noneconomic Attributes

Working with MARR

10.1 The two primary sources of capital are debt and equity. *Debt capital* refers to capital obtained by borrowing from outside the company. *Equity capital* represents funds obtained from within the organization, such as owners' funds, retained earnings, etc.

10.2 The project that is not undertaken due to lack of funds, say B, that has a ROR of $i^*_{B,}$ in effect sets the MARR, because it's rate of return is a lost opportunity rate of return.

10.3 (a) Higher risk - *raise*
(b) Company wants to expand into a competitor's area - *lower*
(c) Higher corporate taxes - *raise*
(d) Limited availability of capital - *raise*
(e) Increased market interest rates - *raise*
(f) Government imposition of price controls – *lower*

10.4 12 - 8 = 4% per year

10.5 Before-tax MARR = 0.15/(1 – 0.38)
= 0.242 (24.2%)

10.6 (a) Effective tax rate = 0.12 + (0.88)(0.22) = 0.314

Before-tax MARR = 0.15/(1 – 0.314)
= 0.218 (21.8%)

(b) Bid amount = 7.2 million/(1 – 0.218)
= $9.2 million

10.7 (a) Bonds are debt financing
(b) Stocks are always equity
(c) Equity
(d) Equity loans are debt financing, like house mortgage loans

10.8 (a) ROR measure: Select projects A, E and C to total $21 million. Opportunity cost is 12.8%, the ROR of project B

PW measure: Select projects A, E, C and D to total \$29 million. Opportunity cost is 12.8%, the ROR of project B

(b) Since 12.8% is less than a MARR of 15%, any project with ROR < 15% should be eliminated initially. Projects B and D should not be considered.

10.9 (a) MARR = WACC + required return = 8% + 4% = 12%. The 3% risk factor is considered after the project is evaluated, not added to the MARR

(b) Evaluate the project and determine the ROR. If it is 15% and Tom rejects the proposal, his MARR is effectively 15% per year.

D-E Mix and WACC

10.10 (a) Calculate the two WACC values.

$$WACC_1 = 0.6(12\%) + 0.4(9\%) = 10.8\%$$
$$WACC_2 = 0.2(12\%) + 0.8(12.5\%) = 12.4\%$$

Use option 1 with a D-E mix of 40%-60%

(b) Let x_1 and x_2 be the maximum costs of debt capital

Option 1: $10\% = 0.6(12\%) + 0.4(x_1)$
$x_1 = [10\% - 0.6(12\%)]/0.4$
$= 7\%$

Debt capital cost would have to decrease from 9% to 7%

Option 2: $10\% = 0.2(12\%) + 0.8(x_2)$
$x_2 = [10\% - 0.2(12\%)]/0.8$
$= 9.5\%$

Debt capital cost would have to decrease from 12.5% to 9.5%

10.11 Debt portion of \$15 million represents 40% of the total

Total amount of financing = 15,000,000/0.40 = \$37,500,000

10.12 Debt: 16 + 30 = \$46 million
Equity: 4 + 12 = \$16 million

% debt = 46/(46 + 16) = 74%
% equity = 16/62 = 26%

D-E mix is 74%-26%

10.13 WACC = cost of debt capital + cost of equity capital

$= (0.4)[0.667(8\%) + 0.333(10\%)] + (0.6)[(0.4)(5\%) + (0.6)(9\%)]$

$= 0.4[8.667\%] + 0.6\ [7.4\%]$

$= 7.91\%$

10.14 Compute the WACC for each D-E mix. For example, the 70-30 D-E mix results in

$$\text{WACC} = 0.7(13\%) + 0.3(7.8\%) = 11.44\%$$

D-E Mix	WACC, %
100-0	14.50
70-30	11.44
65-35	10.53
50-50	9.70
35-65	9.84
20-80	12.48
0-100	12.50

D-E mix of 50%-50% has the lowest WACC value.

10.15 Solve for X, the cost of debt capital

$$\text{WACC} = 10.7\% = 0.8(6\%) + (1\text{-}0.8)(X)$$
$$X = (10.7 - 4.8)/0.2 = 29.5\%$$

The rate of 29.5% for debt capital (loans, bonds, etc.) seems very high.

10.16 Company's equity = 30(0.35) = $10.5 million

Return on equity = (4/10.5)(100%) = 38.1%

10.17 (a) Business: all debt; D-E = 100%-0%
Engineer: 50% debt; D-E is 50%-50%

(b) Business: FW = 30,000(F/P,4%,1)
= 30,000(1.04)
= $31,200

Check is for $31,200

Engineer: FW = 25,000 + 25,000(F/P,7%,1)
= 25,000 + 26,750
= $51,750

Two checks: $25,000 to parents and $26,750 to credit union

(c) Business: 4%
Engineer: 0.5(0%) + 0.5(7%) = 3.5%   or   (1750/50,000)*100 = 3.5%

10.18 First Engineering: D-E mix = 87/(175-87) = 87/88
Basically, a 50-50 D-E mix

Midwest Development: D-E mix = (175-62)/62 = 113/62
Approximately, a 65-35 D-E mix

10.19 Total financing = 3 + 4 + 6 = $13 million

WACC = (3/13)(8%) + (4/13)(9%) + (6/13)(11%)
= 1.85% + 2.77% + 5.07%
= 9.69%

10.20 Before-taxes:

WACC = 0.4(9%) + 0.6(12%) = 10.8% per year

After-tax approximation: Insert Equation [10.4] into the before-tax WACC relation.

After-tax WACC = (equity)(equity rate) + (debt)(before-tax debt rate)($1–T_e$)
= 0.4(9%) + 0.6(12%)(1-0.35)
= 8.28% per year

The tax advantage reduces the WACC from 10.8% to 8.28% per year

10.21 (a) Determine the after-tax cost of debt capital, Equation [10.4], and WACC

After-tax cost of debt capital = 10(1 - 0.36) = 6.4%
After-tax WACC = 0.35(6.4%) + 0.65(14.5%) = 11.665%

Interest charged to revenue for the project:

14.0 million(0.11665) = $1,633,100

(b) After-tax WACC = 0.25(14.5%) + 0.75(6.4%)
= 8.425%

Interest charged to revenue for the project:

14.0 million(0.08425) = $1,179,500

As more and more capital is borrowed, the company risks higher loan rates and owns less and less of itself. Debt capital (loans) becomes more expensive and harder to acquire.

Cost of Debt Capital

10.22 (a) Before-tax cost of debt capital is 8%

(b) Interest = 4,000,000(0.08)
= $320,000

Tax savings = 320,000(0.39)
= $124,800

Before-tax repayment = 4,000,000(1.08) = $4,320,000

After tax repayment = 4,320,000 – 124,800 = $4,195,200

0 = 4,000,000 – 4,195,200(P/F,i*,1)

i* = 0.488 (4.88%)

After-tax cost = 4.88%

(c) Approximation using Equation [10.4] is

After-tax cost of debt capital = 8%(1 - 0.39) = 0.488 (4.88%)

In this case, the approximation is the same as the actual calculated result

10.23 (a) 0 = 2,800,000 – 196,000(P/A,i*,10) – 2,800,000(P/F,i*,10)

i* = 7.0% (RATE function)

Before-tax cost of debt capital is 7.0% per year

(b) NCF is determined using Equation [10.6]

NCF = 196,000(1 - 0.33) = $131,320

0 = 2,800,000 - (131,320)(P/A,i*,10) - 2,800,000

$i^* = 4.69\%$ (RATE function)

After-tax cost of debt capital is 4.69% per year

10.24 Before-tax bond annual interest = 6 million*0.06 = $360,000
Annual bond interest NCF = 360,000(1 – 0.4) = $216,000

Find i* using a PW relation

$$0 = 6{,}000{,}000 - 216{,}000(P/A,i^*,10) - 6{,}000{,}000(P/F,i^*,10)$$
$$i^* = 3.60\% \text{ per year}$$

(Note: The correct answer is also obtained if the before-tax debt cost of 6% is used to estimate the after-tax debt cost of 6%(1 - 0.4) = 3.60% from Equation [10.4]).

10.25 (a) $0 = 19{,}000{,}000 – 1{,}200{,}000(P/A,i^*,15) – 20{,}000{,}000(P/F,i^*,15)$

$i^* = 6.53\%$ (spreadsheet)

Before-tax cost of debt capital is 6.53% per year

(b) Tax savings = 1,200,000(0.29) = $348,000

NCF = 1,200,000 - 348,000 = $852,000

$0 = 19{,}000{,}000 - 852{,}000(P/A,i^*,15) – 20{,}000{,}000$

$i^* = 4.73\%$ (spreadsheet)

After-tax cost of debt capital is 4.73% per year

(c) Before taxes: = RATE(15,-1200000,19000000,-20000000) displays 6.53%
After taxes: = RATE(15,-1200000*(1-0.29),19000000,-20000000) displays 4.73%

10.26 (a) Bank loan

Annual loan payment = 800,000(A/P,8%,8)
= 800,000(0.17401)
= $139,208

Principal payment = 800,000/8 = $100,000
Annual interest = 139,208 – 100,000 = $39,208
Tax saving = 39,208(0.40) = $15,683
Effective interest payment = 39,208 – 15,683 = $23,525
Effective annual payment = 23,525 + 100,000 = $123,525

The AW-based i* relation is:

$$0 = 800{,}000(A/P,i^*,8) - 123{,}525$$
$$(A/P,i^*,8) = 0.15441$$
$$i^* = 4.95\% \qquad \text{(RATE function)}$$

<u>Bond issue</u>

Annual bond dividend = 800,000(0.06) = \$48,000
Tax saving = 48,000(0.40) = \$19,200
Effective bond dividend = 48,000 – 19,200 = \$28,800

The AW-based i* relation is:

$$0 = 800{,}000(A/P,i^*,10) - 28{,}800 - 800{,}000(A/F,i^*,10)$$

$$i^* = 3.60\% \qquad \text{(RATE or IRR function)}$$

Bond financing is cheaper.

(b) Before taxes: bonds cost 6% per year, which is less than the 8% loan. The answer before taxes is the same as that after taxes.

10.27 (a) Face value = $\dfrac{\$2{,}500{,}000}{0.97}$ = \$2,577,320

(b) Bond interest = $\dfrac{0.042(2{,}577{,}320)}{4}$ = \$27,062 every 3 months

Dividend quarterly net cash flow = \$27,062(1 - 0.35) = \$17,590

The rate of return equation per 3-months over 20(4) quarters is:

$$0 = 2{,}500{,}000 - 17{,}590(P/A,i^*,80) - 2{,}577{,}320(P/F,i^*,80)$$

<u>Factor solution</u>:

By trial and error, i* is between 0.5% and 0.75% per quarter. Tables provide values for 75 and 84 years. Use the formula for n = 80

i = 0.5%:
$2{,}500{,}000 - 17{,}590[(1.005)^{80}-1/0.005(1.005)^{80}\,] - 2{,}577{,}320(1/(1.005)^{80}$
$2{,}500{,}000 - 17{,}590[65.8023] - 2{,}577{,}320(0.6710)$
$\$-386.84 < 0$; $i^* > 0.5\%$

i = 0.75%:
$2{,}500{,}000 - 17{,}590[(1.0075)^{80}-1/0.0075(1.0075)^{80}\,] - 2{,}577{,}320(1/(1.0075)^{80}$
$2{,}500{,}000 - 17{,}590[59.9944] - 2{,}577{,}320(0.5500)$

$27.17 > 0; i* < 0.75%

By interpolation,

i* = 0.00734 per quarter (0.734% per quarter)

Annual nominal i* = (0.734)(4) = 2.936% per year

Annual effective i* = $(1.00734)^4 - 1$ = 0.0297 (2.97% per year)

Spreadsheet solution:

Function: = RATE(80,-17590,2500000,-2577320) displays i* = 0.732% per quarter

Nominal i* = (0.732)(4) = 2.928% per year

= EFFECT(2.928%,4) displays 2.96% per year

Cost of Equity Capital

10.28 Debt financing has the lower after-tax cost because interest paid for corporate debt is deductible, but dividends paid to stockholders for equity capital are not.

10.29 (a) Money raised = 2,700,000(54)(0.90)
= $131,220,000

(b) Cost of Equity financing = 3.24/(54)(0.9)
= 0.0666 (6.67%)

10.30 (a) R_e = 1.90/80 + 0.03
= 0.05375 (5.38%)

(b) R_e = 1.90/80*0.95 + 0.01
= 0.035 (3.5%)

10.31 Let x = dividend growth rate
0.08 = 4.76/140 + x
x = 0.046 (4.6%)

10.32 R_e = 0.035 + 0.92(0.05)
= 0.081 (8.1%)

10.33 R_e = 0.032 + 1.41(0.038)
= 0.0856 (8.56%)

10.34 (a) Dividend method: $R_e = DV_1/P + g$

$$= 0.93/18.80 + 0.015$$
$$= 0.0644 \ (6.44\%)$$

(b) CAPM: (The return values are in percent)

$$R_e = R_f + \beta(R_m - R_f)$$
$$= 2.0 + 1.19(4.95 – 2.0)$$
$$= 5.51\%$$

CAPM estimate of cost of equity capital is lower.

(c) Set dividend method $R_e = 0.0551$ and solve for DV_1

$$0.0551 = DV_1/18.80 + 0.015$$
$$DV_1 = 18.80(0.040)$$
$$= \$0.754$$

For any initial dividend less than 75.4¢ per share, the CAPM estimate will be larger.

10.35 Last year CAPM computation: $R_e = 4.0 + 1.10(5.1 – 4.0)$

$$= 4.0 + 1.21 = 5.21\%$$

This year CAPM computation: $R_e = 3.9 + 1.18(5.1 – 3.9)$

$$= 3.9 + 1.42 = 5.32\%$$

Equity costs slightly more this year in part because the company's stock became more volatile based on an increase in beta. Also, the safe return rate decreased 0.1% in the switch from U.S. to Euro bonds.

10.36 Total equity and debt fund is \$15 million.

Equity WACC = (retained earnings fraction)(cost) + (stock fraction)(cost)
= (4/15)(7.4%) + (6/15)(4.8%)
= 3.893%

Debt WACC = (5/15)(9.8%)
= 3.267%

WACC = 3.893 + 3.267
= 7.16%

MARR = WACC + 4%
= 7.16 + 4.0
= 11.16%

Different D-E Mixes

10.37 A large D-E mix over time is not healthy financially because this indicates that the person owns too small of a percentage of his or her own assets (equity ownership) and is risky for creditors and lenders. When the economy is in a 'tight money situation' additional cash and debt capital (loans, credit cards, etc.) will be hard to obtain and very expensive in terms of the interest rate charged.

10.38 (a) Find cost of equity capital using CAPM.

$$R_e = 4\% + 1.05(5\%)$$
$$= 9.25\%$$
$$\text{MARR} = 9.25\%$$

Find i* on 50% equity investment.

$$0 = -5{,}000{,}000 + 1{,}115{,}000(P/A,i^*,6)$$
$$i^* = 9.01\% \qquad \text{(RATE function)}$$

The investment is not economically acceptable since i* < MARR.

(b) Determine WACC and set MARR = WACC. For 50% debt financing at 8%,

$$\text{WACC} = \text{MARR} = 0.5(8\%) + 0.5(9.25\%)$$
$$= 8.625\%$$

The investment is acceptable, since 9.01% > 8.625%.

10.39 <u>100% equity financing</u>

MARR = 8.5% is known. Determine PW at the MARR.

$$PW = -250{,}000 + 30{,}000(P/A,8.5\%,15)$$
$$= -250{,}000 + 30{,}000(8.3042)$$
$$= -250{,}000 + 249{,}126$$
$$= \$-874$$

Since PW < 0, 100% equity does not meet the MARR requirement.

<u>60%-40% D-E financing</u>

Loan principal = 250,000(0.60) = $150,000
Loan payment = 150,000(A/P,9%,15)
= 150,000(0.12406)
= $18,609 per year

Cost of 60% debt capital is 9% for the loan.

$$\begin{aligned} WACC &= 0.4(8.5\%) + 0.6(9\%) \\ &= 8.8\% \\ MARR &= 8.8\% \end{aligned}$$

$$\begin{aligned} \text{Annual NCF} &= \text{project NCF - loan payment} \\ &= \$30{,}000 – 18{,}609 \\ &= \$11{,}391 \end{aligned}$$

$$\begin{aligned} \text{Amount of equity invested} &= 250{,}000 - 150{,}000 \\ &= \$100{,}000 \end{aligned}$$

Calculate PW at the MARR on the basis of the committed equity capital.

$$\begin{aligned} PW &= -100{,}000 + 11{,}391(P/A,8.8\%,15) \\ &= -100{,}000 + 11{,}391(8.1567) \\ &= \$ -7{,}087 \end{aligned}$$

Conclusion: PW < 0; a 60% debt-40% equity mix does not meet the MARR requirement.

Recommendation: Do not invest using either D-E plan, 0%-100% or 60%-40%

10.40 Determine i* for each plan

Plan 1: 80% equity means $480,000 funds are invested. Use a PW-based relation.

$$0 = -480{,}000 + 90{,}000\,(P/A,i^*,7)$$
$$i_1^* = 7.30\% \qquad \text{(RATE function)}$$

Plan 2: 50% equity means $300,000 invested.

$$0 = -300{,}000 + 90{,}000\,(P/A,i^*,7)$$
$$i_2^* = 22.93\% \qquad \text{(RATE function)}$$

Plan 3: 40% equity means $240,000 invested.

$$0 = -240{,}000 + 90{,}000(P/A,i^*,7)$$
$$i_3^* = 32.18\% \qquad \text{(RATE function)}$$

Determine the MARR values.

(a) MARR = 7.5% for all plans

(b) $MARR_1 = WACC_1 = 0.8(7.5\%) + 0.2(10\%) = 8.0\%$
$MARR_2 = WACC_2 = 0.5(7.5\%) + 0.5(10\%) = 8.75\%$

$\text{MARR}_3 = \text{WACC}_3 = 0.4(7.5\%) + 0.6(10\%) = 9.0\%$

(c) $\text{MARR}_1 = (8.00 + 7.5)/2 = 7.75\%$
$\text{MARR}_2 = (8.75 + 7.5)/2 = 8.125\%$
$\text{MARR}_3 = (9.00 + 7.5)/2 = 8.25\%$

Make the decisions using i* values for each plan. The '?'poses the question "Is the plan justified in that i* ≥ MARR?". The decision is no (N) or yes (Y) for each plan.

		Part (a)		Part (b)		Part (c)	
Plan	i*	MARR	$?^+$	MARR	$?^+$	MARR	$?^+$
1	7.30%	7.5%	N	8.00 %	N	7.75%	N
2	22.93	7.5	Y	8.75	Y	8.125	Y
3	32.18	7.5	Y	9.00	Y	8.25	Y

Same decision for all 3 options; plans 2 and 3 are acceptable.

(d) Yes, because plan 1, with 80% equity and i* = 7.3%, is the only plan not acceptable.

10.41 (a) Calculate the two WACC values for financing alternatives 1 and 2

$\text{WACC}_1 = 0.4(9\%) + 0.6(10\%) = 9.6\%$

$\text{WACC}_2 = 0.25(9\%) + 0.75(10.5\%) = 10.125\%$

Use approach 1, with a D-E mix of 40%-60%

(b) Let x_1 and x_2 be the maximum costs of debt capital for each plan, respectively

Alternative 1: $9.5\% = \text{WACC}_1 = 0.4(9\%) + 0.6(x_1)$
$x_1 = 9.83\%$

Debt capital cost must decrease from 10% to 9.83%

Alternative 2: $9.5\% = \text{WACC}_2 = 0.25(9\%) + 0.75(x_2)$
$x_2 = 9.67\%$

Debt capital cost would have to decrease from 10.5% to 9.67%

10.42 MARR = WACC + 12.5%. Total equity and debt fund is \$15 million. Debt capital gets a tax break; equity does not. From Equation [10.4]

After-tax cost of debt = 10.4%(1 - 0.32) = 7.072%

After-tax WACC = equity cost + debt cost
= (4/15)(7.4%) + (6/15)(4.8%) + 5/15(7.072%)
= 1.973 + 1.920 + 2.357
= 6.25%

After-tax MARR = 6.25 + 12.5
= 18.75%

10.43 (a) Stan: Stock value increase: 0.10(20,000) = $2000
Equity value at year-end: $22,000 or a 10% increase

Theresa: Condo value increase: 0.10(100,000) = $10,000
Equity value at year-end: $30,000 or a 50% increase

(b) Stan: Stock value decrease: -0.10(20,000) = $-2000
Equity value at year-end: $18,000 or a 10% decrease

Theresa: Condo value decrease: -0.10(100,000) = $-10,000
Equity value at year-end: $10,000 or a 50% decrease

(c) Under high leverage situations, the gain or loss is multiplied by the leverage factor. If the investment goes down a small amount, the higher leverage loses much more than the unleveraged investment ($2000 loss for Stan vs. a $10,000 loss for Theresa). With gains, the return on equity capital is much larger for the higher-leveraged investment. This is why it is more risky.

Multiple, Non-economic Attributes

10.44 (a) $W_i = 1/6 = 0.1667$

(b) $R_j = \sum_{i=1}^{i=6} 0.1667V_{ij}$

10.45 $\Sigma s_i = 40 + 60 + 70 + 30 + 50$
$= 250$

$W_1 = 40/250 = 0.16$
$W_2 = 60/250 = 0.24$
$W_3 = 70/250 = 0.28$
$W_4 = 30/250 = 0.12$
$W_5 = 50/250 = \underline{0.20}$
1.00

10.46 (a) S = 1 + 2 + 3 +… + 10

$= n(n+1)/2$
$= 10(11)/2$
$= 55$

(b) $W_D = 4/55 = 0.073$

(c) $S = 1 + 2 + 3 + 10 + 5 + \ldots + 10$
$= 61$

$W_D = 10/61 = 0.164$

10.47 Ratings by attribute with 10 for #1

Attribute, i	Importance, S_i	Logic
1	10	Most important (10)
2	2.5	0.5(5) = 2.5
3	5	1/2(10) = 5
4	5	1/2(10) = 5
5	5	2(2.5) = 5
	27.5	

$W_i = s_i/27.5$

Attribute, i	W_i
1	0.364
2	0.090
3	0.182
4	0.182
5	0.182
	1.000

10.48 Ratings by attribute with 100 for most important.

Logic: #1 = 0.90(#5) = 0.90(100) = 90
#2 = 0.10(100) = 10
#3 = 0.30(100) = 30
#4 = 2(#3) = 2(30) = 60
#5 = 100
#6 = 0.80(#4) = 0.80(60) = 48

Attribute	Importance
1	9
2	1

3	3
4	6
5	10
6	4.8
	33.8

W_i = Score/33.8

Attribute	W_i
1	9/33.8 = 0.27
2	1/33.8 = 0.03
3	3/33.8 = 0.09
4	6/33.8 = 0.18
5	10/33.8 = 0.30
6	4.8/33.8 = 0.14

10.49 (a) Calculate W_i = importance score/sum and solve for R_j

Inspector

Attribute, i	Importance score	W_i	$Rj = Wj \times Vij$ 1	2	3
1	20	0.10	5	7	10
2	80	0.40	40	24	12
3	100	0.50	50	20	25
Sum	200		95	51	47

Select alternative 1 since R_1 = 95 is largest.

Manager

Attribute, i	Importance score	W_i	$Rj = Wj \times Vij$ 1	2	3
1	100	0.50	25	35	50
2	80	0.40	40	24	12
3	20	0.10	10	4	5
Sum	200	1.00	75	63	67

Since R_1 = 75 is the largest, select alternative 1

(b) Results are the same, even though the Inspector and Manager rated opposite on factors 1 and 3. The high score on attribute 1 (ROR) by the Manager is balanced by the Inspector's high score on attribute 3 (accuracy).

10.50 (a) Calculate R_j using *manager* scores.

$W_i = \dfrac{\text{Importance score}}{\text{Sum}}$

Attribute, i	Importance by Manager	W_i	R_j A	R_j B
1	80	0.48	0.48	0.43
2	35	0.21	0.07	0.11
3	30	0.18	0.18	0.04
4	20	0.12	0.03	0.12
	165		0.76	0.70

Select proposal A

(b) Calculate R_j using the *team supervisor* scores.

Attribute, i	Importance by Supervisor	W_i	R_j A	R_j B
1	25	0.08	0.08	0.07
2	80	0.27	0.09	0.14
3	100	0.34	0.34	0.07
4	90	0.31	0.08	0.31
	295		0.59	0.59

Either proposal is acceptable

(c) Select A, since PW_A is larger

Conclusion: 2 methods indicate A, but the supervisor's score-basis indicates indifference between A and B.

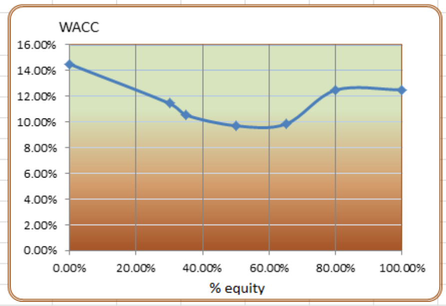
Spreadsheet Exercises

10.51 (a) Top part of the spreadsheet image indicates a D-E mix of 50%-50% to have the lowest WACC at 9.7%. Graph is included.

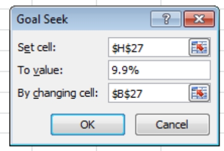
(b) Lower image shows updated WACC values. D-E mix of 35%-65% has lowest WACC of 10.19%. Row 27 is a repeat of the 35%-65% analysis. Goal Seek results in:

Maximum debt percentage of 10.15% to obtain WACC = 9.9%. The D-E mix is 10.15%-89.85%

Plan	Debt capital: Percentage debt	Debt capital: Rate, %	Debt capital: Cost of debt	Equity capital: Percentage equity	Equity capital: Rate, %	Equity capital: Cost of equity	WACC	
1	100.00%	14.50%	14.50%	0.00%	0.00%	0.00%	14.50%	
2	70.00%	13.00%	9.10%	30.00%	7.80%	2.34%	11.44%	
3	65.00%	12.00%	7.80%	35.00%	7.80%	2.73%	10.53%	
4	50.00%	11.50%	5.75%	50.00%	7.90%	3.95%	9.70%	**Lowest**
5	35.00%	9.90%	3.47%	65.00%	9.80%	6.37%	9.84%	
6	20.00%	12.40%	2.48%	80.00%	12.50%	10.00%	12.48%	
7	0.00%	0.00%	0.00%	100.00%	12.50%	12.50%	12.50%	



Plan	Debt capital: Percentage debt	Debt capital: Rate, %	Debt capital: Cost of debt	Equity capital: Percentage equity	Equity capital: Rate, %	Equity capital: Cost of equity	WACC	
1	100.00%	15.50%	15.50%	0.00%	0.00%	0.00%	15.50%	
2	70.00%	14.00%	9.80%	30.00%	7.80%	2.34%	12.14%	
3	65.00%	13.00%	8.45%	35.00%	7.80%	2.73%	11.18%	
4	50.00%	12.50%	6.25%	50.00%	7.90%	3.95%	10.20%	
5	35.00%	10.90%	3.82%	65.00%	9.80%	6.37%	10.19%	**Lowest**
6	20.00%	13.40%	2.68%	80.00%	12.50%	10.00%	12.68%	
7	0.00%	0.00%	0.00%	100.00%	12.50%	12.50%	12.50%	
5	10.15%	10.90%	1.11%	89.85%	9.80%	8.81%	9.91%	



10.52 (a) Project i* = 9.012% for the 50-50 D-E mix, where $5,000,000 in equity is required. (See cells F6 and K6).

Project justified, since i* = 9.012% > MARR = 8.625%

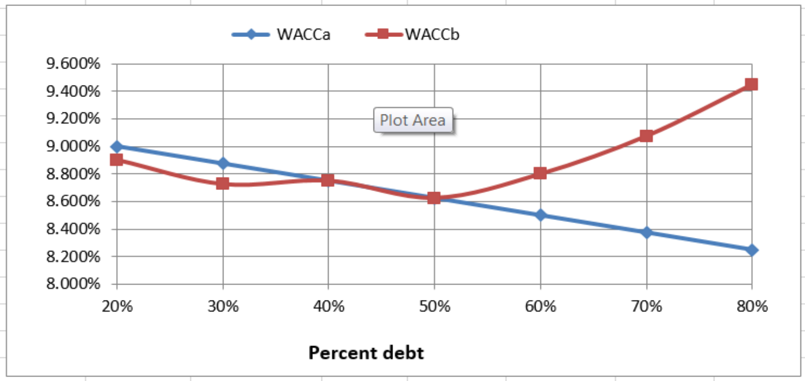
(b) Project i* values on equity basis are shown in columns F and K. The results are the same for both banks and varying loan rates for increasing debt percentages.

First three D-E mixes (equity of 80%, 70%, and 60%) indicate that the project is not justified. For lower equity percentages, the project is justified.

D-E mix		Bank 1					Bank 2				
Debt, %	Equity, %	Loan rate, %	$WACC_a$	Equity Amount, $	Project i* on equity basis	Justified? i* ≥ MARR = $WACC_a$	Loan rate, %	$WACC_b$	Equity Amount, $	Project i* on equity basis	Justified? i* ≥ MARR = $WACC_b$
20%	80%	8.0%	9.000%	8,000,000	-4.882%	No	7.5%	8.900%	8,000,000	-4.882%	No
30%	70%	8.0%	8.875%	7,000,000	-1.279%	No	7.5%	8.725%	7,000,000	-1.279%	No
40%	60%	8.0%	8.750%	6,000,000	3.202%	No	8.0%	8.750%	6,000,000	3.202%	No
50%	50%	8.0%	8.625%	5,000,000	9.012%	Yes	8.0%	8.625%	5,000,000	9.012%	Yes
60%	40%	8.0%	8.500%	4,000,000	17.019%	Yes	8.5%	8.800%	4,000,000	17.019%	Yes
70%	30%	8.0%	8.375%	3,000,000	29.162%	Yes	9.0%	9.075%	3,000,000	29.162%	Yes
80%	20%	8.0%	8.250%	2,000,000	51.058%	Yes	9.5%	9.450%	2,000,000	51.058%	Yes

A value = 1,115,000

Project i* is via RATE function
= RATE(6,-$B$11,value in column E or J)



10.53 Two independent, revenue projects with different lives. Find AW at MARR, select all with AW > 0. Find WACC first.

Equity capital is 40% at a cost of 7.5% per year

Debt capital costs 5% per year, compounded quarterly. Effective after-tax rate is:

Effective after-tax debt cost = $[(1 + 0.05/4)^4 - 1]\ (1 - 0.3)\ (100\%)$
$= 5.095(0.7)$
$= 3.566\%$ per year

WACC = $0.4(7.5\%) + 0.6(3.566\%)$
$= 5.14\%$ per year

MARR = WACC = 5.14%

	A	B	C	D
1	MARR	5.14%	7.14%	
2				
3	Project	W	R	
4	Year	NCF, $	NCF, $	
5	0	-250,000	-125,000	
6	1	48,000	30,000	
7	2	48,000	30,000	
8	3	48,000	30,000	
9	4	48,000	30,000	
10	5	48,000	30,000	
11	6	48,000		
12	7	48,000		
13	8	48,000		
14	9	48,000		
15	10	48,000		
16				
17	AW @ MARR	15,403	1,016	
18	AW @ MARR+2%	12,175	-601	
19				
20	Project i*	14.04%	6.40%	

(a) At MARR = 5.14%, both projects are acceptable (row 17)

(b) Project W is acceptable, since $i^*_W = 14.04\% > \text{MARR} + 2\% = 7.14\%$ (row 20)
Project R is not acceptable, since $i^*_R = 6.40\% < \text{MARR} + 2\% = 7.14\%$

10.54 (a) The spreadsheet shows that A is selected from the <u>manager's</u> scores ($R_A = 0.771$) and that *A is very, very slightly better than B* using the <u>supervisor's</u> scores ($R_A = 0.595$ vs. $R_B = 0.585$).

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Manager		Your Rating, V_{ij}		R_j Value for Manager		Supervisor		Your Rating, V_{ij}		R_j Value for Supervisor	
2	Attribute, i	Importance score	Weight, Wj	A	B	A	B	Importance score	Weight, Wj	A	B	A	B
3	Economics	80	0.48	1.00	0.90	0.485	0.436	25	0.08	1.00	0.90	0.085	0.076
4	Durability	35	0.21	0.35	0.50	0.074	0.106	80	0.27	0.35	0.50	0.095	0.136
5	Safety	30	0.18	1.00	0.20	0.182	0.036	100	0.34	1.00	0.20	0.339	0.068
6	Maintainability	20	0.12	0.25	1.00	0.030	0.121	90	0.31	0.25	1.00	0.076	0.305
7	Total	165	1.00			0.771	0.700	295	1.00			0.595	0.585

(b) Use Goal Seek to force R_B to equal $0.595*1.10 = 0.655$. Your value rating for safety must increase from 0.20 to a minimum of 0.41.

8													
9		Manager		Your Rating, V_{ij}		R_j Value for Manager		Supervisor		Your Rating, V_{ij}		R_j Value for Supervisor	
10	Attribute, i	Importance score	Weight, Wj	A	B	A	B	Importance score	Weight, Wj	A	B	A	B
11	Economics	80	0.48	1.00	0.90	0.485	0.436	25	0.08	1.00	0.90	0.085	0.076
12	Durability	35	0.21	0.35	0.50	0.074	0.106	80	0.27	0.35	0.50	0.095	0.136
13	Safety	30	0.18	1.00	0.20	0.182	0.036	100	0.34	1.00	0.41	0.339	0.138
14	Maintainability	20	0.12	0.25	1.00	0.030	0.121	90	0.31	0.25	1.00	0.076	0.305
15	Total	165	1.00			0.771	0.700	295	1.00			0.595	0.655
16													

Goal Seek
Set cell: $M$15
To value: 0.655
By changing cell: $K$13
OK Cancel

Additional Problems and FE Exam Review Questions

10.55 Answer is (d)

10.56 WACC = equity fraction(cost of equity) + fraction of debt(cost of debt)
$0.10 = 0.60(x) + 0.40(0.04)$
$x = 0.14$ (14%)
Answer is (c)

10.57 % debt = 15/(6 + 3.5 + 15)
= 61.2%

% equity = (6 + 3.5)/(6 + 3.5 + 15)
= 38.8%

D-E mix = 61-39
Answer is (a)

10.58 Answer is (b)

10.59 WACC = (5/10)(13.7%) + (2/10)(8.9%) + (3/10)(7.8%)
= 10.97%
Answer is (c)

10.60 Before-tax ROR = after-tax ROR/(1 - T_e)
= 11.2%/(1 - 0.39)
= 18.36%
Answer is (c)

10.61 Historical WACC = 0.5(11%) + 0.5(9%) = 10%

Let x = cost of equity capital

WACC = (equity fraction)(cost of equity) + (fraction of debt)(cost of debt)

10% = 0.25(x) + 0.75[9%(1.2)]
x = (10 - 8.1)/0.25
= 7.6%
Answer is (a)

10.62 $\Sigma s_i = 55 + 45 + 85 + 30 + 60 = 275$

$W_1 = 55/275 = 0.20$
Answer is (b)

10.63 Answer is (b)

10.64 Sum of scores = 100 + 75 + 50
= 225

Weight = 50/225
= 0.222
Answer is (c)

10.65 R = 0.4(3) + 0.3(7) + 0.2(2) + 0.1(10)
= 4.7
Answer is (d)

10.66 S = 1 + 2 + 3 +… + 10 = 10(11)/2 = 55

Attributes are ranked in decreasing order of importance; score for attribute B is 9.

W_B = 9/55 = 0.1636
Answer is (b)

Solution to Case Study, Chapter 10

There is not always a definitive answer to case study exercises. Here are example responses

EXPANDING A BUSINESS -- DEBT OR EQUITY FINANCING?

1. Set MARR = WACC

WACC = (% equity)(cost of equity) + (% debt)(cost of debt)

Equity: Use Eq. [10.7]

$$R_e = \frac{0.50}{15} + 0.05 = 8.33\%$$

Debt: Interest is tax deductible; use Eqs. [10.5] and [10.6].

Tax savings = (interest)(tax rate)
= (loan payment – principal portion)(tax rate)

Loan payment = 750,000(A/P,8%,10) = \$111,773 per year
Interest = 111,773 – 75,000 = \$36,773
Tax savings = (36,773)(0.35) = \$12,870

Cost of debt capital is i* from a PW relation:

$$0 = \text{loan amount} - (\text{annual payment after taxes})(P/A,i^*,10)$$
$$= 750{,}000 - (111{,}773 - 12{,}870)(P/A,i^*,10)$$

$$(P/A,i^*,10) = 750{,}000 / 98{,}903 = 7.5832$$

$i^* = 5.37\%$ (RATE function)

Plan A(50-50): MARR = $WACC_A = 0.5(5.37) + 0.5(8.33) = 6.85\%$

Plan B(0-100%): MARR = $WACC_B = 8.33\%$

2. A: <u>50–50 D–E financing</u>

   Use relations in case study statement and the results from Question #1.

$$TI = 300{,}000 - 36{,}773 = \$263{,}227$$
$$Taxes = 263{,}227(0.35) = \$92{,}130$$
$$\text{After-tax NCF} = 300{,}000 - 75{,}000 - 36{,}773 - 92{,}130 = \$96{,}097$$

   Find plan i_A^* from AW relation for \$750,000 of equity capital

$$0 = (\text{committed equity capital})(A/P,i_A^*,n) + S(A/F, i_A^*,n) + \text{after tax NCF}$$

$$0 = -750{,}000(A/P,i_A^*,10) + 200{,}000(A/F,i_A^*,10) + 96{,}097$$

   $i_A^* = 7.67\%$ (RATE function)

   Since 7.67% > $WACC_A$ = 6.85%, plan A is acceptable.

   B: <u>0–100 D–E financing</u>

   Use relations is the case study statement

   After tax NCF = 300,000(1–0.35) = \$195,000

   All \$1.5 million is committed. Find i_B^*

$$0 = -1{,}500{,}000(A/P,i_B^*,10) + 200{,}000(A/F,i_B^*,10) + 195{,}000$$

   $i_B^* = 6.61\%$ (RATE function)

Now 6.61% < $WACC_B$ = 8.33%, plan B is rejected.

Recommendation: Select plan A with 50-50 financing.

3. Spreadsheet shows the hard way (develops debt-related cash flows for each year) and the easy way (uses costs of capital from #1) to plot WACC. It is shaped differently than the WACC curve in Figure 10-2.

Chapter 10 - Case Study Question #3 (The hard way)

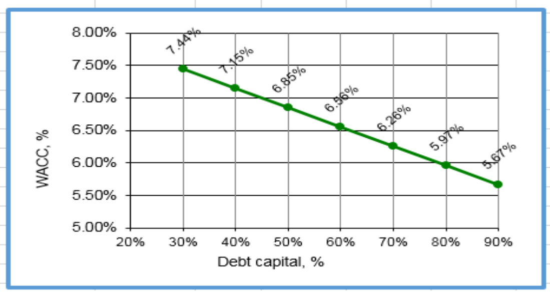
Capital investment	$ 1,500,000
Cost of equity capital	8.33%
Tax rate	35%

		Cost of debt capital					
% debt	Loan amount	Loan payment	Interest amount	Tax savings	Loan cash flow	Cost of debt	WACC
0.00001%	$ 0.15	$ 0.02	$ 0.01	$ 0.00	$ 0.02	5.37%	8.33%
30%	$ 450,000	$ 67,063	$ 22,063	$ 7,722	$ 59,341	5.37%	7.44%
40%	$ 600,000	$ 89,418	$ 29,418	$ 10,296	$ 79,122	5.37%	7.15%
50%	$ 750,000	$ 111,772	$ 36,772	$ 12,870	$ 98,902	5.37%	6.85%
60%	$ 900,000	$ 134,127	$ 44,127	$ 15,444	$ 118,682	5.37%	6.56%
70%	$ 1,050,000	$ 156,481	$ 51,481	$ 18,018	$ 138,463	5.37%	6.26%
80%	$ 1,200,000	$ 178,835	$ 58,835	$ 20,592	$ 158,243	5.37%	5.97%
90%	$ 1,350,000	$ 201,190	$ 66,190	$ 23,166	$ 178,023	5.37%	5.67%

Chapter 10 - Case Study Question #3 (The easy way)

Cost of debt capital = 5.37%
Cost of equity capital = 8.33%

% debt	WACC
0%	8.33%
30%	7.44%
40%	7.15%
50%	6.85%
60%	6.55%
70%	6.26%
80%	5.96%
90%	5.67%



Chapter 11
Replacement and Retention Decisions

Foundations of Replacement

11.1 The defender refers to the currently-owned, in-place asset while the challenger refers to the equipment/process that is under consideration as its replacement.

11.2 Sample reasons why a replacement study might be needed are: (1) reduced performance, (2) obsolescence, and (3) altered requirements.

11.3 The defender's value of P is its *fair market value*. If the asset must be updated or augmented, this cost is added to the first cost. Obtain market value estimates from expert resellers, appraisers, or others familiar with the asset being evaluated.

11.4 (a) The difference, \$10,000 is a sunk cost and must be taken care of using tax laws. It cannot be added to the cost of the solid state equipment.
(b) She is incorrect. The new equipment costs \$20,000.

11.5 (a) Yes, as long as assets similar to the ones under comparison (including the *in-place defender*) are likely to be available in the future.
(b) No, because one or more of the asset's life will not end exactly when the study period ends, rendering the cost estimates wrong.
(c) Yes, as long as assets similar to the ones under comparison (including the *in-place defender*) are likely to be available through the end of the study period.

11.6 P = market value = 85,000 – 10,000(1)
= \$75,000
n = 5 years
AOC = \$36,500 + \$1500t, where t = 0, 1, 2, 3, 4 for the 5-year max life remaining
S = \$85,000 – 10,000(6)
= \$25,000

(Note: P and AOC will carry – signs in the evaluation)

11.7 Defender:
P = market value = 6000(1 – 0.33) = \$4000
AOC = \$78,000 per year
n = 2 years
S = 6000(1 – 0.33) = \$4000

Challenger:
P = \$170,000
AOC = \$54,000 per year
n = 5 years
S = \$20,000

(Note: P and AOC will carry – signs in an evaluation)

11.8 P is market value after 2 years: P = 40,000 – 2(3000) = \$34,000
S is market value after 3 years: S = 40,000 – 3(3000) = \$31,000
AOC, year 3 = 30,000 + 3(1000) = \$33,000
(Note: P and AOC will carry – signs in an evaluation)

11.9 (a) P = 90,000 – 8000(2) = \$74,000
S = 90,000 – 8000(3) = \$66,000
AOC = \$65,000

(b) P = 90,000 – 8000(3) = \$66,000
S = 90,000 – 8000(4) = \$58,000
AOC = \$65,000

Economic Service Life

11.10 (a) The ESL of the defender is 3 years with the lowest AW of \$-85,000

(b) Defender has the lower AW at \$-85,000 for n = 3.

11.11 ESL occurs when the AW is the lowest cost.
ESL for defender is n = 4 years
ESL for challenger is n = 5 years

11.12 Add AW amounts; select lowest AW

Years Retained	Total AW
1	\$-102,000
2	-84,334
3	-84,190
4	-83,857
5	-84,294

Economic service life is n = 4 years.

11.13 (a) Find total AW for years 1, 3 , 4, and 5. Take lowest total and subtract year 2 AW values to get AW of salvage value that will produce same AW as lowest one.

Years Retained	Total AW, $
1	-83,000
2	
3	-77,127
4	-81,006
5	-89,466

Minimum AW of salvage value = -77,127 – (-46,095 – 46,000)
= $14,968

(b) 14,968 = S(A/F,10%,2)
14,968 = S(0.47619)
S = $31,433

11.14 (a) <u>*By hand:*</u>

AW_1 = -600,000(A/P,10%,1) – 92,000 + 450,000(A/F,10%,1)
= -600,000(1.1000) – 92,000 + 450,000(1.0000)
= $-302,000

AW_2 = -600,000(A/P,10%,2) – 92,000 + 300,000(A/F,10%,2)
= -600,000(0.57619) – 92,000 + 300,000(0.47619)
= $-294,857

AW_3 = -600,000(A/P,10%,3) – 92,000 + 150,000(A/F,10%,3)
= -600,000(0.40211) – 92,000 + 150,000(0.30211)
= $-287,952

AW_4 = -600,000(A/P,10%,4) – 92,000
= -600,000(0.31547) – 92,000
= $-281,282

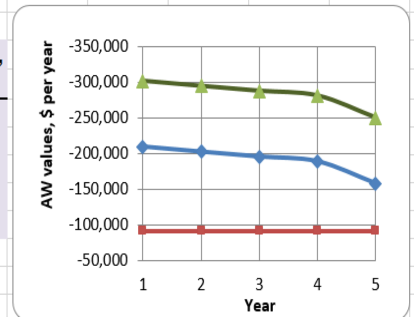
AW_5 = -600,000(A/P,10%,5) – 92,000
= -600,000(0.26380) – 92,000
= $-250,280

ESL is 5 years with AW = $-250,280

(b) *By spreadsheet:* Graph shows CR, AOC and total AW

ESL is 5 years with an AW = $-250,278 per year

	A	B	C	D	E	F
1	Interest rate	10%				
2	First cost	$ 600,000				
3		Salvage	AOC	Capital	AW of AOC,	Total AW,
4	Year	Value, $	$/year	Recovery, $/year	$/year	$/year
5	1	450,000	-92,000	-210,000	-92,000	-302,000
6	2	300,000	-92,000	-202,857	-92,000	-294,857
7	3	150,000	-92,000	-195,952	-92,000	-287,952
8	4	0	-92,000	-189,282	-92,000	-281,282
9	5	0	-92,000	-158,278	-92,000	-250,278
10						
11				ESL is 5 years with AW = $-250,278		
12						



11.15 (a) *By hand:*

$$AW_1 = -180{,}000(A/P,15\%,1) - 84{,}000 + 45{,}000(A/F,15\%,1)$$
$$= -180{,}000(1.1500) - 84{,}000 + 45{,}000(1.0000)$$
$$= \$-246{,}000$$

$$AW_2 = -180{,}000(A/P,15\%,2) - 84{,}000 + 45{,}000(A/F,15\%,2)$$
$$= -180{,}000(0.61512) - 84{,}000 + 45{,}000(0.46512)$$
$$= \$-173{,}791$$

$$AW_3 = -180{,}000(A/P,16\%,3) - [84{,}000(P/A,15\%,2) + 89{,}000(P/F,15\%,3)](A/P,15\%,3) + 45{,}000(A/F,15\%,3)$$
$$= -180{,}000(0.43798) - [84{,}000(1.6257) + 89{,}000(0.6575)](0.43798) + 45{,}000(0.28798)$$
$$= \$-151{,}317$$

$$AW_4 = -180{,}000(A/P,15\%,4) - [84{,}000(P/A,15\%,2) + 89{,}000(P/F,15\%,3) + 94{,}000(P/F,15\%,4)](A/P,15\%,4) + 45{,}000(A/F,15\%,4)$$
$$= \$-141{,}190$$

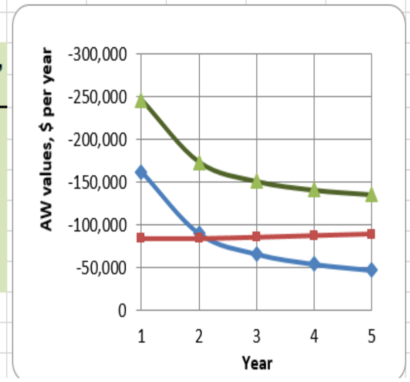
$$AW_5 = -180{,}000(A/P,15\%,5) - [84{,}000(P/A,15\%,2) + 89{,}000(P/F,15\%,3) + 94{,}000(P/F,15\%,4) + 99{,}000(P/F,15\%,5)](A/P,15\%,5) + 45{,}000(A/F,15\%,5)$$
$$= \$-135{,}934$$

ESL is 5 years with AW = $-135,934

(b) *By spreadsheet*: Graph shows CR, AOC and total AW

ESL is 5 years with an AW = $-135,934 per year

	A	B	C	D	E	F
1	Interest rate	15%				
2	First cost	$ 180,000				
3		Salvage	AOC	Capital	AW of AOC,	Total AW,
4	Year	Value, $	$/year	Recovery, $/year	$/year	$/year
5		-180,000	0			
6	1	45,000	-84,000	-162,000	-84,000	-246,000
7	2	45,000	-84,000	-89,791	-84,000	-173,791
8	3	45,000	-89,000	-65,877	-85,440	-151,317
9	4	45,000	-94,000	-54,036	-87,154	-141,190
10	5	45,000	-99,000	-47,023	-88,911	-135,934
11						
12				ESL is 5 years with AW = $-135,934		
13						



11.16 $AW_1 = -70{,}000(A/P,12\%,1) - 75{,}000 + 59{,}500(A/F,12\%,1) = \$-93{,}900$
$AW_2 = -70{,}000(A/P,12\%,2) - 75{,}000 + 50{,}575(A/F,12\%,2) = \$-92{,}563$
$AW_3 = -70{,}000(A/P,12\%,3) - 75{,}000 + 42{,}989(A/F,12\%,3) = \$-91{,}405$
$AW_4 = -70{,}000(A/P,12\%,4) - 75{,}000 + 36{,}540(A/F,12\%,4) = \$-90{,}401$
$AW_5 = -70{,}000(A/P,12\%,5) - 75{,}000 + 31{,}059(A/F,12\%,5) = \$-89{,}530$
$AW_6 = -70{,}000(A/P,12\%,6) - 75{,}000 + 26{,}400(A/F,12\%,6) = \$-88{,}773$

ESL is 6 years with AW = $-88,773 per year

11.17 (a) Solution by hand using regular AW computations

Year	Salvage Value, $	AOC, $ per year
1	100,000	70,000
2	80,000	80,000
3	60,000	90,000
4	40,000	100,000
5	20,000	110,000
6	0	120,000
7	0	130,000

$AW_1 = -150{,}000(A/P,15\%,1) - 70{,}000 + 100{,}000(A/F,15\%,1) = \$-142{,}500$

$AW_2 = -150{,}000(A/P,15\%,2) - [70{,}000 + 10{,}000(A/G,15\%,2)]$
$+ 80{,}000(A/F,15\%,2) = \$-129{,}709$

AW_3 = $-127,489
AW_4 = $-127,792
AW_5 = $-129,009
AW_6 = $-130,608
AW_7 = $-130,552

ESL = 3 years with AW_3 = $-127,489

(b) Spreadsheet below utilizes the annual marginal costs to determine that ESL is 3 years with AW = $-127,489.

	A	B	C	D	E	F	G
1							
2		Market	Loss in MV	Lost interest		MC for	AW of
3	Year	value	for year	MV for year	AOC	year	marginal cost
4	0	$150,000					
5	1	$100,000	$ 50,000	$ 22,500	$ 70,000	$142,500	$ (142,500)
6	2	$ 80,000	$ 20,000	$ 15,000	$ 80,000	$115,000	$ (129,709)
7	3	$ 60,000	$ 20,000	$ 12,000	$ 90,000	$122,000	$ (127,489)
8	4	$ 40,000	$ 20,000	$ 9,000	$100,000	$129,000	$ (127,792)
9	5	$ 20,000	$ 20,000	$ 6,000	$110,000	$136,000	$ (129,009)
10	6	$ -	$ 20,000	$ 3,000	$120,000	$143,000	$ (130,607)
11	7	$ -	$ -	$ -	$130,000	$130,000	$ (130,553)
12							
13							
14			=0.15*$B9			=SUM(C11:E11)	

11.18 (a) The three estimate changes are made in the spreadsheet: increase to $4 million for heating element exchange in year 5; market value retention of only 50% starting with year 5; and, increases of 25% per year in maintenance cost starting in year 5.

Results are significantly different. ESL is now 8 or 9 years, with a flat AW curve for several years.

	A	B	C	D	E	F
1	Interest rate	15%			First cost, $ million	38.00
2		**Market**	**AOC**	**Capital**	**AW of AOC,**	**Total AW,**
3	**Year**	**Value, $**	**$/year**	**Recovery, $/year**	**$/year**	**$/year**
4	1	25.00	-3.40	-18.70	-3.40	-22.10
5	2	18.75	-3.74	-14.65	-3.56	-18.21
6	3	14.06	-4.11	-12.59	-3.72	-16.31
7	4	10.55	-4.53	-11.20	-3.88	-15.08
8	5	5.27	-5.66	-10.55	-4.14	-14.70
9	6	2.64	-11.07	-9.74	-4.93	-14.67
10	7	1.32	-8.84	-9.01	-5.29	-14.30
11	8	0.66	-11.05	-8.42	-5.71	**-14.13**
12	9	0.33	-13.81	-7.94	-6.19	**-14.13**
13	10	0.16	-17.26	-7.56	-6.74	-14.30
14	11	0.08	-21.58	-7.26	-7.34	-14.60
15	12	0.04	-26.97	-7.01	-8.02	-15.03
16						
17		Value retains 50% as of year 5	AOC increases by 25% as of year 5; extra cost is $4M in year 6			

(b) ESL has decreased from 12 to 8 or 9 years (about a 25 to 33% decrease); AW of costs has increased from \$12.32 to \$14.13 million per year, which is an annual increase of 14.7%.

Replacement Study

11.19 In chapter 6, neither asset is currently owned. Here, one is presently in place.

11.20 Determine the ESL and AW for the new challenger. If defender estimates changed, calculate their new ESL and AW values. Select the better of D or C.

11.21 The opportunity-cost approach uses the current market value of the defender as its first cost when considering its replacement because the use of those funds (i.e., funds that remain "invested" in the defender) is foregone if the defender is retained.

11.22 The cash flow approach subtracts the salvage value of the defender from the first cost of the challenger and then calculates the AW of D and C . This is not a good procedure for the following reasons: (1) It violates the equal service requirement if the remaining life of the defender is not equal to the life of the challenger; and, (2) It yields an incorrect value for the annual cost of the challenger (lower than the true value), which could lead to incorrect capital recovery calculations.

11.23 (a) Purchase the challenger at the end of year 2, when its AW will be lower than that of the defender.

(b) Since the challenger has the same AW now as that of the defender for the next two years of retention, purchase the challenger now.

11.24 (a) AW_D = - (12,000 + 25,000)(A/P,10%,3) - 48,000 + 19,000(A/F,10%,3)
= -37,000(0.40211) - 48,000 + 19,000(0.30211)
= \$-57,138

(b) AW_C = -68,000(A/P,10%,3) - 35,000 + 21,000(A/F,10%,3)
= -68,000(0.40211) - 35,000 + 21,000(0.30211)
= \$-55,999

Select the challenger now

11.25 AW_C = -80,000(A/P,12%,3) - 19,000 + 10,000(A/F,12%,3)
= -80,000(0.41635) - 19,000 + 10,000(0.29635)
= \$-49,345

For the defender, the higher trade-in value applies, because that represents the best market value estimate

$$AW_D = -20{,}000(A/P,12\%,3) -15{,}000 - 31{,}000 + 9000(A/F,12\%,3)$$
$$= -20{,}000(0.41635) -15{,}000 - 31{,}000 + 9000(0.29635)$$
$$= \$-51{,}660$$

$AW_C < AW_D$; select the challenger

11.26 (a) $AW_D = -(7000 + 22{,}000)(A/P,10\%,3) - 27{,}000 + 12{,}000(A/F,10\%,3)$
$$= -29{,}000(0.40211) - 27{,}000 + 12{,}000(0.30211)$$
$$= \$-35{,}036$$

$$AW_C = -65{,}000(A/P,10\%,3) - 14{,}000 + 23{,}000(A/F,10\%,3)$$
$$= -65{,}000(0.40211) - 14{,}000 + 23{,}000(0.30211)$$
$$= \$-33{,}189$$

Replace the defender with the challenger

(b) Find AW_C over 2 years and compare with AW_D over 3 years

$$AW_C = -65{,}000(A/P,10\%,2) - 14{,}000 + 23{,}000(A/F,10\%,2)$$
$$= -65{,}000(0.57619) - 14{,}000 + 23{,}000(0.47619)$$
$$= \$-40{,}500$$

Now $AW_C > AW_D$; retain the defender

11.27 Find defender ESL; compare with $AW_C = \$-97{,}000$

$$AW_{D,1} = -37{,}000(A/P,10\%,1) - 85{,}000 + 30{,}000(A/F,10\%,1)$$ –
$$= -37{,}000(1.10) - 85{,}000 + 30{,}000(1.000)$$
$$= \$-95{,}700$$

$$AW_{D,2} = -37{,}000(A/P,10\%,2) - 85{,}000 + 19{,}000(A/F,10\%,2)$$
$$= -37{,}000(0.57619) - 85{,}000 + 19{,}000(0.47619)$$
$$= \$-97{,}271$$

Defender ESL is n = 1 year with $AW_D = \$-95{,}700$
Keep equipment one year and then replace with contractor

11.28 No option for retention of defender D beyond one year

$$AW_D = -9000(A/P,10\%,1) - 192{,}000$$
$$= -9000(1.1000) -192{,}000$$
$$= \$-201{,}900$$

$$AW_C = -320{,}000(A/P,10\%,4) - 68{,}000 + 50{,}000(A/F,10\%,4)$$

$= -320,000(0.31547) – 68,000 + 50,000(0.21547)$
$= \$-158,177$

Replace the defender now with System C

11.29 $AW_D = -25,000(A/P,15\%,3) - 190,000$
$= \$-200,950$

$AW_C = -600,000(A/P,15\%,10) – 70,000 + 50,000(A/F,15\%,10)$
$= \$-187,088$

Select the challenger

Spreadsheet functions:

Defender: = -PMT(15%,3,-25000) -190000 displays AW_D

Challenger: = -PMT(15%,10,-600000,50000) – 70000 displays AW_C

11.30 Determine the ESL for the defender and compare to the AW_C for the same number of years.

By hand:

$AW_{D,1} = -32,000(A/P,10\%,1) – 24,000 + 25,000(A/F,10\%,1)$
$= -32,000(1.10) -24,000 + 25,000$
$= \$-34,200$

$AW_{D,2} = -32,000(A/P,10\%,2) - [24,000 + 1000(A/G,10\%,2)] + 14,000(A/F,10\%,2)$
$= -32,000(0.57619) – [24,000 + 1000(0.4762)] + 14,000(0.47619)$
$= \$-36,247$

$AW_{D,3} = -32,000(A/P,10\%,3) - [24,000 + 1000(A/G,10\%,3)] + 10,000(A/F,10\%,3)$
$= -32,000(0.40211) – [24,000 + 1000(0.9366)] + 10,000(0.30211)$
$= \$-34,783$

ESL for defender is n = 1 year with $AW_D = \$-34,200$

$AW_C = \$-33,000 < AW_D$; replace the defender with the challenger now.

By spreadsheet: Defender ESL is n = 1; select challenger now

	A	B	C	D	E–J
1		Defender			
2	Year	MV, $	M&O, $	AW, $	PMT Function
3	0	-32,000			
4	1	25,000	-24,000	-34,200	= - PMT(10%,$A4,$B$3,$B4) - PMT(10%,$A4,NPV(10%,$C$4:$C4)+0)
5	2	14,000	-25,000	-36,248	= - PMT(10%,$A5,$B$3,$B5) - PMT(10%,$A5,NPV(10%,$C$4:$C5)+0)
6	3	10,000	-26,000	-34,783	= - PMT(10%,$A6,$B$3,$B6) - PMT(10%,$A6,NPV(10%,$C$4:$C6)+0)

11.31 Find ESL of the defender; compare with AW_C over 5 years.

$$
\begin{aligned}
AW_{D,1} &= -8000(A/P,15\%,1) - 50{,}000 + 6000(A/F,15\%,1) \\
&= -8000(1.15) - 44{,}000 \\
&= \$-53{,}200
\end{aligned}
$$

$$
\begin{aligned}
AW_{D,2} &= -8000(A/P,15\%,2) - 50{,}000 + (-3000 + 4000)(A/F,15\%,2) \\
&= -8000\,(0.61512) - 50{,}000 + 1000(0.46512) \\
&= \$-54{,}456
\end{aligned}
$$

$$
\begin{aligned}
AW_{D,3} &= -8000(A/P,15\%,3) - [50{,}000(P/F,15\%,1) + 53{,}000(P/F,15\%,2)](A/P,15\%,3) + (-60{,}000 + 1000)(A/F,15\%,3) \\
&= -8000\,(0.43798) - [50{,}000(0.8696) + 53{,}000(0.7561)](0.43798) - 59{,}000(0.28798) \\
&= -\$57{,}089
\end{aligned}
$$

The ESL is 1 year with $AW_{D,1}$ = \$-53,200

Challenger: Salvage value: 125,000(0.08) = \$10,000
AOC per year: 125,000(0.20) = \$25,000

$$
\begin{aligned}
AW_C &= -125{,}000(A/P,15\%,5) - 25{,}000 + 10{,}000(A/F,15\%,5) \\
&= -125{,}000(0.29832) - 25{,}000 + 10{,}000(0.14832) \\
&= \$-60{,}807
\end{aligned}
$$

Since the ESL $AW_{D,1}$ value is lower than the challenger AW_C, Randall-Rico should keep the defender now and replace it after 3 years.

Replacement Study over a Specified Study Period

11.32 Compare cost of replacement (challenger) with cost of keeping defender (in-place) for 3 more years

$$
\begin{aligned}
AW_C &= -80{,}000(A/P,10\%,3) - 37{,}000 + 20{,}000(A/F,10\%,3) \\
&= -80{,}000(0.40211) - 37{,}000 + 20{,}000(0.30211) \\
&= \$-63{,}127
\end{aligned}
$$

$$
\begin{aligned}
AW_D &= -40{,}000(A/P,10\%,3) - 55{,}000 + 11{,}000(A/F,10\%,3) \\
&= -40{,}000(0.40211) - 55{,}000 + 11{,}000(0.30211) \\
&= \$-67{,}761
\end{aligned}
$$

Replace in-place with challenger now, since $AW_C < AW_D$

11.33 Option 1 (lease) is defender; option 2 is challenger. Find AW of lease over 5-year period

$$
\begin{aligned}
AW_D &= [-180{,}000 - 180{,}000(P/A,15\%,4)](A/P,15\%,5) \\
&= [-180{,}000 - 180{,}000(2.8550)](0.29832) \\
&= \$-207{,}004
\end{aligned}
$$

$$
\begin{aligned}
AW_C &= -1{,}600{,}000(A/P,15\%,5) - 58{,}000 + 220{,}000 + (0.30)(1{,}600{,}000)(A/F,15\%,5) \\
&= -1{,}600{,}000(0.29832) + 162{,}000 + (480{,}000)(0.14832) \\
&= \$-244{,}118
\end{aligned}
$$

Select option 1, continue to lease the land

11.34 (a) Option 1: Keep defender for 2 years and then replace it with challenger for 1 year

$$
\begin{aligned}
AW_1 &= -70{,}000 - (80{,}000 - 70{,}000)(A/F,15\%,3) \\
&= -70{,}000 - 10{,}000(0.28798) \\
&= \$-72{,}880
\end{aligned}
$$

Option 2: Replace now with challenger for all 3 years

$$AW_2 = \$-75{,}000$$

Keep the machine 2 years and then replace with the challenger

(b) The annual worth will be $AW_1 = \$-72{,}880$

11.35 *By hand:*

There are three options for a 2-year study period

Option	Keep defender X	Use challenger Y
A	0 years	2 years
B	1 year	1 year
C	2 years	0 years

$$\begin{aligned} AW_{X,1} &= -82{,}000(A/P,12\%,1) - 30{,}000 + 50{,}000(A/F,12\%,1) \\ &= -82{,}000(1.1200) - 30{,}000 + 50{,}000(1.0000) \\ &= \$-71{,}840 \end{aligned}$$

$$\begin{aligned} AW_{X,2} &= -82{,}000(A/P,12\%,2) - 30{,}000 + 42{,}000(A/F,12\%,2) \\ &= -82{,}000(0.59170) - 30{,}000 + 42{,}000(0.47170) \\ &= \$-58{,}708 \end{aligned}$$

$$\begin{aligned} AW_{Y,1} &= -97{,}000(A/P,12\%,1) - 27{,}000 + 66{,}000(A/F,12\%,1) \\ &= -97{,}000(1.1200) - 27{,}000 + 66{,}000(1.0000) \\ &= \$-69{,}640 \end{aligned}$$

$$\begin{aligned} AW_{Y,2} &= -97{,}000(A/P,12\%,2) - 27{,}000 + 56{,}000(A/F,12\%,2) \\ &= -97{,}000(0.59170) - 27{,}000 + 56{,}000(0.47170) \\ &= \$-57{,}980 \end{aligned}$$

Option	Years for X	Years for Y	AW cash flows, $ per year		Option AW, $ per year
			Year 1	Year 2	
A	0	2	-57,980	-57,980	-57,980
B	1	1	-71,840	-69,640	-70,802
C	2	0	-58,708	-58,708	-58,708

$AW_B = -71{,}840 + (71{,}840 - 69{,}640)(A/F,12\%,2) = \$-70{,}802$

Lowest AW is $-57,980 by selling robot X and purchasing robot Y for the 2 years.

By spreadsheet: Select option A (Buy robot Y now)

	A	B	C	D	E	F	G
1		Option A: X for 0; Y for 2 years					
2	$AW_{Y,2}$	-57,980	= -PMT(12%,2,-97000,56000) - 27000				
3							
4		Option B: X for 1; Y for 1 year					
5	$AW_{X,1}$	-71,840	= -PMT(12%,1,-82000,50000) - 30000				
6	$AW_{Y,1}$	-69,640	= -PMT(12%,1,-97000,66000) - 27000				
7							
8		Option C: X for 2; Y for 0 years					
9	$AW_{X,2}$	-58,708	= -PMT(12%,2,-82000,42000)-30000				
10							
11		AW by year, $		AW for			
12	Option	Year 1	Year 2	option, $			
13	A	-57,980	-57,980	-57,980	Select option A; sell X, buy Y		
14	B	-71,840	-69,640	-70,802			
15	C	-58,708	-58,708	-58,708			

11.36 (a) Two options; (1) upgrade and retain defender for 3 years, or (2) buy challenger now

$$AW_1 = -(40{,}000 + 79{,}000)(A/P,15\%,3) - 85{,}000 + 30{,}000(A/F,15\%,3)$$
$$= -119{,}000(0.43798) - 85{,}000 + 30{,}000(0.28798)$$
$$= \$-128{,}480$$

$$AW_2 = -220{,}000(A/P,15\%,3) - 45{,}000 + 50{,}000(A/F,15\%,3)$$
$$= -220{,}000(0.43798) - 45{,}000 + 50{,}000(0.28798)$$
$$= \$-126{,}957$$

Select option 2; replace the defender now

(b) AW_2 = \$-126,957 for 3 years

(c) AW_1 function: = -PMT(15%,3,-119000,30000) - 85000
AW_2 function: = -PMT(15%,3,-220000,50000) – 45000

(d) AW_2 function for 8 years: = -PMT(15%,8,-220000,10000) – 45000

The display is AW_2 = \$-93,299. This is a reduction of \$33,658 per year in capital recovery from the AW amount of \$-126,957over the 3-year study period in (a).

(Note: This amount is correct since the AOC is constant at \$45,000 per year regardless of the number of years in service.)

11.37 All evaluations are performed at an effective $i = 1\%$ per month

(a) For 1-year study period

$$AW_{K,1} = -160{,}000(A/P,1\%,12) - 7000 + 50{,}000(A/F,1\%,12)$$
$$= -160{,}000(0.08885) - 7000 + 50{,}000(0.07885)$$
$$= \$-17{,}274$$

$$AW_{L,1} = -210{,}000(A/P,1\%,12) - 5000 + 100{,}000(A/F,1\%,12)$$
$$= -210{,}000(0.08885) - 5000 + 100{,}000(0.07885)$$
$$= \$-15{,}774$$

Process L is better

(b) For 2-year study period

$$AW_{K,2} = -160{,}000(A/P,1\%,24) - 7000 + 40{,}000(A/F,1\%,24)$$
$$= -160{,}000(0.04707) - 7000 + 40{,}000(0.03707)$$
$$= \$-13{,}048$$

$$AW_{L,2} = -210{,}000(A/P,1\%,24) - 5000 + 70{,}000(A/F,1\%,24)$$
$$= -210{,}000(0.04707) - 5000 + 70{,}000(0.03707)$$
$$= \$-12{,}290$$

Process L is better

(c) For 3-year study period, repurchase process K machine for 1 year after 24 months

$$AW_{K,3} = -160{,}000(A/P,1\%,36) - 7000$$
$$+ (-160{,}000 + 40{,}000)(P/F,1\%,24)(A/P,1\%,36) + 50{,}000(A/F,1\%,36)$$
$$= -160{,}000(0.03321) - 7000 - 120{,}000(0.7876)(0.03321)$$
$$+ 50{,}000(0.02321)$$
$$= \$-14{,}292$$

$$AW_{L,3} = -210{,}000(A/P,1\%,36) - 5000 + 45{,}000(A/F,1\%,36)$$
$$= -210{,}000(0.03321) - 5000 + 70{,}000(0.02321)$$
$$= \$-10{,}349$$

Process L is still better

11.38 Use the market value estimates in Example 11.3 (Figure 11-3) to calculate CR for n = 6 and n = 12 years for the challenger GH. In \$ million units,

$$n = 6 \text{ years: } CR = -38(A/P,15\%,6) + 5.93(A/F,15\%,6)$$
$$= -38(0.26424) + 5.93(0.11424)$$

= \$-9.36 (\$-9.36 million)

n = 12 years: CR = -38(A/P,15%,12) + 1.06(A/F,15%,12)
= -38(0.18448) + 1.06(0.03448)
= \$-6.97 (\$-6.97 million)

Required revenue to recover \$38 million first cost plus 15% per year is reduced over 25% if the full 12-year life is considered rather than the abbreviated 6-year study period.

Replacement Value

11.39 -RV(A/P,20%,3) – 272,000 + 150,000(A/F,20%,3) = -2,200,000(A/P,20%,3) – 340,000
+ 595,000 + 800,000(A/F.20%,3)

-RV(0.47473) – 272,000 + 150,000(0.27473) = -2,200,000(0.47473) + 255,000
+ 800,000(0.27473)

-RV(0.47473) = -338,832
RV = \$713,735

Probably can't sell presently-owned MRI for anything close to \$713,735. Therefore, keep presently-owned MRI

11.40 (a) Relations to determine RV or minimum trade-in value

-RV(A/P,12%,7) – 27,000 + 40,000(A/F,12%,7) = -370,000(A/P,12%,12)
- 50,000 + 22,000(A/F,12%,12)

-RV(0.21912) – 27,000 + 40,000(0.09912) = -370,000(0.16144)
- 50,000 + 22,000(0.04144)

-RV(0.21912) = -108,821 + 23,035

RV = \$391,502

(b) Spreadsheet functions:
AW for defender: = - PMT(12%,7,\$B\$1,40000) – 27000, if the RV value sought is in cell B1
AW for challenger: = - PMT(12%,12,-370000,22000) - 50000 displays \$-108,821

Use Goal Seek to find RV = \$391,501

11.41 (a) *By hand:*

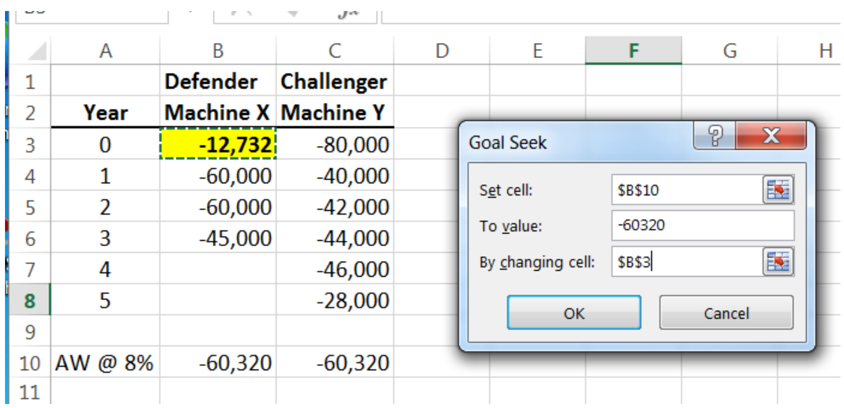
-RV(A/P,8%,3) – 60,000 + 15,000(A/F,8%,3) = -80,000(A/P,8%,5)
 - [40,000 + 2000(A/G,8%,5)] + 20,000(A/F,8%,5)

-RV(0.38803) – 60,000 + 15,000(0.30803) = -80,000(0.25046)
 - [40,000 + 2000(1.8465)] + 20,000(0.17046)

-0.38803 RV = -4941.05
RV = $12,734

(b) *By spreadsheet:*

Estimates must be entered since there is no spreadsheet function for gradients. Use Goal Seek to find RV = $-12,732 (cell B3)



11.42 *By hand:*

-RV(A/P,12%,4) – [40,000 + 2000(A/G,12%,4)] = -150,000(A/P,12%,10)
 - [10,000 + 500(A/G,12%,10)] + 50,000(A/F,12%,10)

-RV(0.32923) – [40,000 + 2000(1.3589)] = -150,000(0.17698)
 - [10,000 + 500(3.5847)] + 50,000(0.05698)

-0.32923RV = -35,490 + 42,718

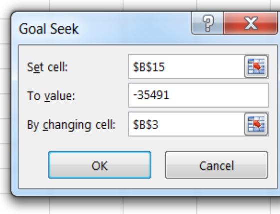
RV = $-21,954

The RV value is a negative amount, meaning that the trade-in amount of $21,954 should be paid by the owner of Machine A in order to buy Machine B.

By spreadsheet:

Note that the RV value found by Goal Seek is positive, indicating that the trade-in value is in favor of the seller of machine B, that is, machine A owner should pay the $21,950 to get rid of A to purchase B.

	A	B	C
1		Defender	Challenger
2	Year	Machine A	Machine B
3	0	21,950	-150,000
4	1	-40,000	-10,000
5	2	-42,000	-10,500
6	3	-44,000	-11,000
7	4	-46,000	-11,500
8	5		-12,000
9	6		-12,500
10	7		-13,000
11	8		-13,500
12	9		-14,000
13	10		35,500
14			
15	AW @ 12%	-35,491	-35,491
16			



11.43 $-RV(A/P,15\%,1) – 53,000 = -226,000(A/P,15\%,10) – 48,000 + 60,000(A/F,15\%,10)$

$-RV(1.15) – 53,000 = -226,000(0.19925) – 48,000 + 60,000(0.04925)$

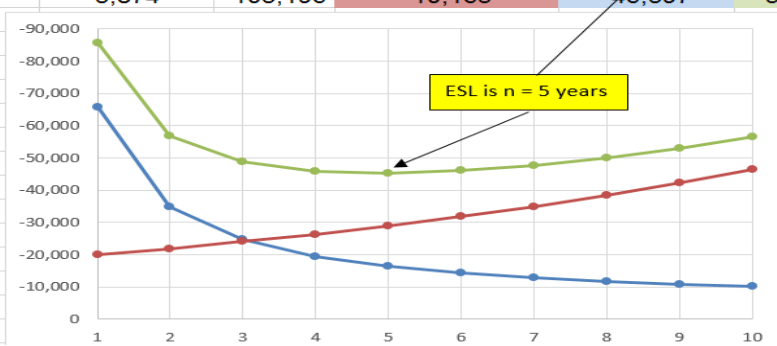
$-1.15RV = -90,076 + 53,000$

Minimum RV = $32,240

Spreadsheet Exercises

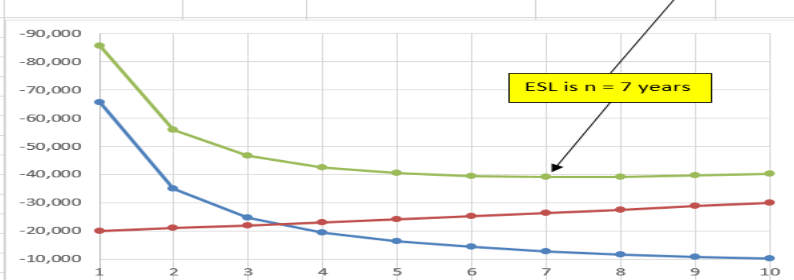
11.44 (a) ESL is n = 5 years with an AW = $-45,363 per year

	A	B	C	D	E	F
1	Interest rate	8%				
2	First cost	$ 70,000				
3		Market	AOC	Capital	AW of AOC,	Total AW,
4	Year	Value, $	$/year	Recovery, $/year	$/year	$/year
5	1	10,000	-20,000	-65,600	-20,000	-85,600
6	2	9,000	-24,000	-34,927	-21,923	-56,850
7	3	8,100	-28,800	-24,667	-24,041	-48,709
8	4	7,290	-34,560	-19,517	-26,376	-45,892
9	5	6,561	-41,472	-16,414	-28,949	-45,363
10	6	5,905	-49,766	-14,337	-31,787	-46,124
11	7	5,314	-59,720	-12,849	-34,917	-47,767
12	8	4,783	-71,664	-11,731	-38,372	-50,103
13	9	4,305	-85,996	-10,861	-42,186	-53,047
14	10	3,874	-103,196	-10,165	-46,397	-56,562



(b) ESL is now n = 7 years and the total AW curve is very flat between about n = 6 to 8, making the ESL less sensitive to changes in estimates.

	A	B	C	D	E	F
1	Interest rate	8%				
2	First cost	$ 70,000				
3		Market	AOC	Capital	AW of AOC,	Total AW,
4	Year	Value, $	$/year	Recovery, $/year	$/year	$/year
5	1	10,000	-20,000	-65,600	-20,000	-85,600
6	2	9,000	-22,000	-34,927	-20,962	-55,888
7	3	8,100	-24,200	-24,667	-21,959	-46,626
8	4	7,290	-26,620	-19,517	-22,993	-42,510
9	5	6,561	-29,282	-16,414	-24,065	-40,479
10	6	5,905	-32,210	-14,337	-25,176	-39,513
11	7	5,314	-35,431	-12,849	-26,325	-39,174
12	8	4,783	-38,974	-11,731	-27,514	-39,246
13	9	4,305	-42,872	-10,861	-28,744	-39,605
14	10	3,874	-47,159	-10,165	-30,015	-40,180



11.45 Spreadsheet verification below: ESL for defender is n = 1; select defender over challenger with AW_C = $-60,806

	A	B	C	D	E	F	G
1	Interest rate	15%					
2	First cost	-8,000		DEFENDER ESL ANALYSIS			
3		Market	AOC	Capital	AW of AOC,	Total AW,	
4	Year	Value, $	$/year	Recovery, $/year	$/year	$/year	
5	1	6,000	-50,000	-3,200	-50,000	-53,200	ESL
6	2	4,000	-53,000	-3,060	-51,395	-54,456	
7	3	1,000	-60,000	-3,216	-53,873	-57,089	
8							
9				CHALLENGER AW ANALYSIS			
10							
11	First cost	-125,000				-60,806	

(a) Write the challenger PMT function in cell-reference format in cell F11

= - PMT($B$1,5,$B$11,-$B$11*0.08) + $B$11*0.2

Use Goal Seek to display the required first cost of
P_C = $-109,364

(b) Use Goal seek to force $AW_{D,1}$ = $-60,806 (cell F5). Required trade-in/market value is now $14,614.

Correct economic decision? No, because the new trade-in value increases the ESL to n = 2 with $AW_{D,2}$ = $-58,524, which is lower that AW_C $-60,806. Analysis is below.

	A	B	C	D	E	F	G
1	Interest rate	15%					
2	Market value	-14,614		DEFENDER ESL ANALYSIS			
3		Market	AOC	Capital	AW of AOC,	Total AW,	
4	Year	Value, $	$/year	Recovery, $/year	$/year	$/year	
5	1	6,000	-50,000	-10,806	-50,000	-60,806	
6	2	4,000	-53,000	-7,129	-51,395	-58,524	ESL
7	3	1,000	-60,000	-6,113	-53,873	-59,986	
8							
9				CHALLENGER AW ANALYSIS			
10							
11	First cost	-125,000				-60,806	

11.46 Process M (upgrade current machinery) is economically better (cell B11)

	A	B	C	D	E	F
1	**Process K**	**2-year study period**				
2						
3	$AW_{K,2}$	-$13,049	= - PMT(1%,24,-160000,40000) - 7000			
4						
5	**Process L**	**2-year study period**				
6						
7	$AW_{L,2}$	-$12,290	= - PMT(1%,24,-210000,70000) - 5000			
8						
9	**Process M**	**Upgrade for 2-year study period**				
10						
11	$AW_{M,2}$	-$11,354	= - PMT(1%,24,-50000,0) - 9000			
12						

Additional Problems and FE Exam Review Questions

11.47 Answer is (a)

11.48 Answer is (d)

11.49 Answer is (d)

11.50 Answer is (b)

11.51 Answer is (c)

11.52 For a 3-year study period, there is no combination of defender and challenger machines that will be as low as $-70,000 by keeping the defender.
Answer is (d).

11.53 Answer is (b)

11.54 Add AW values for first cost, operating cost, and salvage value; select lowest AW of 2 years.
Answer is (a)

Years Retained	Total AW
1	$-102,000
2	-83,334
3	-84,190
4	-84,857
5	-84,294

11.55 -RV(A/P,10%,5) – 120,000 + 40,000(A/F,10%,5) = -670,000(A/P,10%,10) – 94,000 + 60,000(A/F,10%,10)

$-RV(0.26380) – 120,000 + 40,000(0.16380) = -670,000(0.16275) – 94,000$
$+ 60,000(0.06275)$

$RV = \$325,358$
Answer is (c)

11.56 $AW_2 = -32,000(A/P,10\%,2) - 24,000 + 14,000(A/F,10\%,2)$
$= -32,000(0.57619) - 24,000 + 14,000(0.47619)$
$= \$-35,771$
Answer is (c)

11.57 Answer is (a)

11.58 $AW_{D,2} = \$-13,700$ is less than $AW_{C,3} = \$-13,800$
Answer is (c)

Solution to Case Study, Chapter 11

Sometimes, there is not a definitive answer to a case study exercise. Here are example responses.

A PUMPER SYSTEM WITH AN ESL PROBLEM

1. The ESL is 13 years. Year 13 is predicted to require the 4th rebuild; the pump will not be used beyond 13 years anyway.

	A	B	C	D	E	F	G	H	I	J	K
1			#1. Find the ESL								
2									Operating	Cumulative	
3								Year	hours	hours	
4		First cost &		Capital	AW of AOC	Total		1	500	500	
5	Year	rebuild cost	AOC	recovery	and rebuild	AW		2	1500	2000	
6	0	$ (800,000)						3	2000	4000	
7	1	$ -	$ (25,000)	$ (880,000)	$ (25,000)	$ (905,000)		4	2000	6000	Rebuild
8	2	$ -	$ (25,000)	$ (460,952)	$ (25,000)	$ (485,952)		5	2000	8000	
9	3	$ -	$ (25,000)	$ (321,692)	$ (25,000)	$ (346,692)		6	2000	10000	
10	4	$ (150,000)	$ (25,000)	$ (252,377)	$ (57,321)	$ (309,697)		7	2000	12000	Rebuild
11	5	$ -	$ (40,000)	$ (211,038)	$ (54,484)	$ (265,522)		8	2000	14000	
12	6	$ -	$ (46,000)	$ (183,686)	$ (53,384)	$ (237,070)		9	2000	16000	
13	7	$ (180,000)	$ (52,900)	$ (164,324)	$ (72,306)	$ (236,630)		10	2000	18000	Rebuild
14	8	$ -	$ (60,835)	$ (149,955)	$ (71,303)	$ (221,258)		11	2000	20000	
15	9	$ -	$ (69,960)	$ (138,912)	$ (71,204)	$ (210,116)		12	2000	22000	
16	10	$ (216,000)	$ (80,454)	$ (130,196)	$ (85,337)	$ (215,534)		13	2000	24000	Replace
17	11	$ -	$ (92,522)	$ (123,171)	$ (85,725)	$ (208,896)					
18	12	$ -	$ (106,401)	$ (117,411)	$ (86,692)	$ (204,103)					
19	13	$ -	$ (122,361)	$ (112,623)	$ (88,147)	$ (200,769)	ESL				
20											
21		Answer: ESL is 13 years with AW = $-200,769									

2. Required MV = $1,420,983 found using Solver with F12 the target cell and B12 the changing cell. This MV is well above the first cost of $800,000.

	A	B	C	D	E	F	G	H	I	J	K	L
1	#2. Find required market value at end of year 6 to make ESL be n = 6 years											
2									Operating	Cumulative		
3								Year	hours	hours		
4		First cost &		Capital	AW of AOC	Total		1	500	500		
5	Year	rebuild cost	AOC	recovery	and rebuild	AW		2	1500	2000		
6	0	$ (800,000)						3	2000	4000		
7	1	$ -	$ (25,000)	$ (880,000)	$ (25,000)	$ (905,000)		4	2000	6000	Rebuild	
8	2	$ -	$ (25,000)	$ (460,952)	$ (25,000)	$ (485,952)		5	2000	8000		
9	3	$ -	$ (25,000)	$ (321,692)	$ (25,000)	$ (346,692)		6	2000	10000		
10	4	$ (150,000)	$ (25,000)	$ (252,377)	$ (57,321)	$ (309,697)		7	2000	12000	Rebuild	
11	5	$ -	$ (40,000)	$ (211,038)	$ (54,484)	$ (265,522)		8	2000	14000		
12	6	$ 1,420,983	$ (46,000)	$ (183,686)	$ 130,786	$ (52,900)	ESL	9	2000	16000		
13	7	$ -	$ (52,900)	$ (164,324)	$ 111,424	$ (52,900)		10	2000	18000	Rebuild	
14	8	$ -	$ (60,835)	$ (149,955)	$ 96,361	$ (53,594)		11	2000	20000		
15	9	$ -	$ (69,960)	$ (138,912)	$ 84,113	$ (54,799)		12	2000	22000		
16	10	$ -	$ (80,454)	$ (130,196)	$ 73,787	$ (56,409)		13	2000	24000	Replace	
17	11	$ -	$ (92,522)	$ (123,171)	$ 64,813	$ (58,358)						
18	12	$ -	$ (106,401)	$ (117,411)	$ 56,806	$ (60,604)						
19	13	$ -	$ (122,361)	$ (112,623)	$ 49,500	$ (63,123)						
20												
21	Answer: The market value would be extremely high at $1.42 million to make ESL be 6 years.											
22		This is substantially more than the pump cost new at $800,000.										

3. Solver yields the base AOC = $-201,983 in year 1 with increases of 15% per year. The rebuild cost in year 4 (after 6000 hours) is $150,000. This AOC series is huge compared to the estimated AOC of $25,000 (years 1 – 4).

	A	B	C	D	E	F	G	H	I	J	K
1	#3. Find the base AOC to make ESL be n = 6 years; no rebuild done										
2									Operating	Cumulative	
3		AOC, $/year	-$201,982.83					Year	hours	hours	
4		First cost &		Capital	AW of AOC	Total		1	500	500	
5	Year	rebuild cost	AOC	recovery	and rebuild	AW		2	1500	2000	
6	0	$ (800,000)						3	2000	4000	
7	1	$ -	$ (201,983)	$ (880,000)	$ (201,983)	$ (1,081,983)		4	2000	6000	
8	2	$ -	$ (232,280)	$ (460,952)	$ (216,410)	$ (677,363)		5	2000	8000	
9	3	$ -	$ (267,122)	$ (321,692)	$ (496,520)	$ (818,212)		6	2000	10000	Sell
10	4	$ -	$ (307,191)	$ (252,377)	$ (247,990)	$ (500,367)					
11	5	$ -	$ (353,269)	$ (211,038)	$ (265,235)	$ (476,273)					
12	6	$ -	$ (406,260)	$ (183,686)	$ (283,513)	$ (467,199)	ESL				
13	7		$ (467,199)	$ (164,324)	$ (302,874)	$ (467,199)					
14											
15		Answer: This is also not very reasonable. The AOC base in year 1 would have to be very large									
16			at $201,982 per year to force ESL to be 6 years.								

4. Compare the results in #2 and #3 with that in #1 and comment on them.

Chapter 12
Independent Projects with Budget Limitation

Capital Rationing and Independent Projects

12.1 1. Several independent projects are identified with cash flow estimates for each
2. Each project is selected entirely or not at all
3. A budgetary constraint is identified.
4. Objective is to maximize return on available funds

12.2 A *contingent project*: has a condition placed on its acceptance or rejection while a *dependent project* is accepted or rejected based on the decision about another project

12.3 (a) $2^5 = 32$; (b) $2^8 = 256$; (c) $2^{13} = 8192$; (d) Reduced by only 1 in all cases

12.4 DN, A, B, C, AB, AC, BC, ABC

12.5 There are a total of $2^6 = 64$ possible bundles; only 9 are within budget constraint of $31,000 as follows:

Bundle	Total PW, $
Q	2,000
P	6,000
QP	8,000
M	11,000
MQ	13,000
MP	17,000
L	29,000
LQ	31,000
MPQ	19,000

12.6

(a) Acceptable	(b) Non-acceptable
DN	2
1	1,2
3	1,4
4	2,4
1,3	1,2,3
2,3	1,2,4
3,4	1,3,4
	2,3,4

1,2,3,4

12.7	Acceptable:	Non-acceptable
	DN	2
	1	1,5
	3	1,2
	4	1,2,5
	5	1,2,3
	1,3	1,4,5
	1,4	1,3,5
	3,4	1,3,4,5
	3,5	2,3
	4,5	2,4
	2,4,5	2,5
	1,3,4	1,2,4
	3,4,5	1,2,4,5
	2,3,4,5	2,3,4
		2,3,5
		1,2,3,4
		1,2,3,5
		1,2,3,4,5

Selecting Independent Projects using PW Analysis

12.8 (a) Select the bundle with the largest positive PW value that does not violate the budget limit of $50,000. Select bundle 3.

(b) The leftover $6000 is assumed to be invested elsewhere at a return of 15% per year.

12.9 Eliminate projects 1 and 4, since PW < 0

(a) Select projects 2, 3 and 5 with PW > 0 at 18%

(b) Of 2^5 = 32 bundles, list acceptable bundles and PW values. Select project 5 with largest PW of $9800.

	A	B	C
1	**Project bundle**	**Investment, $**	**PW, $**
2	DN	0	0
3	2	-25,000	8500
4	3	-20000	500
5	5	-52,000	9800
6	2,3	-45,000	9000

12.10 (a) <u>*By hand*</u>

Sample calculations of PW for bundles

$$PW_A = -25{,}000 + 6000(P/A,15\%,4) + 4000(P/F,15\%,4)$$
$$= -25{,}000 + 6000(2.8550) + 4000(0.5718)$$
$$= \$-5583$$

$$PW_{A,B} = -55{,}000 + 15{,}000(P/A,15\%,4) + 3000(P/F,15\%,4)$$
$$= -55{,}000 + 15{,}000(2.8550) + 3000(0.5718)$$
$$= \$-10{,}460$$

Bundle	Proposals	PW at 15%, $
1	A	-5583
2	B	-4877
3	C	4261
4	A,B	-10,460
5	A,C	-1322
6	DN	0

Select bundle 3 (project C) with the largest, and only, PW > 0

(b) <u>*By spreadsheet:*</u> Select bundle 4 (Project C) with the only PW > 0

	A	B	C	D	E	F	G
1	MARR =	15%					
2							
3	Bundle	1	2	3	4	5	6
4	Projects	DN	A	B	C	AB	AC
5	Year						
6	0	0	-25,000	-30,000	-50,000	-55,000	-75,000
7	1	0	6,000	9,000	15,000	15,000	21,000
8	2	0	6,000	9,000	15,000	15,000	21,000
9	3	0	6,000	9,000	15,000	15,000	21,000
10	4	0	10,000	8,000	35,000	18,000	45,000
11							
12							
13							
14							
15							
16	PW @ 15%	0	-5,583	-4,877	4,260	-10,460	-1,323

= NPV($B$1,C7:C15)+C6

12.11 (a) <u>*By hand:*</u>

Of the 2^4 = 16 possible bundles, 7 are within the $800,000 limit:
DN, R1, S2, T3, U4, R1 & S2, and R1 & T3. (Values are in $1000 units.)

$$PW_{DN} = \$0$$

$$PW_{R1} = -200 + (150 - 50)(P/A,20\%,5)$$
$$= -200 + 100(2.9906)$$
$$= \$99.06 \qquad (\$99{,}060)$$

$$PW_{S2} = -400 + (450 - 200)(P/A,20\%,5)$$
$$= -400 + 250(2.9906)$$
$$= \$347.650 \qquad (\$347,650)$$

$$PW_{T3} = -500 + (520 - 300)(P/A,20\%,5)$$
$$= -500 + 220(2.9906)$$
$$= \$157.932 \qquad (\$157,932)$$

$$PW_{U4} = -700 + (770 - 400)(P/A,20\%,5)$$
$$= -700 + 370(2.9906)$$
$$= \$406.522 \qquad (\$406,522)$$

$$PW_{R1,S2} = 99.060 + 347.650$$
$$= \$446.710 \qquad (\$446,710)$$

$$PW_{R1,T3} = 99.060 + 157.932$$
$$= \$256.992 \qquad (\$256,992)$$

Select product lines R1 and S2 with the largest PW = $446,710

(b) *By spreadsheet:*

$800,000 limit: Select product lines R1 and S2 with PW = $446,714 (column G)

$900,000 limit: Either products R1 and U4 or S2 and T3 can be selected as their PW values are (essentially) the same at $505,588 (columns I and J)

	A	B	C	D	E	F	G	H	I	J
1	MARR =	20%		Cash flows are in $1000 units						
2										
3	Bundle	1	2	3	4	5	6	7	8	9
4	Products	DN	R1	S2	T3	U4	R1,S2	R1,T3	R1,U4	S2,T3
5	Year									
6	0	0	-200	-400	-500	-700	-600	-700	-900	-900
7	1	0	100	250	220	370	350	320	470	470
8	2	0	100	250	220	370	350	320	470	470
9	3	0	100	250	220	370	350	320	470	470
10	4	0	100	250	220	370	350	320	470	470
11	5		100	250	220	370	350	320	470	470
12										
13										
14										
15										
16	PW @ 20%	0	99.061	347.653	157.935	406.526	446.714	256.996	505.588	505.588
17							(a) Selection		(b) Selection is	
18							@ $800000		basically the same @	
19							limit		$900,000 limit	

12.12 Develop bundles with less than $240,000 investment; select the one with the largest PW.

Bundle	Projects	Investment, $	NCF, $/Year	PW, $
1	A	-100,000	50,000	166,746
2	B	-125,000	24,000	3,038
3	C	-120,000	75,000	280,118

4	E	-220,000	39,000	-11,939
5	F	-200,000	82,000	237,462
6	AB	-225,000	74,000	169,784
7	AC	-220,000	125,000	446,864
8	DN	0	0	0

PW_1 = -100,000 + 50,000(P/A,10%,8)
= -100,000 + 50,000(5.3349)
= $166,746

PW_2 = -125,000 + 24,000(P/A,10%,8)
= -125,000 + 24,000(5.3349)
= $3,038

PW_3 = -120,000 + 75,000(P/A,10%,8)
= -120,000 + 75,000(5.3349)
= $280,118

PW_4 = -220,000 + 39,000(P/A,10%,8)
= -220,000 + 39,000(5.3349)
= $-11,939

PW_5 = -200,000 + 82,000(P/A,10%,8)
= -200,000 + 82,000(5.3349)
= $237,462

All other PW values are obtained by adding the respective PW for bundles 1 through 5.

Conclusion: Select PW = $446,864, which is bundle 7 (projects A and C) with $220,000 total investment.

12.13 (a) Budget = $800,000i = 10%   6 viable bundles

Bundle	Projects	NCF_{j0}, $	NCF_{jt}, $	SV, $	PW at 10%, $
1	X	-250,000	50,000	45,000	-60,770
2	Y	-300,000	90,000	-10,000	-21,539
3	Z	-550,000	150,000	100,000	-6,215
4	XY	-550,000	140,000	35,000	-82,309
5	XZ	-800,000	200,000	145,000	-66,985
6	DN	0	0	0	0

$PW_j = NCF_j(P/A,10\%,4) + S(P/F,10\%,4) - NCF_{j0}$

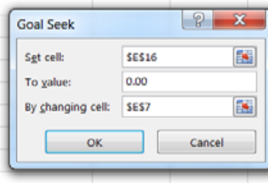
Since no bundle has PW > 0, select the ‘Do nothing’ project.

(b) Set $PW_3 = 0$ and determine NCF_3

$$PW_3 = 0 = NCF_3(P/A,10\%,4) + 100{,}000(P/F,10\%,4) – 550{,}000$$
$$0 = NCF_3(3.1699) + 100{,}000(0.6830) – 550{,}000$$
$$NCF_3 = \$151{,}961$$

(c) Part (a): Since no PW > 0. Select DN, bundle 1 (no projects)
Part (b): Use Goal Seek to determine NCF_3 = \$151,962 to obtain $PW_4 = 0$ (cell E7)

	A	B	C	D	E	F	G
1	MARR =	10%					
2							
3	Bundle	1	2	3	4	5	6
4	Projects	DN	X	Y	Z	XY	XZ
5	Year						
6	0	0	-250,000	-300,000	-550,000	-550,000	-800,000
7	1	0	50,000	90,000	151,962	140,000	200,000
8	2	0	50,000	90,000	151,962	140,000	200,000
9	3	0	50,000	90,000	151,962	140,000	200,000
10	4	0	95,000	80,000	251,962	175,000	345,000
11							
12					= E$7 + 100000		
13							
14							
15							
16	PW @ 10%	0	-60,771	-21,542	0	-82,313	-66,990



12.14 Two assumptions are (1) every project will last for the period of the longest-lived project, and (2) reinvestment of any positive net cash flows is assumed to be at the MARR from the time they are realized until the end of the longest-lived project.

12.15 <u>By hand:</u> Determine the PW for each project

PW_A = -1,500,000 + 360,000(P/A,10%,8) = \$420,564
PW_B = -3,000,000 + 600,000(P/A,10%,10) = \$686,760
PW_C = -1,800,000 + 620,000(P/A,10%,5) = \$550,296
PW_D = -2,000,000 + 520,000(P/A,10%,5) = \$-28,784 (not acceptable)

<u>By spreadsheet</u>: Enter the following to display the project PW value.

A: = -PV(10%,8,360000)-1500000 Display: \$420,573
B: = -PV(10%,10,600000)-3000000 Display: \$686,740
C: = -PV(10%,5,620000)-1800000 Display: \$550,288
D: = -PV(10%,5,520000)-2000000 Display: \$-28,791 (not acceptable)

Formulate acceptable bundles from the $2^4 = 16$ possibilities, without both B and C, and select projects with largest total PW of a bundle.

(a) With b = \$4 million, select projects A and C with PW = \$970,860.

Bundle	Investment $ million	PW, $ (by hand)
DN	0	0
A	-1.5	420,564
B	-3.0	686,760
C	-1.8	550,296
A,C	-3.3	970,860

(b) With b = \$5.5 million, select projects A and B with PW = \$1,107,324

Bundle	Investment $ Million	PW, $
DN	0	0
A	-1.5	420,564
B	-3.0	686,760
C	-1.8	550,296
A,B	-4.5	1,107,324
A,C	-3.3	970,860

(c) With no-limit, select all with PW > 0. Select projects A and B (B & C cannot both be chosen).

12.16 (a) b = \$710,000 i = 10% n_j = 4 or 6 years 5 viable bundles

Bundle, j	Projects	NCF_{j0}, $	NCF_{jt}, $	S, $	PW, $
1	Do nothing	0	0	0	0
2	A	-140,000	50,000	45,000	49,230
3	B	-300,000	90,000	-10,000	-21,539
4	C	-590,000	150,000	100,000	119,745
5	AB	-440,000	140,000	35,000	27,691

$PW_j = NCF_j(P/A,10\%,4) + S(P/F,10\%,4) - NCF_{j0}$

Select bundle 4 (Project C) with largest PW of \$119,745

(b) Develop the spreadsheet so that the annual NCF values can be adjusted using Goal Seek. Functions are shown below.

	A	B	C	D	E	F
1	MARR =	0.1		Limit	710000	
2						
3	Bundle	1	2	3	4	5
4	Projects	DN	A	B	C	AB
5	Year					
6	0	0	-140000	-300000	-590000	-440000
7	1	0	50000	90000	150000	140000
8	2	0	= C$7	= D$7	=E$7	=F$7
9	3	0	= C$7	= D$7	=E$7	=F$7
10	4	0	= C$7 + 45000	= D$7-10000	=E$7	=F$7 + 35000
11	5	0			=E$7	
12	6	0			=E$7+100000	
13						
14						
15						
16	PW @ 10%	0	= NPV($B$1,C7:C15)+C6	= NPV($B$1,D7:D15)+D6	= NPV($B$1,E7:E15)+E6	= NPV($B$1,F7:F15)+F6

Repeatedly apply Goal Seek to find NCF values that force the PW values to equal that of bundle 4, namely PW_3 = $119,736, per the spreadsheet PW value for bundle 3. The spreadsheet for bundle 1 (project A) is shown below. NCF values for all bundles resulting from Goal Seek applications are as follows:

NCF_A = $72,243 compared to the estimated $50,000
NCF_B = $134,569 compared to the estimated $90,000
NCF_{AB} = $169,039 compared to the estimated $140,000

	A	B	C	D	E	F
1	MARR =	10%		Limit	710,000	
2						
3	Bundle	1	2	3	4	5
4	Projects	DN	A	B	C	AB
5	Year					
6	0	0	-140,000	-300,000	-590,000	-440,000
7	1	0	72,243	90,000	150,000	140,000
8	2	0	72,243	90,000	150,000	140,000
9	3	0	72,243	90,000	150,000	140,000
10	4	0	117,243	80,000	150,000	175,000
11	5	0			150,000	
12	6	0			250,000	
13						
14						
15						
16	PW @ 10%	0	119,736	-21,542	119,736	27,687

Goal Seek
Set cell: $C$16
To value: 119736
By changing cell: $C$7
OK Cancel

12.17 (a) For b = $25,000 only 4 bundles of the 32 possibilities are viable.

Bundle	Projects	Investment, $	PW at 12%, $
1	S	-15,000	8,540
2	M	-10,000	3,000
3	E	-25,000	10
4	SM	-25,000	11,540

Select projects S and M with PW = $11,540 and $25,000 invested.

(b) With b = $49,000, 5 additional bundles are viable.

Bundle	Projects	Investment, $	PW at 12%, $
5	H	-40,000	15,350
6	SA	-41,000	20,640
7	SE	-40,000	8,550

8	AM	-36,000	15,100
9	ME	-35,000	3,010

Select projects S and A with PW = \$20,640 and \$41,000 invested.

(c) Select all projects since they each have PW > 0 at 12% per year.

12.18 (a) <u>By hand:</u> The bundles and PW values are determined at MARR = 8% per year.

Bundle	Projects	Initial Investment, \$M	NCF, \$ per year	Life, years	PW at 8%, \$
1	1	-1.5	360,000	8	568,776
2	2	-3.5	600,000	10	526,060
3	3	-1.8	520,000	5	276,204
4	4	-2.0	820,000	4	715,922
5	1,3	-3.3	880,000	1-5	844,980
			360,000	6-8	

12.18 (cont.)

Bundle	Projects	Initial Investment, \$M	NCF, \$ per year	Life, years	PW at 8%, \$
6	1,4	-3.5	1,180,000	1-4	1,284,698
			360,000	5-8	
7	3,4	-3.8	1,340,000	1-4	992,126
			520,000	5	

Select PW = \$1.285 million for projects 1 and 4 with \$3.5 million invested.

(b) <u>By spreadsheet:</u> Set up a spreadsheet for all 8 bundles. Bundle 8 is DN with PW = 0. Select projects 1 and 4 with the largest PW = \$1,284,734 and invest \$3.5 million.

	A	B	C	D	E	F	G	H
1	MARR =	8%						
2								
3	Bundle	1	2	3	4	5	6	7
4	Projects	1	2	3	4	1,3	1,4	3,4
5	Year	Net cash flows (NCF), \$1000 per year						
6	0	-1,500	-3,500	-1,800	-2,000	-3,300	-3,500	-3,800
7	1	360	600	520	820	880	1,180	1,340
8	2	360	600	520	820	880	1,180	1,340
9	3	360	600	520	820	880	1,180	1,340
10	4	360	600	520	820	880	1,180	1,340
11	5	360	600	520		880	360	520
12	6	360	600			360	360	
13	7	360	600			360	360	
14	8	360	600			360	360	
15	9		600					
16	10		600					
17	PW, \$1000	568.790	526.049	276.209	715.944	844.999	1,284.734	992.153

12.19 Budget limit b = \$15,000 MARR = 12% per year

Bundle	Projects	Investment	NCF for years 1-5, \$	PW at 12%, \$
1	1	\$-5,000	1000,1700,2400, 3000,3800	3019
2	2	- 7,000	500,500,500, 500,10500	476
3	3	- 9,000	5000,5000,2000	874
4	4	-10,000	0,0,0,17000	804
5	1,2	-12,000	1500,2200,2900, 3500, 14300	3496
6	1,3	-14,000	6000,6700,4400, 3000,3800	3893
7	1,4	-15,000	1000,1700,2400, 20000,3800	3823
8	DN	0	0	0

Since PW_6 = \$3893 is largest, select bundle 6, which is projects 1 and 3.

12.20 (a) Spreadsheet solution for Problem 12.19 as stated. Projects 1 and 3 are selected with PW = \$3893 with \$14,000 invested.

	A	B	C	D	E	F	G	H	I
1	MARR =	12.0%							
2									
3	Bundle	1	2	3	4	5	6	7	8
4	Projects	1	2	3	4	1,2	1,3	1,4	DN
5	Year	Net cash flows, NCF, \$ per year							
6	0	-5,000	-7,000	-9,000	-10,000	-12,000	-14,000	-15,000	0
7	1	1,000	500	5,000	0	1,500	6,000	1,000	0
8	2	1,700	500	5,000	0	2,200	6,700	1,700	0
9	3	2,400	500	2,000	0	2,900	4,400	2,400	0
10	4	3,000	500		17,000	3,500	3,000	20,000	0
11	5	3,800	10,500			14,300	3,800	3,800	0
12	PW Value	3,019	477	874	804	3,496	3,893	3,823	0

(b) Spreadsheet solution for 14 viable bundles (6 additional) indicates selection of bundle 14 (projects 1, 3, and 4) with PW = \$4697 and \$24,000 invested.

	A	B	C	D	E	F	G	H	I
1	MARR =	12.0%		Limit	\$25,000				
2									
3	Bundle	1	2	3	4	5	6	7	8
4	Projects	1	2	3	4	1,2	1,3	1,4	DN
5	Year	Net cash flows, NCF, \$ per year							
6	0	-5,000	-7,000	-9,000	-10,000	-12,000	-14,000	-15,000	0
7	1	1,000	500	5,000	0	1,500	6,000	1,000	0
8	2	1,700	500	5,000	0	2,200	6,700	1,700	0
9	3	2,400	500	2,000	0	2,900	4,400	2,400	0
10	4	3,000	500		17,000	3,500	3,000	20,000	0
11	5	3,800	10,500			14,300	3,800	3,800	0
12	PW Value	3,019	477	874	804	3,496	3,893	3,823	0
13									
14	Bundle	9	10	11	12	13	14		
15	Projects	2,3	2,4	3,4	1,2,3	1,2,4	1,3,4		
16	Year	Net cash flows, NCF, \$ per year							
17	0	-16,000	-17,000	-19,000	-21,000	-22,000	-24,000		
18	1	5,500	500	5,000	6,500	1,500	6,000		
19	2	5,500	500	5,000	7,200	2,200	6,700		
20	3	2,500	500	2,000	4,900	2,900	4,400		
21	4	500	17,500	17,000	3,500	20,500	20,000		
22	5	10,500	10,500	0	14,300	14,300	3,800		
23	PW Value	1,350	1,280	1,678	4,370	4,300	4,697		

12.21 (a) Spreadsheet shows the solution. Select projects 1 and 2 for an investment of \$3.0 million and PW = \$199,496.

	A	B	C	D	E	F	G
1	MARR =	12.5%					
2							
3	Bundle	1	2	3	4	5	6
4	Projects	1	2	3	1,2	1,3	DN
5	Year	Net cash flows, NCF, \$ per year					
6	0	-900,000	-2,100,000	-1,000,000	-3,000,000	-1,900,000	0
7	1	250,000	385,000	200,000	635,000	450,000	0
8	2	245,000	390,000	250,000	635,000	495,000	0
9	3	240,000	395,000	312,500	635,000	552,500	0
10	4	235,000	400,000	390,625	635,000	625,625	0
11	5	230,000	405,000	488,281	635,000	718,281	0
12	6	225,000	410,000		635,000	225,000	0
13	7		415,000		415,000	0	0
14	8		420,000		420,000	0	0
15	9		425,000		425,000	0	0
16	10		430,000		430,000	0	0
17	PW Value	69,691	129,805	109,614	199,496	**179,305**	0

(b) The Goal Seek target cell is D17 to equal \$199,496 by changing cell D7. Result is a reduced year-one NCF requirement for project 3 to \$136,705. However, with these changes for project 3, the best selection is now projects 1 and 3 with PW = \$269,187.

	A	B	C	D	E	F	G
1	MARR =	12.5%	Limit =	$3 million			
2							
3	Bundle	1	2	3	4	5	6
4	Projects	1	2	3	1,2	1,3	DN
5	Year			Net cash flows, NCF, $			
6	0	-900,000	-2,100,000	-1,000,000	-3,000,000	-1,900,000	0
7	1	250,000	385,000	**136,705**	635,000	386,705	0
8	2	245,000	390,000	170,881	635,000	415,881	0
9	3	240,000	395,000	213,602	635,000	453,602	0
10	4	235,000	400,000	267,002	635,000	502,002	0
11	5	230,000	405,000	333,753	635,000	563,753	0
12	6	225,000	410,000	333,753	635,000	558,753	0
13	7	0	415,000	333,753	415,000	333,753	0
14	8	0	420,000	333,753	420,000	333,753	0
15	9	0	425,000		425,000	0	0
16	10	0	430,000		430,000	0	0
17	PW Value	69,691	129,805	199,496	199,496	**269,187**	0
18							
19							
20							

Reduced NCF series

Linear Programming and Capital Budgeting

12.22 To develop the 0-1 ILP formulation, first calculate PW_E, since it was not included in Table 12-2. All amounts are in \$1000 units.

$$
\begin{aligned}
PW_E &= -21{,}000 + 9500(P/A,15\%,9) \\
&= -21{,}000 + 9500(4.7716) \\
&= \$24{,}330
\end{aligned}
$$

The linear programming formulation is:

$$\text{Maximize } Z = 3694x_1 - 1019\,x_2 + 4788\,x_3 + 6120\,x_4 + 24{,}330\,x_5$$

Constraints: $10{,}000x_1 + 15{,}000\,x_2 + 8000\,x_3 + 6000\,x_4 + 21{,}000\,x_5 < 20{,}000$

$$x_k = 0 \text{ or } 1 \text{ for } k = 1 \text{ to } 5$$

(a) *For b = \$20 million:* The spreadsheet solution uses the template in Figure 12-5. MARR is set to 15% and a constraint is set to \$20,000 in Solver. Projects C and D are selected (row 19) for a \$14,000 investment with Z = \$10,908 (cell I2), as in Example 12.1. (Note: Set the solving method to Simplex LP)

MARR = 15%							
						Maximum Z =	$ 10,908
Projects	A	B	C	D	E		
Year	Net cash flows, NCF, $ per year						
0	-10,000	-15,000	-8,000	-6,000	-21,000		
1	2,870	2,930	2,680	2,540	9,500		
2	2,870	2,930	2,680	2,540	9,500		
3	2,870	2,930	2,680	2,540	9,500		
4	2,870	2,930	2,680	2,540	9,500		
5	2,870	2,930	2,680	2,540	9,500		
6	2,870	2,930	2,680	2,540	9,500		
7	2,870	2,930	2,680	2,540	9,500		
8	2,870	2,930	2,680	2,540	9,500		
9	2,870	2,930	2,680	2,540	9,500		
10							
11							
12							
Projects selected	0	0	1	1	0	0	
PW value at MARR, $	3,694	-1,019	4,788	6,120	24,330		
Contribution to Z, $	0	0	4,788	6,120	0		
Investment, $	0	0	8,000	6,000	0	Total =	$ 14,000

Solver Parameters
Set Objective: $I$2
To: Max Min
By Changing Variable Cells: $B$19:$G$19
Subject to the Constraints:
$B$19:$G$19 = binary
$I$22 <= 20000

(b) *b = $13 million:* Reset the budget constraint to b = $13,000 in Solver and obtain a new solution to select only project D with Z = $6120 and only $6000 of the $13,000 invested.

(c) *b = $30 million:* When the constraint is reset to $30,000, projects D and E are selected with the maximum Z = $30,450 and $27,000 invested.

12.23 Use the Figure 12-5 template at 8% with an investment limit of $4 million. Select projects 1 and 4 with $3.5 million invested and Z ≈ $1.285 million.

MARR =	8%							
Projects	1	2	3	4	5	6		
Year	Net cash flows, NCF, $						Maximum Z =	$1,284,734
0	-1,500,000	-3,500,000	-1,800,000	-2,000,000				
1	360,000	600,000	520,000	820,000				
2	360,000	600,000	520,000	820,000				
3	360,000	600,000	520,000	820,000				
4	360,000	600,000	520,000	820,000				
5	360,000	600,000	520,000					
6	360,000	600,000						
7	360,000	600,000						
8	360,000	600,000						
9		600,000						
10		600,000						
11								
12								
Projects selected	1	0	0	1	0	0		
PW value at MARR	568,790	526,049	276,209	715,944	0	0		
Contribution to Z	568,790	0	0	715,944	0	0		
Investment	1,500,000	0	0	2,000,000	0	0	Total =	$3,500,000

Solver Parameters
Set Objective: $I$5
To: Max Min Value Of:
By Changing Variable Cells: $B$19:$G$19
Subject to the Constraints:
$B$19:$G$19 = binary
$I$22 <= 4000000
Make Unconstrained Variables Non-Negative
Select a Solving Method: GRG Nonlinear
Solving Method
Select the GRG Nonlinear engine for Solver Problems that engine for linear Solver Problems, and select the Evolution non-smooth.

12.24 Enter the NCF values from Problem 12.21 into the capital budgeting template and a constraint of b = \$3,000,000 into Solver. Select projects 1 and 2 for Z = \$199,496 with all \$3 million invested.

	A	B	C	D	E	F	G	H	I
1	MARR =	13%							
2									
3									
4	Projects	1	2	3	4	5	6		
5	Year	Net cash flows, NCF, $						Maximum Z =	$ 199,496
6	0	-900,000	-2,100,000	-1,000,000					
7	1	250,000	385,000	200,000					
8	2	245,000	390,000	250,000					
9	3	240,000	395,000	312,500					
10	4	235,000	400,000	390,625					
11	5	230,000	405,000	488,281					
12	6	225,000	410,000						
13	7		415,000						
14	8		420,000						
15	9		425,000						
16	10		430,000						
17	11								
18	12								
19	Projects selected	1	1	0	0	0	0		
20	PW value at MARR,	69,691	129,805	109,614	0	0	0		
21	Contribution to Z, $	69,691	129,805	0	0	0	0		
22	Investment, $	900,000	2,100,000	0	0	0	0	Total =	$ 3,000,000

Solver Parameters
Set Objective: $I$5
To: Max Min Value Of:
By Changing Variable Cells: $B$19:$G$19
Subject to the Constraints:
$B$19:$G$19 = binary
$I$22 <= 3000000

12.25 *Linear programming model:* In \$1000 units,

$$\text{Maximize } Z = 501x_1 + 261x_2 + 202x_3 + 481x_4$$

$$\text{Constraints: } 500x_1 + 700\,x_2 + 900\,x_3 + 1000\,x_4 \le 1600$$

$$x_k = 0 \text{ or } 1 \text{ for } k = 1 \text{ to } 4$$

Spreadsheet solution: Enter all estimates on a spreadsheet with a b = \$1600 constraint in Solver to obtain the answer:

Select projects 1 and 4 with Z = \$982,000 and \$1.5 million invested

	A	B	C	D	E	F	G	H	I	J
1	MARR =	5%								
2										
3										
4	Projects	1	2	3	4	5	6			
5	Year	Net annual savings, $1000 per year						Maximum Z =	$ 982	
6	0	-500	-700	-900	-1,000					
7	1	100	50	500	0					
8	2	170	50	500	0					
9	3	240	50	200	0					
10	4	300	50		1,800					
11	5	380	1,000							
12	6									
13	7									
14	8									
15	9									
16	10									
17	11									
18	12									
19	Projects selected	1	0	0	1	0	0			
20	PW value at MARR, $1000	501	261	202	481	0	0			
21	Contribution to Z, $1000	501	0	0	481	0	0			
22	Investment, $1000	500	0	0	1,000	0	0	Total =	$ 1,500	

Solver Parameters
Set Objective: $I$5
To: Max Min Value
By Changing Variable Cells: $B$19:$G$19
Subject to the Constraints:
$B$19:$G$19 = binary
$I$22 <= 1600

12.26 Build a spreadsheet and use Solver repeatedly at increasing values of b to find the vendor combinations that maximize the value of Z; then develop a scatter graph.

	A	B	C	D	E	F	G	H	I
1	MARR =	5%							
2	Vendors	1	2	3	4	5	6		
3	Year	Net annual savings, $1000 per year						Z =	$1,243
4	0	-500	-700	-900	-1,000				
5	1	100	50	500	0				
6	2	170	50	500	0				
7	3	240	50	200	0				
8	4	300	50		1,800				
9	5	380	1,000						
10	Projects selected	1	1	0	1	0	0		
11	PW value at MARR, $1000	501	261	202	481	0	0		
12	Contribution to Z, $1000	501	261	0	481	0	0		
13	Investment, $1000	500	700	0	1,000	0	0	Total =	$2,200

Investment Limit, $1000	Value of Z, $1000	Vendor(s) Selected
500	501	1
750	501	1
1000	501	1
1250	762	1, 2
1500	982	1, 4
1750	982	1, 4
2000	982	1, 4
2250	1243	1, 2, 4
2500	1243	1, 2, 4

Other Ranking Measures

12.27 (a) IROR: 0 = -750,000 + 135,000(P/A,i*,10)
i* = 12.4%

PI = 135,000(P/A,12%,10)/│-750,000│
= 135,000(5.6502)/750,000
= 1.02

PW = -750,000 + 135,000(P/A,12%,10)
= -750,000 + 135,000(5.6502)
= $12,777

(b) By all three measures, since IROR > 12%; PI > 1.0 and PW > 0 at MARR = 12%

(c) The function = RATE(10,135000,-750000) displays 12.4148% as the breakeven i*.

No, all values of MARR above or below the breakeven will generate the same decision for all three measures.

12.28 (a) Select projects A, B and C with a total $55,000 investment.

(b) Overall ROR = [30,000(23.3%) + 10,000(19.0%) + 15,000(17.3%)
+ 5,000(12.0%)]/60,000
= 20.1%

12.29 <u>IROR:</u>

$$0 = -400{,}000 + (192{,}000 - 75{,}000)(P/A,i^*,5) + 80{,}000(P/F,i^*,5)$$
$$i^* = 18.09\%$$

<u>PI:</u>

$$\text{PW of NCF} = (192{,}000 - 75{,}000)(P/A,12\%,5) + 400{,}000(0.20)(P/F,12\%,5)$$
$$= (192{,}000 - 75{,}000)(3.6048) + 80{,}000(0.5674)$$
$$= \$467{,}154$$

PW of first cost = -400,000

$$PI = 467{,}154/400{,}000$$
$$= 1.17$$

12.30 (a) $PI_1 = 900{,}000(P/A,10\%,7)/4{,}000{,}000$
$= 900{,}000(4.8684)/4{,}000{,}000$
$= 1.10$

$PI_2 = 1{,}900{,}000(P/A,10\%,10)/7{,}000{,}000$
$= 1{,}900{,}000(6.1446)/7{,}000{,}000$
$= 1.67$

$PI_3 = 2{,}900{,}000(P/A,10\%,15)/17{,}000{,}000$
$= 2{,}900{,}000(7.6061)/17{,}000{,}000$
$= 1.30$

$PI_4 = 3{,}600{,}000(P/A,10\%,10)/15{,}000{,}000$
$= 3{,}600{,}000(6.1446)/15{,}000{,}000$
$= 1.47$

$PI_5 = 5{,}000{,}000(P/A,10\%,8)/30{,}000{,}000$
$= 5{,}000{,}000(5.3349)/30{,}000{,}000$
$= 0.89$ (eliminated since $PI < 1.0$)

Projects in PI order	2	4	3	1
Cumulative Investment, $1000	7,000	22,000	39,000	43,000

Select projects 2 and 4; invest $22,000,000.

(b) The function = -PV(10%,8,5000)/30000 displays $PI_5 = 0.89$, which is unacceptable since it is < 1.0.

12.31 *<u>By hand:</u>*

(a) Find IROR for each project, rank by decreasing IROR and then select projects within the budget constraint of $120,000.

For X: $0 = -30{,}000 + 9000(P/A,i^*,10)$
$i^* = 27.3\%$

For Y: $0 = -15{,}000 + 4{,}900(P/A,i^*,10)$
$i^* = 30.4\%$

For Z: $0 = -45{,}000 + 11{,}100(P/A,i^*,10)$
$i^* = 21.0\%$

For A: $0 = -70{,}000 + 19{,}000(P/A,i^*,10)$
$i^* = 24.0\%$

For B: $0 = -40{,}000 + 10{,}000(P/A,i^*,10)$
$i^* = 21.4\%$

Projects in IROR order	Y	X	A	B	Z
Cumulative Investment, $1000	15,000	45,000	115,000	155,000	200,000

(b) Overall ROR = [15,000(30.4%) + 30,000(27.3%) + 70,000(24.0%)
+ 5,000(15.0%)]/120,000
= 25.3%

By spreadsheet:

(a) Select projects Y, X, and A with a total investment of $115,000 (column G)

	A	B	C	D	E	F	G
1				i* order usng SORT function			
2	i* using RATE function		Initial	on column B values		Initial	Cumulative
3	Project	i*	Investment, $	Project	i*	Investment, $	Investment, $
4	X	27.3%	30,000	Y	30.4%	15,000	15,000
5	Y	30.4%	15,000	X	27.3%	30,000	45,000
6	Z	21.0%	45,000	A	24.0%	70,000	115,000
7	A	24.0%	70,000	B	21.4%	40,000	155,000
8	B	21.4%	40,000	Z	21.0%	45,000	200,000

(b) Same as for hand solution: Overall ROR = 25.3% per year

12.32 (a) *By hand:* Find IROR for each project; select highest ones within budget constraint of $100 million.

For W: $0 = -12{,}000 + 5000(P/A,i^*,3)$
$i^* = 12.0\%$

For X: $0 = -25{,}000 + 7{,}300(P/A,i^*,4)$

$i^* = 6.5\%$

For Y: $0 = -45{,}000 + 12{,}100(P/A,i^*,6)$
$i^* = 15.7\%$

For Z: $0 = -60{,}000 + 9000(P/A,i^*,8)$
$i^* = 4.2\%$

Only two projects (W and Y) have $i^* \geq$ MARR = 12%.

Select Y and W with total investment of $57 million.

<u>*By spreadsheet:*</u> Select Y and W after ranking (row 12); invest $57 million

	A	B	C	D	E	F	G	H	I	J	K	L
1		Annual Savings, $M per year						Annual Savings, $M per year				
2	Year	W	X	Y	X		Year	Y	W	X	X	Y, W
3	0	-12.0	-25.0	-45.0	-60.0		0	-45.0	-12.0	-25.0	-60.0	-57.0
4	1	5.0	7.3	12.1	9.0		1	12.1	5.0	7.3	9.0	17.1
5	2	5.0	7.3	12.1	9.0		2	12.1	5.0	7.3	9.0	17.1
6	3	5.0	7.3	12.1	9.0		3	12.1	5.0	7.3	9.0	17.1
7	4		7.3	12.1	9.0		4	12.1		7.3	9.0	12.1
8	5			12.1	9.0		5	12.1			9.0	12.1
9	6			12.1	9.0		6	12.1			9.0	12.1
10	7				9.0		7				9.0	0.0
11	8				9.0		8				9.0	0.0
12	IROR, i*	12.0%	6.5%	15.7%	4.2%		i*	15.7%	12.0%	6.5%	4.2%	15.1%
13							Cum Inv, $M	45.0	57.0	82.0	142.0	
14												
15		IROR value using IRR function						Ordered by IROR value				

(b) Find i^* of Y and W

NCF, year 0: $-57.0 million
NCF, years 1-3: $17.1 million
NCF, years 4-6: $12.1 million

$0 = -57 + 17.1(P/A,i^*,3) + 12.1(P/A,i^*,3)(P/F,i^*,3)$
$i^* = 15.1\%$ (IRR function, column L)

(c) The $43 million not committed makes MARR = 12% elsewhere.

Overall ROR = [57,000(15.1) + 43,000(12.0)]/100,000
= 13.8%

Since 13.8% > MARR = 12%, the selection of Y and W is acceptable.

12.33 (a) $PI_A = 4000(P/A,10\%,10)/18{,}000$
$= 4000(6.1446)/18{,}000$
$= 1.37$

PI_B = 2800(P/A,10%,10)/15,000
= 2800(6.1446)/15,000
= 1.15

PI_C = 12,600(P/A,10%,10)/35,000
= 12,600(6.1446)/35,000
= 2.21

PI_D = 13,000(P/A,10%,10)/60,000
= 13,000(6.1446)/60,000
= 1.33

PI_E = 8000(P/A,10%,10)/50,000
= 8000(6.1446)/50,000
= 0.98

Rank order by PI	C	A	D	B
Cumulative Investment, $1000	35	53	113	128

Select projects C, A, and D; invest $113,000. E is eliminated with PI < 1.0

(b)

Rank order by IROR	C	A	D	B
Cumulative Investment, $1000	35	53	113	128

Select C, A, and D; invest a total of $113,000. E is eliminated with IROR < 10%

(c) Ranked order by PW value is C, D and A

(d) No, projects selected are the same using PI, IROR and PW ranking.

12.34 The IROR, PI, and PW values are shown in the table below. Sample calculations for project F are:

IROR: 54,000/200,000 = 27.0%
PI: [54,000/0.25)]/200,000 = 1.08
PW: -200,000 + 54,000/0.25 = $16,000

Projects G and K can be eliminated since their IROR, PI and PW are not acceptable.

Project	First Cost, $	Annual Income, $ per year	IROR, %	PI	PW, $
F	-200,000	54,000	27.0	1.08	16,000
G	-120,000	21,000	17.5	0.70	-36,000
H	-250,000	115,000	46.0	1.84	210,000
I	-370,000	205,000	55.4	2.22	450,000

J	-50,000	26,000	52.0	2.08	54,000
K	-9000	2,100	23.3	0.93	-600

(a) *b = $700,000*

(1) IROR: Projects selected are I, J, and H with $670,000 invested

IROR rank	I	J	H	F
Cum Inv, $1000	370	420	670	870

(2) PI: Projects selected are I, J, and H with $670,000 invested

PI rank	I	J	H	F
Cum Inv, $1000	370	420	670	870

(3) PW: Projects selected are I, H and J with $670,000 invested

PW rank	I	H	J	F
Cum Inv, $1000	370	620	670	870

(b) *b = $600,000*

(1) IROR: Projects selected are I and J with $420,000 invested
(2) PI: Projects selected are I and J with $420,000 invested
(3) PW: Project selected is only I with $370,000 invested

(Note: The PW-based selection is different for the reduced budget of $600,000)

Additional Problems and FE Exam Review Problems

12.35 Answer is (c)

12.36 Answer is (d)

12.37 PW of NCF = (170,000 – 80,000)(P/A,10%,5) + 60,000(P/F,10%,5)
= (170,000 – 80,000)(3.7908) + 60,000(0.6209)
= $378,426

PI = 378,426/400,000
= 0.95
Answer is (a)

12.38 Select the bundle with the highest positive PW value that does not violate the budget limit of $45,000. Select bundle 4.
Answer is (d)

12.39 There are 7 possible bundles under the $35,000 limit: P, Q, R, T, PR, and PT. Largest PW = $23,800 is for projects P and T at an initial investment of $32,000.
Answer is (b)

12.40 Maximum number of bundles = $2^4 = 16$
Answer is (d)

12.41 There are $2^4 = 16$ possible bundles. Considering the selection restriction and the $400,000 limitation, the viable bundles are:

Acceptable Projects	Investment
DN	$ 0
2	150
3	75
4	235
2, 3	225
2, 4	385
3, 4	310

Not acceptable bundles: 1, 12, 13, 14, 123, 124, 234, 134, 1234
Answer is (b)

12.42 Answer is (a)

12.43 Answer is (c)

12.44 Answer is (a)

Chapter 13
Breakeven and Payback Analysis

Breakeven Analysis for a Project

13.1 (a) $89x = 5{,}000{,}000 + 45x$
$44x = 5{,}000{,}000$
$x = 113{,}636$

(b) At 100,000 units: Profit = revenue - cost
$= 89(100{,}000) - [5{,}000{,}000 + 45(100{,}000)]$
$= \$-600{,}000$ (loss)

At 200,000 units: Profit $= 89(200{,}000) - [5{,}000{,}000 + 45(200{,}000)]$
$= \$3{,}800{,}000$ (profit)

13.2 For breakeven, 0 = revenue – cost
$0 = 50x - [40{,}000 + (3000/100)x]$
$20x = 40{,}000$
$x = 2000$ units per year

13.3 Let x = days per year to breakeven
$0 = -48{,}000(A/P,8\%,5) + 2000(A/F,8\%,5) - 65x + 300x$
$0 = -48{,}000(0.25046) + 2000(0.17046) - 65x + 300x$
$235x = 11{,}681.16$
$x = 49.7$ days/year

13.4 (a) Let x = miles per year to breakeven
$0 = -68{,}000(A/P,10\%,5) + 36{,}000(A/F,10\%,5) - 0.50x + 0.61x$
$0 = -68{,}000(0.26380) + 36{,}000(0.16380) - 0.50x + 0.61x$
$0.11x = 12{,}041.60$
$x = 109{,}469$ miles/year

(b) No. days = 109,469/600
= 182 days/year

13.5 Let x = selling price
$0 = 4000x - [800{,}000 + 290(4000)]$
$x = \$490$/unit

13.6 $Q_{BE} = 500{,}000/(250-200)$
$= 10{,}000$ units per year
$1.20Q_{BE} = (1.20)10{,}000 = 12{,}000$ units

Profit = 50(12,000) – 500,000 = $100,000 per year

13.7 $0 = -150{,}000{,}000(A/P,10\%,10) + 12{,}500(250)X^{0.5}$
$= -150{,}000{,}000(0.16275) + 3{,}125{,}000X^{0.5}$
$X^{0.5} = 24{,}412{,}500/3{,}125{,}000$
$= 7.812$

$X = 61\%$

13.8 (a) Let x = selling price per unit

$0 = 12{,}000x - [160{,}000 + 400(12{,}000)]$
$= 12{,}000x - 4{,}960{,}000$
$x = \$413.33$

(b) $400{,}000 = 12{,}000x - 4{,}960{,}000$
$x = 446.67$ (447 valves per year)

13.9 Savings = 0.25(20) = $5.00
Cost/mile = 2.50/30 = $0.083/mile
Breakeven roundtrip = 5.00/0.083 = 60.24 miles
One way = 60.24/2 = 30.12 miles

13.10 Let x = cost/ton
$0 = 25{,}000x - [300{,}000 + 12(25{,}000)]$
$x = \$24$ per ton

13.11 (a) Let x = hours per month billed to breakeven
$15{,}000 = -900{,}000(A/P,1\%,120) - 1{,}100{,}000 + 1{,}500{,}000(A/F,1\%,120) + 10(90)x$
$15{,}000 = -900{,}000(0.01435) - 1{,}100{,}000 + 1{,}500{,}000(0.00435) + 10(90)x$
$900x = 1{,}121{,}390$
$x = 1246$ hours/month

(b) Billable hours per professional = 1246/10 = 124.6 hours

(c) Total hours per month = 260(8)/12 = 173.3

Billable percent = 124.6/173.3
= 0.719 (72%)

13.12 (a) First express all variable costs In terms of *cost per unit* and then find breakeven production rate.

$$\text{Variable production cost} = 1{,}700{,}000/40{,}000 = \$42.50$$

$$\text{Variable selling expenses} = 96{,}000/40{,}000 = \$2.40$$

$$\begin{aligned}\text{Breakeven production} &= \text{income - cost}\\ &= 70x - (240{,}000 + 42.50x + 2.40x)\\ 25.1x &= 240{,}000\\ x &= 9562 \text{ units per year}\end{aligned}$$

(b) $$\begin{aligned}\text{Profit} &= 40{,}000(70) - [(240{,}000 + 40{,}000(42.50) + 40{,}000(2.40)]\\ &= 2{,}800{,}000 - 2{,}036{,}000\\ &= \$764{,}000\end{aligned}$$

(c) $$\begin{aligned}1{,}000{,}000 &= 70x - [(240{,}000 + x(42.50) + x(2.40)]\\ x &= 49{,}402\end{aligned}$$

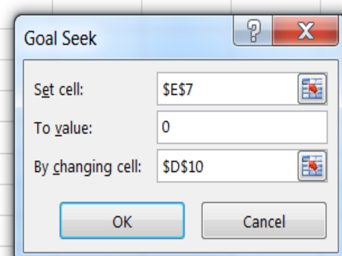
13.13 (a) <u>*By hand:*</u> Set PW of revenue equal to PW of expenses

$$4000(43{,}000 - 25{,}000)(P/A,12\%,3) + G(43{,}000 - 25{,}000)(P/G,12\%,3) = 250{,}000{,}000 - 0.20(250{,}000{,}000)(P/F,12\%,3)$$

$$\begin{aligned}4000(18{,}000)(2.4018) + G(18{,}000)(2.2208) &= 250{,}000{,}000 - 50{,}000{,}000(0.7118)\\ 172{,}929{,}600 + 39{,}974.4G &= 214{,}410{,}000\\ G &= 1038 \text{ more cars per year}\end{aligned}$$

(b) <u>*By spreadsheet:*</u>

	A	B	C	D	E
1					Revenue -
2	Year	Production	Revenue	Expenses	Expenses
3	0			-250,000,000	-250,000,000
4	1	4000	172,000,000	-100,000,000	72,000,000
5	2	5038	216,619,544	-125,941,595	90,677,949
6	3	6075	261,239,088	-101,883,191	159,355,897
7	PW at 12%		512,204,026	-512,204,026	0
8					
9					
10		Increase per year, G		1037.7	
11					



Breakeven Analysis between Alternatives

13.14 x = number of units per year

$$10{,}000 + 50x = 22{,}800 + 10x$$
$$x = 320$$

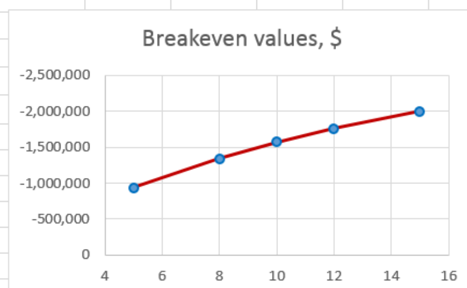
13.15 (a)

$$-2{,}300{,}000(A/P,8\%,20) - 486 = -x(A/P,8\%,10) – 774$$
$$-2{,}300{,}000(0.10185) - 486 = -x(0.14903) – 774$$
$$x = \$1{,}569{,}932$$

(b) Develop the AW relations using the PMT function while allowing the n value for asphalt to vary from 5 to 15 years. Set the initial cost at a constant, e.g., $-1,000,000, and use Goal Seek successively to determine the breakeven value. Plot n versus breakeven asphalt cost.

	A	B	C	D–F	G
1					
2				PMT function	Value, $
3	**Concrete:**			= -PMT(8%,20,-2300000) -486	-234,746
4					
5	**Asphalt:**	Life	Cost, $		
6		5	-934,182	= -PMT(8%,$B6,$C6) - 774	-234,746
7		8	-1,344,553	= -PMT(8%,$B7,$C7) - 774	-234,746
8		10	-1,569,971	= -PMT(8%,$B8,$C8) - 774	-234,746
9		12	-1,763,231	= -PMT(8%,$B9,$C9) - 774	-234,746
10		15	-2,002,678	= -PMT(8%,$B10,$C10) - 774	-234,746

Breakeven cost after Goal Seek is applied



13.16 (a) Calculate BTU/dollar for gasoline and set equal to cost for ethanol

$$115{,}600/3.50 = 75{,}670/x$$
$$x = \$2.29$$

(b) Energy in E85 $= 0.85(75{,}670) + 0.15(115{,}600)$
$= 81{,}660$ BTU/gallon

$$115{,}600/3.50 = 81{,}660/x$$
$$x = \$2.47$$

13.17

$$90{,}000(A/P,8\%,5) + 450x = 800x$$
$$90{,}000(0.25046) + 450x = 800x$$
$$x = 64.4 \text{ days}$$

13.18 Set annual costs equal to each other and solve for FC, the fixed cost

$$160{,}000 + 50(1000) = FC + (200/10)(1000)$$
$$FC = \$190{,}000$$

13.19 *By hand*

(a) Set annual costs equal to each other and solve for A, annual maintenance cost

$$1{,}025{,}000(A/P,8\%,3) + 355{,}000 = 3{,}525{,}000(A/P,8\%,10) + A$$
$$1{,}025{,}000(0.38803) + 355{,}000 = 3{,}525{,}000(0.14903) + A$$
$$A = \$227{,}400$$

(b) Annual gravel road maintenance = 355,000(1.30) = \$461,500; find n for the gravel road

$$1{,}025{,}000(A/P,8\%,n) + 461{,}500 = 3{,}525{,}000(A/P,8\%,10) + 227{,}400$$
$$(A/P,8\%,n) = [3{,}525{,}000(0.14903) + 227{,}400 - 461{,}500]/1{,}025{,}000$$
$$= 0.28413$$

n is between 4 and 5 years. By interpolation, n = 4.3 years

By spreadsheet

See the spreadsheet for explanation to display answers: (a) \$227,405 per year, and (b) 4.3 years using the PMT function to find A and Goal Seek to find n

	A	B	C	D	E	F	G	H	I	J
1	Gravel				Paved					
2	n, years =	4.3			n, years =	10				
3	Maintenance	355,000			Maintenance	227,405				
4	A, \$/year	\$752,734				\$752,734				
5										
6	(a) Use Goal Seek to change cell F3 to reach breakeven at the A value in cell B3								Answer in F3	\$ 227,405
7										
8	(b) Use Goal Seek to change cell B2 to reach breakeven at the A value in cell F3, with									
9		maintenance = 355,000(1.3) = \$461,500 per year							Answer In B2	4.3

13.20 (a) Let x = days per year to just breakeven

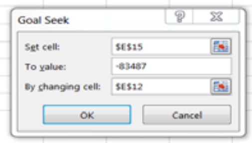
$$-190{,}000(A/P,10\%,10) - 40{,}000 + 70{,}000(A/F,10\%,10) - 260x = -1100x - 180x$$
$$-190{,}000(0.16275) - 40{,}000 + 70{,}000(0.06275) - 260x = 1280x$$
$$1020x = 66{,}530$$
$$x = 65.22 \text{ days/year}$$

The need must be 66 or more days per year to justify purchase

(b) Using Goal Seek is easier. Set up the annual cost using PMT with the breakeven days of 65.22 for the purchase option and 100 for the rent option. Changing cell is E12 and the display is:

Required rental cost is lower than $655 per day to justify rental

	A	B	C	D	E
1		**Purchase**			**Rent**
2	Days/year	65.22		days/year	65.22
3	Cost/day	260		Cost/day	1100
4				Driver/day	180
5					
6	A, $/year	-83,487			-83,482
7					
8		**Before Goal Seek**			
9					
10		**Purchase**			**Rent**
11	Days/year	65.22		days/year	100
12	Cost/day	260		Cost/day	655
13				Driver/day	180
14					
15	A, $/year	-83,487			-83,487
16	Function	= - PMT(10%,10,-190000,70000) - 40000 - 260*$B$11			= - E12*$E$11 - E13*$E$11
17					
18		**After Goal Seek**			



13.21 Let x = square yards per year to breakeven

$$-109{,}000 - 2.75x = -225{,}000(A/P,8\%,15) - 13x$$
$$-109{,}000 - 2.75x = -225{,}000(0.11683) - 13x$$
$$10.25x = 82{,}713$$
$$x = 8070 \text{ square yards/year}$$

13.22 Let x = gallons per day to breakeven

$$-465 - (485/720{,}000)x = -328 - (1280/950{,}000)x$$
$$0.00067376x = 137$$
$$x = 203{,}336 \text{ gallons per day}$$

13.23 (a) Fixed cost, C: $FC_C = -500{,}000(A/P,10\%,5) + 0.25(500{,}000)(A/F,10\%,5)$
$$= -500{,}000(0.2638) + 0.25(500{,}000)(0.1638)$$
$$= \$-111{,}425 \text{ per year}$$

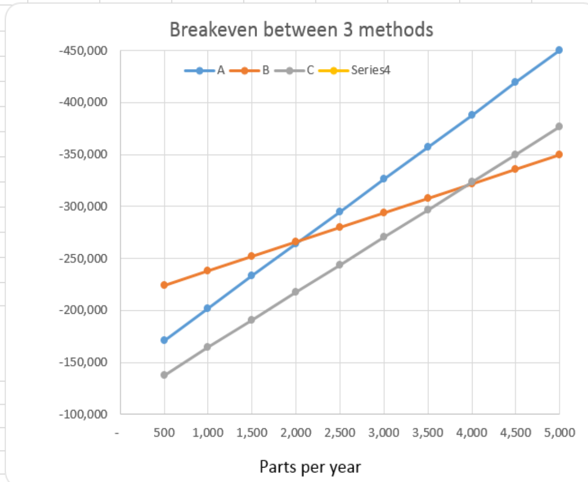
Method A has a higher fixed cost and higher variable cost than Method C. Eliminate A and find breakeven between B and C

Let x = number of parts per year

$$-210{,}000 - 28x = -111{,}425 - 53x$$
$$x = 3943 \text{ parts/year}$$

(b) Plot annual cost curves. Breakeven is only between B and C at slightly less than 4000 parts/year.

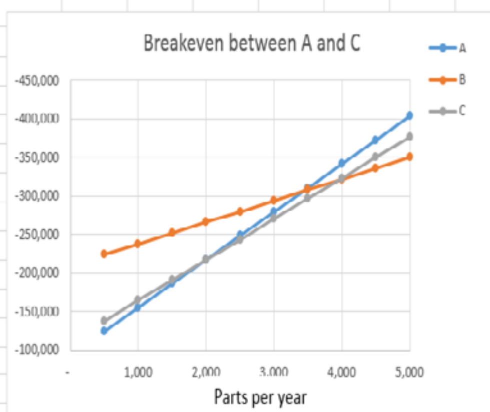
	Method		
Parts/year	A	B	C
500	-171,000	-224,000	-137,924
1,000	-202,000	-238,000	-164,424
1,500	-233,000	-252,000	-190,924
2,000	-264,000	-266,000	-217,424
2,500	-295,000	-280,000	-243,924
3,000	-326,000	-294,000	-270,424
3,500	-357,000	-308,000	-296,924
4,000	-388,000	-322,000	-323,424
4,500	-419,000	-336,000	-349,924
5,000	-450,000	-350,000	-376,424
Sample function: 500 parts/year	= -140000 - 62*$A3	= -210000 - 28*$A3	= - PMT(10%,5, -500000,125000) - 53*$A3



(c) Make the fixed cost for method A the variable (cell A14). Use Goal Seek to force the annual costs to be equal at 2000. The new annual fixed cost for method A is $-93,424. Plot all three curves. Now, breakeven conditions are:

Parts per year	Method
Less than 2000	A
Between 2000 and ~3900	C
More than ~3900	B

	Method		
Parts/year	A	B	C
500	-124,424	-224,000	-137,924
1,000	-155,424	-238,000	-164,424
1,500	-186,424	-252,000	-190,924
2,000	-217,424	-266,000	-217,424
2,500	-248,424	-280,000	-243,924
3,000	-279,424	-294,000	-270,424
3,500	-310,424	-308,000	-296,924
4,000	-341,424	-322,000	-323,424
4,500	372,424	336,000	349,924
5,000	-403,424	-350,000	-376,424
Fixed cost, $/yr	-93,424		



13.24 Let x = ads per year

$-21x = -42{,}000(A/P,10\%,3) - 55{,}000 + 2000(A/F,10\%,3) - 10x$

$-21x = -42{,}000(0.40211) - 55{,}000 + 2000(0.30211) - 10x$

$-11x = -71{,}284$

$x = 6480$

13.25 (a) Set $AW_A = AW_B$, with P_B = first cost of B. The final term in AW_B removes the repainting cost in year 12 only.

$$
\begin{aligned}
AW_A &= AW_B \\
-250{,}000(A/P,12\%,4) - 3000 &= -P_B(A/P,12\%,12) - 5000(A/F,12\%,2) + 5000(A/F,12\%,12) \\
-250{,}000(0.32923) - 3000 &= -P_B(0.16144) - 5000(0.47170) + 5000(0.04144) \\
-85{,}308 &= -P_B(0.16144) - 2151 \\
-83{,}157 &= -P_B(0.16144) \\
P_B &= \$515{,}095
\end{aligned}
$$

(b) Set $P_B = -700{,}000$ and solve for P_A

$$
\begin{aligned}
AW_A &= AW_B \\
-P_A(A/P,12\%,4) - 3000 &= -700{,}000(A/P,12\%,12) - 5000(A/F,12\%,2) + 5000(A/F,12\%,12) \\
-P_A(0.32923) - 3000 &= -700{,}000(0.16144) - 5000(0.47170) + 5000(0.04144) \\
-P_A(0.32923) &= -112{,}159 \\
P_A &= \$340{,}671
\end{aligned}
$$

13.26 Let x = days per year

$$
\begin{aligned}
-125{,}000(A/P,12\%,8) + 5000(A/F,12\%,8) - 2000 - 40x &= -45(125 + 20x) \\
-125{,}000(0.2013) + 5000(0.0813) - 2000 - 40x &= -45(125 + 20x) \\
-21{,}131 &= -860x \\
x &= 24.6
\end{aligned}
$$

At least 25 days per year are needed to justify the purchase.

13.27 Let R_3 = production rate in year 3

$$
\begin{aligned}
-40{,}000 - 50R_3 &= -70{,}000 - 12R_3 \\
38R_3 &= 30{,}000 \\
R_3 &= 789.5 \qquad \text{(790 units)}
\end{aligned}
$$

13.28 Let T = number of tons/year. Solve relation $AW_1 = AW_2$ for T

Variable costs (VC) for each machine:

VC_1: $24T/10 = 2.4T$ $\qquad$ VC_2: $2(24)T/6 = 8T$

$$
\begin{aligned}
-123{,}000(A/P,7\%,10) - 5000 - 2.4T &= -70{,}000(A/P,7\%,6) - 2500 - 8T \\
(8-2.4)T &= -70{,}000(0.20980) - 2500 \\
&\quad + 123{,}000(0.14238) + 5000 \\
5.6T &= 5327 \\
T &= 951 \text{ tons/year}
\end{aligned}
$$

If tonnage is less than breakeven, select machine 2 since the slope is steeper. At

1500 tons, select machine 1.

13.29 (a) Solve relation Revenue - Cost = 0 for Q = number of filters per year

$$50Q - [200{,}000(A/P,6\%,5) + 25{,}000 + 20Q] = 0$$
$$30Q = 200{,}000(0.23740) + 25{,}000$$
$$Q = 72{,}480/30$$
$$= 2416 \text{ filters per year}$$

At 5000 units, make the filters in-house

(b) Solve the relation $AW_{buy} = AW_{make}$ for Q = number of filters per year.

$$(50-30)Q = -72{,}480 + (50-20)Q$$
$$Q = 7248 \text{ filters per year}$$

Since 5000 < 7248, to *buy* is the correct choice.

(c) *Make*: 5000 at \$20 each.

$$\text{Profit} = 5000(50-20) - 72{,}480$$
$$= \$77{,}520$$

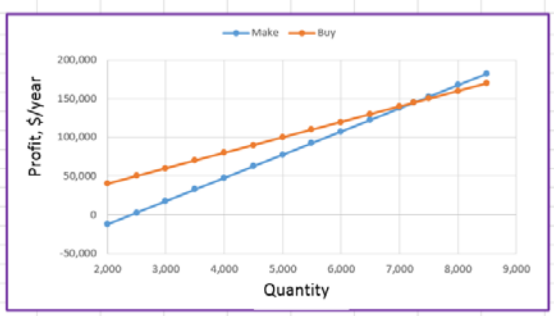
Buy: 5000 at \$30 each

$$\text{Profit} = 5000(50-30)$$
$$= \$100{,}000$$

Profit is higher for buying

(d) The spreadsheet below verifies the answers above

	Profit, $ per year		
Quantity	Make	Buy	
2,000	-12,480	40,000	
2,500	2,520	50,000	
3,000	17,520	60,000	
3,500	32,520	70,000	
4,000	47,520	80,000	
4,500	62,520	90,000	
5,000	77,520	100,000	Part (c)
5,500	92,520	110,000	
6,000	107,520	120,000	
6,500	122,520	130,000	
7,000	137,520	140,000	
7,248	144,960	144,960	Part (b)
7,500	152,520	150,000	
8,000	167,520	160,000	
8,500	182,520	170,000	



Payback Analysis

13.30 Payback analysis should be used only as a supplemental analysis tool because it only determines the time necessary to recover the initial investment at a stated return ($i \geq 0\%$). Therefore, payback does not recognize cash flows beyond the payback period. An alternative with increasing cash flows that makes the rate of return increase may not be selected when another alternative (possibly with a lower rate of return) has a shorter payback period.

13.31 $0 = -70{,}000 + (14{,}000 - 1850)(P/A,10\%,n_p)$

$(P/A,10\%,n_p) = 5.76132$

From the 10% interest rate table, n_p is very close to 9 years.

Spreadsheet function: = NPER(10%,12150,-70000) displays 9.006 years

13.32 (a) Develop the PW relation and solve for n_p

$$-245{,}000 + (92{,}000 - 38{,}000)(P/A,15\%,n_p) + 245{,}000(0.15)(P/F,15\%,n_p) = 0$$

Solve by trial and error:

Try n = 7: $-245{,}000 + (92{,}000 - 38{,}000)(4.1604) + 36{,}750(0.3759) = 0$

$-6524 < 0$

Try n = 8: $-245{,}000 + (92{,}000 - 38{,}000)(4.4873) + 36{,}750(0.3269) = 0$

$+9327 > 0$

Therefore, $n_p \approx 8$ years

(b) Spreadsheet function: = NPER(15%,54000,-245000,0.15*245000) displays 7.4 years

13.33 $0 = -39{,}000 + (13{,}500 - 6000)(P/A,10\%,n_p)$

$(P/A,10\%,n_p) = 5.2000$

From 10% interest tables, n_p is between 7 and 8 quarters

13.34 (a) Develop the W relation and solve for n_p

$$-280{,}000(A/P,10\%,n_p) + 50{,}000 - 15{,}000 = 0$$

$$(A/P,10\%,n_p) = 0.125$$

From 10% tables, $n \approx 17$ years

(b) Spreadsheet function: = NPER(10%,35000,-280000) displays 16.9 years

13.35 $42{,}000 + 1000n_p = 25{,}000(F/P,10\%,n_p)$

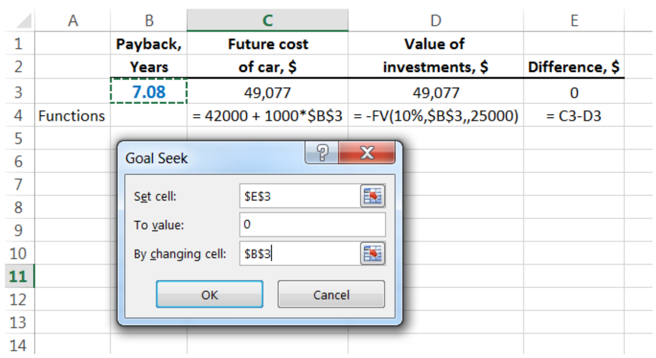
(a) <u>*By trial and error:*</u>

Try n = 7: 49,000 > 48,718

Try n = 8: 50,000 < 53,590

By linear interpolation, n_p = 7.08 years

(b) *By spreadsheet:* Use Goal Seek to force the difference to be zero by changing cell B3. Payback displayed is n_p = 7.08 years. Must wait 8 years to buy the car.



13.36 (a) i = 0%: n_p = 200,000/(90,000 – 50,000)

= 5 years

i = 12%: -200,000 + (90,000 -50,000)(P/A,12%,n_p) = 0

(P/A,12%,n_p) = 5.0000

From 12% interest tables, value is slightly over 8 years. Round to n_p = 8 years

Spreadsheet function: = NPER(12%,40000,-200000) displays 8.08 years

(b) Develop the relation Revenue = Cost and solve for X_{BE} = gallons per year

i = 0%; n_p = 5 years: $10X_{BE}$ = 200,000/5

X_{BE} = 4000 gallons per year

i = 12%; n_p = 8 years: $10X_{BE}$ = 200,000(A/P,12%,8)

X_{BE} = 20,000(0.2013)

= 4026 gallons per year

13.37 (a) i = 0%: *Semi-automatic:*

n = 40,000/10,000

= 4 years

Automatic:

n = 90,000/15,000

= 6 years

Select only the semi-automatic machine

(b) $i = 10\%$: *Semi-automatic:*

$$-40{,}000 + 10{,}000(P/A,10\%,n_p) = 0$$
$$(P/A,10\%,n_p) = 4.0000$$

From 10% interest tables, n_p is between 5 and 6 years.
Therefore, $n_p = 6$ years.

Automatic:

$$-90{,}000 + 15{,}000(P/A,10\%,n_p) = 0$$
$$(P/A,10\%,n_p) = 6.0000$$

From 10% interest tables, n_p is between 9 and 10 years.
Therefore, $n_p = 10$ years.

Select neither alternative since n_p values are both > 5 years

13.38 Let n_p = number of years until payback

$$0 = -130{,}000 + (75{,}000 - 45{,}000)n_p$$
$$30{,}000n_p = 130{,}000$$
$$n_p = 4.3 \text{ years}$$

13.39 Let n_p = number of months. Sample relation for equivalent worth per month for \$15,000 OC is: = - PMT(0.5%,\$D3,-90000,) + 22000 - \$A3

Apply Goal Seek for each OC estimate to set equivalent worth to zero and display the respective n_p value.

	A	B	C	D
1	Operating	Equiv. worth,		Payback,
2	cost, \$/month	\$/month		n_p months
3	15,000	\$0		13.3
4	16,000	\$0		15.6
5	17,000	\$0		18.9
6	18,000	\$0		23.9
7	19,000	\$0		32.6
8	20,000	\$0		51.1

13.40 $-250{,}000 - 500n_p + 250{,}000(1 + 0.0075)^{np} = 100{,}000$

Try n = 50: 88,239 < 100,000
Try n = 60: 111,420 > 100,000

$n_p = 55.1$ months or 4.6 years

13.41 (a) *Payback:*

A: $0 = -300{,}000 + 60{,}000(P/A,8\%,n_p)$

$(P/A,8\%,n_p) = 5.0000$

n_p is between 6 and 7 years

B: $0 = -300{,}000 + 10{,}000(P/A,8\%,n_p) + 15{,}000(P/G,8\%,n_p)$

Try $n = 7$: $0 > -37{,}573$

Try $n = 8$: $0 < +24{,}558$

n_p is between 7 and 8 years

Select A

(b) *Present worth:*

A: $PW = -300{,}000 + 60{,}000(P/A,8\%,10)$

$= -300{,}000 + 60{,}000(6.7101)$

$= \$102{,}606$

B: $PW = -300{,}000 + 10{,}000(P/A,8\%,10) + 15{,}000(P/G,8\%,10)$

$= -300{,}000 + 10{,}000(6.7101) + 15{,}000(25.9768)$

$= \$156{,}753$

Select B

Income for B increases rapidly in later years, which is not accounted for in payback analysis, which selected A.

(c) A sample spreadsheet solution follows. Answers are:

Payback: Select A with $n_p = 6.6$ years

PW: Select B with $PW_B > PW_A$

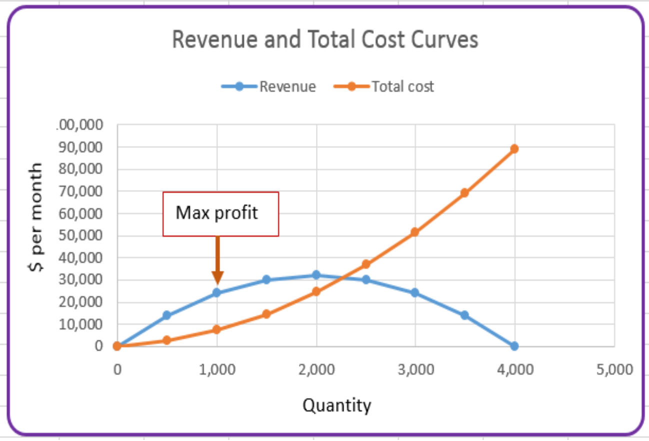
	A	B	C	D	E	F	G	H
1		**Payback analysis**				**Present worth analysis**		
2	Alternative	n_p, years	Function			Alternative	PW, $	
3	A	6.6	= NPER(8%,60000,-300000)			A	102,605	
4						B	156,753	
5	B	Year	CF, $/year	PW, $			**Select B**	
6		0	-300,000					
7		1	10,000	-290,741				
8		2	25,000	-269,307				
9		3	40,000	-237,554				
10		4	55,000	-197,127				
11		5	70,000	-149,487				
12		6	85,000	-95,922				
13		7	100,000	-37,573	**Payback is between 7 and 8 years**			
14		8	115,000	24,558				
15		9	130,000	89,590				
16		10	145,000	156,753	**Select A**			
17								

Spreadsheet Exercises

13.42 (a) Plot shows maximum quantity at approximately 1000 units. Profit estimate is \$16,790 per month.

Breakeven points Q_{BE} are at approximately 0 and 2300 units.

	A	B	C	D
1		Coefficient		
2		Q^2	Q	Constant
3	Revenue	-0.008	32	
4	Total cost	0.005	2.2	10
5				
6	Quantity	Revenue	Total cost	Profit
7	0	0	10	-10
8	500	14,000	2,360	11,640
9	1,000	24,000	7,210	16,790
10	1,500	30,000	14,560	15,440
11	2,000	32,000	24,410	7,590
12	2,500	30,000	36,760	-6,760
13	3,000	24,000	51,610	-27,610
14	3,500	14,000	68,960	-54,960
15	4,000	0	88,810	-88,810



(b) Profit = R – TC = (-.008-.005) Q^2 + (32-2.2)Q - 10
= -.013Q^2 + 29.8Q - 10

Q_p = -b/2a = -29.8/2(-.013)
= 1146 units

Profit = -b^2/4a + c = -29.8^2 / 4(-.013) - 10
= \$17,068

If Q = 1146 is entered into the spreadsheet column A, the profit is displayed as \$17,068

13.43 Let R = revenue for years 2 through 8. Set up PW = 0 relation.

PW = Revenue – costs
0 = 50,000(P/F,10%,1) + R(P/A,10%,7)(P/F,10%,1)
-150,000 + 10,000(P/F,10%,8) – 42,000(P/A,10%,8)

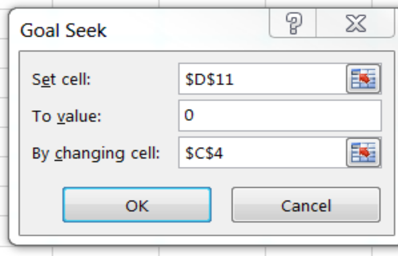
$$R = \frac{-50{,}000(0.9091) + 150{,}000 - 10{,}000(0.4665) + 42{,}000(5.3349)}{(4.8684)(0.9091)}$$
= 323,946/4.4259
= \$73,193 per year

Spreadsheet solution uses Goal Seek to find R = \$73,195 with the remaining revenue cells set equal to this value.

Year	Cost	Revenue	NCF, \$/year
0	-150,000		-150,000
1	-42,000	50,000	8,000
2	-42,000	73,195	31,195
3	-42,000	73,195	31,195
4	-42,000	73,195	31,195
5	-42,000	73,195	31,195
6	-42,000	73,195	31,195
7	-42,000	73,195	31,195
8	-42,000	83,195	41,195
PW			0

Changing cell for Goal Seek

Includes salvage value



13.44 (a) Current: Q_{BE} = 400,000/(14-10) = 100,000 units

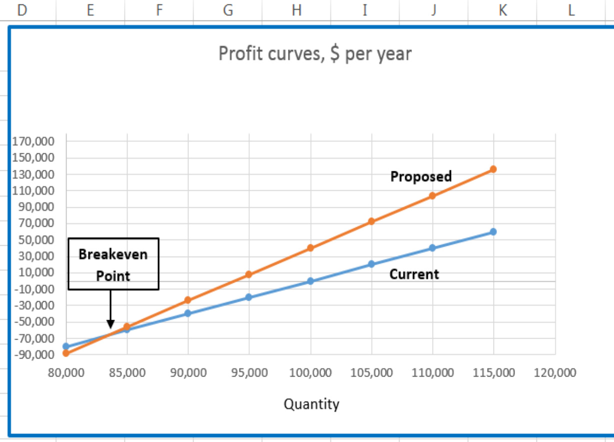
(b) New: Q_{BE} = 600,000/[16-48(0.2)] = 93,750 units

13.45 Current: Profit = 14Q -400,000 - 10Q = 4Q -400,000
New: Profit = 16Q - 600,000 - 9.60Q = 6.4Q - 600,000

Profit curves cross at approximately 83,000 units. Note that the profit is negative for both systems at this point.

System	Revenue, \$ per unit	Fixed cost, \$
Current	4.00	400,000
Proposed	6.40	600,000

Quantity, Q units	Profit, \$ per year Current	Proposed
80,000	-80,000	-88,000
85,000	-60,000	-56,000
90,000	-40,000	-24,000
95,000	-20,000	8,000
100,000	0	40,000
105,000	20,000	72,000
110,000	40,000	104,000
115,000	60,000	136,000



13.46 Solve the relation $AW_I = AW_O$ for N = number of tests per year

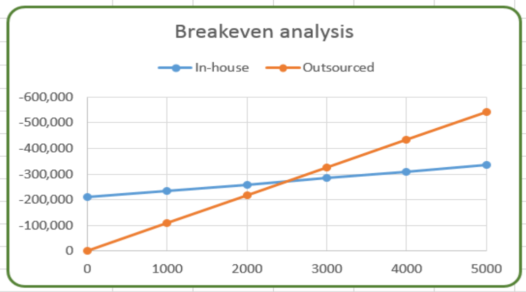
-125,000(A/P,5%,8) – 190,000 -25N = -100N – 25N(F/A,5%,3)(A/F,5%,8)
[75 + 25(3.1525)(0.10472)]N = 125,000(0.15472) + 190,000

$$83.25N = 209{,}340$$
$$N = 2514 \text{ tests per year}$$

13.47 Spreadsheet used to calculate AW values for each N value. Functions are written in cell reference format for sensitivity analysis.

Breakeven occurs at approximately 2500 tests per year.

	A	B	C	D	E
1	**In-house**			**Outsourced**	
2	Initial cost, $	-125,000		Cost, Years 1-5	100
3	Fixed cost, $/year	-190,000		Extra, years 6-8	25
4	Cost, $/sample	25			
5					
6		**AW values, $ per year**			
7	**Tests, N**	**In-house**	**Outsourced**		
8	0	-209,340	0		
9	1000	-234,340	-108,253		
10	2000	-259,340	-216,507		
11	3000	-284,340	-324,760		
12	4000	-309,340	-433,014		
13	5000	-334,340	-541,267		



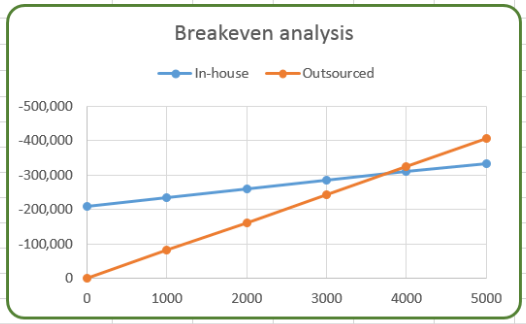
	A	B	C
17		**Sample functions**	
18	**Tests, N**	**In-house**	**Outsourced**
19	0	= -PMT(5%,8,$B$2) + $B$3 - $B$4*$A8	= - $E$2*$A8 - PMT(5%,8,,FV(5%,3,$E$3))*$A8
20	1000	= -PMT(5%,8,$B$2) + $B$3 - $B$4*$A9	= - $E$2*$A9 - PMT(5%,8,,FV(5%,3,$E$3))*$A9

13.48 <u>By hand:</u> It will raise the breakeven point. Outsourcing will cost \$75, increasing to \$93.75, which is an \$18.75 increase per sample in years 6-8. Resolve for N.

$$-125{,}000(A/P,5\%,8) - 190{,}000 - 25N = -75N - 18.75N(F/A,5\%,3)(A/F,5\%,8)$$
$$[50 + 18.75(3.1525)(0.10472)]N = 125{,}000(0.15472) + 190{,}000$$
$$56.19N = 209{,}340$$
$$N = 3726 \text{ tests per year}$$

<u>By spreadsheet:</u> Simply change the entries in the cost cells for outsourced. New breakeven is approximately 3700 tests per year, as calculated above

	A	B	C	D	E
1	In-house			Outsourced	
2	Initial cost, $	-125,000		Cost, Years 1-5	75
3	Fixed cost, $/year	-190,000		Extra, years 6-8	18.75
4	Cost, $/sample	25			
5					
6		AW values, $ per year			
7	Tests, N	In-house	Outsourced		
8	0	-209,340	0		
9	1000	-234,340	-81,190		
10	2000	-259,340	-162,380		
11	3000	-284,340	-243,570		
12	4000	-309,340	-324,760		
13	5000	-334,340	-405,950		



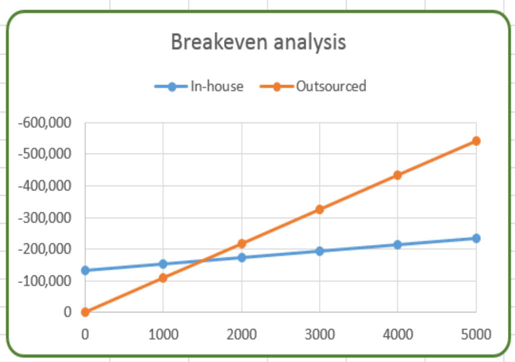
13.49 It will decrease the breakeven point.

By hand:

$$-125{,}000(A/P,5\%,8) - 115{,}000 - 20N = -100N - 25N(F/A,5\%,3)(A/F,5\%,8)$$
$$[80 + 25(3.1525)(0.10472)]N = 125{,}000(0.15472) + 115{,}000$$
$$88.25N = 134{,}340$$
$$N = 1522 \text{ tests per year}$$

By spreadsheet: Enter new fixed cost and sample cost for In-house and reenter $100, with $25 extra for outsourced option. New breakeven is approximately 1500 tests per year, as calculated above.

	A	B	C	D	E
1	In-house			Outsourced	
2	Initial cost, $	-125,000		Cost, Years 1-5	100
3	Fixed cost, $/year	-115,000		Extra, years 6-8	25
4	Cost, $/sample	20			
5					
6		AW values, $ per year			
7	Tests, N	In-house	Outsourced		
8	0	-134,340	0		
9	1000	-154,340	-108,253		
10	2000	-174,340	-216,507		
11	3000	-194,340	-324,760		
12	4000	-214,340	-433,014		
13	5000	-234,340	-541,267		



Additional Problems and FE Exam Review Questions

13.50 Production < breakeven point; select alternative with higher slope
Answer is (a)

13.51 Let x = number of cars serviced per year

-400,000(A/P,10%,15) + (0.10)(400,000) (A/F,10%,15) – 300x = 720x
-400,000(0.13147) + (0.10)(400,000) (0.03147) – 300x = -720x
420x = 51,329
x = 122 cars per year
Answer is (b)

13.52 Set equations equal to each other and solve for Q_{BE}
$2Q_{BE}$ = 23,000(0.16275) - 4000(0.06275) - 8000(0.31547) + 3000
Q_{BE} = 1984
Answer is (a)

13.53 Both the fixed cost and variable cost of Method X are higher than those of Y. Therefore, X can never be favored.
Answer is (a)

13.54 Amount of gasoline to drive there and back = 54/18
= 3 gallons

Cost of 3 gallons = 3(3.60)
= $10.80

Must save $10.80 for 22 gallons of gasoline:
Required savings per gallon = 10.80/22
= $0.49

Cost of gasoline = 3.60 – 0.49
= $3.11
Answer is (d)

13.55 Answer is (b)

13.56 Cost/yd^3 = [2(76) + 580]/160 = $4.58
Answer is (d)

13.57 VC = 22.50(4)(8)/1000
= $0.72
Answer is (c)

13.58 Let VC_{IoT} = variable cost of IoT-based process

40,000 + 30(4000) = 80,000 + VC_{IoT}(4000)
160,000 – 80,000 = VC_{IoT}(4000)
VC_{IoT} = 20
Answer is (b)

13.59 $-320{,}000 + (98{,}000-40{,}000)(P/A,20\%,n_p) = 0$

$$(P/A,20\%,n_p) = 5.5172$$

There is no P/A factor this large for any n value, that is, $n \rightarrow \infty$. From observation, the net income of \$58,000 per year < \$64,000 interest per year; the investment will never pay off.

Answer is (d)

13.60 Breakeven = 500,000/(250 – 200)

= 10,000

Breakeven + 10% = 10,000(1.10) = 11,000

Profit per 100 units = 250(11,000) - 500,000 - 200(11,000)

Answer is (b)

13.61 Let x = number of ft^2

$$-1400(A/P,10\%,3) - 2.03x = -3.25x$$

$$-1400(0.40211) - 2.03x = -3.25x$$

$$x = 461 \text{ ft}^2$$

Answer is (a)

13.62 VC = 40(4)/8

= \$20 per km

Answer is (c)

Solution to Case Study, Chapter 13

Sometimes, there is not a definitive answer to a case study exercise. Here are example responses.

WATER TREATMENT PLANT PROCESS COSTS

1. Savings = 40 hp * 0.75 kw/hp * 0.12 $/kwh * 24 hr/day * 30.5 days/mth ÷ 0.90
   = $2928 per month

2. A decrease in the efficiency of the aerator motor renders the selected alternative of "sludge recirculation only" *more* attractive, because the cost of aeration would be higher, and, therefore the net savings from its discontinuation would be greater.

3. If the cost of lime increased by 50%, the lime costs for "sludge recirculation only" and "neither aeration nor sludge recirculation" would increase by 50% to $393 and $2070, respectively. Therefore, the cost difference would *increase*.

4. If the efficiency of the sludge recirculation pump decreased from 90% to 70%, the net savings between alternatives 3 and 4 would *decrease*. This is because the $262 saved by not recirculating with a 90% efficient pump would increase to a monthly savings of $336 by not recirculating with a 70% efficient pump.

5. If hardness removal were discontinued, the extra cost for its removal (column 4 in Table 13-1) would be zero for all alternatives. The favored alternative under this scenario would be alternative 4 (neither aeration nor sludge recirculation) with a total savings of $2,471 – 469 = $2002 per month.

6. If the cost of electricity decreased to 8¢/kwh, the aeration only and sludge recirculation only monthly costs would be $244 and $1952, respectively. The net savings for alternative 2 would then be $-1605, alternative 3 would save $845, and alternative four would save $347. Therefore, the best alternative continues to be number 3.

7. (a) For alternatives 1 and 2 to breakeven, the total savings would have to be equal to the total extra cost of $1,849. Thus,

$$1{,}849/30.5 = (5)(0.75)(x)(24)/0.90$$
$$x = 60.6 \text{ cents per kwh}$$

(b)
$$1107/30.5 = (40)(0.75)(x)(24)/0.90$$
$$x = 4.5 \text{ cents per kwh}$$

(c)
$$1{,}849/30.5 = (5)(0.75)(x)(24)/0.90 + (40)(0.75)(x)(24)/0.90$$
$$x = 6.7 \text{ cents per kwh}$$

Chapter 14
Effects of Inflation

Adjusting for Inflation

14.1 Inflated dollars are converted into constant-value dollars by dividing by one plus the inflation rate per period for however many periods are involved.

14.2 Cost will double in 10 years when the value of the money has decreased by exactly one half. From Eq. [14.3]

$$1(1 + f)^{10} = 2$$
$$(1 + f) = 2^{0.1}$$
$$= 1.0718$$

$$f = 0.0718 \qquad \text{(7.18\% per year)}$$

14.3 (a) There is no difference.

(b) The then-current or future dollars have been divided by $(1 + f)^n$ to obtain constant-value dollars.

14.4 Future amount $= 10{,}000(1 + 0.07)^{10}$
$= \$19{,}672$

14.5 Today's purchasing power $= 1{,}000{,}000/(1.05)^{30}$
$= \$231{,}377$

14.6 (a) Future cost $= 106{,}000(1.03)^2$
$= €112{,}455$

(b) Constant-value cost = €106,000

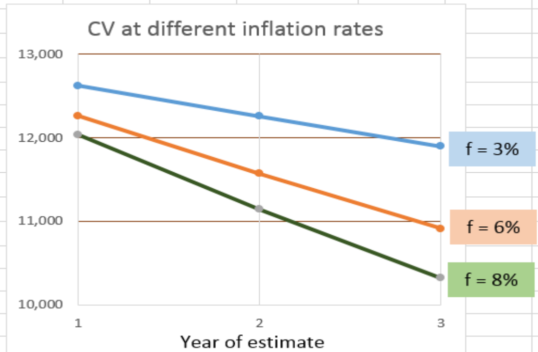
14.7 (a) $CV_{0,}$ year 1 estimate $= 13{,}000/(1 + 0.06)^1 = \$12{,}264$

$CV_{0,}$ year 2 estimate $= 13{,}000/(1 + 0.06)^2 = \$11{,}570$

$CV_{0,}$ year 3 estimate $= 13{,}000/(1 + 0.06)^3 = \$10{,}915$

(b) Use PV functions at 3, 6 and 8%

	CV in year 0, $		
Year	f = 3%	f = 6%	f = 8%
1	12,621	12,264	12,037
2	12,254	11,570	11,145
3	11,897	10,915	10,320



14.8 (a) Freshman year cost = $19{,}548(1.07)^3 = \$23{,}947$

(b) Second year cost = $(19{,}548 + 4000)(1.07)^4 = \$30{,}867$

14.9 (a) $5550(1 + f)^2 = 5730$

$1 + f = (1.03243)^{0.5}$

$f = 0.0161$ (1.61% per year)

(b) Award in 2018 = $5730(1 + 0.025)^2 = \$6020.08$

14.10 $62{,}100(1 + f)^{10} = 93{,}500$

$1 + f = (1.50564)^{0.1}$

$f = 0.0418$ (4.18% per year)

14.11 (a) CV purchasing power = $500{,}000/(1.03)^{27}$ = £225,095

(b) = -PV(3%,27,,500000) displays the answer £225,095

(c) Sample answers, depending upon the exchange rate (as of March 2016)
USD: £225,095(1.42) = $319,635
Euro: £225,095(1.29) = €290,373

14.12 (a) CV = $1{,}000{,}000/(1.04)^{40} = \$208{,}289$

(b) Future dollars = $1{,}000{,}000(1.04)^{40} = \$4{,}801{,}021$

14.13 (a) Cost; year 10 = $1000(1.10)^5\,(1.0)^5$
= $1610.51

(b) Cost; year 10, 5% per year = $1000(1 + 0.05)^{10}$ = \$1628.89

The cost is *not the same* because there are different inflation rates on different costs in each year over the 10 years

14.14 $23{,}930{,}909 = 1{,}000{,}000(1 + f)^{103}$

$(1 + f)^{103} = 23.9309$

$1 + f = 23.9309^{0.00971}$

$f = 0.0313$ (3.13% per year)

14.15 $7984 = 10{,}000(1 + f)^{20}$

$(1 + f)^{20} = 0.7984$

$(1 + f) = 0.7984)^{0.05}$

$f = -0.0112$ (-1.12% per year)

14.16 (a) CO_2 discharge = $36.9(1 + 0.025)^6$

= 42.79 gigatons (4.279×10^{10} tons)

(b) Total % increase = [(42.79 – 36.9)/36.9](100)

= 15.96%

14.17 (a) Total per month = 2699(1.015)(1.021)(1.027) = \$2872.53

(b) CV purchasing power = $2872.53/(1.02)^3$ = \$2706.85

Since \$2706.85 > \$2699, the CV purchasing power is slightly more.

14.18 Increase is f = 100% per day

Cost after 7 days = $1.25(1+1)^7$ = \$160

Present Worth Calculations with Inflation

14.19 Some examples of what Jake could do (there are many others):

1. Use savings and pay off the car loan immediately and do not apply for another loan for some time.
2. Use his savings over the 6-month period to pay off the car loan, and moderate his practices to take out one loan and pay premiums as required
3. Considering his salary cut, a new job may be necessary; then pay off the car loan in 6-months.

14.20 $0.25 = 0.20 + f + 0.20f$

$1.20f = 0.05$

$f = 0.0417$ (4.2% per year)

14.21 $0.40 = i + 0.09 + i(0.09)$
$1.09i = 0.31$
$i = 0.284$ (28.4% per year)

14.22 $0.40 = i + 0.08 + (i)(0.08)$
$1.08i = 0.32$
$i = 0.296$ (29.6% per year)

14.23 $i_f = 0.04 + 0.07 + (0.04)(0.07)$
$= 0.113$ (11.3% per year)

14.24 $0.03 = i + 0.04 + (i)(0.04)$
$1.04i = - 0.01$
$i = - 0.0096$ (- 0.96% per year)

Minus sign shows a negative real return rate. Inflation is higher than the market MARR.

14.25 <u>By hand:</u>

(a) Inflation not considered uses i = 10%

PW = -150,000 – 60,000(P/A,10%,5) + 0.20(150,000)(P/F,10%,5)
= -150,000 - 60,000(3.7908) + 30,000(0.6209)
= $-358,821

(b) Inflation considered uses i_f

$i_f = 0.10 + 0.07 + (0.10)(0.07)$
$= 17.7\%$

PW = -150,000 – 60,000(P/A,17.7%,5) + 0.20(150,000)(P/F,17.7%,5)
= -150,000 - 60,000(3.1485) + 30,000(0.44271)
= $-325,630

<u>By spreadsheet:</u> PW values and associated functions are shown.

	A	B	C	D	E
1	Inflation not accounted for			Inflation accounted for	
2	i =	10%		i_f =	17.7%
3	Function				= 0.1 + 0.07 + 0.1*0.07
4					
5	**PW, $**	**-358,820**			**-325,631**
6	Function	= - PV(B2,5,-60000,30000) - 150000			= - PV(E2,5,-60000,30000) - 150000

14.26 (a) Use i_f for future dollars and i for CV dollars

$i_f = 0.10 + 0.06 + (0.10)(0.06) = 0.166 \quad (16.6\%)$

$$\begin{aligned} PW &= 16{,}000 + 40{,}000(P/F,16.6\%,3) + 12{,}000(P/F,16.6\%,4) + 26{,}000(P/F,10\%,7) \\ &= 16{,}000 + 40{,}000(0.63082) + 12{,}000(0.54101) + 26{,}000(0.51316) \\ &= \$61{,}067 \end{aligned}$$

(b) Convert CF in year 7 to future dollars and develop NPV function

	A	B	C
1	i_f value =	16.60%	= 0.1 + 0.06 + 0.1*0.06
2			
3	Year	Future CF, $	Function
4	0	16,000	
5	1	0	
6	2	0	
7	3	40,000	
8	4	12,000	
9	5	0	
10	6	0	
11	7	39,094	= 26000*(1.06)^7
12			
13	PW value	61,067	= NPV($B$1,B5:B11) + B4

14.27 *<u>By hand</u>*:

Method 1: $PW = -10{,}000 + 2000(P/F,20\%,1) + 5000(P/A,20\%,3)(P/F,20\%,1)$
$= -10{,}000 + 2000(0.8333) + 5000(2.1065)(0.8333)$
$= \$443.33$

Method 2: Must convert all NCF to CV dollars and the inflated rate i_f to a real rate i

Real i: $0.20 = i + 0.05 + (i)(0.05)$
$1.05i = 0.15$
$i = 0.1429 \quad (14.29\%)$

$$\begin{aligned} PW &= -10{,}000 + [2000/(1.05)^1](P/F,14.29\%,1) + \\ &\quad + [5000/(1.05)^2](P/F,14.29\%,2) + [5000/(1.05)^3](P/F,14.29\%,3) \\ &\quad + [5000/(1.05)^4](P/F,14.29\%,4) \\ &= -10{,}000 + 1666.60 + 3471.96 + 2893.19 + 2410.90 \\ &= \$442.65 \quad \text{(rounding error)} \end{aligned}$$

By spreadsheet: Convert to CV; sample function for year 2: = \$B7/(1 + \$B\$2)^\$A7

	A	B	C
1	i value =	14.29%	
2	f value =	5.00%	
3		NCF, \$1000	
4	Year	Future	CV
5	0	-10,000	-10,000.00
6	1	2,000	1,904.76
7	2	5,000	4,535.15
8	3	5,000	4,319.19
9	4	5,000	4,113.51
10			
11	PW, \$1000		443.67

14.28 (a) Use real i = 10%

$$\begin{aligned} PW &= 81{,}000(P/F,10\%,2) \\ &= 81{,}000(0.8264) \\ &= \$\ 66{,}938 < \$68{,}000 \end{aligned}$$

Purchase later for \$81,000

(b) Use $i_f = 0.10 + 0.05 + (0.10)(0.05) = 0.155$ (15.5% per year)

$$\begin{aligned} PW &= 81{,}000(P/F,15.5\%,2) \\ &= 81{,}000[1/(1 + 0.155)^2] \\ &= 81{,}000(0.7496) \\ &= \$60{,}719 < \$68{,}000 \end{aligned}$$

Purchase later for \$81,000

14.29 (a) $$\begin{aligned} PW_A &= -31{,}000 - 28{,}000(P/A,10\%,5) + 5000(P/F,10\%,5) \\ &= -31{,}000 - 28{,}000(3.7908) + 5000(0.6209) \\ &= \$-134{,}038 \end{aligned}$$

$$\begin{aligned} PW_B &= -48{,}000 - 19{,}000(P/A,10\%,5) + 7000(P/F,10\%,5) \\ &= -48{,}000 - 19{,}000(3.7908) + 7000(0.6209) \\ &= \$-115{,}679 \end{aligned}$$

Select machine B

Spreadsheet function for PW_A: = - PV(10%,5,-28000,5000) - 31000
PW_B: = - PV(10%,5,-19000,7000) - 48000

(b) $i_f = 0.10 + 0.03 + (0.10)(0.03) = 0.133$ (13.3%)

$$\begin{aligned} PW_A &= -31{,}000 - 28{,}000(P/A,13.3\%,5) + 5000(P/F,13.3\%,5) \\ &= -31{,}000 - 28{,}000(3.4916) + 5000(0.5356) \end{aligned}$$

= $-126,087

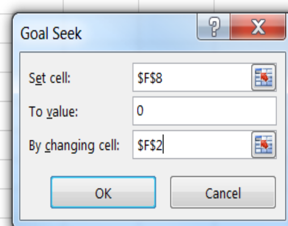
PW_B = -48,000 – 19,000(P/A,13.3%,5) + 7000(P/F,13.3%,5)
= -48,000 – 19,000(3.4916) + 7000(0.5356)
= $-110,591

Select machine B

Spreadsheet function for PW_A: = - PV(13.3%,5,-28000,5000) - 31000
PW_B: = - PV(13.3%,5,-19000,7000) – 48000

(c) Maintain i = 10% for B; use Goal Seek to force PW difference to be 0 while changing the i for A (cell F2).
A real i of 17.7% and an inflation-adjusted return of 21.2% are required for breakeven.

	A	B	C	D	E	F	G
1	**Machine**	**A**	**B**		**Machine**	**A**	**B**
2	i value	10.0%	10.0%		i value	17.7%	10.0%
3	f value	3%	3%		f value	3%	3%
4	Market i value	13.3%	13.3%		Market i valu	21.2%	13.300%
5							
6	PW for A, $	-126,088			PW for A, $	-110,592	
7	PW for B, $	-110,592			PW for B, $	-110,592	
8	Difference	-15,496			Difference	0	
9							
10	Before Goal Seek				After Goal Seek		
11							



14.30 i_f = 0.12 + 0.03 + (0.12)(0.03) = 15.36%

CC_X = -18,500,000 – 25,000/0.1536
= $-18,662,760

For alternative Y, first find AW and then divide by i_f

AW_Y = -9,000,000(A/P,15.36%,10) – 10,000 + 82,000(A/F,15.36%,10)
= -9,000,000(0.20199) – 10,000 + 82,000(0.04839)
= $-1,823,942

CC_Y = 1,823,942/0.1536
= $-11,874,622

Select alternative Y

14.31 Future amount required: $F = 2{,}000{,}000(1 + 0.04)^{23}$
= $4,929,431

Deposit now: $P = 4{,}929{,}431(P/F,8\%,23)$
$= 4{,}929{,}431(0.1703)$
$= \$839{,}482$

14.32 (a) The \$2.1 million are then-current dollars. Use i_f to find PW

$i_f = 0.15 + 0.03 + (0.15)(0.03) = 0.1845$ (18.45%)

$PW_{Lear} = 2{,}100{,}000(P/F,18.45\%,3)$
$= 2{,}100{,}000[(1/(1 + 0.1845)^3]$
$= \$1{,}263{,}613$

(b) He should buy the used Learjet, since \$1.1 million is lower than PW_{Lear}

14.33 $i_f = 0.15 + 0.035 + (0.15)(0.035) = 0.19025$ (19.025%)

$PW_{IWS} = (2{,}500{,}000 + 100{,}000)(P/F,19.025\%,2)$
$= 2{,}600{,}000[(1/(1 + 0.19025)^2]$
$= 2{,}600{,}000(0.70587)$
$= \$1{,}835{,}262$

$PW_{AG} = \$1{,}700{,}000$

Select AG Enterprises

14.34 (a) Find present worth of all three plans

Method 1: $PW_1 = \$450{,}000$

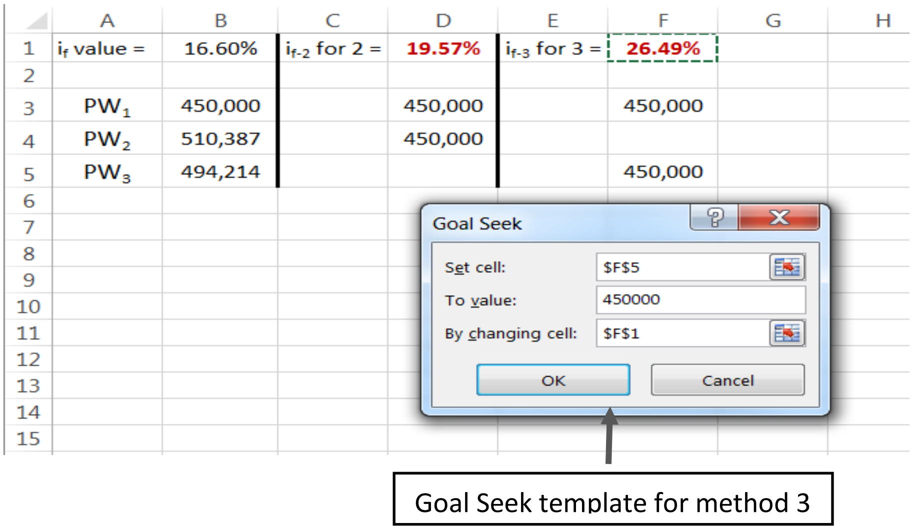
Method 2: $i_f = 0.10 + 0.06 + (0.10)(0.06) = 0.166$ (16.6%)

$PW_2 = 1{,}100{,}000(P/F,16.6\%,5)$
$= 1{,}100{,}000(0.46399)$
$= \$510{,}389$

Method 3: $PW_3 = 200{,}000 + 400{,}000(P/F,16.6\%,2)$
$= 200{,}000 + 400{,}000(0.73553)$
$= \$494{,}214$

Select payment method 2 with the highest PW value

(b) Rates are different for method 2 and 3, but they must both equal $PW_1 = \$450{,}000$. Use Goal Seek for each of method 2 and 3 to obtain $i_{f\text{-}2} = 19.57$ % and $i_{f\text{-}3} = 26.49\%$



Future Worth and Other Calculations with Inflation

14.35 Cost, year 5 = $140{,}000(1 + 0.03)^3(1 + 0.05)$
$= \$160{,}631$

14.36 (a) Cost, year 1 = $300{,}000(1 + 0.04)^1$
$= \$312{,}000$

Cost, year 2 = $300{,}000(1 + 0.04)^2$
$= \$324{,}480$

Cost, year 3 = $300{,}000(1 + 0.04)^3$
$= \$337{,}459$

(b) Case 1

14.37 (a) Find F, then deflate the amount by dividing by $(1 + f)^n$

$$F = 5000(F/P,15\%,17) + 8000(F/P,15\%,14) + 9000(F/P,15\%,13) + 15{,}000(F/P,15\%,10) + 16{,}000(F/P,15\%,6) + 20{,}000$$
$$= 5000(10.7613) + 8000(7.0757) + 9000(6.1528) + 15{,}000(4.0456) + 16{,}000(2.3131) + 20{,}000$$
$$= \$283{,}481$$

Purchasing power = $283{,}481/(1.03)^{17} = \$171{,}511$

(b) Purchasing power in CV dollars = \$61,762, which is less than the total amount deposited of \$73,000

	A	B
1	**Year**	**Deposit, $**
2	0	5,000
3	1	-
4	2	-
5	3	8,000
6	4	9,000
7	5	-
8	6	-
9	7	15,000
10	8	-
11	9	-
12	10	-
13	11	16,000
14	12	-
15	13	-
16	14	-
17	15	-
18	16	-
19	17	20,000
20	F value at 6%, $	120,306
21	Purchashing power at 4%, $	**61,762**
22	Total deposited, $	**73,000**

14.38 $740{,}000 = 625{,}000(F/P,f,5)$

$(F/P,f,5) = 1.184$

$(1 + f)^5 = 1.184$

$f = (1.184)^{0.2} - 1$

$= 0.0344$ (3.44% per year)

14.39 (a) $F = 10{,}000(F/P,10\%,5)$

$= 10{,}000(1.6105)$

$= \$16{,}105$

(b) Purchasing power $= 16{,}105/(1 + 0.05)^5$

$= \$12{,}619$

(c) $i_f = i + 0.05 + (i)(0.05)$

$0.10 = i + 0.05 + (i)(0.05)$

$1.05i = 0.05$

$i = 0.0476$ (4.76%)

or use Equation [14.9]

$i = (0.10 - 0.05)/(1 + 0.05)$

$= 0.0476$ (4.76%)

14.40 $F = P[(1 + i)(1 + f)(1 + g)]^n$

(a) $F = 145{,}000[(1 + 0.08)(1 + 0.04)(1 + 0.03)]^3$
$= 145{,}000(1.54840)$
$= \$224{,}518$

(b) $F = 145{,}000[(1 + 0.08)(1 + 0.04)(1 + 0.03)]^8$
$= 145{,}000(3.20889)$
$= \$465{,}289$

14.41 (a) $1{,}030{,}000 = 653{,}000(1 + f)^{18}$
$(1 + f)^{18} = 1.57734$
$f = (1.57734)^{1/18} - 1$
$= 0.0256$ (2.56%)

(b) The market rate is f + 5%.

$i_f = 0.03 + 0.05 = 0.08$

$F = 1{,}030{,}000(1.08)^6$
$= \$1{,}634{,}481$

14.42 Account will have to grow at rate of i_f

$i_f = 0.10 + 0.04 + (0.10)(0.04) = 0.144$ (14.4%)

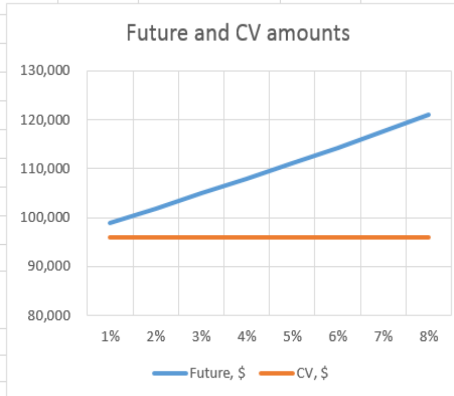
Amount required: $F = 422{,}000(F/P,14.4\%,15)$
$= 422{,}000(7.52299)$
$= \$3{,}174{,}701$

Function: = - FV(0.1+0.04+0.1*0.04,15,,422000) displays $3,174,701

14.43 (a) In CV dollars, the cost will be the same as today, $96,000, for all inflation rates.

(b) Future dollars line grows at $(1+f)^3$

	A	B	C
1	Inflation, %	Future, $	CV, $
2	1%	98,909	96,000
3	2%	101,876	96,000
4	3%	104,902	96,000
5	4%	107,987	96,000
6	5%	111,132	96,000
7	6%	114,338	96,000
8	7%	117,604	96,000
9	8%	120,932	96,000
10			
11	Sample function for 2%	= 96000*(1+$A3)^3	= $B3/(1+$A3)^3



14.44 Purchasing power = $1{,}800{,}000/(1 + 0.038)^{20}$
= \$853,740

14.45 Since future cost is a CV amount, future amount will increase by real i of 5%; inflation of 4% is not used in the calculation.

F = 40,000(F/P,5%,3)
= 40,000(1.1576)
= \$46,304

14.46 (a) Future amount is equal to the return at the market interest rate

F = 0.005(15,200,000)(F/A,9%,7)
= 76,000(9.2004)
= \$699,230

(b) Buying power = $699{,}230/(1 + 0.05)^7$
= 699,230(0.7107)
= \$496,943

14.47 (a) Required future amount is equal to a return of i_f on its investment

$i_f = (0.10 + 0.04) + 0.03 + (0.1 + 0.04)(0.03) = 0.1742$ (17.42%)

Required future: F = 1,000,000(F/P,17.42%,4)
= 1,000,000(1.90094)
= \$1,900,940

Company will get more; make the investment

(b) Set F relation equal to \$2.5 million; find i_f; calculate MARR and then i

$$
\begin{aligned}
2{,}500{,}000 &= 1{,}000{,}000(F/P,i_f,4) \\
&= 1{,}000{,}000(1 + i_f)^4 \\
(1 + i_f) &= 2.5^{0.25} \\
i_f &= 1.2574 - 1 \\
&= 0.2574 \qquad (25.74\%)
\end{aligned}
$$

$$
\begin{aligned}
0.2574 &= \text{MARR} + 0.03 + (\text{MARR})(0.03) \\
\text{MARR} &= 0.2574 - 0.03)/1.03 \\
&= 0.2208
\end{aligned}
$$

$$
\begin{aligned}
i &= \text{MARR} - 0.04 \\
&= 0.2208 - 0.04 \\
&= 0.18.08 \qquad (18.08\%)
\end{aligned}
$$

If the real interest rate i paid on capital exceeds 18.08% per year, the investment is not economically justified.

Capital Recovery with Inflation

14.48 $i_f = 0.15 + 0.06 + (0.15)(0.06) = 0.219 \quad (21.9\%)$

$$
\begin{aligned}
AW &= 183{,}000(A/P,21.9\%,5) \\
&= 183{,}000(0.34846) \\
&= \$63{,}768
\end{aligned}
$$

Function: = - PMT(0.15+0.06+0.15*0.06,5,183000) displays the same AW value

14.49 (a) In constant value dollars, use i = 12% to recover the investment

$$
\begin{aligned}
A &= 40{,}000{,}000(A/P,12\%,10) \\
&= 40{,}000{,}000(0.17698) \\
&= \$7{,}079{,}200
\end{aligned}
$$

(b) In future dollars, use i_f to recover the investment

$i_f = 0.12 + 0.07 + (0.12)(0.07) = 0.1984 \quad (19.84\%)$

$$
\begin{aligned}
A &= 40{,}000{,}000(A/P,19.84\%,10) \\
&= 40{,}000{,}000(0.23723) \\
&= \$9{,}489{,}200
\end{aligned}
$$

14.50 (a) To maintain purchasing power, use f to find future dollars.

$$F = 5{,}000{,}000(F/P,5\%,4)$$
$$= 5{,}000{,}000(1.2155)$$
$$= \$6{,}077{,}500$$

(b) Use market rate to find A

$$A = 6{,}077{,}500(A/F,10\%,4)$$
$$= 6{,}077{,}500(0.21547)$$
$$= \$1{,}309{,}519 \text{ per year}$$

(c) Function: = - PMT(10%,4,,-FV(5%,4,,5000000)) displays A = $1,309,530

14.51 (a) $i_f = 0.10 + 0.028 + (0.10)(0.028) = 0.1308$ (13.08% per year)

$$A = 12{,}000(F/A,13.08\%,20)(A/P,13.08\%,10)$$
$$= 12{,}000(81.7076)(0.18488)$$
$$= \$181{,}273 \text{ per year}$$

(b) Function: = - PMT(0.1+0.028+0.1*0.028,10,-FV(0.1+0.028+0.1*0.028,20,12000)) displays A = $181,272 directly

14.52 Use market interest rate (i_f) to calculate AW in then-current dollars

$$AW = 750{,}000(A/P,10\%,5)$$
$$= 750{,}000(0.26380)$$
$$= \$197{,}850$$

14.53 Find amount needed at 2% inflation rate and then find A using market rate.

$$F = 45{,}000(1 + 0.02)^3$$
$$= 45{,}000(1.06121)$$
$$= \$47{,}754.45$$

$$A = 47{,}754.45\ (A/F,8\%,3)$$
$$= 47{,}754.45\ (0.30803)$$
$$= \$14{,}710$$

14.54 Use 15% rate to determine cost of expansion, then find A using market rate.

$$F = 50{,}000{,}000(F/P,15\%,3)$$
$$= 50{,}000{,}000(1.5209)$$
$$= \$76{,}045{,}000$$

$$A = 76{,}045{,}000\ (A/F,10\%,3)$$

$= 76{,}045{,}000\ (0.30211)$
$= \$22{,}973{,}955$ per year

Function: = - PMT(10%,3,,- FV(15%,3,,50000000)) displays A = $22,973,942

14.55 (a) Use i_f (market interest rate) to find AW

$AW = 50{,}000(0.08) + 5000 = \$9000$

(b) For CV dollars, first find P using i (real interest rate); then find A using i_f

$0.08 = i + 0.04 + i(0.04)$
$i = 0.0385 \qquad (3.85\%)$

$PW = 50{,}000 + 5000/0.0385$
$= \$179{,}870$

$AW = 179{,}870(A/P,8\%,3)$
$= 179{,}870(0.38803)$
$= \$69{,}79$

14.56 <u>*By hand:*</u>

(a) For CV dollars, use i = 12% per year

$AW_A = -150{,}000(A/P,12\%,5) – 70{,}000 + 40{,}000(A/F,12\%,5)$
$= -150{,}000(0.27741) – 70{,}000 + 40{,}000(0.15741)$
$= \$-105{,}315$

$AW_B = -1{,}025{,}000(0.12) – 5{,}000$
$= \$-128{,}000$

Select machine A

(b) For then-current dollars, use i_f

$i_f = 0.12 + 0.07 + (0.12)(0.07) = 0.1984 \qquad (19.84\%)$

$AW_A = -150{,}000(A/P,19.84\%,5) – 70{,}000 + 40{,}000(A/F,19.84\%,5)$
$= -150{,}000(0.3332) – 70{,}000 + 40{,}000(0.1348)$
$= \$-114{,}588$

$AW_B = -1{,}025{,}000(0.1984) – 5{,}000$
$= \$-208{,}360$

Select machine A, now by a larger margin

Spreadsheet: Select A in both cases; the difference in AW values is larger for future dollars

	A	B	C	D	E
1		**CV dollars, $**		**Future dollars, $**	
2	Rate, %	12%		19.84%	
3		A	B	A	B
4	**AW value, $**	**-105,315**	**-128,000**	**-114,588**	**-208,360**
5	PMT function	= - PMT(12%,5,-150000,40000) - 70000	= -1025000*0.12 - 5000	= - PMT(19.84%,5,-150000,40000) - 70000	= -1025000*0.1984 - 5000
6					
7	Selection	**Select A**		**Select A**	

Additional Problems and FE Exam Review Questions

14.57 $i_f = 0.12 + 0.07 + (0.12)(0.07) = 0.1984$ (19.84%)
Answer is (d)

14.58 Answer is (c)

14.59 Answer is (b)

14.60 Answer is (c)

14.61 $33{,}015 = 29{,}350(1 + f)^3$
$(1 + f)^3 = 1.1249$
$(1 + f) = 1.1249^{0.333}$
$f = 4.0\%$
Answer is (b)

14.62 Answer is (c)

14.63 $F = 1000(F/P,5\%,35)$
$= 1000(5.5160)$
$= \$5516$
Answer is (c)

14.64 $i_f = 0.04 + 0.03 + (0.04)(0.03) = 0.0712$ (7.12%)

$P = 50{,}000[1/(1 + 0.0712)^6] = \$33{,}094$

Answer is (c)

14.65 i_f per month $= 0.01 + 0.01 + (0.01)(0.01) = 0.0201$ (2.01%)

Nominal per year = 2.01(12) = 24.12%
Answer is (d)

14.66 Cost = 85,000(F/P,4%,3)
= 85,000(1.12486)
= $95,613
Answer is (a)

14.67 Find accumulated amount in future dollars, then divide by inflation rate

F = 6000(F/A,10%,40)
= 6000(442.5926)
= $2,655,556

Buying power = $2,655,556/(1 + 0.05)^{40}$
= $377,210
Answer is (a)

Engineering Economy, 8th edition
Leland Blank and Anthony Tarquin

Chapter 15
Cost Estimation and Indirect Cost Allocation

Understanding Cost Estimation

15.1 (1) scoping/feasibility, (2) order of magnitude, (3) partially designed, (4) design 60-100% complete, (5) detailed estimate.

15.2 Rent: AOC
Direct labor: AOC
Equipment overhaul: FC
Overhead: AOC
Supplies: AOC
Insurance: AOC
Equipment cost: FC
Utility cost: AOC
Installation: FC
Delivery charges: FC
Salaried personnel: AOC
Periodic safety training: AOC

15.3 The top-down (design-to-cost) strategy works best

15.4 Calculate taxes (A), make bids (E), pay bonuses (A), determine profit or loss (A), predict sales (E), set prices (A), evaluate proposals (E), distribute resources (E), plan production (E), and set goals (E)

15.5 Bottom-up: Input = cost estimates; Output = required price
Top-down: Input = competitive price; Output = cost estimates

15.6 Project staff (D), Audit and legal (I), Utilities (I), Rent (I), Raw materials (D), Equipment training (D), Project supplies (D), Labor (D), Administrative staff (I), Miscellaneous office supplies (I), Quality assurance (I), IT department (I), scheduled maintenance (D), shared software packages (I)

15.7 (a) License plate (I)
(b) Drivers license (I)
(c) Gasoline (D)
(d) Highway toll fee (I, since it is usually an option to choose a non-toll route)
(e) Oil change (D)
(f) Repairs after collision (I)
(g) Gasoline tax (D, since it is a part of the direct cost of gas)
(h) Monthly loan payment (I)
(i) Annual inspection fee (I)
(j) Garage rental (I)

15.8 *Order-of magnitude estimates* and they should be within ±20% of the actual cost.

Unit Costs

15.9 (a) Cost = 98.23(50,000) = $4,911,500

(b) Max size = 4,000,000/98.23 = 40,721 ft^2
Max cost = 4,000,000/50,000 = $80 per ft^2

15.10 Cost = 69.18(570) = $39,433

15.11 (a) Medium: Cost = 3,457,500/30,000 = $115.25 per ft^2

(b) Low: 3,111,750/30,000 = $103.73 per ft^2
High: 4,321,875/30,000 = $144.06 per ft^2

Percent increase = [(144.06 – 103.73)/103.73] (100) = 38.9%

15.12 Property cost: (100 ×150)(2.50) = $37,500
House cost: (50 × 46)(0.75)(125) = $215,625
Furnishings: (6)(3,000) = $18,000

Total cost = $271,125

15.13 (a) Crew cost per day = 8[26.70 + 29.30 + 5(24.45) + 33.95] = $1697.60

(b) Cost per cubic yard = 1697.60/165 = $10.29 per cubic yard

(c) Cost for 550 cubic yards = 10.29(550) = $5659

15.14 Cost in Hawaii = 1,350,000 (2.19/0.83) = $3,562,048

Difference = 3,562,048 - 1,350,000 = $2,212,048

15.15 Cost/liter = (23.02)(1.32*0.65*1.28*0.25*1.53) = $9.67 per liter

15.16 (a) Cost_A = 130,000(180) = $23,400,000

Cost_B

Type	Area	Unit cost	Estimated cost
Classroom	39,000	$125	$4.8750 million
Lab	52,000	185	9.6200 million
Office	39,000	110	4.2900 million
Furnishings-labs	32,500	150	4.8750 million

Furnishings-other	97,500	25	2.4375 million
			$26.0975 million

(b) % increase = (26,097,500 - 23,400,000)/ 23,400,000
= 0.115 (11.5% higher for B)

Cost Indexes

15.17 Cost = 185,000(245.9/210.0) = $216,626

15.18 Index in 2011 was 227.2
Index in 2014 was 236.5
Index in 2015 was 238.0

Difference from 2104 to 2015 was 1.5

Estimated index in 2018 is 238.0 + 3(1.5) = 242.5

$C_{2018} = C_{2011}(I_{2018}/I_{2011})$
$= 800(242.5/227.2)$
$= \$854$

15.19 Table 15.3 index in 2010 was 217.0

$C_{now} = 87,200(273/217)$
$= \$109,703$

15.20 (a) Time between 2009 and 2016 is 7 years

Actual inflation rate: $5167 = 3423(1 + f)^7$
$f = (5167/3423)^{1/7} - 1$
$f = 0.0606$ (6.06% per year)

(b) Projected inflation rate: $4098 = 3423(1 + f)^7$
$f = (4098/3423)^{1/7} - 1$
$f = 0.0261$ (2.61% per year)

Difference = 6.06 – 2.61
= 3.45%

15.21 Let I_t = index value

$83,400 = 67,900 (I_t/1457.4)$
$I_t = 83,400(1457.4)/67,900$
$= 1790.1$

15.22 PCI for 2010: 550.8/5.508 = 100
PCI for 2014: 579.8/5.508 = 105.3
PCI for 2015: 556.8/5.508 = 101.1

Alternate solution:
2014 value: To have a PCI value of 100 in year 2010, divide the 2014 value by 550.8
PCI for 2014 = 579.8/550.8 *100 = 105.3
PCI for 2015 = 556.8/550.8 *100 = 101.1

15.23 $C_{2012} = 0.25(809.2 \text{ million})$
$= \$202.3 \text{ million}$

Index value in 2012 was 8570

$C_{2017} = 202.3(9324/8570)$
$= \$220.1 \text{ million}$

15.24 Let i = decimal increase from 1926 to 2011, which is 85 years

$1490.2 = 100(F/P,i,85)$
$14.902 = (1+i)^{85}$
$(1+i) = 1.03229$
$i = 0.0323$ (3.23 % per year)

15.25 At 2% per quarter, annual increase = $(1 + 0.02)^8 - 1 = 0.1717$ (17.17%)

Index value = 100(1.1717)
= 117.17

15.26 Mechanical % in 2006: 2,511,893/13,136,431 = 0.1912 (19.12%)

(a) Mechanical cost in 2015: 15,700,000(0.1912) = \$3,001,840

(b) Mechanical cost in 2015: 15,700,000(0.1912)(1.20) = \$3,602,208

15.27 (a) $\text{Cost}_{10} = 85{,}000\,(1+0.02)^3\,(1+0.05)^7 = \$126{,}924$

(b) $126{,}924 = 85{,}000(I_{10}/1203)$
$I_{10} = 1796.35$

15.28 (a) Use Equation [15.2] to predict prices with 2008 as the C_0 (\$35.82 for B-E and \$44.60 for WTI). For example, prediction for WTI for 2011 is

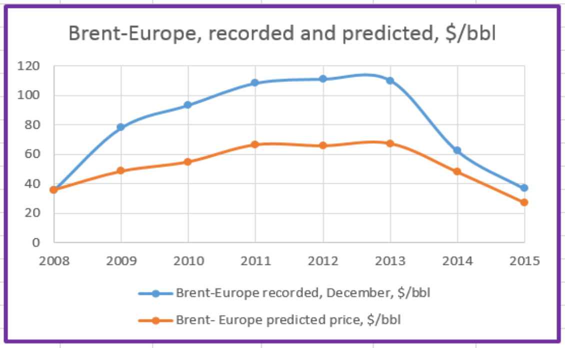
$$C_{2011} = C_{2008}(I_{2011}/I_{2008})$$
$$= 44.60(194.4/105.1)$$

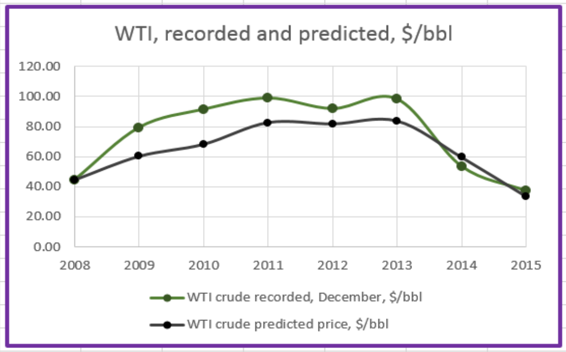
= \$82.50

(b) Graph shows year versus recorded and predicted Brent-Europe prices

(c) Graph shows year versus recorded and predicted WTI prices
(d) WTI crude prices are tracked more closely

Year	Global Crude Oil Index, Annual	Brent-Europe recorded, December, \$/bbl	Brent- Europe predicted price, \$/bbl	WTI crude recorded, December, \$/bbl	WTI crude predicted price, \$/bbl
2008	105.1	35.82	35.82	44.60	44.60
2009	141.9	77.91	48.36	79.39	60.22
2010	160.6	93.23	54.74	91.38	68.15
2011	194.4	108.09	66.26	98.83	82.50
2012	192.3	110.80	65.54	91.83	81.60
2013	196.8	109.95	67.07	98.17	83.51
2014	140.2	62.16	47.78	53.45	59.49
2015	79.3	36.61	27.03	37.13	33.65





Cost-capacity and Factor Methods

15.29 The cost index bases the estimate on cost differences over time for a *specified value* of a variable, while a CER estimates costs between *different values* of a design variable.

15.30 $C_2 = 153{,}200(0.75/2)^{0.58} = \$86{,}736$

15.31 The value of the exponent will be 1.0

15.32 From Table 15-6, cost-capacity exponent for compressors is 0.32

$$\begin{aligned} C_2 &= 14{,}000(600/250)^{0.32} \\ &= 14{,}000(1.323) \\ &= \$18{,}527 \end{aligned}$$

15.33
$$\begin{aligned} 3{,}000{,}000 &= 550{,}000\,(100{,}000/6000)^x \\ 5.4545 &= (16.6667)^x \\ \log 5.4545 &= x \log 16.667 \end{aligned}$$

$$0.7367 = 1.2218\,x$$
$$x = 0.60$$

15.34 (a) $450{,}000 = 200{,}000(60{,}000/35{,}000)^x$
$$2.25 = 1.7143^x$$
$$\log 2.25 = x \log 1.7143$$
$$x = 1.504$$

(b) Since $x > 1.0$, there is dis-economy of scale, so that the percentage increase in cost is *greater* than the percentage increase in size, which is the opposite of most situations.

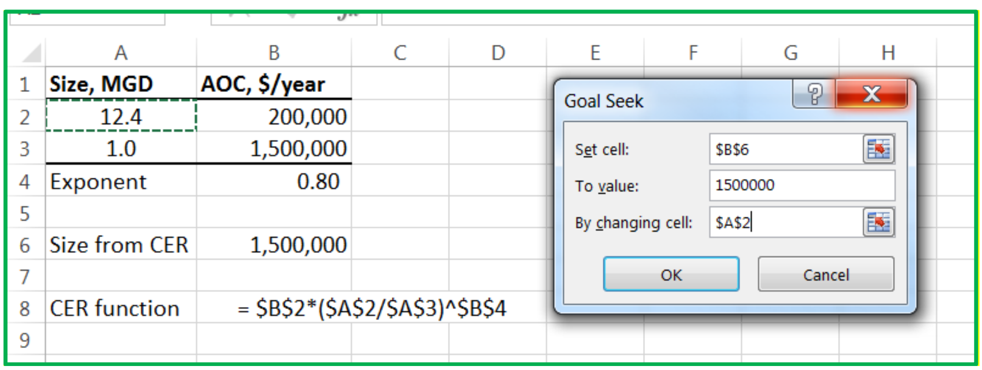
15.35 In $ million units,

$$250 = 55(600/Q_1)^{0.67}$$
$$4.5454 = (600/Q_1)^{0.67}$$
$$\log 4.5454 = 0.67(\log 600 - \log Q_1)$$
$$0.6576 - 1.8614 = -\,0.67 \log Q_1$$
$$-1{,}7967 = -\log Q_1$$
$$Q_1 = 62.6 \text{ MW}$$

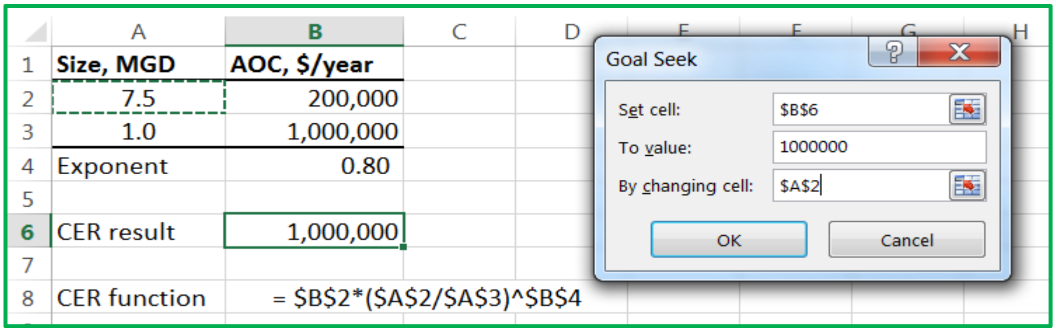
15.36 (a) In $ million units,

$$1.5 = 0.2(Q_2/1)^{0.80}$$
$$\log 7.5 = 0.8(\log Q_2 - \log 1)$$
$$1.09383 = \log Q_2$$
$$Q_2 = 12.4 \text{ MGD}$$

(b) Spreadsheet image is below with Goal Seek template set with AOC = $1.5 million.
Answer is 12.4 MGD



(c) Spreadsheet image is below with Goal Seek template set with AOC = $1.0 million.
Answer is 12.4 MGD



15.37 Use Equation [15.3] and Table 15-3

$C_2 = 60{,}000\ (4/1)^{0.24}\ (1490.2/1244.5)$
$= 50{,}000\ (1.181)(1.0157)$
$= \$100{,}206$

15.38 C_2 in 1999 $= 160{,}000\ (1000/200)^{0.35}$
$= \$281{,}034$

C_2 in 2018 $= 281{,}034\ (1.35/1)$
$= \$379{,}396$

15.39 Find I_{2016} in Equation [15.4]

$10{,}200 = 3750(2)^{0.89}\ (I_{2016}/1490.2)$
$I_{2016} = 2187.2$

15.40 $C_T = 2.25(2{,}300{,}000)$
$= \$5{,}175{,}000$

15.41 (a) $h = 55.4/17.8 = 3.11$

(b) h is a multiplier that accounts for costs such as construction, maintenance, labor, materials and all indirect components.

15.42 $2{,}300{,}000 = (1 + 1.55 + 0.43)C_E$
$C_E = \$771{,}812$

15.43 (a) $h = 1 + 1.52 + 0.31 = 2.83$

$C_T = 2.83\ (1{,}600{,}000)$
$= \$4{,}528{,}000$

(b) $h = 1 + 1.52 = 2.52$

$C_T = [1{,}600{,}000(2.52)](1.31)$

= \$5,281,920

15.44 Apply Equation [15.6]. Monetary units are in \$ million

$h = 1 + 0.2 + 0.5 + 0.25 = 1.95$

C_T in 2004: 1.75 (1.95) = \$3.41 (\$3.41 million)

Update to now with the cost index

C_T now: 3.41 (3713/2509) = 3.41(1.48) = \$5.05 million

15.45 (a) $h = 1 + 0.30 + 0.30 = 1.60$

Let f_I be the indirect cost factor

$$430{,}000 = [250{,}000\,(1.60)]\,(1 + f_I)$$
$$(1+ f_I) = 430{,}000/[250{,}000(1.60)]$$
$$(1+ f_I) = 1.075$$
$$f_I = 0.075$$

The indirect cost factor used is much lower than 0.30.

(b) $C_T = 250{,}000[1.60](1.30)$
= \$520,000

Indirect Cost (IDC) Rates and Allocation

15.46 Total direct labor hours = 2000 + 8000 + 5000
= 15,000 hours

Indirect cost rate = 36,000/15,000 = \$2.40 per hour

Allocation:
Dept A = 2000(2.40) = \$4800

Dept B = 8000(2.40 = \$19,200

Dept C = 5000(2.40) = \$12,000

15.47 (a) North: Miles basis; rate = 300,000/350,000 = \$0.857 per mile
South: Labor basis; rate = 200,000/20,000 = \$10 per hour
Midtown: Labor basis; rate = 450,000/64,000 = \$7.03 per hour

(b) Allocation:
North: 275,000(0.857) = \$235,675
South: 31,000(10) = \$310,000
Midtown: 55,500(7.03) = \$390,165

Percent allocated = (235,675 + 310,000 + 390,165)/1.2 million × 100% = 78%

15.48 Rate for CC100 = 25,000/800 = \$31.25 per hour
Rate for CC110 = 50,000/200 = \$250 per hour
Rate for CC120 = 75,000/1200 = \$62.50 per hour
Rate for CC190 = 100,000/1600 = \$62.50 per hour

15.49 (a) From Equation [15.8], estimated basis level = allocated IDC / IDC rate

Month	Basis Level	Basis
February	2800/1.40 = 2000	Space
March	3400/1.33 = 2556	Direct labor costs
April	3500/1.37 = 2555	Direct labor costs
May	3600/1.03 = 3495	Space
June	6000/0.92 = 6522	Material costs

(b) The only way the rate could decrease is by switching the allocation basis from month to month. If a single allocation basis had been used throughout, the rate would have to generally increase for each basis. For example, if space had been used for each month, the monthly rates would have been:

Month	Rate
February	2800/2000 = \$1.40 per ft^2
March	3400/2000 = \$1.70 per ft^2
April	3500/3500 = \$1.00 per ft^2
May	3600/3500 = \$1.03 per ft^2
June	6000/3500 = \$1.71 per ft^2

15.50 Determine AW for *Make* and *Buy* alternatives. *Make* has annual indirect costs.

<u>Hand solution</u>: Make: Indirect cost computation

Dept	Rate, \$ (1)	Usage (2)	Annual cost, \$ (3) = (1)(2)
X	2.40	450,000	1,080,000
Y	0.50	850,000	425,000
Z	20.00	4500	90,000
\$/year			1,595,000

AW_{make} = -3,000,000(A/P,12%,6) + 500,000(A/F,12%,6) – 1,500,000 – 1,595,000

$= -3{,}000{,}000(0.24323) + 500{,}000(0.12323) -3{,}095{,}000$
$= \$-3{,}763{,}075$

$AW_{buy} = -3{,}900{,}000 – 300{,}000(A/G,12\%,6)$
$= -3{,}900{,}000 – 300{,}000(2.1720)$
$= \$-4{,}551{,}600$

Select *make* alternative

Spreadsheet solution: Select *make* alternative.

	A	B	C	D	E	F
1		MAKE			BUY	
2					Year	Cost
3		Indirect cost computation			1	-3,900,000
4	Dept	Rate	Usage	Indirect cost	2	-4,200,000
5	X	2.40	$450,000	$ 1,080,000	3	-4,500,000
6	Y	0.50	$850,000	$ 425,000	4	-4,800,000
7	Z	20.00	4500	$ 90,000	5	-5,100,000
8				$ 1,595,000	6	-5,400,000
9	PW	-$15,471,490			PW	-$18,713,540
10	AW	-$3,763,064			AW	-$4,551,614

ABC Method of IDC Allocation

15.51 Determine the rates by basis, then distribute the $1,000,000

Basis	Total usage	Rate
Direct material cost	$51,300	$19.49/$
Previous build-time	1395 work-hours	716.85/work-hour
New build-time	1260 work-hours	793.65/work-hour

Example allocation for **New York**:

Direct material cost: 19.49(20,000) = $389,800
Previous build time: 716.85(400) = $286,740
New build time: 793.65(425) = $337,301

	Allocation by each basis, $		
	Material cost	Previous build-time	New build-time
NY	389,800	286,740	337,301
VA	247,523	297,493	281,746
TN	362,514	415,773	380,952
Total	$999,837	$1,000,006	$999,999

15.52 Total budget = 19 pumps × \$20,000/pump = \$380,000

(a) Total service trips = 190 + 55 + 38 + 104 = 387

Allocation/Trip = 380,000/387 = \$981.91

Station ID	Service Trips/year	IDC Allocation, \$
Sylvester	190	190(981.91) = 186,563
Laurel	55	55(981.91) = 54,005
7^{th} St	38	38(981.91) = 37,313
Spicewood	104	104(981.91) = 102,119
		\$380,000

(b)

Station ID	Number of pumps	Allocation at \$20,000/pump
Sylvester	5	100,000
Laurel	7	140,000
7^{th} St	3	60,000
Spicewood	4	80,000
	19	\$380,000

Sylvester and Spicewood went down; Laurel went up by 2.6 times; 7^{th} St went up 60%.

15.53 (a) Activity is the department at each hub that lose or damage baggage.
(b) Cost driver is the number of bags handled, some of which are lost or damaged.

15.54 Total bags handled = 4,835,900

IDC allocation rate = 667,500/4,835,900 = \$0.13803 per bag handled (~13.8¢/bag)

Use 13.8¢ per bag checked and handled

	Bags handled	Allocation \$
HUA	2,490,000	343,620
DFW	1,582,400	218,371
SAT	763,500	105,363

15.55 Compare last year's allocation based on flight traffic with this year's based on baggage traffic using a rate of \$0.138/bag. Significant change took place, especially at SAT.

Hub	Last year; flight basis, \$	This year; baggage basis, \$	Percent change
HUA	330,000	343,620	+ 4.13%
DFW	187,500	218,371	+16.46
SAT	150,000	105,363	-29.76

15.56 (a) Rate = 1,000,000/16,500 guests = \$60.61 per guest

Allocation = number of guests × rate

	Site			
	1	2	3	4
Guests	3500	4000	8000	1000
Allocation, \$	212,135	242,440	484,880	60,610

(b) Guest-nights = (guests)(length of stay)

Total guest-nights = (3500)(3.0) + … + (1000)(4.75) = 35,250

Rate = 1,000,000/35,250 = \$28.37 per guest-night

	Site			
	1	2	3	4
Guest-nights	10,500	10,000	10,000	4750
Allocation, \$	297,885	283,700	283,700	134,757

(c) The actual indirect allocation to sites 3 and 4 (\$60,610 vs. \$134,757) are significantly different by the two methods.

(d) Another basis could be guest-dollars, that is, total amount of money a guest spends.

Additional Problems and FE Exam Review Questions

15.57 Percentage of total cost = 40/119 = 33.6%
Answer is (c)

15.58 C_2 = 700,000(10,000/6059)
= \$1,155,306
Answer is (b)

15.59 89,750 = 75,000(I_2/307)
I_2 = 367.4
Answer is (a)

15.60 $C_2 = 42{,}000(300/200)^{0.64}$
= \$54,444
Answer is (d)

15.61 $\text{Cost}_{\text{now}} = 16{,}000\ (1364/1192)\ (2)^{0.65}$
= \$28,729
Answer is (c)

15.62 $120{,}000 = 80{,}000(2)^x$
$\log 1.5 = x \log 2$
$0.1761 = 0.31013x$
$x = 0.585$
Answer is (b)

15.63 $C_T = (1 + 1.36 + 0.31)(650{,}000) = \$1{,}735{,}500$
Answer is (c)

15.64 $C_T = 3.96(390{,}000)$
$= \$1{,}544{,}400$
Answer is (d)

15.65 Allocation = (900 + 1300)(2000)
= \$4.4 million
Percent allocated = 4.4/19.56 million
= 0.225 (22.5%)
Answer is (a)

15.66 Answer is (d)

15.67 $C_T = [(1 + 1.82)(650{,}000)](1.31)$
$= \$2{,}401{,}230$
Answer is (b)

15.68 Answer is (b)

Chapter 16
Depreciation Methods

Fundamentals of Depreciation

16.1 Depreciation is a tax-allowed deduction; it reduces the tax burden.

16.2 *Unadjusted basis* is the installed cost of an asset and includes all costs incurred in preparing an asset for use. *Adjusted basis* is the amount of the basis remaining after some depreciation has been charged.

16.3 (a) *Book value* is the remaining capital investment after depreciation charges have been subtracted. *Market value* is the amount of money that can be realized if the asset is sold on the open market.
(b) A hi-tech computer workstation may have a market value that is much lower than its book value due to rapidly advancing technology.

16.4 *Productive life* – Time the asset is actually expected to provide useful service.
Tax recovery period – Time *allowed by tax laws* to depreciate the asset's value to salvage (or zero).
Book recovery period – Time used on *company accounting books* for depreciation to salvage (or zero)

16.5 B = 18,000 + 300 + 1200 = $19,500
S = 0
n = 7 years

16.6 B = 780,000 + 4300 + 6400 = $790,700
n = 15 years
S = 0.10(780,000) = $78,000

16.7 (a) BV_2 = 100,000 – (40,000 + 24,000)
= $36,000

(b) BV_3 = 100,000 – (40,000 + 24,000 +14,000) = $22,000

Difference = 22,000 - 20,000 = $2000

Market value is lower

(c) Percent written off = [(40,000 + 24,000 + 14,000)/100,000](100%)
= 78%

16.8 Tax depreciation: D_t = Rate*BV_{t-1}
Book depreciation: D_t = Rate*40,000

	Tax Depreciation		Book Depreciation	
Year, t	D_t	BV_t	D_t	BV_t
0		40,000		40,000
1	16,000	24,000	10,000	30,000
2	9,600	14,400	10,000	20,000
3	5,760	8,640	10,000	10,000
4	3,456	5,184	10,000	0

16.9 Quoting Publication 946, 2015 version:

(a) "Depreciation is an annual income tax deduction that allows you to recover the cost or other basis of certain property over the time you use the property. It is an allowance for the wear and tear, deterioration, or obsolescence of the property."

(b) "An estimated value of property at the end of its useful life. Not used under MACRS."

(c) General Depreciation System (GDS) and Alternative Depreciation System (ADS). The recovery period and method of depreciation are the primary differences.

(d) The following cannot be MACRS depreciated: intangible property; films and video tapes and recordings; certain property acquired in a nontaxable transfer; and property placed into service before 1987.

(e) Depreciation *begins* when property is placed into service, when it is ready and available for a specific use, whether in a business activity, an income-producing activity, a tax-exempt activity, or a personal activity. Even if not using the property, it is in service when it is ready and available for its specific use.
Depreciation *ends* when property is retired from service, even if its cost or other basis is not fully recovered.

(f) A taxpayer can elect to recover all or part of the cost of certain qualifying property, up to a limit, by deducting it in the year the property is placed in service. The taxpayer can elect the Section 179 Deduction instead of recovering the cost through depreciation deductions.

Straight Line Depreciation

16.10 d_t = 1/n = 1/5 = 0.20 (20% per year)

16.11 B = $400,000

$S = 0.1(300{,}000) = \$30{,}000$

$D_t = (400{,}000 - 30{,}000)/8$
$\quad = \$46{,}250$ per year t $\qquad (t = 1, \ldots, 8)$

$BV_4 = 400{,}000 – 4(46{,}250) = \$215{,}000$

16.12 (a) $D_t = \dfrac{12{,}000 – 2000}{7} = \$1428.57$

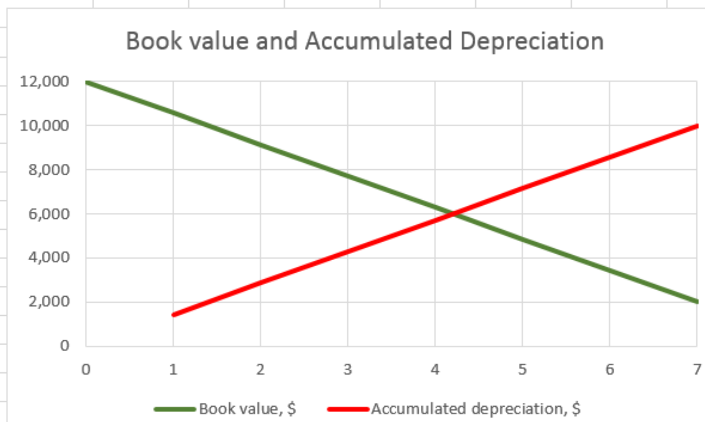
(b) Function: = SLN(12000,2000,7) displays $1428.57 per year

(c) $BV_3 = 12{,}000 – 3(1428.57) = \$7712.29$

(d) $d = 1/n = 1/7 = 0.14286$

(e) Graph of accumulated D_t and BV_t $(t = 1, \ldots, 7)$ is shown below.

Year	Book value, $	Annual depreciation, $	Accumulated depreciation, $
0	12,000.00		
1	10,571.43	1,428.57	1,428.57
2	9,142.86	1,428.57	2,857.14
3	7,714.29	1,428.57	4,285.71
4	6,285.71	1,428.57	5,714.29
5	4,857.14	1,428.57	7,142.86
6	3,428.57	1,428.57	8,571.43
7	2,000.00	1,428.57	10,000.00



16.13 Depreciation charge is the same each year

$D = (170{,}000 – 20{,}000)/3 = \$50{,}000$
$BV_2 = 170{,}000 – 2(50{,}000) = \$70{,}000$

16.14 (a) Salvage value = book value at end of recovery period, year 4

$S = 65{,}000 – 2(27{,}500) = \$10{,}000$

(b) $(B – 10{,}000)/4 = 27{,}500$
$B = \$120{,}000$

16.15 (a) Depreciation is constant and determined from the change in book value

$$D = 296{,}000 - 224{,}000 = 72{,}000/\text{year}$$

Since the asset has a 5-year life, 2 more years of depreciation will reduce BV to the salvage value S.

$$S = 224{,}000 - 2(72{,}000) = \$80{,}000$$

(b) $(B - 80{,}000)/5 = 72{,}000$

$$B = \$440{,}000$$

16.16 (a) $D = (50{,}000 - 5000)/4 = \$11{,}250$

Accumulated depreciation = 11,250(3) = $33,750

(b) Depreciation remaining = $BV_3 - S$

$$BV_3 = 50{,}000 - 3(11{,}250) = \$16{,}250$$

Depreciation remaining = 16,250 – 5000 = $11,250

16.17 (a) $D = (750{,}000 - 50{,}000)/10 = \$70{,}000$

BV_6 = capital investment remaining
= 750,000 - 6(70,000)
= $330,000

(b) Loss = BV_6 - selling price
= 330,000 – 175,000
= $155,000

(c) With depreciation charge of $70,000 per year, determine when BV = $260,000

$$BV_7 = 750{,}000 - 7(70{,}000) = \$260{,}000$$

Should have depreciated the machine only one more year.

Declining Balance Depreciation

16.18 (a) <u>*By hand:*</u>

DDB: $D_2 = (2/5)(500{,}000)(3/5)^1 = \$120{,}000$

150% DB: $d = 1.5/5 = 0.3$

$$D_2 = (0.3)(500{,}000)(0.7)^1$$
$$= \$105{,}000$$

$$\text{SL: } D_2 = (500{,}000 - 50{,}000)/5$$
$$= \$90{,}000$$

(b) *By spreadsheet:*

	A	B	C
1	**Method**	**Function**	**D_2, $**
2	DDB	= DDB(500000,50000,5,2)	120,000
3			
4	DB	= DDB(500000,50000,5,2,1.5)	105,000
5			
6	SL	= SLN(500000,50000,5)	90,000

16.19 (a) $d = 2/5 = 0.40$

$$D_2 = 0.40(78{,}000)(1 - 0.40)^1$$
$$= \$18{,}720$$

$$D_4 = 0.40(78{,}000)(1 - 0.40)^3$$
$$= \$6739$$

(b) $$BV_2 = 78{,}000(1 - 0.40)^2$$
$$= \$28{,}080$$

$$BV_4 = 78{,}000(1 - 0.40)^4$$
$$= \$10{,}109$$

16.20 $d = 2/5 = 0.40$

$$25{,}000 = B(1 - 0.4)^3$$
$$B = \$115{,}740$$

16.21 Substitute 0.25B for BV; $d = 2/5 = 0.4$; find t

$$0.25B = B(1 - 0.4)^t$$
$$\log 0.25 = t(\log 0.6)$$
$$t = 2.71 \text{ years}$$

16.22 $d = 2/5 = 0.4$

% remaining after 5 years $= (1 - 0.4)^5 = 0.078$ (7.8%)

16.23 $d = 2/5 = 0.40$

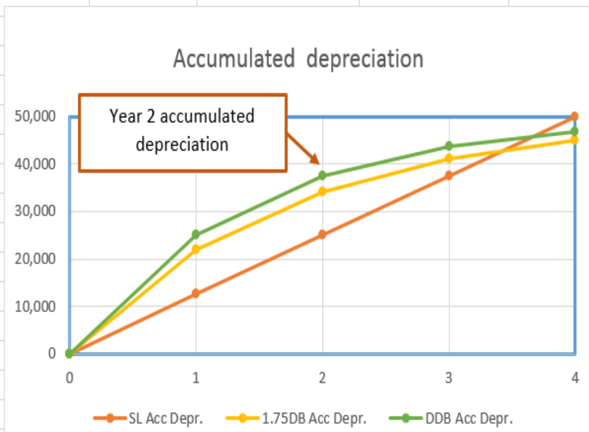
$$BV_3 = 40{,}000(1 - 0.40)^3$$

= $8640

Difference = 8640 – 4000 = $4640

16.24 Largest accumulated depreciation in year 2 is for DDB method: $37,500

	A	B	C	D	E	F	G
1		Straight line (SL)		1.75 Declining balance (1.75 DB)		Double declining balance (DDB)	
2	Year	Annual Depr.	SL Acc Depr.	Annual Depr.	1.75DB Acc Depr.	Annual Depr.	DDB Acc Depr.
3	0		0		0		0
4	1	12,500	12,500	21,875	21,875	25,000	25,000
5	2	12,500	**25,000**	12,305	**34,180**	12,500	**37,500**
6	3	12,500	37,500	6,921	41,101	6,250	43,750
7	4	12,500	50,000	3,893	44,994	3,125	46,875
8							
9	Function	SLN		DDB with d = 1.75		DDB	



MACRS Depreciation and Recovery Periods

16.25 (a) Personal property recovery periods are 3,5,7,10,15, or 20 years
(b) Real property recovery periods are 27.5 or 39 years
(c) Salvage value is always assumed to be zero
(d) Assets with no identifiable MACRS class have a 7 year recovery period

16.26 (a) The half-year convention assumes that all property is placed in service at the midpoint of the tax year of installation.
(b) The half-year convention is evident in two places: the first and last years of the MACRS depreciation rates table. In year one, the MACRS rate is limited to one-half of the allowed DB or DDB rate. Also, the last year's rate is one-half of the previous year's rate.

16.27 Recovery period is 15 years (Table 16-4)

$D_3 = 0.0855(770{,}000) = \$65{,}835$

$BV_3 = B$ - total depreciation through year 3
$= 770{,}000(1 - 0.050 - 0.095 - 0.0855)$
$= \$592{,}515$

16.28 3 years: $BV_3 = 400{,}000 - 400{,}000(0.20 + 0.32 + 0.192)$
$= \$115{,}200$

5 years: $BV_5 = 400{,}000(0.0576)$
$= \$23{,}040$

6 years: MACRS always depreciates to BV = 0 in its last year

16.29 (a) Total depreciation for 3 years = 100% – 57.6%
= 42.4%

Look in MACRS rate table for column that has total depreciation of 42.4% for first three years

Check n = 7: Sum = 14.29 +24.49 +17.49 = 56.27% (too high)
Check n = 10: Sum = 10.0 +18.0 +14.4 = 42.4 OK

Recovery period is n = 10 years

(b) Function = VDB(140000,0,10,MAX(0,4-1.5),MIN(10,4-0.5),2) displays the value
$D_4 = \$16{,}128$

16.30 (a) MACRS: $BV_3 = 300{,}000 – 300{,}000(0.20 + 0.32 + 0.192)$
$= \$86{,}400$

DDB: $d = 2/5 = 0.4$
$BV_3 = 300{,}000(1 – 0.4)^3$
$= \$64{,}800$

DDB provides faster write-off for 3 years by 86,400 – 64,800 = \$21,600

(b) Schedules show that DDB has a lower BV_3 of \$64,800. Same difference of \$21,600.

	A	B	C	D	E	F
1		MACRS			DDB	
2	Year	Rate	Depreciation	Book value	Depreciation	Book value
3	0			300,000		300,000
4	1	0.2	60,000	240,000	120,000	180,000
5	2	0.32	96,000	144,000	72,000	108,000
6	3	0.192	57,600	**86,400**	43,200	**64,800**
7	4	0.1152	34,560	51,840	4,800	60,000
8	5	0.1152	34,560	17,280	0	60,000
9	6	0.0576	17,280	0		

(c) Depreciation is limited by the DDB function to ensure that S = \$60,000 is maintained.

16.31 (a) From MACRS depreciation rate table, $d_3 = 0.192$

B = 14,592/0.192 = \$76,000

(b) D_1 = \$15,200 and D_4 = \$8755

	A	B	C	D
1		MACRS		
2	Year	Rate	Depreciation	Book value
3	0			76,000
4	1	0.2	15,200	60,800
5	2	0.32	24,320	36,480
6	3	0.192	14,592	21,888
7	4	0.1152	8,755	13,133
8	5	0.1152	8,755	4,378
9	6	0.0576	4,378	0

16.32 Use rates for real property with n = 39 years

D_1 = 0.01391(3,400,000) = \$47,294
D_2 to D_{10} = 0.02564(3,400,000) = \$87,176

Total depreciation for 10 years = 47,294 + 9(87,176) = \$831,878
BV_{10} = 3,400,000 – 831,878
= \$2,568,122

Anticipated selling price is 1.5(BV_{10}) = \$3,852,183

Fairfield hopes to sell it for \$452,183 more than they paid for it.

16.33 (a) MACRS in United States: n = 5, BV_6 = \$0

Over-depreciated by \$100,000

Selling price – BV_6 = 100,000 – 0 = \$100,000

SL in Malaysia: n = 10

D = (750,000 – 150,000)/10 = \$60,000 per year

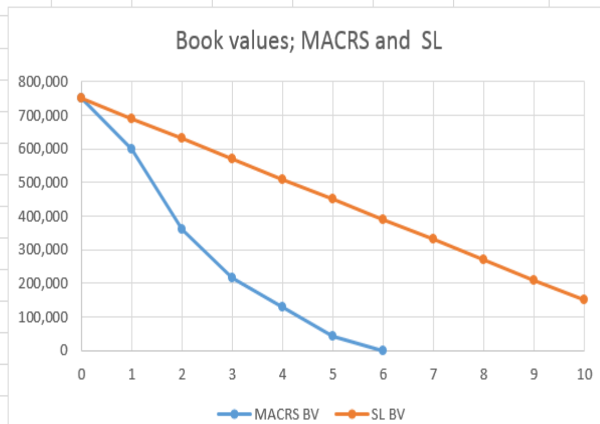
BV_6 = 750,000 – 6(60,000) = \$390,000

Under-depreciated by \$290,000

Selling price – BV_6 = 100,000 – 390,000 = \$-290,000

(b) MACRS goes to BV_6 = \$0 and SL stops at BV_{10} = \$150,000. BV_6 values are highlighted.

	A	B	C	D	E	F
1			MACRS		SL	
2	Year	Rate	Depreciation	MACRS BV	Depreciation	SL BV
3	0			750,000		750,000
4	1	0.2	150,000	600,000	60,000	690,000
5	2	0.32	240,000	360,000	60,000	630,000
6	3	0.192	144,000	216,000	60,000	570,000
7	4	0.1152	86,400	129,600	60,000	510,000
8	5	0.1152	86,400	43,200	60,000	450,000
9	6	0.0576	43,200	0	60,000	390,000
10	7				60,000	330,000
11	8				60,000	270,000
12	9				60,000	210,000
13	10				60,000	150,000
14						



16.34 SL: D_3 = [80,000 - 0.25(80,000)]/5
= \$12,000 per year

BV_3 = 80,000 – 3(12,000)
= \$44,000

MACRS: BV_3 = 80,000 – 80,000(0.20 + 0.32 + 0.192)
= \$23,040

Difference = 44,000 – 23,040 = \$20,960

MACRS has a lower BV after 3 years by \$20,960

16.35 (a) MACRS: $d_3 = 0.1440$; sum of rates for 3 years is 0.4240

$D_3 = 0.1440(1{,}000{,}000) = \$144{,}000$

$BV_3 = 1{,}000{,}000 – 0.4240(1{,}000{,}000) = \$576{,}000$

(b) DDB: $d = 2/15 = 0.13333$

$D_3 = 0.13333(1{,}000{,}000)(1 - 0.13333)^2 = \$100{,}146$

$BV_3 = 1{,}000{,}000(1 - 0.1333)^3 = \$650{,}970$

(c) ADS SL: $d = 1/15 = 0.06666$ years 2 through 15; ½ that for years 1 and 16.

$D_3 = 0.06666(1{,}000{,}000) = \$66{,}660$

$BV_3 = 1{,}000{,}000 -2.5(66{,}660) = \$833.350$

Depletion

16.36 Depreciation applies to assets that can be replaced. Depletion is applicable only to natural resources.

16.37 (a) $CD_t = 2{,}100{,}000/350{,}000 = \$6.00$ per ounce

Cost depletion, 3 years = 6.00(175,000)
= \$1,050,000

(b) Remaining investment = 2,100,000 – 1,050,000
= \$1,050,000

Remaining silver = 100,000 ounces

New $CD_t = 1{,}050{,}000/100{,}000$
= \$10.50 per ounce

16.38 Percentage depletion for coal is 10% of gross income, provided it does not exceed 50% of taxable income (TI).

Year	Gross* Income, \$	% depletion @10%	50% of TI	Allowed depletion
1	336,826	33,683	70,000	33,683
2	526,050	52,605	70,000	52,605
3	807,437	80,744	70,000	70,000

*tons x \$/ton

16.39 CD_t = 500,000/200 = \$2500 per million board feet

Year 1: There is no depletion deduction in year 1 because no lumber will be harvested until year 2.

Year 2: Cost depletion = 2500(20)
= \$50,000

16.40 Percentage depletion = 0.20(GI) = 700,000
GI = \$3,500,000

Let N = number of barrels

$$\begin{aligned} GI &= N \times \text{price} \\ 3{,}500{,}000 &= N \times 40 \\ N &= 87{,}500 \\ \text{Reserves} &= 87{,}500/0.01 \\ &= 8{,}750{,}000 \text{ barrels} \end{aligned}$$

16.41 (a) CD_t = 2,900,000/100 = \$29,000 per 1000 tons

Annual cost depletion = volume × \$29,000

Year	Volume, 1000 tons	Cost depletion, $ per year
1	10	290,000
2	9	261,000
3	15	435,000
4	15	435,000
5	18	522,000
Total	67	\$1,943,000

(b) No, cost depletion is limited by total first cost. Since \$1.943 million < \$2.9 million, all 5 years of depletion are acceptable.

16.42 Cost depletion: Remaining investment = 35.0 – 24.8 million = \$10.2 million

New cost depletion: CD_t = 10.2 million/8000 = \$1275 per 100 tons
Cost depletion, year 11: 720(1275) = \$918,000

Percentage depletion: Rate is 10% of GI for coal; GI range is \$6,125,000 to \$8,500,000

Percentage depletion, year 11, minimum: 0.1(6,125,000) = \$612,500
maximum: 0.1(8,500,000) = \$850,000

Additional Problems and FE Exam Review Questions

16.43 Answer is (b)

16.44 Depreciation is same for all years in straight line method.

$D_3 = [100{,}000 - 0.15(100{,}000)]/5 = \$17{,}000$
Answer is (a)

16.45 $D = \dfrac{50{,}000 - 10{,}000}{5} = \$8000$ per year

$BV_3 = 50{,}000 - 3(8{,}000) = \$26{,}000$
Answer is (d)

16.46 $d = 2/5 = 0.4$

$D_2 = (0.4)(28{,}000)(1 - 0.4)^{2-1} = \$6720$
Answer is (c)

16.47 $d = 2/8 = 0.25$

$BV_2 = 30{,}000(1 - 0.25)^2 = \$16{,}875$
Answer is (c)

16.48 DDB: $d = 2/n = 2/8 = 0.25$

$$\begin{aligned} \text{Market value} &= 1.20BV_4 = 1.20B(1\text{-}d)^4 \\ &= 1.2(500{,}000)(0.75)^4 \\ &= \$189{,}844 \end{aligned}$$

Answer is (d)

16.49 $BV = 150{,}000 - 150{,}000(0.10 + 0.18 + 0.144) = \$86{,}400$
Answer is (a)

16.50 MACRS salvage value is always \$0
Answer is (a)

16.51 Answer is (b)

16.52 Answer is (c)

16.53 $CD_t = 210{,}000{,}000/700{,}000 = \$300$ per ounce

Cost depletion charge = 300(35,000) = \$10.5 million
Answer is (c)

16.54 Factor = (70,000 - 20,000)/25,000 = 2.00 per tree
Depletion = 2.00(7000) = $14,000
Answer is (d)

16.55 Percentage: GI = 65,000(40) = $2.6 million

Depletion charge = 0.05(2.6 million)
= $130,000

Cost: CD_t = $1.28 per ton
Depletion charge = (65,000)(1.28)
= $83,200

Answer is (b)

APPENDIX PROBLEMS

Sum-of-Years-Digits Depreciation

16A.1 *By hand:* SUM = 36; use SYD rates for (B - S) = €10,000

t	d_t	D_t, €	BV_t, €
1	8/36	2,222.22	9777.78
2	7/36	1,944.44	7833.33
3	6/36	1,666.67	6166.67
4	5/36	1,388.89	4777.78
5	4/36	1,111.11	3666.67
6	3/36	833.33	2833.33
7	2/36	555.56	2277.78
8	1/36	277.78	2000.00

Spreadsheet:

	A	B	C	D
1				
2	**Year, t**	**SYD rate**	**Depr, €**	**BV, €**
3	0			12,000.00
4	1	0.22	2222.22	9,777.78
5	2	0.19	1944.44	7,833.33
6	3	0.17	1666.67	6,166.67
7	4	0.14	1388.89	4,777.78
8	5	0.11	1111.11	3,666.67
9	6	0.08	833.33	2,833.33
10	7	0.06	555.56	2,277.78
11	8	0.03	277.78	2,000.00
12				
13	Function	(9-year)/36	SYD(B,S,8,t)	

16A.2 (a) B = \$150,000; n = 10; S = \$15,000 and SUM = 55

$$D_2 = \frac{10 - 2 + 1}{55}(150{,}000 - 15{,}000) = \$22{,}091$$

$$BV_2 = 150{,}000 - \left[\frac{2(10 - 1 + 0.5)}{55}\right](150{,}000 - 15{,}000) = \$103{,}364$$

$$D_7 = \frac{10 - 7 + 1}{55}(150{,}000 - 15{,}000) = \$9818$$

$$BV_7 = 150{,}000 - \left[\frac{7(10 - 3.5 + 0.5)}{55}\right](150{,}000 - 15{,}000) = \$29{,}727$$

(b) Same results as in part (a) for years 2 and 7

	A	B	C
1			
2	**Year, t**	**Depr, \$**	**BV, \$**
3	0		150,000
4	1	24,545	125,455
5	**2**	**22,091**	**103,364**
6	3	19,636	83,727
7	4	17,182	66,545
8	5	14,727	51,818
9	6	12,273	39,545
10	**7**	**9,818**	**29,727**
11	8	7,364	22,364
12	9	4,909	17,455
13	10	2,455	15,000
14			
15	Function	SYD	

16A.3 B = \$400,000; n = 6 and S = 0.15(400,000) = \$60,000

(a) Use Equation. [16A.2] and S = 21

$$BV_3 = 400{,}000 - \left[\frac{3(6 - 1.5 + 0.5)}{21}\right](400{,}000 - 60{,}000) = \$157{,}143$$

(b) By Equation [16A.3] and t = 4:

$$d_4 = \frac{6 - 4 + 1}{21} = 3/21 = 1/7$$

$$\begin{aligned} D_4 &= d_4(B - S) \\ &= (3/21)(400{,}000 - 60{,}000) \\ &= \$48{,}571 \end{aligned}$$

Unit-of-Production Depreciation

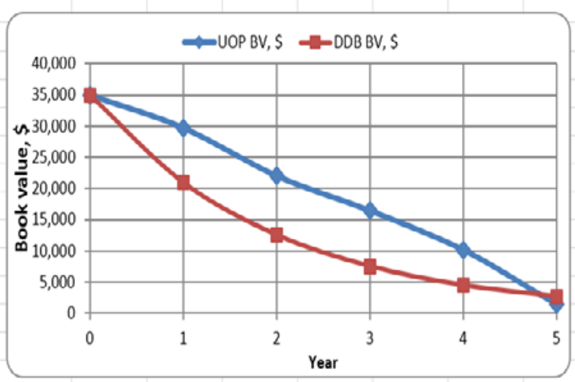
16A.4 D_t = (tests per year t/10,000)(70,000)

Year, t	Number of tests	D_t, \$	BV_t, \$
1	3810	26,670	43,330
2	2720	19,040	24,290
3	5390	24,290*	0

*D_3 = 5390/10,000(70,000) = \$37,730 is too large; only the remaining BV = \$24,290 can be charged in year 3.

16A.5 DDB method does depreciate faster, but UOP, in this case, did depreciate slightly more of the first cost (\$33,600 vs. \$32,278).

	UOP method			DDB method	
Year	Mileage	Depr, \$	UOP BV, \$	Depr, \$	DDB BV, \$
0			35,000		35,000
1	15,000	5,250	29,750	14,000	21,000
2	22,000	7,700	22,050	8,400	12,600
3	16,000	5,600	16,450	5,040	7,560
4	18,000	6,300	10,150	3,024	4,536
5	25,000	8,750	1,400	1,814	2,722
Totals	96,000	33,600		32,278	



Switching Methods

16A.6 B = $45,000 n = 5 S = $3000 i = 18%

Compute the D_t for each method and select the larger value to maximize PW_D.

For DDB, d = 2/5 = 0.4. By Equation [16A.6], $BV_5 = 45{,}000(1 - 0.4)^5 = 3499 > 3000$

Switching is advisable. Remember to consider S = $3000 in Equation [16A.8].

Year,	DDB Method		Switching to SL method	Larger
t	D, Eq. [16A.7]	BV	D, Eq. [16A.8]	Depreciation
0		$45,000		
1	$18,000	27,000	$8,400	$18,000 (DDB)
2	10,800	16,200	6,000	10,800 (DDB)
3	6,480	9,720	4,400	6,480 (DDB)
4	3,888	5,832	3,360	3,888 (DDB)
5	2,333	3,499*	2,832	2,832 (SL)

*BV_5 will be $3000 exactly when SL depreciation of $2832 is applied in year 5.

$BV_5 = 5832 - 2832 = \$3000$

The switch to SL occurs in year 5 and the PW of depreciation is:

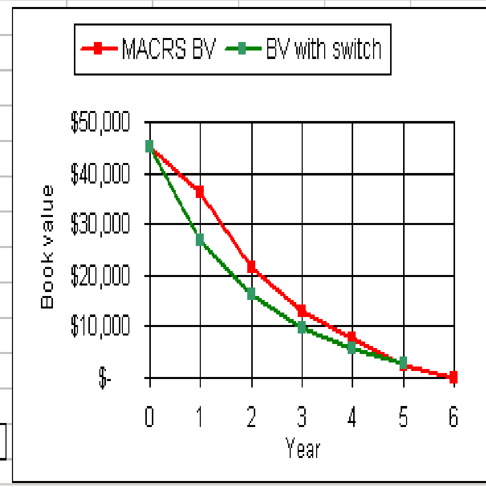
$PW_D = 18{,}000(P/F,18\%,1) + \ldots + 2{,}832(P/F,18\%,5) = \$30{,}198$

16A.7 Develop a spreadsheet for the DDB-to-SL switch using the VDB function (column B) and MACRS rates or VDB function, plus PW_D for both methods.

		Switching DDB-to-SL		MACRS	
Year	Depr.	BV with switch	d rate	Depr.	MACRS BV
0		$ 45,000			$ 45,000
1	$ 18,000	$ 27,000	0.2	$ 9,000	$ 36,000
2	$ 10,800	$ 16,200	0.32	$ 14,400	$ 21,600
3	$ 6,480	$ 9,720	0.192	$ 8,640	$ 12,960
4	$ 3,888	$ 5,832	0.1152	$ 5,184	$ 7,776
5	$ 2,832	$ 3,000	0.1152	$ 5,184	$ 2,592
6			0.0576	$ 2,592	$ -
PW of Depr.	$30,198			$29,128	

=NPV(18%,B5:B9)

=VDB($C$4,3000,5,$A8,$A9,2,FALSE)



Were switching allowed in the US, it would give only a slightly higher PW_D = \$30,198 compared to MACRS PW_D = \$29,128.

16A.8 175% DB: $d = 1.75/10 = 0.175$ for $t = 1$ to 5

$BV_t = 110{,}000(0.825)^t$

Sample function: = DDB(110000,10000,10,$A4,1.75)

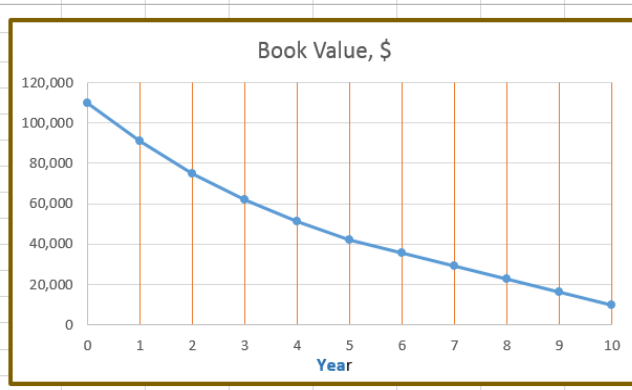
SL: $D_t = (BV_5 - 10{,}000)/5 = (42{,}040 - 10{,}000)/5 = \$6408$ for $t = 6$ to 10

$BV_t = BV_5 - t(6408)$

Sample function: = SLN($D$8,10000,5)

PW_D at 12% = \$64,210

	175% DB	SL	
Year	Depr., $	Depr., $	BV, $
0			110,000
1	19,250	0	90,750
2	15,881	0	74,869
3	13,102	0	61,767
4	10,809	0	50,958
5	8,918	0	42,040
6	0	6,408	35,632
7	0	6,408	29,224
8	0	6,408	22,816
9	0	6,408	16,408
10	0	6,408	10,000
Function	DDB	SLN	
PW of Depr, $			64,210



16A.9 (a) Use Equation [16A.6] for DDB with $d = 2/25 = 0.08$

$BV_{25} = 255{,}000(1 - 0.08)^{25} = \$31{,}713 < \$50{,}000$

No, the switch should not be made

(b) $255{,}000(1\text{-}d)^{25} > 50{,}000$

$1 - d > [\ 50{,}000/255{,}000]^{1/25}$

$1 - d > (0.19608)^{0.04} = 0.93691$
$d < 1 - 0.93691 = 0.06309$

If $d < 0.063$ the switch is advantageous.

Let x = numerator in Equation [16.5]

$d = 0.063 = x/25$
$x = 1.575$

This is a 157.5% DB depreciation, which is less than 200% for the DDB method.

MACRS Rates

16A.10 *By hand:* Verify that the rates are the following with $d = 0.40$

t	1	2	3	4	5	6
d_t	0.20	0.32	0.192	0.1152	0.1152	0.0576

d_1: $d_{DB,1} = 0.5d = 0.20$

d_2: By Eq. [16A.15] for DDB:
$d_{DB,2} = 0.4(1 - 0.2) = 0.32$ (selected)

By Eq. [16A.16] for SL:

$d_{SL,2} = 0.8/4.5 = 0.178$

d_3: DDB: $d_{DB,3} = 0.4(1 - 0.2 - 0.32) = 0.192$ (selected)

SL: $d_{SL,2} = 0.48/3.5 = 0.137$

d_4: DDB: $d_{DB,4} = 0.4(1 - 0.2 - 0.32 - 0.192) = 0.1152$

SL: $d_{SL,4} = 0.288/2.5 = 0.1152$ (select either)

Switch to SL occurs in year 4

d_5: Use the SL rate, n = 5

$d_{SL,5} = 0.1728/1.5 = 0.1152$

d_6: $d_{SL,6}$ is the remainder or 1/2 the d_5 rate.

$$d_{SL,6} = 1 - \sum_{t=1}^{5} d_t = 1 - (0.2 + 0.32 + 0.192 + 0.1152 + 0.1152)$$

$= 0.0576$

By spreadsheet: Function and value spreadsheet are shown. Use Equation [16A.16] to calculate SL depreciation each year

	A	B	C	D	E	F
1		**DDB with MACRS rules**		**SL**		**BV with**
2	**Year**	**Depr.**	**BV**	**Depr., $**	**Selected**	**switch**
3	0		1.0000			1.0000
4	1	0.2000	0.8000	0.2000	**0.2000**	0.8000
5	2	0.3200	0.4800	0.1778	**0.3200**	0.4800
6	3	0.1920	0.2880	0.1371	**0.1920**	0.2880
7	4	0.1152	0.1728	0.1152	**0.1152**	0.1728
8	5	0.0691	0.1037	0.1152	**0.1152**	0.0576
9	6	0.0346	0.0691		**0.0576**	0.0000
10	Total				1.0000	

	A	B	C	D	E	F
1		**DDB with MACRS rules**		**SL**		**BV with**
2	**Year**	**Depr.**	**BV**	**Depr., $**	**Selected**	**switch**
3	0		1			1
4	1	= DDB(1,0,5,1)/2	= C3-B4	= SLN($C3,0,5)	**= MAX($B4,$D4)**	= $F3 - $E4
5	2	= DDB(C4,0,5,1)	= C4-B5	= SLN($C4,0,5-$A5+1.5)	**= MAX($B5,$D5)**	= $F4 - $E5
6	3	= DDB(C5,0,5,1)	= C5-B6	= SLN($C5,0,5-$A6+1.5)	**= MAX($B6,$D6)**	= $F5 - $E6
7	4	= DDB(C6,0,5,1)	= C6-B7	= SLN($C6,0,5-$A7+1.5)	**= MAX($B7,$D7)**	= $F6 - $E7
8	5	= DDB(C7,0,5,1)	= C7-B8	= SLN($C7,0,5-$A8+1.5)	**= MAX($B8,$D8)**	= $F7 - $E8
9	6	= B8/2	= C8-B9		**= E8/2**	= $F8 - $E9
10	Total				=SUM(E4:E9)	

16A.11 B = $30,000 n = 5 years d = 0.40

Find BV_3 using d_t rates derived from Equations [16A.11] through [16A.13].

t = 1: $d_1 = 1/2(0.4) = 0.2$

$D_1 = 30{,}000(0.2) = \$6000$

$BV_1 = \$24{,}000$

t = 2: For DDB depreciation, use Eq. [16A.12]

$d = 0.4$

$D_{DB} = 0.4(24{,}000) = \$9600$

$BV_2 = 24{,}000 - 9600 = \$14{,}400$

For SL, if switch is better, in year 2, by Eq. [16A.13].

$$D_{SL} = \frac{24{,}000}{5-2+1.5} = \$5333$$

Select DDB; it is larger.

t = 3: For DDB, apply Eq. [16A.12] again.

$D_{DB} = 14{,}400(0.4) = \$5760$

$BV_3 = 14{,}400 - 5760 = \$8640$

For SL, Eq. [16A.13]

$$D_S = \frac{14{,}400}{5-3+1.5} = \$4114$$

Select DDB.

Conclusion: When sold for \$5000, $BV_3 = \$8640$. Therefore, there is a loss of \$3640 relative to the MACRS book value.

NOTE: If Table 16.2 rates are used, cumulative depreciation in % for 3 years is:

$20 + 32 + 19.2 = 71.2\%$

$30{,}000(0.712) = \$21{,}360$

$BV_3 = 30{,}000 - 21{,}360 = \$8640$

16A.12 Determine MACRS depreciation for n = 7 using Equations [16A.11] through [16A.13] and apply them to B = \$50,000. (S) indicates the selected method and amount.

DDB	SL
t = 1: $d = 1/7 = 0.143$ $D_{DB} = \$7150$ (S) $BV_1 = \$42{,}850$	$D_{SL} = 0.5(1/7)(50{,}000)$ $= \$3571$
t = 2: $d = 2/7 = 0.286$ $D_{DB} = \$12{,}255$ (S)	$D_{SL} = \frac{42{,}850}{7-2+1.5} = \$6592$

$BV_2 = \$30,595$

t = 3: d = 0.286 $\quad D_{SL} = \dfrac{30,595}{7–3+1.5} = \$5563$

$D_{DB} = \$8750$ (S)

$BV_3 = \$21,845$

t = 4: d = 0.286 $\quad D_{SL} = \dfrac{21,845}{7–4+1.5} = \$4854$

$D_{DB} = \$6248$ (S)

$BV_4 = 15,597$

t = 5: d = 0.286 $\quad D_{SL} = \dfrac{15,597}{7–5+1.5} = \$4456$

$D_{DB} = \$4461$ (S)

$BV_5 = \$11,136$

t = 6: d = 0.286 $\quad D_{SL} = \dfrac{11,136}{7–6+1.5} = \$4454$ (S)

$D_{DB} = \$3185$

(Use SL hereafter) $\quad BV_6 = \$6682$

t = 7: $\quad D_{SL} = \dfrac{6682}{7–7+1.5} = \$4454$

$BV_7 = \$2228$

t = 8: $\quad D_{SL} = \$2228$

$BV_8 = 0$

The depreciation amounts sum to \$50,000

Year	Depr., \$	Year	Depr., \$
1	7150	5	4461
2	12,255	6	4454
3	8750	7	4454
4	6248	8	2228

16A.13 (a) The SL rates with the half-year convention for n = 3 are:

Year	d rate	Formula
1	0.167	1/2n
2	0.333	1/n
3	0.333	1/n
4	0.167	1/2n

(b)

Year, t	1	2	3	4	PW_D, \$
MACRS D_t, \$	26,664	35,560	11,848	5928	61,253
SL alternative D_t, \$	13,360	26,640	26,640	13,360	56,915

The MACRS PW_D is larger by \$4338

Chapter 17
After-Tax Economic Analysis

Terminology and Basic Tax Computations

17.1 NOI = Net Operating Income; GI = Gross Income; T_e = Effective Tax Rate: NOPAT = Net Operating Profit After Taxes; TI = Taxable Income; R = Revenue; OE = Operating Expenses; EBIT = Earnings Before Interest and Income Taxes

17.2 (a) From Table 17.1, marginal tax rate = 39%

(b) Taxes = 0.15(50,000) + 0.25(75,000 – 50,000) + 0.34(100,000 - 75,000) + 0.39(250,000 – 100,000)
= \$80,750

or

Taxes = 22,250 + 0.39(250,000 – 100,000)
= \$80,750

(c) Average tax rate = (80,750/250,000)(100%) = 32.3%

17.3 (a) Net operating profit after taxes; (b) Taxable income; (c) Depreciation;
(d) Operating expense; (e) Taxable income; (f) Taxable income;
(g) Operating expense; (h) Gross income

17.4 (a) In \$1 million units,

NOI = 51.3 = GI – 23.6
GI = \$74.9 (\$74.9 million)

(b) EBIT = NOI
= \$51.3 million

17.5 TI = EBIT – D
= 21.4 – 9.5 million
= \$11.9 million

17.6 $T_e = 0.07 + (1 – 0.07)(0.35)$
$= 0.3955$ (39.55%)

17.7 (a) From Table 17.1, TI is in the range of \$100,000 to \$335,000

72,000 = 22,250 + 0.39(TI – 100,000)
0.39TI = 88,750
TI = $227,564

(b) Average tax rate = T = 72,000/227,564 = 0.316 (31.6%)

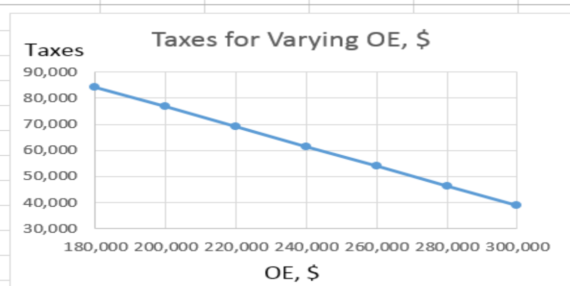
(c) NOPAT = TI(1 – T)
= 227,564(1 – 0.316)
= $155,654

17.8 (a) TI = GI – OE – depreciation
= 450,000 – 230,000 – 48,000
= $172,000

Taxes = 172,000(0.38) = $65,360

(b) Use the TI relation with varying OE amounts to plot taxes. It is linear.

	A	B	C
1	**OE, $**	**TI, $**	**Taxes, $**
2	180,000	222,000	84,360
3	200,000	202,000	76,760
4	220,000	182,000	69,160
5	240,000	162,000	61,560
6	260,000	142,000	53,960
7	280,000	122,000	46,360
8	300,000	102,000	38,760
9			
10	Function	= GI-OE-D	= T*TI
11			



17.9 (a) T_e = 9.8 + (1 – 0.098)(31%) = 37.76%
TI = 4.9 – 2.1 – 1.4 = $1.4 million

Tax estimate = 1,400,000(0.3776) = $528,640

(b) 528,640/4,900,000 × 100% = 10.8%

17.10 (a) Company ABC:
TI = Gross income - Expenses - Depreciation
= (1,500,000 + 31,000) – 754,000 – 148,000
= $629,000
Taxes = 113,900 + 0.34(629,000 – 335,000)
= $213,860

Company XYZ:
TI = (820,000 + 25,000) – 591,000 – 18,000

$= \$236,000$

$\text{Taxes} = 22,250 + 0.39(236,000 – 100,000)$

$= \$75,290$

(b) ABC: $(213,860/1,500,000)*100\% = 14.26\%$

XYZ: $(75,290/820,000)*100\% = 9.18\%$

(c) ABC:

$\text{Taxes} = (TI)(T_e) = 629,000(0.34) = \$213,860$

$\%\ \text{error} = 0\%$

<u>XYZ:</u>

$\text{Taxes} = (TI)(T_e) = 236,000(0.39) = \$92.040$

$\%\ \text{error} = [(92,040 – 75,290)/75,290]*100\%$

$= 22.2\%$ over estimate

17.11 (a) $T_e = 0.076 + (1 – 0.076)(0.34) = 0.3902$

$TI = 6.5\ \text{million} – 4.1\ \text{million} = \$2.4\ \text{million}$

$\text{Taxes} = 2,400,000(0.3902) = \$936,480$

(b) $NOPAT = TI(1\text{-}T_e) = 2,400,000(0.6098) = \$1,463,520$

Reduction in OE is determined by setting NOPAT = $2 million

$2,000,000 = TI(0.6098)$

$TI = \$3,279,764$

$\text{OE reduction} = 3,279,764 – 2,400,000$

$= \$879,764$

17.12 (a) Federal taxes = $13,750 + 0.34(5000) = \$15,450$ (using Table 17-1 rates)

Average federal rate = $(15,450/80,000)(100\%)$

$= 19.31\%$

(b) Effective tax rate $= [0.06 + (1 – 0.06)(0.1931)]100\%$

$= 24.15\%$

(c) Total taxes using effective rate = $80,000(0.2415)$

$= \$19,320$

(d) State: $80,000(0.06) = \$4800$

Federal: $80{,}000[0.1931(1 - 0.06)] = 80{,}000(0.1815)$
$= \$14{,}520$

17.13 (a) $T_e = 0.06 + (1 - 0.06)(0.23)$
$= 0.2762$

(b) Reduced $T_e = 0.9(0.2762)$
$= 0.2486$

Set x = required provincial rate
$0.2486 = x + (1 - x)(0.23)$
$x = 0.0186/0.77$
$= 0.0242$ (2.42%)

(c) Since $T_e = 22\%$ is lower than the current federal rate of 23%, no provincial tax could be levied, plus an interest-free grant of 1% of TI, or \$70,000, would have to be made available.

17.14 IRS Publication 17 for 2015 is used for this solution.

	A	B	C	D	E	F	G	H	I
1					FAMILY A				
2	Salary	Dividends	Other	Exemptions	Deductions	TI	Taxes*	% of TI	% of Income
3	65,000	8,000	0	-20,000	-12,000	41,000	5,228	12.75%	7.16%
4									
5	*From Publicaton 17 for 2015: Taxes = 1845 + 0.15(41000 - 18450) = \$5228								
6									
7					FAMILY B				
8	Salary	Dividends	Other	Exemptions	Deductions	TI	Taxes*	% of TI	% of Income
9	290,000	58,000	14,000	-12,000	-25,000	325,000	82,779	25.47%	22.87%
10									
11	*From Publication 17 for 2015: Taxes = 51,577.50 + 0.33*(325,000 - 230,450) = \$82,779								

17.15 Tax rates for 2015 from IRS Publication 17 are below. Your tax amounts and plot will vary when the current tax rates are applied. The percentage of GI spent on taxes has varied widely as shown in the plot.

	If taxable income is over	but not over	The tax is	Of the amount over
Married/Filing jointly and qualifying widow(er)s	\$ 0	\$18,450	\$0.00 + 10%	\$ 0
	18,450	74,900	1,845.00 + 15%	18,450
	74,900	151,200	10,312.50 + 25%	74,900
	151,200	230,450	29,387.50 + 28%	151,200
	230,450	411,500	51,577.50 + 33%	230,450
	411,500	464,850	111,324.00 + 35%	411,500
	464,850	—	129,996.50 + 39.6%	464,850

	A	B	C	D	E	F	G	H	I	J	K	L
1	Year	Salaries			Other	Gross	Personal	Itemized	TI	2015 Marginal	Taxes	% of GI
2		Joyce	Vincent	Dividends	Income	Income	Exemptions	Deduction		Bracket, %		
3	1	69	61	5		135	7	8	120.0	25	21.5875	15.99
4	2	71	65	6		142	7	10	125.0	25	22.8375	16.08
5	3	75	72	6	5	158	7	10	141.0	25	26.8375	16.99
6	4	80	78	7	10	175	7	11	157.0	25	30.8375	17.62
7	5	25	79	7	12	123	14	11	98.0	25	16.0875	13.08
8	6	25	83	7	10	125	14	11	100.0	25	16.5875	13.27
9	7	25	85	8	8	126	14	12	100.0	25	16.5875	13.16
10	8	27	90	8	5	130	14	12	104.0	25	17.5875	13.53
11	9	28	92	8		128	16	14	98.0	25	16.0875	12.57
12	10	30	0	4		34	20	10	4.0	10	0.4	1.18
13	11	70	20	4		94	20	10	64.0	15	8.6775	9.23
14	12	80	20	5		105	20	8	77.0	25	10.8375	10.32
15	13	90	20	5		115	20	8	87.0	25	13.3375	11.60
16	14	95	60	6		161	20	10	131.0	25	24.3375	15.12
17	15	100	62	10		172	22.5	12	137.5	25	25.9625	15.09
18	16	105	65	15		185	22.5	14	148.5	25	28.7125	15.52
19	17	107	70	20		197	22.5	16	158.5	28	31.4315	15.96
20	18	110	75	15		200	22.5	20	157.5	28	31.1515	15.58

CFBT and CFAT

17.16 CFBT does not include life of asset, depreciation, and tax rate

17.17 NOPAT = GI – OE – D – Taxes

CFAT = GI – OE – P + S

The NOPAT expression deducts depreciation outside of the TI and tax computation. The CFAT expression removes the capital investment (or adds salvage) but does not consider depreciation, since it is a noncash flow item.

17.18 Use TI = GI – OE – D and CFBT = GI – OE = TI + D

Taxes = $TI(T_e)$ = 120,000(0.35) = \$42,000

CFAT = CFBT – taxes
= TI + D – taxes
= 120,000 + 133,350 – 42,000
= \$211,350

17.19 CFBT = [750,000 – 400,000(0.36)]/(1 – 0.36)
= \$946,875

17.20 Solve for GI

CFBT = CFAT + taxes
GI – OE = CFAT + (GI – OE – D)(T_e)

GI = [CFAT + OE(1 - T_e) - DT_e]/ (1 - T_e)
= [2,500,000 + 900,000(0.736) – 900,000(0.264)]/0.736

= \$3,973,913

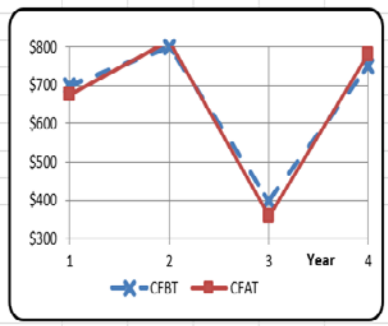
17.21 *By hand:* Missing values are shown in **bold.**

$CFBT_2 = 950 – 150 = \$800$
$D_2 = 0.4445(1900) = \$845$
$D_4 = 0.0741(1900) = \$141$
$TI_3 = 600 – 200 – 281 = \$119$
$Taxes_2 = -45(0.35) = \$-16$
$CFAT_3 = 400 – 42 = \$358$

Year	GI	E	P and S	CFBT	D	TI	Taxes	CFAT
0			-1900	-1900				-1900
1	800	-100	0	700	633	67	23	677
2	950	-150	0	**800**	**845**	-45	**-16**	816
3	600	-200	0	400	281	**119**	42	**358**
4	300	-250	700	750	**141**	-91	-32	782

Spreadsheet: Missing values are shown in **bold.** Functions for year 4 are detailed.

	A	B	C	D	E	F	G	H	I
1	Year	GI	OE	P and S	CFBT	D	TI	Taxes	CFAT
2	0			-1,900	-1,900		0	0	-1,900
3	1	800	-100	0	700	633	67	23	677
4	2	950	-150	0	**800**	**845**	-45	**-16**	816
5	3	600	-200	0	400	281	**119**	42	**358**
6	4	300	-250	700	750	**141**	-91	-32	782
7	Totals					1,900			
8									
9	Functions for year 4				= \$B6+\$C6+\$D6	= VDB(1900,0,3,MAX(0,\$A6-1.5),MIN(3,\$A6-0.5),2)	= \$B6+\$C6-\$F6	= G6 *0.35	= B6 +C6+D6 -H6



17.22 *By hand:* $CFAT = GI – OE – P + S – (GI – OE –D)T_e$

(a) P & S = 0
D = 200,000(0.0741) = \$14,820
CFAT = 100,000 – 50,000 – (100,000 – 50,000 – 14,820)(0.40)
= \$35,928

(b) S = \$20,000
D = 200,000(0.0741) = \$14,820
CFAT = 100,000 – 50,000 + 20,000 - (100,000 – 50,000 – 14,820)(0.40)
= \$55,928

(c) S = \$20,000 D = 0
CFAT = 100,000 – 50,000 + 20,000 - (100,000 – 50,000)(0.40)
= \$50,000

Spreadsheet: MACRS $d_4 = 0.0741$ SL $d_4 = 0$
Same answers as above using Table 17-2 template

	A	B	C	D	E	F	G	H	I	J
1	Part	Year	GI	OE	S	CFBT	MACRS D	TI	Taxes	CFAT
2	**(a)**	4	100,000	-50,000	0	50,000	14,820	35,180	14,072	35,928
3										
4	**(b)**	4	100,000	-50,000	20,000	70,000	14,820	35,180	14,072	55,928
5										
6		Year	GI	OE	S	CFBT	SL D	TI	Taxes	CFAT
7	**(c)**	4	100,000	-50,000	20,000	70,000	0	50,000	20,000	50,000

17.23 All monetary units are in $1 million

$T_e = 0.065 + (1 - 0.065)(0.35) = 0.39225$

(a) CFAT = GI – OE – TI(T_e)
= 48 – 28 – (48 - 28 - 8.2)(0.39225)
= 20 – 11.8(0.39225)
= $15.37 ($15.37 million)

(b) Taxes = (48 – 28 – 8.2)(0.39225)
= 11.8(0.39225)
= $4.62855 ($4,628,550)

% of revenue = (4.628/48)100% = 9.64%

(c) NOPAT = TI(1 - T_e) = 11.8(1 - 0.39225)
= $7.17145 ($7,171,450)

17.24 CFBT = CFAT + taxes
GI – E = CFAT + (GI – E – D)(T_e)

Solve for GI to obtain a general relation for each year t:
$GI_t = [CFAT + E(1- T_e) - DT_e]/ (1- T_e)$

Where: CFAT = $2.5 million
$T_e = 8\% + (1- 0.08)(20\%) = 26.4\%$
$1- T_e = 0.736$

Year 1: GI_1 = [2.5 million + 650,000(0.736) – 650,000(0.264)]/0.736
= $3,813,587

Year 2: GI_2 = [2.5 million + 900,000(0.736) – 900,000(0.264)]/0.736
= $3,973,913

Year 3: GI_3 = [2.5 million + 1,150,000(0.736) – 1,150,000(0.264)]/0.736
= $4,134,239

17.25 _By hand:_

Method A: Years 1-5, Depreciation = (100,000 – 10,000)/5 = $18,000
CFBT = 35,000 – 15,000 = $20,000
Taxes = (20,000 – 18,000)(0.34) = $680
CFAT = 20,000 – 680 = $19,320

AW_A = -100,000(A/P,7%,5) + 19,320 + 10,000(A/F,7%,5)
= $-3330

Method B: Years 1-5, Depreciation = (150,000 – 20,000)/5 = $26,000
CFBT = 45,000 – 6,000 = $39,000
Taxes = (39,000 – 26,000)(0.34) = $4420
CFAT = 39,000 – 4420 = $34,580

AW_B = -150,000(A/P,7%,5) + 34,580 + 20,000(A/F,7%,5)
= $1474

Method B is selected, the same as when MACRS is detailed.

Spreadsheet: Select Method B.

	A	B	C	D	E	F	G	H	I
1					METHOD A				
2	Year	GI	OE	P and S	CFBT	SL D	TI	Taxes	CFAT
3	0	0	0	-100,000	-100,000	0	0	0	-100,000
4	1	35,000	-15,000		20,000	18,000	2,000	680	19,320
5	2	35,000	-15,000		20,000	18,000	2,000	680	19,320
6	3	35,000	-15,000		20,000	18,000	2,000	680	19,320
7	4	35,000	-15,000		20,000	18,000	2,000	680	19,320
8	5	35,000	-15,000	10,000	30,000	18,000	2,000	680	29,320
9	Totals					90,000			
10	AW @ 7%								-3,330
11									
12					METHOD B				
13	Year	GI	OE	P and S	CFBT	SL D	TI	Taxes	CFAT
14	0	0	0	-150,000	-150,000	0	0	0	-150,000
15	1	45,000	-6,000		39,000	26,000	13,000	4,420	34,580
16	2	45,000	-6,000		39,000	26,000	13,000	4,420	34,580
17	3	45,000	-6,000		39,000	26,000	13,000	4,420	34,580
18	4	45,000	-6,000		39,000	26,000	13,000	4,420	34,580
19	5	45,000	-6,000	20,000	59,000	26,000	13,000	4,420	54,580
20	Totals					130,000			
21	AW @ 7%								1,474

Depreciation Effects on Taxes

17.26 Depreciation is a 100% deduction from TI when corporate taxes are calculated. Thus, one is equivalent to the other from an economic viewpoint.

17.27 D_{SL} = (180,000 – 30,000)/5 = $30,000
D_{MACRS} = 180,000 (0.32) = $57,600

Difference in taxes = (57,600 – 30,000)(0.36)
= \$9936 (less taxes paid for MACRS)

17.28 (a) Spreadsheet shows tax curves. There is no depreciation allowed after n = 6 for SL and after n = 7 for DDB (due to the S value)

(b) Total taxes are the same at \$168,000.
PW_{tax} is slightly smaller for the DDB method, even though n is larger by 2 years.

METHOD SL, n = 6

Year	B and S	CFBT	D	BV	TI	SL Taxes	CFAT
0	-280,000	-280,000	0	280,000	0	0	-280,000
1	0	100,000	40,000	240,000	60,000	18,000	82,000
2	0	100,000	40,000	200,000	60,000	18,000	82,000
3	0	100,000	40,000	160,000	60,000	18,000	82,000
4	0	100,000	40,000	120,000	60,000	18,000	82,000
5	0	100,000	40,000	80,000	60,000	18,000	82,000
6	40,000	100,000	40,000	40,000	60,000	18,000	122,000
7	0	100,000	0	40,000	100,000	30,000	70,000
8	0	100,000	0	40,000	100,000	30,000	70,000
Totals			240,000			168,000	
PW @ 8%						116,925	

METHOD DDB, n = 8

Year	B and S	CFBT	D	BV	TI	DDB Taxes	CFAT
0	-280,000	-280,000	0	280,000	0	0	-280,000
1	0	100,000	70,000	210,000	30,000	9,000	91,000
2	0	100,000	52,500	157,500	47,500	14,250	85,750
3	0	100,000	39,375	118,125	60,625	18,188	81,813
4	0	100,000	29,531	88,594	70,469	21,141	78,859
5	0	100,000	22,148	66,445	77,852	23,355	76,645
6	40,000	100,000	16,611	49,834	83,389	25,017	114,983
7	0	100,000	9,834	40,000	90,166	27,050	72,950
8	0	100,000	0	40,000	100,000	30,000	70,000
Totals			213,555			168,000	
PW @ 8%						114,179	

17.29 Did you guess correctly that SL with n = 4 years will be slightly lower in PW of taxes? The shorter recovery period overrides the accelerated write-off in this case.

	A	B	C	D	E	F	G	H	I	J	K
1					I. SL, n = 4						
2	Year	GI - OE	P and S	CFBT	D rate	D	BV	TI	Taxes	CFAT	
3	0		-100	-100			100	0	0	-100	
4	1	50	0	50	0.25	25	75	25	7.50	42.50	
5	2	50	0	50	0.25	25	50	25	7.50	42.50	
6	3	50	0	50	0.25	25	25	25	7.50	42.50	
7	4	50	0	50	0.25	25	0	25	7.50	42.50	
8	5	50	0	50	0.00	0	0	50	15.00	35.00	
9	6	50	0	50	0.00	0	0	50	15.00	35.00	
10	Totals				1.00	100			60.00		
11	PW @ 10%								41.55		
12	Function				d = 0.25	SLN			0.3TI	CFBT-Taxes	
13					II. MACRS, n = 5						
14	Year	GI - OE	P and S	CFBT	D rate	D	BV	TI	Taxes	CFAT	
15	0		-100	-100			100.00	0	0	-100	
16	1	50	0	50	0.2000	20.00	80.00	30.00	9.00	41.00	
17	2	50	0	50	0.3200	32.00	48.00	18.00	5.40	44.60	
18	3	50	0	50	0.1920	19.20	28.80	30.80	9.24	40.76	
19	4	50	0	50	0.1152	11.52	17.28	38.48	11.54	38.46	
20	5	50	0	50	0.1152	11.52	5.76	38.48	11.54	38.46	
21	6	50	0	50	0.0576	5.76	0.00	44.24	13.27	36.73	
22	Totals				1.0000	100.00			60.00		
23	PW @ 10%								42.13		Slightly higher than SL
24	Function				Table 16-2	100D rate			0.3TI	CFBT-Taxes	
25					III. DDB, n = 6						
26	Year	GI - OE	P and S	CFBT	D rate	D	BV	TI	Taxes	CFAT	
27	0		-100	-100			100.00	0	0	-100	
28	1	50	0	50	0.3333	33.33	66.67	16.67	5.00	45.00	
29	2	50	0	50	0.2222	22.22	44.44	27.78	8.33	41.67	
30	3	50	0	50	0.1481	14.81	29.63	35.19	10.56	39.44	
31	4	50	0	50	0.0988	9.88	19.75	40.12	12.04	37.96	
32	5	50	0	50	0.0658	6.58	13.17	43.42	13.02	36.98	
33	6	50	0	50	0.0439	4.39	8.78	45.61	13.68	36.32	
34	Totals				0.9122	91.22			62.63		
35	PW @ 10%								43.40		Slightly higher than MACRS
36	Function				$d(1-d)^{t-1}$	DDB			0.3TI	CFBT-Taxes	
37											

17.30 (a) _Recovery over 3 years._ SL depreciation is 60,000/3 = \$20,000 per year

$$
\begin{aligned}
\text{Year 1-3: Taxes} &= (GI - OE - D)(T_e) \\
&= (32{,}000 - 10{,}000 - 20{,}000)(0.31) \\
&= \$620
\end{aligned}
$$

$$
\begin{aligned}
\text{Year 4-6: Taxes} &= (GI - OE)(T_e) \\
&= (32{,}000 - 10{,}000)(0.31) \\
&= \$6820
\end{aligned}
$$

Total taxes = 3(620) + 3(6820) = \$22,320

$$
\begin{aligned}
PW_{tax} &= 620(P/A,12\%,3) + 6820(P/A,12\%,3)(P/F,12\%,3) \\
&= 620(2.4018) + 6820(2.4018)(0.7118) \\
&= \$13{,}149
\end{aligned}
$$

Recovery over 6 years. SL depreciation is 60,000/6 = \$10,000 per year

$$
\begin{aligned}
\text{Year 1-6: Taxes} &= (GI – OE – D)(T_e) \\
&= (32{,}000 - 10{,}000 - 10{,}000)(0.31)
\end{aligned}
$$

$= \$3720$

Total taxes = 6(3720) = \$22,320

$PW_{tax} = 3720(P/A,12\%,6)$
$= 3720(4.1114)$
$= \$15,294$

Recovery in 3 years has a lower PW_{tax} value; total taxes are the same for both.

(b) *Spreadsheet:* Solution follows with only the functions shown.

	A	B	C	D	E	F	G	H	I	J
1					Recovery over 3 years			Recovery over 6 years		
2	Year	GI	Expenses	P or S	Depr.	TI	Taxes	Depr.	TI	Taxes
3	0			-66000						
4	1	32000	-10000		= SLN(65000,5000,3)	= B4+C4-E4	=F4*0.31	= SLN(65000,5000,6)	= B4+C4-H4	= I4*0.31
5	2	32000	-10000		= SLN(65000,5000,3)	= B5+C5-E5	=F5*0.31	= SLN(65000,5000,6)	= B5+C5-H5	= I5*0.31
6	3	32000	-10000		= SLN(65000,5000,3)	= B6+C6-E6	=F6*0.31	= SLN(65000,5000,6)	= B6+C6-H6	= I6*0.31
7	4	32000	-10000		0	= B7+C7-E7	=F7*0.31	= SLN(65000,5000,6)	= B7+C7-H7	= I7*0.31
8	5	32000	-10000		0	= B8+C8-E8	=F8*0.31	= SLN(65000,5000,6)	= B8+C8-H8	= I8*0.31
9	6	32000	-10000	5000	0	= B9+C9-E9	=F9*0.31	= SLN(65000,5000,6)	= B9+C9-H9	= I9*0.31
10	Total				=SUM(E4:E9)	=SUM(F4:F9)	=SUM(G4:G9)	=SUM(H4:H9)	=SUM(I4:I9)	=SUM(J4:J9)
11	PW of taxes @ 12%						**= NPV(12%,G4:G9)**			**= NPV(12%,J4:J9)**
12										

(c) *Spreadsheet functions:*

Function for 3-year recovery: = -PV(12%,3,620) + PV(12%,3,,PV(12%,3,6820))
displays \$13,148

Function for 6-year recovery: = -PV(12%,6,3720) displays \$15,294

17.31 (a) In \$1000 units for monetary values.

SL: D = (20 – 0)/3 = \$6.667 $T_e = 0.40$

Year	GI	P	OE	D	TI	Taxes	CFAT
0	–	-20	–	**–**	**–**	**–**	**-20.0000**
1	8		-2	**6.667**	**-0.667**	**-0.2667**	**6.2667**
2	15		-4	**6.667**	**4.333**	**1.7333**	**9.2667**
3	12	0	-3	**6.667**	**2.333**	**0.9332**	**8.0668**
4	10	0	-5	**-**	**5.000**	**2.0000**	**3.0000**

(b) In \$1000 units for monetary values.

MACRS rates $T_e = 0.40$

Sample: year 1: D = 20(0.3333) = \$6.666
TI = 8 - 2 - 6.666 = \$- 0.666
Taxes = - 0.666(0.40) = \$- 0.266
CFAT = 8 - 2 - (- 0.266) = \$6.266

Year	GI	P	OE	**D**	**TI**	**Taxes**	**CFAT**
0	–	-20	–	**–**	**–**	**–**	**-20.000**
1	8		-2	**6.666**	**-0.666**	**-0.266**	**6.266**
2	15		-4	**8.890**	**2.110**	**0.844**	**10.156**
3	12	0	-3	**2.962**	**6.038**	**2.415**	**6.585**
4	10	0	-5	**1.482**	**3.518**	**1.407**	**3.593**

17.32 Find the difference between PW of CFBT and CFAT at $T_e = 0.40$ and i = 10%

Year	CFBT, \$	d	Depr., \$	TI, \$	Taxes, \$	CFAT, \$
1	10,000	0.20	1,800	8,200	3,280	6,720
2	10,000	0.32	2,880	7,120	2,848	7,152
3	10,000	0.192	1,728	8,272	3,309	6,691
4	10,000	0.1152	1,037	8,963	3,585	6,415
5	5,000	0.1152	1,037	3,963	1,585	3,415
6	5,000	0.0576	518	4,482	1,793	3,207

PW_{CFBT} = 10,000(P/A,10%,4) + 5000(P/A,10%,2)(P/F,10%,4) = \$37,626
PW_{CFAT} = 6720(P/F,10%,1) + … + 3207(P/F,10%,6) = \$25,359

Cash flow lost to taxes is \$12,267 in PW terms.

17.33 All monetary terms are in \$1000 units
CFAT = GI - OE - P + S - Taxes
NOPAT = TI - Taxes

Scenario 1: Uniform write-off
Sample for year 2: CFAT = 15 – 4 – [(15 – 4 - 6)(0.32)] = 9.40
NOPAT = 5 – 1.6 = 3.40

Year	GI	OE	P	D	TI	Taxes	CFAT	NOPAT
0	–	–	-30	–	–	–	-30.00	
1	8	-2	-	6	0	0.00	6.00	0.00
2	15	-4	-	6	5	1.60	9.40	3.40
3	12	-3	-	6	3	0.96	8.04	2.04
4	10	-5	-	6	-1	-0.32	5.32	-0.68
Total								4.76

(a) Total NOPAT = \$4.76 **(\$4760)**

(b) PW_{tax} = 1.6(P/F,6%,2) + 0.96(P/F,6%,3) - 0.32(P/F,6%.4)
= \$1.9765 **(\$1977)**

Scenario 2: Accelerated write-off

Year	GI	OE	P	D	TI	Taxes	CFAT	NOPAT
0	–	–	-30	–	–	–	-30.0	
1	8	-2	-	6.000	0	0.0	6.000	0.000
2	15	-4	-	9.600	1.400	0.448	10.552	0.952
3	12	-3	-	5.760	3.240	1.037	7.963	2.203
4	10	-5	-	3.456	1.544	0.494	4.506	1.050
Total								4.205

(a) Total NOPAT = \$4.205 **(\$4205)**

(b) PW_{tax} = 0.448(P/F,6%,2) + 1.037(P/F,6%,3) + 0.494(P/F,6%.4)
= \$1.6607 **(\$1661)**

Conclusion: A larger NOPAT and lower PW_{tax} are better economically. Scenario 1 (uniform depreciation) is the choice for the NOPAT measure; however, scenario 2 (accelerated depreciation) is the choice for the PW_{tax} criterion.

Depreciation Recapture and Capital Gains (Losses)

17.34 It is important in an after-tax replacement analysis, because a 'sacrifice' trade-in value may offer a sizeable tax savings in the year of replacement.

17.35 BV_2 = 120,000 – 120,000(0.3333 + 0.4445) = \$26,664

SP = \$60,000; DR is present
DR = 60,000 – 26,664 = \$33,336
D_2 = 120,000(0.4445) = \$53,340

TI_2 = 1,400,000 – 500,000 – 53,340 + 33,336 = \$879,996

$Taxes_2$ = 879,996(0.35) = \$307,999

17.36 TI will increase by DR, since MACRS $BV_5 = 0$

$\Delta TI = DR = SP - BV_5$ = 0.25(400,000) – 0 = \$100,000

Taxes will increase by $\Delta TI(T_e) = 100{,}000T_e$

$T_e = 0.065 + (1 - 0.065)(0.35) = 0.39225$

Tax increase = 100,000(0.39225) = \$39,225

17.37 (a) Land is not depreciable.

CG = TI = 0.15(2,600,000) = \$390,000
Taxes = 390,000(0.30) = \$117,000

(b) SP = \$10,000
BV_5 = 155,000(0.0576) = \$8928
DR = SP – BV_5
= 10,000 – 8928 = \$1072
Taxes = DR(T_e) = 1072(0.30) = \$322

(c) SP = 0.2(150,000) = \$30,000
BV_7 = \$0
DR = SP – BV_7 = \$30,000
Taxes = 30,000(0.3) = \$9000

17.38 Total MACRS depreciation: 20% + 32% + 19.2% = 71.2%

BV_3 = 300,000 – 300,000(0.712)
= \$86,400

Selling price of \$80,000 < BV_3. There is a capital loss

CL = 86,400 – 80,000 = \$6400

17.39 (a) CG = 285,000 – 240,000
= \$45,000

DR = 240,000(0.2 + 0.32 + 0.192)
= \$170,880

(b) Taxes are shown in column I. TI for year 3 must include CG and DR as fully taxable.

	A	B	C	D	E	F	G	H	I	J
1	Year	GI	OE	P and S	CFBT	MACRS 5-year D	BV	TI*	Taxes	CFAT
2	0			-240,000	-240,000		240,000	0	0	-240,000
3	1	100,500	-50,000	0	50,500	48,000	192,000	2,500	700	49,800
4	2	100,500	-50,000	0	50,500	76,800	115,200	-26,300	-7,364	57,864
5	3	100,500	-50,000	285,000	335,500	46,080	69,120	220,300	61,684	273,816
6	Totals									
7										
8	* TI_3 = GI - OE - D + DR + CG = 100,500 - 50,000 - 46,080 + (285,000-240,000) + (240,000-69,120) = \$220,300									

17.40 Land: $CG = \$45,000$
Building: $CL = \$45,000$
Asset 1: $DR = 18,500 – 15,500 = \$3000$
Asset 2: $DR = 10,000 - 5,000 = \$5,000$
$CG = 10,500 - 10,000 = \$500$

17.41 (a) $BV_2 = 28,500 – 28,500(0.3333 + 0.4445) = \$6333$

$CL = 6333 – 5000 = \$1333$

(b) Capital losses can only be used to offset capital gains. This will reduce taxes on the gains. If there are no gains, carry-forward and carry-back allowances may apply.

17.42 Thomas omitted the \$100,000 DR in year 4. If included, in \$1000 units, the CFAT is

$$
\begin{aligned}
CFAT_4 &= 275 + 100 – (275\text{-}250\text{+}100)(0.52) \\
&= 275 + 100 – 65 \\
&= \$310
\end{aligned}
$$

$$
\begin{aligned}
PW &= -1000 + 262(P/A,5\%,3) + 310(P/F,5\%,4) \\
&= -1000 + 262(2.7232) + 310(0.8227) \\
&= \$-31.485 \qquad (\$-31,485)
\end{aligned}
$$

Economically, Thomas made a wrong recommendation, since PW < 0 at 5% per year.

After-Tax Economic Evaluation

17.43 (a) Before-tax ROR: $0 = -750,000 + 260,000(P/A,i^*,3) + 187,500(P/F,i^*,3)$

$i^* = 12.60\%$

(b) Approximate after-tax ROR = (before-tax ROR) $(1\text{-}T_e)$
$= 12.60(1 – 0.37)$
$= 7.94\%$

17.44 Effective tax rate $= 0.06 + (1 – 0.06)(0.35) = 0.38$

Before-tax ROR $= 0.09/(1 - 0.389) = 0.147$

17.45
$$
\begin{aligned}
0.08 &= 0.12(1 - T_e) \\
1 - T_e &= 0.667 \\
T_e &= 0.333 \qquad (33.3\%)
\end{aligned}
$$

17.46 After-tax ROR $= 24(1\text{-} 0.35) = 15.6\%$

17.47 Calculate taxes using Table 17-1 rates; the average tax rate T_e; then after-tax ROR

$$\begin{aligned}\text{Income taxes} &= 113{,}900 + 0.34(8{,}950{,}000 - 335{,}000)\\ &= 113{,}900 + 2{,}929{,}100\\ &= \$3{,}043{,}000\end{aligned}$$

$$\text{Average tax rate} = \text{taxes /TI} = 3{,}043{,}000/8{,}950{,}000 = 0.34$$

$$T_e = 0.05 + (1 - 0.05)(0.34) = 0.373$$

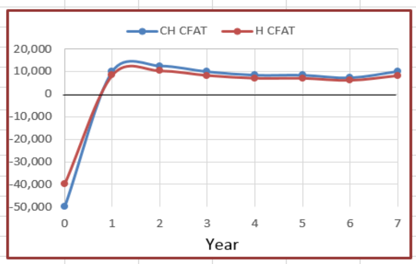
$$\begin{aligned}\text{After-tax ROR} &= (\text{before-tax ROR})(1\text{-}T_e)\\ &= 0.22(1 - 0.373)\\ &= 0.138\end{aligned}$$

A before-tax ROR of 22% is equivalent to an after-tax ROR of 13.8%

17.48 (a) Develop two tables similar to those shown.
Conclusion: PW_A = \$-201 and PW_B = \$1284. Select Hoister (H)

(b) Plot of CFAT values shows they track closely.

	A	B	C	D	E	F	G	H	I
3				CreteHelper (CH)					
4	Year	P and S	CFBT	D rate	MACRS D	BV	TI	Taxes	CH CFAT
5	0	-50,000	-50,000	0.0000	0	50,000	0	0	-50,000
6	1	0	10,000	0.2000	10,000	40,000	0	0	10,000
7	2	0	10,000	0.3200	16,000	24,000	-6,000	-2,400	12,400
8	3	0	10,000	0.1920	9,600	14,400	400	160	9,840
9	4	0	10,000	0.1152	5,760	8,640	4,240	1,696	8,304
10	5	0	10,000	0.1152	5,760	2,880	4,240	1,696	8,304
11	6	0	10,000	0.0576	2,880	0	7,120	2,848	7,152
12	7	4,000	10,000	0	0	0	10,000	4,000	10,000
13	Totals			1.0000	50,000			8,000	
14	PW @ 8%								-201
15									
16				Hoister (H)					
17	Year	B and S	CFBT	D rate	MACRS D	BV	TI	Taxes	H CFAT
18	0	-40,000	-40,000	0.0000	0	40,000	0	0	-40,000
19	1	0	8,500	0.2000	8,000	32,000	500	200	8,300
20	2	0	8,500	0.3200	12,800	19,200	-4,300	-1,720	10,220
21	3	0	8,500	0.1920	7,680	11,520	820	328	8,172
22	4	0	8,500	0.1152	4,608	6,912	3,892	1,557	6,943
23	5	0	8,500	0.1152	4,608	2,304	3,892	1,557	6,943
24	6	0	8,500	0.0576	2,304	0	6,196	2,478	6,022
25	7	3,000	8,500	0	0	0	8,500	3,400	8,100
26	Totals			1.0000	40,000			7,800	
27	PW @ 8%								1,284



17.49   System A: Depreciation = 150,000/3 = \$50,000

Years 1 to 3:   TI = 60,000 – 50,000 = \$10,000
Taxes = 10,000(0.35) = \$3500
CFAT = 60,000 – 3500 = \$56,500

$$\begin{aligned}AW_A &= -150{,}000(A/P,6\%,3) + 56{,}500\\ &= -150{,}000(0.37411) + 56{,}500\\ &= \$384\end{aligned}$$

System B: Depreciation = 85,000/5 = \$17,000

Years 1 to 5: TI = 20,000 – 17,000 = \$3,000
Taxes = 3,000(0.35) = \$1050
CFAT = 20,000 – 1050 = \$18,950

For year 5 only, when B is sold for 10% of first cost:

DR = 85,000(0.10) = \$8500
DR taxes = 8500(0.35) = \$2975

$$AW_B = –85{,}000(A/P,6\%,5) + 18{,}950 + (8500 – 2975)(A/F,6\%,5)$$
$$= –85{,}000(0.23740) + 18{,}950 + 5525(0.17740)$$
$$= -\$249$$

Select system A

17.50 For a 12% after-tax return, find n in a PW relation.

$$–78{,}000 + 18{,}000(P/A,12\%,n) – 1000(P/G,12\%,n) = 0$$

For n = 8 years: –78,000 + 18,000(4.9676) – 1000(14.4714) = -\$3055
For n = 9 years: –78,000 + 18,000(5.3282) – 1000(17.3563) = \$551

n = 8.85 years

Keep the equipment for 8.85 (or 9 rounded off) years

17.51 (a) <u>By hand:</u> **Alternative X**

Year	P and S	GI - OE	D	TI	Taxes	CFAT
0	-8000	-	-	-	-	-8000
1		3500	2666	834	333	3167
2		3500	3556	-56	-22	3522
3		3500	1185	2315	926	2574
4	0	0	593	-593	-237	237

$$PW_X = -8000 + 3167(P/F,8\%,1) + 3522(P/F,8\%,2) + 2574(P/F,8\%,3) + 237(P/F,8\%,4)$$
$$= \$169$$

Alternative Y

Year	P and S	GI - OE	D	TI	Taxes	CFAT
0	-13,000	-	-	-	-	-13,000
1		5000	4333	667	267	4733
2		5000	5779	-779	-311	5311
3		5000	1925	3075	1230	3770
4	0	0	963	-963	-385	385
4	2000	-	-	2000	800	1200

PW_Y = -13,000 + 4733(P/F,8%,1) + 5311(P/F,8%,2) + 3770(P/F,8%,3) + 385(P/F,8%,4) + 1200(P/F,8%,4)
= $93

. Select alternative X

(b) *Spreadsheet:* Select X with the larger PW value. Note handling of $2000 salvage for Y in year 4

	A	B	C	D	E	F	G	H	I
3				Alternative X					
4	Year	P and S	CFBT	D rate	MACRS D	BV	TI	Taxes	X CFAT
5	0	-8,000	-8,000	0.0000	0	8,000	0	0	-8,000
6	1	0	3,500	0.3333	2,666	5,334	834	333	3,167
7	2	0	3,500	0.4445	3,556	1,778	-56	-22	3,522
8	3	0	3,500	0.1481	1,185	593	2,315	926	2,574
9	4	0	0	0.0741	593	0	-593	-237	237
10	Totals			1.0000	8,000			1,000	
11	PW @ 8%								169
12									
13				Alternative Y					
14	Year	B and S	CFBT	D rate	MACRS D	BV	TI	Taxes	Y CFAT
15	0	-13,000	-13,000	0.0000	0	13,000	0	0	-13,000
16	1	0	5,000	0.3333	4,333	8,667	667	267	4,733
17	2	0	5,000	0.4445	5,779	2,889	-779	-311	5,311
18	3	0	5,000	0.1481	1,925	963	3,075	1,230	3,770
19	4	0	0	0.0741	963	0	-963	-385	385
20	4	2000	2000		0	0	2,000	800	1,200
21	Totals				13,000				
22	PW @ 8%*								94
23									
24	* Function for PW must consider $1200 in year 4 separate from NPV function.								

17.52 Method G: Years 1-5: CFBT = 35,000 – 15,000 = $20,000
SL D = 90,000/5 = $18,000
Taxes = (20,000 – 18,000)(0.34) = $680
CFAT = 20,000 – 680 = $19,320

AW_G = -100,000(A/P,7%,5) + 19,320 + 10,000(A/F,7%,5)
= $-3330

Method H: Years 1-5: CFBT = 45,000 – 6,000 = $39,000
SL D = 130,000/5 = $26,000
Taxes = (39,000 – 26,000)(0.34) = $4420
CFAT = 39,000 – 4420 = $34,580

AW_H = -150,000(A/P,7%,5) + 34,580 + 20,000(A/F,7%,5)
= $1474

Method H is selected; the same as with MACRS.

17.53 (a) Function for PW_A: = -PV(14%,10,-3000,3000) – 15000 displays PW_A = $-29,839

Function for PW_B: = -PV(14%,10,-1500,5000) – 22000 displays PW_A = $-28,475

Select B with a slightly higher PW value.

(b) All AOC estimates generate tax savings; GI estimates are equal.

<u>Machine A</u>

Annual depreciation = (15,000 - 3,000)/10 = $1200
Tax savings = (AOC + D)(0.5) = 4200(0.5) = $2100
CFAT = -3000 + 2100 = $-900

PW_A = -15,000 - 900(P/A,7%,10) + 3000(P/F,7%,10)
= -15,000 - 900(7.0236) + 3000(0.5083)
= $-19,796

<u>Machine B</u>

Annual depreciation = (22,000 – 5000)/10 = $1700
Tax savings = (1500 + 1700)(0.50) = $1600
CFAT = –1500 + 1600 = $100

PW_B = –22,000 + 100(P/A,7%,10) + 5000(P/F,7%,10)
= –22,000 + 100(7.0236) + 5000(0.5083)
= $–18,756

Again, select B with a slightly higher PW value.

(c) Again, select machine B. All methods give the same conclusion

	A	B	C	D	E	F	G	H	I
1				**Machine A**					
2	Year	P and S	AOC/CFBT	D rate	MACRS D	BV	TI	Tax saving	A CFAT
3	0	-15,000	-15,000	0.0000	0	15,000	0	0	-15,000
4	1	0	-3,000	0.2000	3,000	12,000	-6,000	-3,000	0
5	2	0	-3,000	0.3200	4,800	7,200	-7,800	-3,900	900
6	3	0	-3,000	0.1920	2,880	4,320	-5,880	-2,940	-60
7	4	0	-3,000	0.1152	1,728	2,592	-4,728	-2,364	-636
8	5	0	-3,000	0.1152	1,728	864	-4,728	-2,364	-636
9	6	0	-3,000	0.0576	864	0	-3,864	-1,932	-1,068
10	7	0	-3,000	0.0000	0	0	-3,000	-1,500	-1,500
11	8	0	-3,000	0.0000	0	0	-3,000	-1,500	-1,500
12	9	0	-3,000	0.0000	0	0	-3,000	-1,500	-1,500
13	10	0	-3,000	0.0000	0	0	-3,000	-1,500	-1,500
14	10	3000					3,000	-1,500	1,500
15									
16	PW @ 7%								**-$18,536**
17									
18				**Machine B**					
19	Year	P and S	AOC/CFBT	D rate	MACRS D	BV	TI	Taxes	B CFAT
20	0	-22,000	-22,000	0.0000	0	22,000	0	0	-22,000
21	1	0	-1,500	0.2000	4,400	17,600	-5,900	-2,950	1,450
22	2	0	-1,500	0.3200	7,040	10,560	-8,540	-4,270	2,770
23	3	0	-1,500	0.1920	4,224	6,336	-5,724	-2,862	1,362
24	4	0	-1,500	0.1152	2,534	3,802	-4,034	-2,017	517
25	5	0	-1,500	0.1152	2,534	1,267	-4,034	-2,017	517
26	6	0	-1,500	0.0576	1,267	0	-2,767	-1,384	-116
27	7	0	-1,500	0.0000	0	0	-1,500	-750	-750
28	8	0	-1,500	0.0000	0	0	-1,500	-750	-750
29	9	0	-1,500	0.0000	0	0	-1,500	-750	-750
30	10	0	-1,500	0.0000	0	0	-1,500	-750	-750
31	10	5000					5000	-2,500	2,500
32	PW @ 7%*								**-$16,850**
33									
34	*** Function for PW of CFAT must consider $2500 in year 10 separate from NPV function.**								

By hand, if needed:

MACRS with n = 5 and a DR in year 10, which is a tax, not a tax savings.

Tax savings = (AOC + D)(0.5), years 1-6

CFAT = -AOC + tax savings, years 1-10.

Machine A

Year 10 has a DR tax of 3,000(0.5) = $1500

Year	P or S	AOC	Depr.	Tax savings	CFAT
0	$-15,000	-	-	-	$-15,000
1		$3000	$3000	$3000	0
2		3000	4800	3900	900
3		3000	2880	2940	-60
4		3000	1728	2364	-636
5		3000	1728	2364	-636
6		3000	864	1932	-1068
7		3000	0	1500	-1500
8		3000	0	1500	-1500
9		3000	0	1500	-1500
10		3000	0	1500	-1500
10	3000	-	-	–1500	1500

$$PW_A = -15{,}000 + 0 + 900(P/F,7\%,2) + \ldots - 1{,}500(P/F,7\%,9) = \$-18{,}536$$

<u>Machine B</u>

Year 10 has a DR tax of 5,000(0.5) = $2,500

Year	P or S	AOC	Depr	Tax savings	CFAT
0	$–22,000	-	-	-	$–22,000
1		$1500	$4400	$2950	1450
2		1500	7040	4270	2770
3		1500	4224	2862	1362
4		1500	2534	2017	517
5		1500	2534	2017	517
6		1500	1268	1384	–116
7		1500	0	750	–750
8		1500	0	750	–750
9		1500	0	750	–750
10		1500	0	750	–750
10	5000	-	-	–2500	2500

PW_B = -22,000 + 1450(P/F,7%,1) + ... + 2500(P/F,7%,10) = $-16,850

After-Tax Replacement

17.54 (a) For a *capital loss*, it is the difference between sales price and the asset's book value. For a *capital gain*, it is the difference between the sales price and the unadjusted basis (first cost) of the asset.

(b) The AW of the *challenger* is affected in year 0 by the capital gains tax. If it is a capital loss, the netting of losses against gains can affect AW.

17.55 TI, year 2 = -70,000 - 49,960 = -119,960

Taxes, year 2 = -119,960(0.35) = $-41,986 (tax savings)

CFAT, year 2 = -70,000 + 41,986
= $-28,014

17.56 (a) Defender: CL = BV_2 - sales price = [300,000 – 2(60,000)] - 100,000
= $-80,000

The CL of $-80,000 by the defender will result in tax consequences as follows:

Taxes = -80,000(0.35) = $-28,000

This represents a *tax savings* for the *challenger* in year 0.

$CFAT_0$, challenger = $-420,000 + 28,000 = $-392,000

$CFAT_0$, defender = $-100,000

(b) Defender, years 1-3: TI = -120,000 – 60,000 = $-180,000

Taxes = 180,000(0.35) = $-63,000
CFAT = -120,000 - (-63,000) = $-57,000

Challenger, years 1-3: TI = -30,000 -140,000 = $-170,000
Taxes = -170,000(0.35) = $-59,500
CFAT = -30,000 - (-59,500) = $29,500

(c) AW_D = -100,000(A/P,15%,3) – 57,000
= -100,000(0.43798) – 57,000
= $-100,798

AW_C = -392,000(A/P,15%,3) + 29,500
= -392,000(0.43798) + 29,500
= $-142,188

Conclusion: Keep the defender

(d) Spreadsheet shows CFAT and AW values; keep the defender.

	A	B	C	D	E	F	G
1			DEFENDER				
2	Year	OE, $	B, $	Depr., $	TI, $	Taxes, $	CFAT, $
3	0		-100,000				-100,000
4	1	-120,000		60,000	-180,000	-63,000	-57,000
5	2	-120,000		60,000	-180,000	-63,000	-57,000
6	3	-120,000		60,000	-180,000	-63,000	-57,000
7	AW @ 15%						**-100,798**
8							
9			CHALLENGER				
10	Year	OE, $	B, $	Depr., $	TI, $	Taxes, $	CFAT, $
11	0		-420,000		-80,000	-28,000	-392,000
12	1	-30,000		140,000	-170,000	-59,500	29,500
13	2	-30,000		140,000	-170,000	-59,500	29,500
14	3	-30,000		140,000	-170,000	-59,500	29,500
15	AW @ 15%						**-142,187**

17.57 Find after-tax PW of costs over *4-year study period*. DR is involved on the defender trade.
<u>By hand:</u>

<u>Defender</u>:
SL depreciation = (45,000 - 5000)/8 = $5000

Annual tax = (-OE – Depr)(T_e)
= (-7000 – 5000)(0.35)
= $-4200 (tax savings)

CFAT = CFBT – taxes
= -7000 – (-4200)

= $-2800

PW_D = -35,000 + 5000(P/F,12%,4) – 2800(P/A,12%,4)
= -35,000 + 5000(0.6355) – 2800(3.0373)
= $-40,327

<u>Challenger:</u>

MACRS depreciation over n = 5, but only 4 years apply

Defender trade depreciation recapture must be included.
Defender BV_3 = 45,000 – 3(5000) = $30,000
SP = $35,000
DR = SP – BV = 5,000
Tax on DR = 5,000(0.35) = $1750

Challenger first cost = -24,000 - 1750 = $-25,750
MACRS depreciation is based on $24,000 first cost

Year	OE	P and S	Rate	Depr.	TI	Taxes	CFAT
0		-25,750					-25,750
1	-8000		0.3333	8,000	-16,000	-5,600	-2,400
2	-8000		0.4445	10,668	-18,668	-6,534	-1,466
3	-8000		0.1481	3,554	-11,554	-4,044	-3,956
4	-8000	0	0.0741	1,778	-9,778	-3,422	-4,578

PW_C = -25,750 – 2400(P/F,12%,1) - … - 4578(P/F,12%,4)
= $-34,787

Conclusion: Select the challenger with a lower PW of cost

Spreadsheet: Same decision; select the challenger

	DEFENDER					
Year	OE, $	P and S, $	Depr., $	TI, $	Taxes, $	CFAT, $
0		-35,000				-35,000
1	-7,000		5,000	-12,000	-4,200	-2,800
2	-7,000		5,000	-12,000	-4,200	-2,800
3	-7,000		5,000	-12,000	-4,200	-2,800
4	-7,000	5,000	5,000	-12,000	-4,200	2,200
PW @ 12%						-40,327

	CHALLENGER					
Year	OE, $	P and S, $	Depr., $	TI, $	Taxes, $	CFAT, $
0		-25,750		5,000	1,750	-25,750
1	-8,000		8,000	-16,000	-5,600	-2,400
2	-8,000		10,668	-18,668	-6,534	-1,466
3	-8,000		3,554	-11,554	-4,044	-3,956
4	-8,000	0	1,778	-9,778	-3,422	-4,578
PW @ 12%						-34,787

17.58 Challenger: Determine AW_C and compare it with AW_D = $2100.
Defender has DR on trade since BV = 0 now.

DR = SP – BV = 25,000 – 0 = $25,000
Tax on DR = 25,000(0.3) = $7500
Challenger first cost = -75,000 – 7500 = $-82,500
SL depreciation = (75,000 - 15,000)/10 = $6000 per year

CFAT, years 1-10 = CFBT – (CFBT - D)(T_e)
= 15,000 – (15,000 – 6000)(0.3)
= $12,300

AW_C = -82,500(A/P,8%,10) +15,000(A/F,8%,10) + 12,300
= -82,500(0.14903) + 15,000(0.06903) + 12,300
= $1040

Retain the defender; it has a larger AW value.

17.59 Defender

Original life estimate was 12 years.

Annual SL depreciation = 450,000 /12 = $37,500

Annual tax savings = (37,500 + 160,000)(0.32) = $63,200

AW_D = -50,000(A/P,10%,5) – 160,000 + 63,200
= -50,000(0.2638) – 96,800
= $–109,990

Challenger

Book value of D = 450,000 – 7(37,500)
= \$187,500

CL from sale of D = BV_7 – Market value
= 187,500 – 50,000
= \$137,500

Tax savings from CL, year 0 = 137,500(0.32)
= \$44,000

Challenger annual SL depreciation = $\frac{700{,}000 - 50{,}000}{10}$
= \$65,000

Annual tax savings = (65,000 +150,000)(0.32)
= \$68,800

Challenger DR when sold in year 8 = \$0

AW_C = (–700,000 + 44,000)(A/P,10%,10) + 50,000(A/F,10%,10) – 150,000 + 68,800
= –656,000(0.16275) + 50,000(0.06275) – 81,200
= \$-184,827

Select the defender. Decision was incorrect since D has a lower AW value of costs.

17.60 (a) *By hand:* Lives are set at 5 (remaining) for the defender and 8 years for the challenger.

Defender

Annual depreciation = $\frac{28{,}000 - 2000}{10}$ = \$2600
Annual tax savings = (2600 + 1200)(0.06) = \$228

AW_D = -15,000(A/P,6%,5) + 2000(A/F,6%,5) – 1200 + 228
= -15,000(0.2374) + 2000(0.1774) – 1200 + 228
= \$– 4178

Challenger

DR from sale of D = Market value – BV_5
= 15,000 – [28,000 – 5(2600)] = 0

Challenger annual depreciation = $\dfrac{15{,}000 - 3000}{8}$ = \$1500

Annual tax saving = (1,500 + 1,500)(0.06) = \$180

Challenger DR, year 8 = 3000 – 3000 = 0

$AW_C = -15{,}000(A/P,6\%,8) + 3000(A/F,6\%,8) - 1500 + 180$
$= -15{,}000(0.16104) + 3000(0.10104) - 1320$
$= \$-3432$

Select the challenger

(b) *Spreadsheet:* With n = 5, AW_D = \$-4178; with n = 8, AW_C = \$-3432.
Select the challenger

	DEFENDER						
Year	OE, \$	P and S, \$	Depr., \$	BV, \$	TI, \$	Taxes, \$	CFAT, \$
0		-15,000		28,000			-15,000
1	-1,200		2,600	25,400	-3,800	-228	-972
2	-1,200		2,600	22,800	-3,800	-228	-972
3	-1,200		2,600	20,200	-3,800	-228	-972
4	-1,200		2,600	17,600	-3,800	-228	-972
5	-1,200	2,000	2,600	15,000	-3,800	-228	1,028
AW @ 6%							-4,178
	CHALLENGER						
Year	OE, \$	P and S, \$	Depr., \$	BV, \$	TI, \$	Taxes, \$	CFAT, \$
0		-15,000		15,000	0	0	-15,000
1	-1,500		1,500	13,500	-3,000	-180	-1,320
2	-1,500		1,500	12,000	-3,000	-180	-1,320
3	-1,500		1,500	10,500	-3,000	-180	-1,320
4	-1,500		1,500	9,000	-3,000	-180	-1,320
5	-1,500		1,500	7,500	-3,000	-180	-1,320
6	-1,500		1,500	6,000	-3,000	-180	-1,320
7	-1,500		1,500	4,500	-3,000	-180	-1,320
8	-1,500	3,000	1,500	3,000	-3,000	-180	1,680
AW @ 6%							-3,432

(c) $AW_D = -15{,}000(A/P,12\%,5) + 2000(A/F,12\%,5) - 1200$
$= -15{,}000(0.27741) + 2000(0.15741) - 1200$
$= \$-5046$

$AW_C = -15{,}000(A/P,12\%,8) + 3000(A/F,12\%,8) - 1500$
$= -15{,}000(0.2013) + 3000(0.0813) - 1500$
$= \$-4276$

Select the challenger. The before-tax and after-tax decisions are the same.
Functions: AW_D: = -PMT(12%,5,-15000,2000) – 1200 displays \$-5046 per year
AW_C: = -PMT(12%,8,-15000,3000) – 1500 displays \$-4276 per year

17.61 (a) Study period is set at 5 years. The only option is the defender for 5 years and the challenger for 5 years.

Defender

First cost = Sale + Upgrade = 15,000 + 9000 = $24,000

Upgrade SL depreciation = $3000 year (years 1-3 only)
OE, years 1-5: = $6000
Tax saving, years 1-3: = (6000 + 3000)(0.4) = $3600
Tax savings, year 4-5: = 6000(0.4) = $2,400
Actual cost, years 1-3: = 6000 – 3600 = $2400
Actual cost, years 4-5: = 6000 – 2400 = $3600

AW_D = -24,000(A/P,12%,5) - 2400 - 1200(F/A,12%,2)(A/F,12%,5)
= -24,000(0.27741) - 2400 - 1200(2.12)(0.15741)
= $-9458

Challenger

DR on defender = $15,000
DR tax = $6000
First cost + DR tax = $46,000
Depreciation = 40,000/5 = $8,000
Expenses = $7,000 (years 1-5)
Tax savings = (8000 + 7000)(0.4) = $6,000
Actual OE = 7000 – 6000 = $1000 (years 1-5)

AW_C = –46,000(A/P,12%,5) – 1000
= –46,000(0.27741) – 1000
= $–13,761

Retain the defender since the AW of cost is smaller.

(b) AW_C will become less costly, but the revenue from the challenger's sale between $2000 to $4000 will be reduced by the 40% tax on DR in year 5.

Economic Value Added

17.62 (a) The EVA shows the monetary worth added to a corporation by an alternative.

(b) The EVA estimates can be used directly in public reports (e.g., to stockholders). EVA shows corporate worth contribution, not just cash flows after taxes.

17.63 BV_1 = 300,000 – 300,000(0.20) = $240,000

$$\begin{aligned} EVA &= NOPAT - MARR(BV_1) \\ &= 70{,}000 - (0.15)(240{,}000) \\ &= \$34{,}000 \end{aligned}$$

17.64 Find BV_{t-1} and solve for NOPAT and TI, year 2, and solve for OE

$BV_1 = 550{,}000 - 550{,}000(0.3333) = \$366{,}685$

Year 2 results:

$$\begin{aligned} EVA = 28{,}000 &= NOPAT - (0.14)(366{,}685) \\ NOPAT &= \$79{,}336 \\ 79{,}336 &= TI(1 - T_e) = TI(1 - 0.35) \\ TI &= \$122{,}055 \end{aligned}$$

$$\begin{aligned} D_2 &= 550{,}000(0.4445) = \$244{,}475 \\ TI_2 &= GI - OE - D \\ 122{,}055 &= 500{,}000 - OE - 244{,}475 \\ OE &= \$133{,}470 \end{aligned}$$

17.65 The spreadsheet verifies that the AW values are the same. Note the difference in the patterns of the CFAT and EVA series. CFAT shows a big cost in year 0 and positive cash flows thereafter. EVA shows nothing in year 0 and after 2 years the value-added terms turn positive, indicating a positive contribution to the corporation's worth.

Year	GI	OE	P	D	TI	Taxes	CFAT	NOPAT	BV	Cost of invested capital	EVA
0			-300,000				-300,000		300,000		0
1	200,000	-80,000		99,990	20,010	7,004	112,997	13,007	200,010	-15,000	-1,994
2	200,000	-80,000		133,350	-13,350	-4,673	124,673	-8,678	66,660	-10,001	-18,678
3	200,000	-80,000		44,430	75,570	26,450	93,551	49,121	22,230	-3,333	45,788
4	200,000	-80,000		22,230	97,770	34,220	85,781	63,551	0	-1,112	62,439
AW @ 5%							$20,328				$20,328
Functions, Yr 4								= F7 - G7	= J6 - E7	= -0.05*J6	= I7 + K7

17.66 Depreciation is SL: Hong Kong: 4.2 million/8 = \$525,000
Vietnam: 3.6 million/5 = \$720,000

The Vietnam supplier indicates a larger AW of EVA; however, the difference is small given the size of the order.

(Note: The CFAT series and AW of CFAT are shown for information only.)

	A	B	C	D	E	F	G	H	I	J	K
1					HONG KONG						
2	Year	GI - OE	P	D	TI	NOPAT	BV	Inv Cap Cost	EVA		CFAT
3	0		-4,200,000				4,200,000				-4,200,000
4	1	1,500,000		525,000	975,000	682,500	3,675,000	-336,000	346,500		1,207,500
5	2	1,800,000		525,000	1,275,000	892,500	3,150,000	-294,000	598,500		1,417,500
6	3	2,100,000		525,000	1,575,000	1,102,500	2,625,000	-252,000	850,500		1,627,500
7	4	2,400,000		525,000	1,875,000	1,312,500	2,100,000	-210,000	1,102,500		1,837,500
8	5	2,700,000		525,000	2,175,000	1,522,500	1,575,000	-168,000	1,354,500		2,047,500
9	6	3,000,000		525,000	2,475,000	1,732,500	1,050,000	-126,000	1,606,500		2,257,500
10	7	3,300,000		525,000	2,775,000	1,942,500	525,000	-84,000	1,858,500		2,467,500
11	8	3,600,000		525,000	3,075,000	2,152,500	0	-42,000	2,110,500		2,677,500
12				4,200,000					$1,127,328		$1,127,328
13											
14											
15					VIETNAM						
16	Year	GI - OE	P	D	TI	NOPAT	BV	Inv Cap Cost	EVA		CFAT
17	0		-3,600,000				3,600,000				-3,600,000
18	1	1,500,000		720,000	780,000	546,000	2,880,000	-288,000	258,000		1,266,000
19	2	1,800,000		720,000	1,080,000	756,000	2,160,000	-230,400	525,600		1,476,000
20	3	2,100,000		720,000	1,380,000	966,000	1,440,000	-172,800	793,200		1,686,000
21	4	2,400,000		720,000	1,680,000	1,176,000	720,000	-115,200	1,060,800		1,896,000
22	5	2,700,000		720,000	1,980,000	1,386,000	0	-57,600	1,328,400		2,106,000
23	6	3,000,000		0	3,000,000	2,100,000	0	0	2,100,000		2,100,000
24	7	3,300,000		0	3,300,000	2,310,000	0	0	2,310,000		2,310,000
25	8	3,600,000		0	3,600,000	2,520,000	0	0	2,520,000		2,520,000
26				3,600,000					$1,224,312		$1,224,312
27											
28								AW of EVA:			
29								= -PMT(8%,8,NPV(8%,I18:I25))			
30											

17.67 (a) Column L shows the EVA each year. Use Eq. [17.23} to calculate EVA.

(b) The AW_{EVA} of $338,000 is calculated on the spreadsheet.

(c) Use Goal Seek to change cell A1 from 12% to 8.51% per year

	A	B	C	D	E	F	G	H	I	J	K	L
1	12%	= i	# years =	6			in $1000					
2	35%	= T_e										
3											Interest on	
4											Invested	
5	Year	GI	OE	P and S	D rate	D	BV	TI	Taxes	NOPAT	Capital	EVA
6	0			-3,000			3,000					
7	1	2,700	-1,000		0.10	300	2,700	1,400	490	910	360	550
8	2	2,600	-1,050		0.20	600	2,100	950	333	618	324	294
9	3	2,500	-1,100		0.20	600	1,500	800	280	520	252	268
10	4	2,400	-1,150		0.20	600	900	650	228	423	180	243
11	5	2,300	-1,200		0.20	600	300	500	175	325	108	217
12	6	2,200	-1,250		0.10	300	0	650	228	423	36	387
13	AW @ 12%											$338

Value-Added Tax

17.68 A sales tax is collected when the goods or services are bought by the end-user, while value-added taxes are collected at every stage of the production/distribution process.

17.69 (a) Tax collected by vendor B = 130,000(0.25) = $32,500

(b) Tax sent by vendor B = amount collected – amount paid to vendor A
= 32,500 – 60,000(0.25) = \$17,500

(c) Amount collected by Treasury = 250,000(0.25) = \$62,500

17.70 VAT by supplier C = 620,000(0.125) = \$77,500

17.71 Taxes paid to supplier A = 350,000(0.04) = \$14,000

Ajinkya kept none of the VAT due to supplier A.

17.72 VAT paid = 350(0.04) + 870(0.125) + 620(0.125) + 90(0.213) + 50(0.326)
= \$235,720

17.73 In \$1000 units,
Average VAT rate = VAT paid/value of goods and services
= 235.720/(350 + 870 + 620 + 90 +50)
= 235.720/1980
= 0.1191 (11.91%)

17.74 Taxes sent = amount collected – amount paid
= 9,200,000(0.15) – 235,720 (from problem 17.72)
= \$1,144,280

17.75 VAT collected = sent by suppliers + sent by Ajinkya
= 235,720 + 1,144.280
= \$1,380,000

or VAT collected = 9,200,000(0.15)
= \$1,380,000

ADDITIONAL PROBLEMS AND FE EXAM PRACTICE PROBLEMS

17.76 Before-tax ROR = After-tax ROR/(1- T_e)
= 11.4%/(1 - 0.39)
= 18.69%
Answer is (c)

17.77 T_e = 0.07 + (1 – 0.07)(0.36) = 0.4048 (40.5%)
Answer is (a)

17.78 Answer is (c)

17.79 Answer is (c)

17.80 Answer is (d)

17.81 Answer is (a)

17.82 Answer is (d)

17.83 Tax difference = (160,000,000 – 120,000,000)(0.50) = $20,000,000
Answer is (b)

17.84 NOI = 360,000 – 76,000 – 7000 – 110,000 – 29,000
= $138,000
Answer is (c)

17.85 The sale results in DR = $16,000, which is an increase in TI.
Tax increase = 16,000(0.36) = $5760
Answer is (b)

17.86 Taxes = (155,000 + 4,000 - 12,000)(0.28)
= 147,000 (0.28)
= $41,160
Answer is (b)

17.87 CFAT = GI – OE – TI(T_e)
26,000 = 30,000 – TI(0.40)
TI = (30,000 – 26,000)/0.40 = $10,000

Taxes = TI(T_e) = 10,000(0.40) = $4000

TI = (GI - OE – D)
10,000 = (30,000 – D)
D = $20,000
Answer is (d)

17.88 BV_5 = 100,000(0.0576) = $5760
DR = 22,000 – 5760 = $16,240
Tax on DR = 16,240(0.30) = $4872
Cash flow = 22,000 – 4872 = $17,128
Answer is (b)

Solution to Case Study, Chapter 17

There is not always a definitive answer to case study exercises. Here are example responses.

AFTER-TAX ANALYSIS FOR BUSINESS EXPANSION

1. The next two spreadsheets perform an analysis of the four D-E mix scenarios

	A	B	C	D	E	F	G	H	I	J	K
1				0% debt and 100% equity financing						Capital =	$ 1,500,000
2			Debt financing (loan)		Equity	MACRS			Taxes		
3	Year	GI - E	Interest[1]	Principal	investment	rate	Depr.	TI	@ 35%	CFAT	
4	0				($1,500,000)	-				($1,500,000)	
5	1	$600,000	$0	$0		0.2000	$300,000	$300,000	$105,000	$495,000	
6	2	$600,000	$0	$0		0.3200	$480,000	$120,000	$42,000	$558,000	
7	3	$600,000	$0	$0		0.1920	$288,000	$312,000	$109,200	$490,800	
8	4	$600,000	$0	$0		0.1152	$172,800	$427,200	$149,520	$450,480	
9	5	$600,000	$0	$0		0.1152	$172,800	$427,200	$149,520	$450,480	
10	6	$600,000			$0	0.0576	$86,400	$513,600	$179,760	$420,240	
11	Totals					1.0000	$1,500,000		$735,000	$1,365,000	
12	PW at 10%									$604,513	
13	(1) Interest plus principal = $ debt/5 + ($ debt)(0.06)										
14											
15				50% debt and 50% equity financing							
16			Debt financing (loan)		Equity	MACRS			Taxes		
17	Year	GI - E	Interest[1]	Principal	investment	rate	Depr.	TI	@ 35%	CFAT	
18	0				($750,000)	-				($750,000)	
19	1	$600,000	($45,000)	($150,000)		0.2000	$300,000	$255,000	$89,250	$315,750	
20	2	$600,000	($45,000)	($150,000)		0.3200	$480,000	$75,000	$26,250	$378,750	
21	3	$600,000	($45,000)	($150,000)		0.1920	$288,000	$267,000	$93,450	$311,550	
22	4	$600,000	($45,000)	($150,000)		0.1152	$172,800	$382,200	$133,770	$271,230	
23	5	$600,000	($45,000)	($150,000)		0.1152	$172,800	$382,200	$133,770	$271,230	
24	6	$600,000			$0	0.0576	$86,400	$513,600	$179,760	$420,240	
25	Totals					1.0000	$1,500,000		$656,250	$1,218,750	
26	PW at 10%									$675,015	
27											
28				There are three worksheets for this case study solution							

	A	B	C	D	E	F	G	H	I	J	K
1				70% debt and 30% equity financing						Capital =	$ 1,500,000
2			Debt financing (loan)		Equity	MACRS			Taxes		
3	Year	GI - E	Interest	Principal	investment	rate	Depr.	TI	@ 35%	CFAT	
4	0				($450,000)	-				($450,000)	
5	1	$600,000	($63,000)	($210,000)		0.2000	$300,000	$237,000	$82,950	$244,050	
6	2	$600,000	($63,000)	($210,000)		0.3200	$480,000	$57,000	$19,950	$307,050	
7	3	$600,000	($63,000)	($210,000)		0.1920	$288,000	$249,000	$87,150	$239,850	
8	4	$600,000	($63,000)	($210,000)		0.1152	$172,800	$364,200	$127,470	$199,530	
9	5	$600,000	($63,000)	($210,000)		0.1152	$172,800	$364,200	$127,470	$199,530	
10	6	$600,000			$0	0.0576	$86,400	$513,600	$179,760	$420,240	
11	Totals					1.0000	$1,500,000		$624,750	$1,160,250	
12	PW at 10%									$703,215	
13											
14											
15				90% debt and 10% equity financing							
16			Debt financing (loan)		Equity	MACRS			Taxes		
17	Year	GI - E	Interest	Principal	investment	rate	Depr.	TI	@ 35%	CFAT	
18	0				($150,000)	-				($150,000)	
19	1	$600,000	($81,000)	($270,000)		0.2000	$300,000	$219,000	$76,650	$172,350	
20	2	$600,000	($81,000)	($270,000)		0.3200	$480,000	$39,000	$13,650	$235,350	
21	3	$600,000	($81,000)	($270,000)		0.1920	$288,000	$231,000	$80,850	$168,150	
22	4	$600,000	($81,000)	($270,000)		0.1152	$172,800	$346,200	$121,170	$127,830	
23	5	$600,000	($81,000)	($270,000)		0.1152	$172,800	$346,200	$121,170	$127,830	
24	6	$600,000			$0	0.0576	$86,400	$513,600	$179,760	$420,240	
25	Totals					1.0000	$1,500,000		$593,250	$1,101,750	
26	PW at 10%									$731,416	

Note: Column B, E used instead of OE for operating expenses.

Conclusion: The 90% debt option has the largest PW at 10%. As mentioned in the chapter, the largest D-E financing option will always offer the largest return on the invested equity capital. But, too high of D-E mixes are risky.

2. Subtract 2 different equity CFAT totals.

For 30% and 10%:
(1,160,250 – 1,101,750) = $58,500

Divide by 2 to get the change per 10% equity increase.
58,500/2 = $29,250

Conclusion: Total CFAT increases by $29,250 for each 10% increase in equity financing.

3. This happens because as less of Pro-Fence's own (equity) funds are committed to the Victoria site, the larger the loan principal.

4. Use the EVA series as an estimate of contribution to Pro-Fence's bottom line through time.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Exercise #4) EVA for 50%-50% financing												
2				50% debt and 50% equity financing				Capital =	$ 1,500,000			Interest on	
3			Debt financing (loan)		Equity	MACRS		Book value		Taxes		invested	
4	Year	GI - E	Interest[1]	Principal	investment	rate	Depr.	BV	TI	@ 35%	NPAT	capital[1]	EVA
5	0				($750,000)	-		$ 1,500,000					
6	1	$600,000	($45,000)	($150,000)		0.2000	$300,000	$ 1,200,000	$255,000	$89,250	$165,750	$150,000	$15,750
7	2	$600,000	($45,000)	($150,000)		0.3200	$480,000	$ 720,000	$75,000	$26,250	$48,750	$120,000	($71,250)
8	3	$600,000	($45,000)	($150,000)		0.1920	$288,000	$ 432,000	$267,000	$93,450	$173,550	$72,000	$101,550
9	4	$600,000	($45,000)	($150,000)		0.1152	$172,800	$ 259,200	$382,200	$133,770	$248,430	$43,200	$205,230
10	5	$600,000	($45,000)	($150,000)		0.1152	$172,800	$ 86,400	$382,200	$133,770	$248,430	$25,920	$222,510
11	6	$600,000			$0	0.0576	$86,400	$ -	$513,600	$179,760	$333,840	$8,640	$325,200
12	Totals					1.0000	$1,500,000			$656,250			
13	PW at 10%												$493,633
14	AW @ 10%												$113,342
15													
16	(1) Interest at 10% is calculated on the basis of $1.5 million, not the smaller amount of equity capital committed.												

Equations used to determine the EVA use NOPAT (or NPAT) and interest on invested capital.

EVA = NPAT – interest on invested capital (column M)

NPAT = TI – taxes

(Interest on invested capital)$_t$ = i(BV in the previous year)
= 0.10(BV_{t-1})

Note: BV on the entire \$1.5 million in depreciable assets is used to determine the interest on invested capital.

Conclusion: The added business in Victoria should turn positive the third year and remain a contributor to the business after that, as indicated by the EVA values. Plus, the AW of EVA at the required 10% return is positive (AW = \$113, 342).

Chapter 18
Sensitivity Analysis and Staged Decisions

Sensitivity to Parameter Variation

18.1 \$290,000: AW = -850,000(A/P,20%,5) + 290,000
= -850,000(0.33438) + 290,000
= \$5777 (ROR > 20%)

\$325,000: AW = -850,000(A/P,20%,5) + 325,000
= -850,000(0.33438) + 325,000
= \$40,777 (ROR > 20%)

The decision to invest is not sensitive to the revenue estimate range.

18.2 (a) *By hand:*
Invest now: FW = -80,000(F/P,20%,5) + 26,000(F/A,20%,5)
= -80,000(2.4883) + 26,000(7.4416)
= \$-5582 (< 20% per year)

Invest 1 year from now: FW = -80,000(F/P,20%,4) + 31,000(F/A,20%,4)
= -80,000(2.0736) + 31,000(5.3680)
= \$520 (> 20% per year)

Invest 2 years from now: FW = -80,000(F/P,20%,3) + 37,000(F/A,20%,3)
= -80,000(1.7280) + 37,000(3.6400)
= \$-3560 (< 20% per year)

Timing will affect the return requirement of 20%; invest 1 year from now.

(b) *Spreadsheet:* Same result using the FV function = - FV(20%,n,savings,-80000)

Investment Year	Estimated Savings, \$/yr	FW, \$
0	26,000	**-5,584**
1	31,000	**520**
2	37,000	**-3,560**

18.3 (a) $PW_{138,000}$ = -500,000 + 138,000(P/A,15%,5)
= -500,000 + 138,000(3.3522)

$= \$-37{,}396$ (ROR < 15%)

$PW_{165{,}000} = -500{,}000 + 165{,}000(P/A{,}15\%{,}5)$
$= -500{,}000 + 165{,}000(3.3522)$
$= \$53{,}113$ (ROR > 15%)

The decision to invest is sensitive to the revenue estimates

(b) Function = PMT(15%,5,-500000) displays $149,158 per year

18.4 (a) $AW_{current} = \$-62{,}000$

$AW_{10{,}000} = -64{,}000(A/P{,}15\%{,}3) – 38{,}000 + 10{,}000(A/F{,}15\%{,}3)$
$= -64{,}000(0.43798) – 38{,}000 + 10{,}000(0.28798)$
$= \$-63{,}151$ (> $-62,000)

$AW_{14{,}500} = -64{,}000(A/P{,}15\%{,}3) – 38{,}000 + 14{,}500(A/F{,}15\%{,}3)$
$= -64{,}000(0.43798) – 38{,}000 + 14{,}500(0.28798)$
$= \$-61{,}855$ (< $-62,000)

$AW_{18{,}000} = -64{,}000(A/P{,}15\%{,}3) – 38{,}000 + 18{,}000(A/F{,}15\%{,}3)$
$= -64{,}000(0.43798) – 38{,}000 + 18{,}000(0.28798)$
$= \$-60{,}847$ (< $-62,000)

The decision is sensitive

(b) Replace when AW < $-62,000 for an S between $10,000 and $14,500. Set up the spreadsheet and use Goal Seek to change the cell containing salvage estimates such that the function = - PMT(15%,3,-64000,salvage) - 38000 displays -62,000. Answer is $13,996.

18.5 $PW_{low} = -80{,}000 + 10{,}000(P/F{,}8\%{,}6) + 10{,}000(P/A{,}8\%{,}6)$
$= -80{,}000 + 10{,}000(0.6302) + 10{,}000(4.6229)$
$= \$–27{,}469$

$PW_{avg} = -80{,}000 + 10{,}000(P/F{,}8\%{,}6) + 16{,}000(P/A{,}8\%{,}6)$
$= -80{,}000 + 10{,}000(0.6302) + 16{,}000(4.6229)$
$= \$268$

$PW_{high} = -80{,}000 + 10{,}000(P/F{,}8\%{,}6) + 20{,}000(P/A{,}8\%{,}6)$
$= -80{,}000 + 10{,}000(0.6302) + 20{,}000(4.6229)$
$= \$18{,}760$

The $10,000 revenue estimate is the only one that *does not* favor the purchase. The average and high estimates do favor purchase

18.6 (a) $PW_{Lease} = -30(1000) - 30(1000)(P/A,20\%,2)$
$= -30(1000) - 30(1000)(1.5278)$
$= \$-75,834$

$PW_{Build,70} = -80,000 - 70(1000) + 120,000(P/F,20\%,3)$
$= -150,000 + 120,000(0.5787)$
$= -\$80,556$

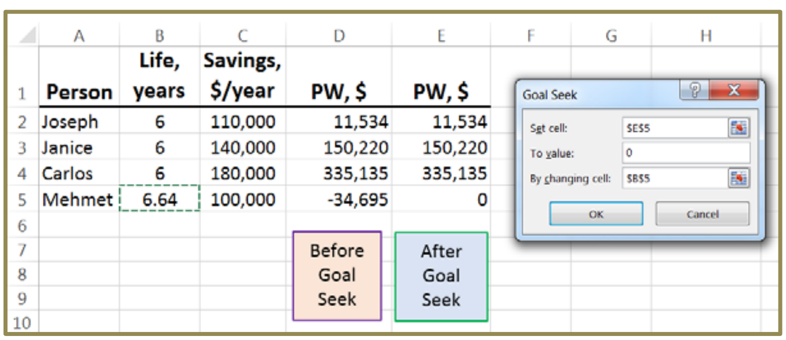
$PW_{Build,\ 63} = -80,000 - 63(1000) + 120,000(P/F,20\%,3)$
$= -143,000 + 120,000(0.5787)$
$= -\$73,556$

The lease option is less expensive if the building cost is \$70; lease is more expensive for the \$63 per m^2 build option.

(b) Functions: Lease: = - PV(20%,2,-30000) -30000 displays \$-75,833
Build, 70: = - PV(20%,3,,120000) - 80000 – 70000 displays \$-80,556
Build, 63: = - PV(20%,3,,120000) - 80000 – 63000 displays \$-73,556

18.7 (a) The first three estimates indicate that the equipment should be purchased; Mehmet's does not (PW = \$-34,695)

(b) Use Goal Seek to change the life (cell B5) from 6 to 6.64 years so that PW = 0 (cell E5)



	A	B	C	D	E
1	Person	Life, years	Savings, \$/year	PW, \$	PW, \$
2	Joseph	6	110,000	11,534	11,534
3	Janice	6	140,000	150,220	150,220
4	Carlos	6	180,000	335,135	335,135
5	Mehmet	6.64	100,000	-34,695	0
7–9				Before Goal Seek	After Goal Seek

18.8 (a) <u>*By hand:*</u>
$AW_1 = -10,000(A/P,i\%,8) - 600 - 100(A/F,i\%,8) - 1750(P/F,i\%,4)(A/P,i\%,8)$
$AW_2 = -17,000(A/P,i\%,12) - 150 - 300(A/F,i\%,12) - 3000(P/F,i\%,6)(A/P,i\%,12)$

Calculate AW at each MARR value. The decision is sensitive to MARR, changing at MARR = 6%.

MARR	AW_1	AW_2	Selection
4%	$-2318	$-2234	2
6%	$-2444	$-2448	1
8%	$-2573	$-2673	1

(b) *Spreadsheet:* The PMT functions are shown; AW values are the same as by hand.

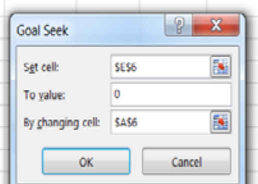
	A	B	C
1	MARR, %	AW_1, $ per year	AW_2, $ per year
2	0.04	= -PMT(A2,8,-10000,-100) - 600 - PMT(A2,8,-PV(A2,4,,-1750))	= -PMT(A2,12,-17000,-300) - 150 - PMT(A2,12,-PV(A2,6,,-3000))
3	0.06	= -PMT(A3,8,-10000,-100) - 600 - PMT(A3,8,-PV(A3,4,,-1750))	= -PMT(A3,12,-17000,-300) - 150 - PMT(A3,12,-PV(A3,6,,-3000))
4	0.08	= -PMT(A4,8,-10000,-100) - 600 - PMT(A4,8,-PV(A4,4,,-1750))	= -PMT(A4,12,-17000,-300) - 150 - PMT(A4,12,-PV(A4,6,,-3000))

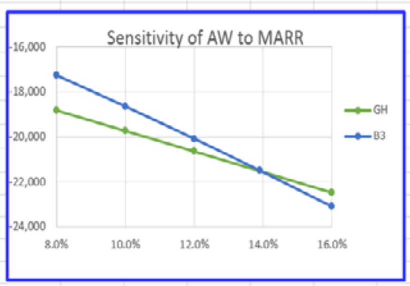
18.9 (a) AW relations are written for MARR values between 8% and 16%

$$AW_1 = -50{,}000(A/P,i,4) - 6{,}000 + 30{,}000(A/F,i,4) - 17{,}000(P/F,i,2)(A/P,i,4)$$
$$AW_2 = -100{,}000(A/P,i,12) - 1{,}500 - 30{,}000(P/F,i,6)(A/P,i,12)$$

(b) Selection changes between MARR values of 14% and 16%. Graph and Goal Seek determine the MARR breakeven point at 13.9% per year.

	A	B	C	D	E
1		AW, $ per year			
2	MARR, %	GH	B3	Selection	Difference
3	8.0%	-18,839	-17,278	B3	1,561
4	10.0%	-19,742	-18,662	B3	1,080
5	12.0%	-20,647	-20,097	B3	549
6	13.9%	-21,511	-21,511	GH	0
7	16.0%	-22,462	-23,111	GH	-648





18.10 $AW_{contract}$ = \$-165,000

$$
\begin{aligned}
AW_{high} &= -250{,}000(A/P,15\%,3) - 75{,}000 + 100{,}000(A/F,15\%,3) \\
&= -250{,}000(0.43798) - 75{,}000 + 100{,}000(0.28798) \\
&= \$-155{,}697 \qquad (< \$-165{,}000)
\end{aligned}
$$

$$
\begin{aligned}
AW_{low} &= -250{,}000(A/P,15\%,3) - 75{,}000 + 10{,}000(A/F,15\%,3) \\
&= -250{,}000(0.43798) - 75{,}000 + 10{,}000(0.28798) \\
&= \$-181{,}615 \qquad (> \$-165{,}000)
\end{aligned}
$$

Decision is sensitive to salvage value.

Total should estimate the salvage more closely before choosing between purchase and subcontractor.

18.11 Required AW < \$-6.1 million

$$
\begin{aligned}
7\%: AW &= -12{,}000{,}000(A/P,7\%,5) - 3{,}100{,}000 + 2{,}000{,}000(A/F,7\%,5) \\
&= -12{,}000{,}000(0.24389) - 3{,}100{,}000 + 2{,}000{,}000(0.17389) \\
&= \$-5{,}678{,}907 \qquad (< \$-6{,}100{,}000;\ \text{acceptable})
\end{aligned}
$$

$$
\begin{aligned}
10\%: AW &= -12{,}000{,}000(A/P,10\%,5) - 3{,}100{,}000 + 2{,}300{,}000(A/F,10\%,5) \\
&= -12{,}000{,}000(0.26380) - 3{,}100{,}000 + 2{,}300{,}000(0.16380) \\
&= \$-5{,}888{,}836 \qquad (< \$-6{,}100{,}000;\ \text{acceptable})
\end{aligned}
$$

$$
\begin{aligned}
15\%: AW &= -12{,}000{,}000(A/P,15\%,5) - 3{,}100{,}000 + 2{,}500{,}000(A/F,15\%,5) \\
&= -12{,}000{,}000(0.29832) - 3{,}100{,}000 + 2{,}500{,}000(0.14832) \\
&= \$-6{,}309{,}040 \qquad (> \$-6{,}100{,}000;\ \text{not acceptable})
\end{aligned}
$$

The decision is sensitive, since AW at 15% exceeds maximum of AW = \$-6,100,000

18.12 Start family now:

$$
\begin{aligned}
FW &= 50{,}000(F/A,10\%,5)(F/P,10\%,20) + 10{,}000(F/A,10\%,20) \\
&= 50{,}000(6.1051)(6.7275) + 10{,}000(57.2750) \\
&= \$2{,}626{,}353 \qquad (< \$3{,}000{,}000)
\end{aligned}
$$

Child 1 year from now:

$$
\begin{aligned}
FW &= 50{,}000(F/A,10\%,6)(F/P,10\%,19) + 10{,}000(F/A,10\%,19) \\
&= 50{,}000(7.7156)(6.1159) + 10{,}000(51.1591) \\
&= \$2{,}870{,}953 \qquad (< \$3{,}000{,}000)
\end{aligned}
$$

Child 2 years from now:

$$
\begin{aligned}
FW &= 50{,}000(F/A,10\%,7)(F/P,10\%,18) + 10{,}000(F/A,10\%,18) \\
&= 50{,}000(9.4872)(5.5599) + 10{,}000(45.5992) \\
&= \$3{,}093{,}386 \qquad (> \$3{,}000{,}000)
\end{aligned}
$$

Their retirement goal <u>is sensitive</u> to when they start a family; wait 2 (or more) years

18.13 (a) *By hand:*

$$
\begin{aligned}
AW_{cont} &= -140{,}000(A/P,15\%,5) - 31{,}000 + 25{,}000(A/F,15\%,5) \\
&= -140{,}000(0.29832) - 31{,}000 + 25{,}000(0.14832) \\
&= \$-69{,}057
\end{aligned}
$$

The lowest cost for batch will occur when its life is longest, 10 years

At n = 10:
$$
\begin{aligned}
AW_{batch} &= -80{,}000(A/P,15\%,10) - 52{,}000 + 10{,}000(A/F,15\%,10) \\
&= -80{,}000(0.19925) - 52{,}000 + 10{,}000(0.04925) \\
&= \$-67{,}448 \qquad (< \$-69{,}057;\text{ acceptable})
\end{aligned}
$$

Try n = 9:
$$
\begin{aligned}
AW_{batch} &= -80{,}000(A/P,15\%,9) - 52{,}000 + 10{,}000(A/F,15\%,9) \\
&= -80{,}000(0.20957) - 52{,}000 + 10{,}000(0.05957) \\
&= \$-68{,}170 \qquad (< \$-69{,}057;\text{ acceptable})
\end{aligned}
$$

Try n = 8:
$$
\begin{aligned}
AW_{batch} &= -80{,}000(A/P,15\%,8) - 52{,}000 + 10{,}000(A/F,15\%,8) \\
&= -80{,}000(0.22285) - 52{,}000 + 10{,}000(0.07285) \\
&= \$-69{,}100 \qquad (> \$-69{,}057;\text{ marginally not acceptable})
\end{aligned}
$$

The batch system will be less expensive than continuous flow if it lasts *at least 8 years*

(b) *Spreadsheet:* An expected life of slightly over 8 years is required to select the batch process.

	A	B	C
1		**Continuous**	**Batch**
2	**Life, years**	**AW, $/year**	**AW, $/year**
3	3		-84,158
4	4		-78,019
5	5	**-69,056**	-74,382
6	6		-71,997
7	7		-70,325
8	8		**-69,100**
9	9		**-68,170**
10	10		**-67,448**

18.14 P = first cost

$$
\begin{aligned}
PW &= -P + (60{,}000 - 5000)(P/A,10\%,5) \\
&= -P + 55{,}000(3.7908) \\
&= -P + 208{,}494
\end{aligned}
$$

Percent variation	P value, $	PW, $
-25	-150,000	58,494
-20	-160,000	48,494

-10	-180,000	28,494
0	-200,000	8,494
10	220,000	-11,506
20	240,000	-31,506
25	250,000	-41,506

Sensitive at +10% increase in first cost when PW goes negative

18.15 R = revenue

PW = -200,000 + R(P/A,10%,5) - 5000(P/A,10%,5)
= -200,000 + R(3.7908) - 5000(3.7908)
= -218,954 + 3.7908R

Percent variation	R value, $	PW, $
-25	45,000	-48,368
-20	48,000	-36,996
-10	54,000	-14,251
0	60,000	8,494
10	66,000	31,239
20	72,000	53,984
25	75,000	65,356

Sensitive at an R of -10% variation in revenue when PW goes negative

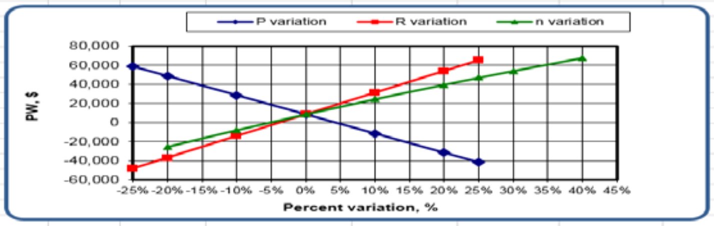
18.16 n = life PW = -200,000 + (60,000 - 5000)((P/A,10%,n)

Percent variation	n, years	PW, $
-20	4.0	-25,656
-10	4.5	-8,175
0	5.0	8,494
10	5.5	24,386
20	6.0	39,541
25	6.3	46,849
30	6.5	53,987
40	7.0	67,762

Sensitive at life variation of -10% when PW goes negative.

18.17 Spreadsheet and plot is for all three parameters: P, R and n. Variations in P and R have about the same effect on PW in opposite directions, and slightly more effect than variation in n.

	A	B	C	D	E	F	G	H	I	J
1					Current PW computation		PW values for sensitivity chart			
2		P value	R value	n value	Variation	PW	% variation	P variation	R variation	n variation
3		-150000	45000		-25%		-25%	58,493	-48,369	
4		-160000	48000	4.0	-20%	-25,657	-20%	48,493	-36,996	-25,657
5		-180000	54000	4.5	-10%	-8,175	-10%	28,493	-14,251	-8,175
6		-200000	60000	5.0	0%	8,493	0%	8,493	8,493	8,493
7		-220000	66000	5.5	10%	24,386	10%	-11,507	31,238	24,386
8		-240000	72000	6.0	20%	39,539	20%	-31,507	53,983	39,539
9		-250000	75000	6.3	25%	46,849	25%	-41,507	65,355	46,849
10				6.5	30%	53,987	30%			53,987
11				7.0	40%	67,763	40%			67,763



18.18 (a) PW calculates the amount you should be willing to pay now. Plot PW versus ±30% changes in (1), (2) and (3) on one graph.

(1) V = face value; r is 4% per 6-month period

$$PW = V(P/F,4\%,20) + 450(P/A,4\%,20)$$
$$= V(0.4564) + 6116$$

(2) b = dividend rate; r is 4% per 6-month period

$$PW = 10{,}000(P/F,4\%,20) + (10{,}000/2)(b)(P/A,4\%,20)$$
$$= 10{,}000(0.4564) + b(5000)(13.5903)$$
$$= 4564 + b(67{,}952)$$

(3) r = nominal rate per 6-month period

$$PW = 10{,}000(P/F,r,20) + 450(P/A,r,20)$$

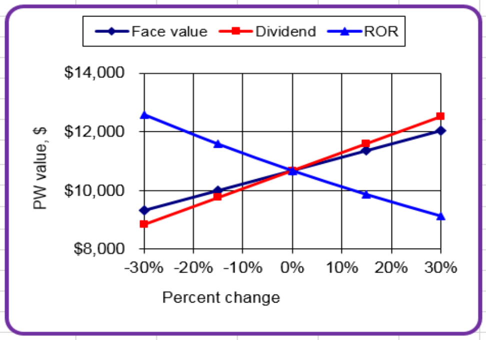
Estimates and changes

% change	Face value	Dividend	ROR
-30%	$ 7,000	$ 315.00	2.8%
-15%	$ 8,500	$ 382.50	3.4%
0%	$ 10,000	$ 450.00	4.0%
15%	$ 11,500	$ 517.50	4.6%
30%	$ 13,000	$ 585.00	5.2%

Present worth analysis*

% change	PW for Face value	PW for Dividend	PW for ROR
-30%	$ 9,310	$ 8,845	$ 12,577
-15%	$ 9,995	$ 9,762	$ 11,578
0%	$ 10,680	$ 10,680	$ 10,680
15%	$ 11,364	$ 11,597	$ 9,871
30%	$ 12,049	$ 12,514	$ 9,142

***PV function used. For example:**
in B17: = - PV(4%,20,450,7000)
in D21: = - PV(5.2%,20,450,10000)



(b) Amount paid is 10,000(1.05) = $10,500

For 0% change, PW = $10,680. Therefore, *$180 less* was paid than the investor was willing to pay to make a nominal 8% per year, compounded semiannually.

Three-Estimate Sensitivity Analysis

18.19 <u>Plan 1 - Lease</u>

<u>Opt: $0.40 per ton (AOC = $2000)</u>

AW = -60,000(A/P,12%,5) - 0.40(100)(50)
= -60,000(0.27741) - 2000
= $-18,645

<u>ML: $0.50 per ton (AOC = $2500)</u>

AW = -60,000(A/P,12%,5) - 0.50(100)(50)
= -60,000(0.27741) - 2,500
= $-19,145

<u>Pess: $0.95 per ton (AOC = $3750)</u>

AW = -60,000(A/P,12%,5) – 0.95(100)(50)
= -60,000(0.27741) – 3750
= $-21,395

<u>Plan 2 – Rental</u>

AW = -15,000 - 50(8)(15.00) = $-21,000 per year

Plan 1 (lease -- optimistic and most likely) are better than rental. However, the lease-pessimistic AW ($-21,395) is slightly higher than the rental option ($-21,000).

18.20 (a) <u>*By hand:*</u>

$$PW_{Pess} = -75{,}000 + 10{,}000(P/A,8\%,6) + 9000(P/F,8\%,6)$$
$$= -75{,}000 + 10{,}000(4.6229) + 9000(0.6302)$$
$$= \$-23{,}099$$

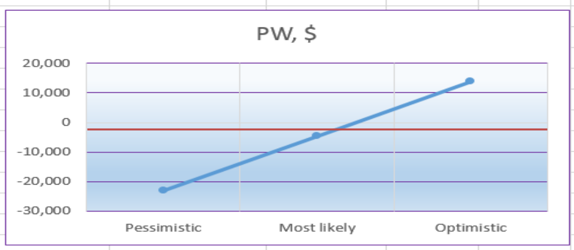
$$PW_{Ml} = -75{,}000 + 14{,}000(P/A,8\%,6) + 9000(P/F,8\%,6)$$
$$= -75{,}000 + 14{,}000(4.6229) + 9000(0.6302)$$
$$= \$-4608$$

$$PW_{Opt} = -75{,}000 + 18{,}000(P/A,8\%,6) + 9000(P/F,8\%,6)$$
$$= -75{,}000 + 18{,}000(4.6229) + 9000(0.6302)$$
$$= \$13{,}884$$

The equipment purchase is justified only under the optimistic estimate of revenue made by Tyler.

(b) <u>*Spreadsheet:*</u> Same results. Use the PV function to display the PW values.

Strategy	Saving, $/year	PW, $
Pessimistic	10,000	-23,100
Most likely	14,000	-4,608
Optimistic	18,000	13,883



18.21 $AW_{cont} = \$-155{,}000$

$$AW_{Opt} = -240{,}000(A/P,20\%,5) - 70{,}000 + 30{,}000(A/F,20\%,5)$$
$$= -240{,}000(0.33438) - 70{,}000 + 30{,}000(0.13438)$$
$$= \$-146{,}220 \quad (< \$-155{,}000;\ \text{purchase equipment})$$

$$AW_{ML} = -240{,}000(A/P,20\%,5) - 85{,}000 + 30{,}000(A/F,20\%,5)$$
$$= -240{,}000(0.33438) - 85{,}000 + 30{,}000(0.13438)$$
$$= \$-161{,}220 \quad (> \$-155{,}000;\ \text{do not purchase equipment})$$

$$AW_{Pess} = -240{,}000(A/P,20\%,5) - 120{,}000 + 30{,}000(A/F,20\%,5)$$
$$= -240{,}000(0.33438) - 120{,}000 + 30{,}000(0.13438)$$
$$= \$-196{,}220 \quad (> \$-155{,}000;\ \text{do not purchase equipment})$$

Optimistic estimate favors purchase; most likely and pessimistic estimates do not.

Function: = - PMT(20%,5,-240000,30000) – AOC_estimate will display the correct AW

18.22 PW_6 = -40,000 + 3500(P/A,10%,6) + 36,000(P/F,10%,6)
= -40,000 + 3500(4.3553) + 36,000(0.5645)
= $-4,434

PW_{10} = -40,000 + 3500(P/A,10%,10) + 49,000(P/F,10%,10)
= -40,000 + 3500(6.1446) + 49,000(0.3855)
= $396

PW_{15} = -40,000 + 3500(P/A,10%,15) + 55,000(P/F,10%,15)
= -40,000 + 3500(7.6061) + 55,000(0.2394)
= $-212

The PW is sensitive to the investment period; invest for 10 years

Function format: = - PV(10%,investment_years,3500,lump_sum) – 40000 displays PW

18.23 AW_{Opt} = -120,000(A/P,10%,10) - [10,000 + 1000(A/G,10%,10)] + 40,000
= -120,000(0.16275) - [10,000 + 1000(3.7255)] + 40,000
= $6745

AW_{ML} = -120,000(A/P,10%,10) - [10,000 + 3000(A/G,10%,10)] + 40,000
= -120,000(0.16275) - [10,000 + 3000(3.7255)] + 40,000
= $-706.50

AW_{Pess} = -120,000(A/P,10%,10) - [10,000 + 5000(A/G,10%,10)] + 40,000
= -120,000(0.16275) - [10,000 + 5000(3.7255)] + 40,000
= $-8158

The decision to expand the BIM is sensitive to gradient increases.

18.24 (a) Tabulated estimates

Location	Investment, $	Market value, $	NCF, $/year	Life, years	MARR, %
Miami					
Pess	-100,000	75,000	15,000	22	8
ML	-100,000	100,000	15,000	20	10
Opt	-100,000	150,000	15,000	16	15
Houston					
Pess	-110,000	82,500	19,000	22	8
ML	-110,000	110,000	19,000	20	10
Opt	-110,000	165,000	19,000	16	15

(b) Calculations use the PV function. Plots are for MARR, life and market values in table above.

MARR analysis

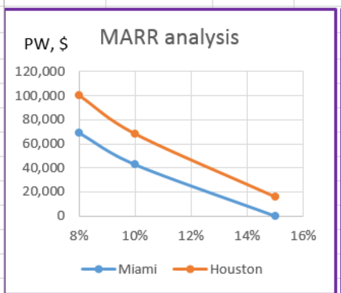
MARR	PW, $ Miami	PW, $ Houston
8%	68,727	100,145
10%	42,568	68,109
15%	0	15,648

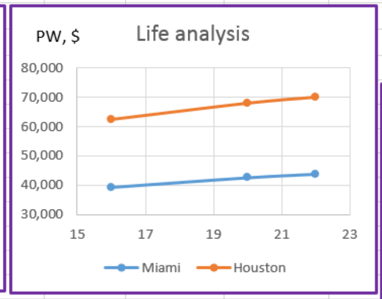
Life analysis

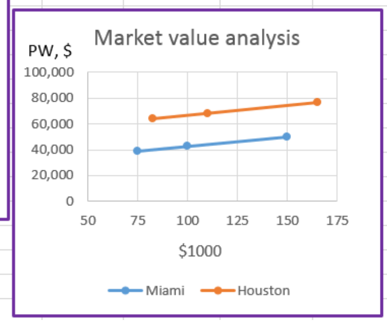
Life	PW, $ Miami	PW, $ Houston
22	43,858	70,172
20	42,568	68,109
16	39,119	62,590

Market value analysis

Value	PW, $ Miami	PW, $ Houston
75,000	38,852	
100,000	42,568	
150,000	50,000	
82,500		64,021
110,000		68,109
165,000		76,284







(c) Observing the PW values, Miami always has a lower PW value, so it is not acceptable; Houston is always the winner.

Expected Value

18.25 E(time) = (0.35)(10 + 20) + 0.15(30 + 70) = 25.5 seconds

18.26 E(X) = 18,000(0.35) + 24,000(0.41) + 29,000(0.13) + 0.11(-5,000)
= $19,360

18.27 $E(\text{flow}_N) = 0.15(100) + 0.75(200) + 0.10(300) = 195$ bbl/day

$E(\text{flow}_E) = 0.35(100) + 0.15(200) + 0.45(300) + 0.05(400) = 220$ bbl/day

18.28 E(FW) = 0.15(200,000 – 25,000) + 0.7(40,000) = $54,250

18.29

n	1	2	3	4
Y	3	9	27	81

E(Y) = 3(0.4) + 9(0.3) + 27(0.233) + 81(0.067) = 15.618

18.30 E(Income) = 1/12[500,000(4) + 600,000(2)+ 700,000(1) + 800,000(2) + 900,000(3)]
= 8,200,000/12
= $683,333

18.31 (a) The subscripts identify the series by probability.

$$\begin{aligned} PW_{0.5} &= -5000 + 1000(P/A,20\%,3) \\ &= -5000 + 1000(2.1065) \\ &= \$-2894 \end{aligned}$$

$$\begin{aligned} PW_{0.2} &= -6000 + 500(P/F,20\%,1) + 1500(P/F,20\%,2) + 2000(P/F,20\%,3) \\ &= -6000 + 500(0.8333) + 1500(0.6944) + 2000(0.5787) \\ &= \$-3384 \end{aligned}$$

$$\begin{aligned} PW_{0.3} &= -4000 + 3000(P/F,20\%,1) + 1200(P/F,20\%,2) - 800(P/F,20\%,3) \\ &= -4000 + 3000(0.8333) + 1200(0.6944) - 800(0.5787) \\ &= \$-1130 \end{aligned}$$

$$\begin{aligned} E(PW) &= (PW_{0.5})(0.5) + (PW_{0.2})(0.2) + (PW_{0.3})(0.3) \\ &= -2894(0.5) - 3384(0.2) - 1130(0.3) \\ &= \$-2463 \end{aligned}$$

(b) $$\begin{aligned} E(AW) &= E(PW)(A/P,20\%,3) \\ &= -2463(0.47473) \\ &= \$-1169 \end{aligned}$$

18.32 ***<u>Certificate of Deposit</u>***

Rate of return, i* = 2.35% (from problem statement)

<u>Stocks</u>

<u>Stock 1</u>: $-6000 + 250(P/A,i^*,4) + 6800(P/F,i^*,5) = 0$
$i^* = 5.80\%$

Function: = RATE(5,250,-6000,6550)

<u>Stock 2</u>: $-6000 + 600(P/A,i^*,4) + 4000(P/F,i^*,5) = 0$
$i^* = 1.61\%$

Function: = RATE(5,600,-6000,3400)

$E(i^*) = 5.80(0.5) + 1.61(0.5) = 3.71\%$

<u>Real Estate</u>

<u>Prob. = 0.3</u>

$-6,000 - 425(P/A,i^*,4) + 9500(P/F,i^*,5) = 0$
$i^* = 4.80\%$

Function: = RATE(5,-425,-6000,9925)

Prob. 0.5

$-6000 + 7200(P/F,i^*,5) = 0$
$(P/F,i^*,5) = 0.6944$
$i^* = 3.71\%$

Function: = RATE(5,,-6000,7200)

Prob. 0.2

$-6000 + 500(P/A,i^*,4) + 100(P/G,i^*,4) + 5200(P/F,i^*,5) = 0$
$i^* = 6.48\%$

Function: = IRR(B2:B7) for cash flows entered in cells B2 through B7

$E(i^*) = 4.80(0.3) + 3.71(0.5) + 6.48(0.2) = 4.59\%$

Invest in real estate for the highest E(i*) of 4.59% per year.

Decision Trees

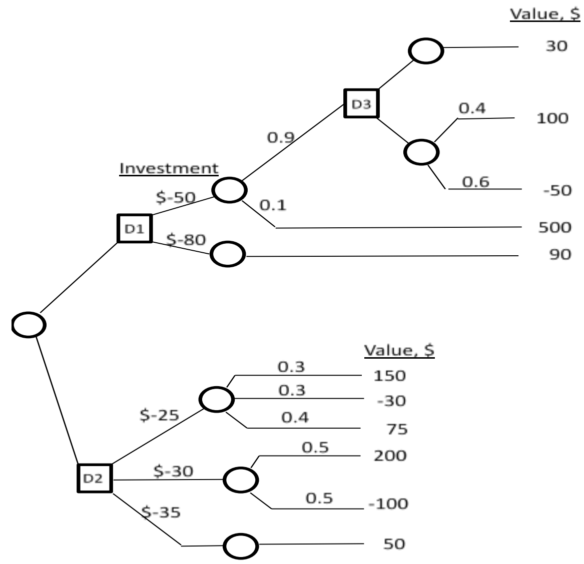
18.33 Compute the expected value for each outcome and select the maximum at D3.

Top node: 0.4(55) + 0.30(-30) + 0.30(-10) = 10.0

Bottom node: 0.6(-7) + 0.4(0) = -4.2

Indicate 10.0 and –4.2 in ovals and select the top branch with E(value) = 10.0

18.34 Maximize the value at each decision node



D3: Top: E(value) = \$30
Bottom: E(value) = 0.4(100) + 0.6(-50) = \$10

Select top at D3 for \$30

D1: Top: 0.9(D3 value) + 0.1(final value)
0.9(30) + 0.1(500) = \$77

At D1, value = E(value) - investment

Top: 77 - 50 = \$27 (larger)
Bottom: 90 - 80 = \$10

Select top at D1 for \$27

D2: Top: E(value) = 0.3(150 - 30) + 0.4(75) = \$66
Middle: E(value) = 0.5(200 - 100) = \$50
Bottom: E(value) = \$50

At D2, value = E(value) - investment

Top: 66 - 25 = \$41 (largest)
Middle: 50 - 30 = \$20
Bottom: 50 - 35 = \$15

Select top at D2 for \$41

Conclusion: Select D2 path and choose top branch ($25 investment)

18.35 Calculate the E(PW) in year 3 and select the largest expected value. In $1000 units,

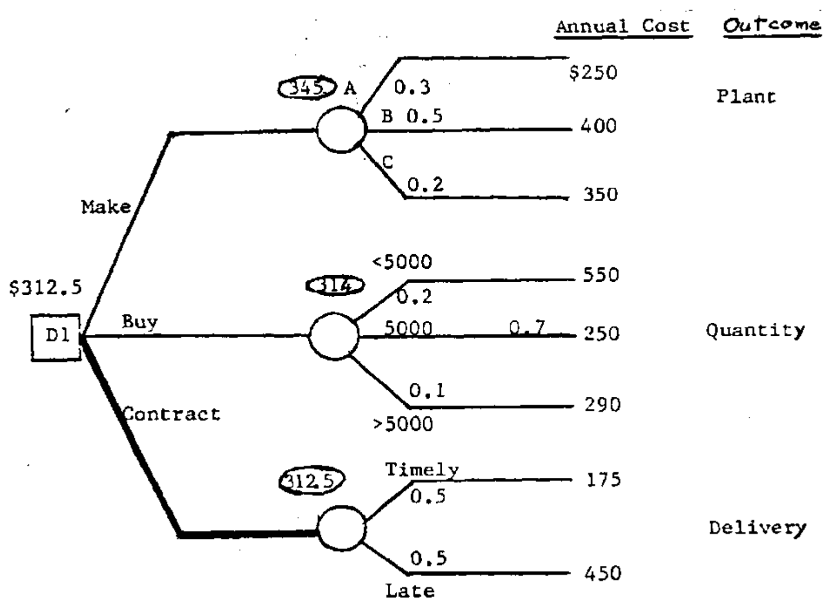
$$
\begin{aligned}
E(\text{PW of D6,X}) &= -200 + 0.7[50(P/A,15\%,3)] + 0.3[40(P/F,15\%,1) \\
&\quad +30(P/F,15\%,2) + 20(P/F,15\%,3)] \\
&= -98.903 \qquad (\$-98{,}903)
\end{aligned}
$$

$$
\begin{aligned}
E(\text{PW of D6,Y}) &= -75 + 0.45[30(P/A,15\%,3) + 10(P/G,15\%,3)] \\
&\quad +0.55[30(P/A,15\%,3)] \\
&= 2.816 \qquad (\$2816)
\end{aligned}
$$

$$
\begin{aligned}
E(\text{PW of D6,Z}) &= -250 + 0.7[190(P/A,15\%,3) - 20(P/G,15\%,3)] \\
&\quad + 0.3[-30(P/A,15\%,3)] \\
&= 4.120 \qquad (\$4{,}120)
\end{aligned}
$$

Select decision branch Z; it has the largest E(PW)

18.36 (a) Select the minimum E(cost) alternative. Monetary values are in $1000 units



Make: $\qquad E(\text{plant cost}) = 0.3(-250) + 0.5(-400) + 0.2(-350)$
$= \$-345 \qquad (\$-345{,}000)$

Buy: $\qquad E(\text{quantity cost}) = 0.2(-550) + 0.7(-250) + 0.1(-290)$
$= \$-314 \qquad (\$-314{,}000)$

Contract: $\qquad E(\text{delivery cost}) = 0.5(-175 - 450)$
$= \$-312.5 \qquad (\$-312{,}500)$

Select the <u>contract alternative</u> since the E(delivery cost) is the lowest at $-312,500

(b) Let ΔP = change in probability is placed in cell A2 of a spreadsheet.

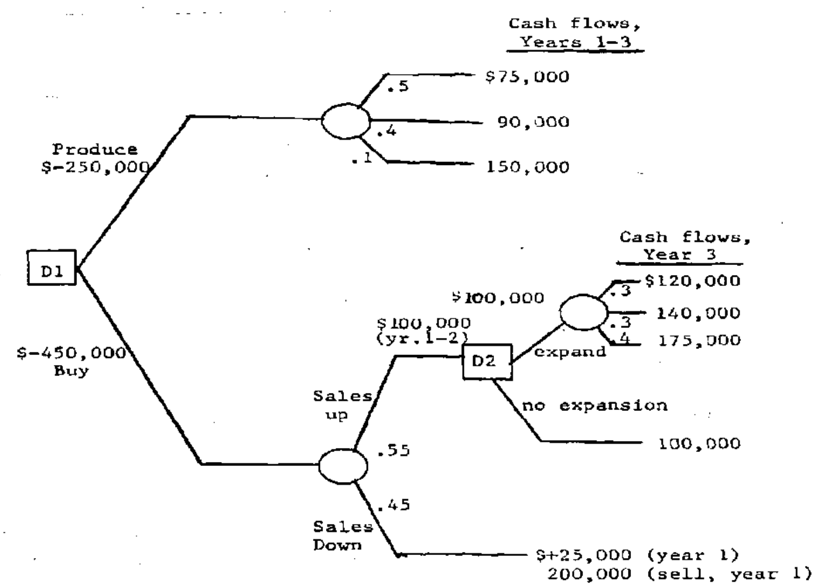
Function: = ((0.2 - 0.5*$A$2)*-550) + ((0.7+$A$2)*-250) + ((0.1 - 0.5*$A$2)*-290)

Use Goal Seek to set the function = $-312.5 and change cell A2. Display is 0.0088.

New probabilities are:

P(< 5000, pay premium) = 0.2 - 0.5(0.0088) = 0.1956
P(5000 available) = 0.7 + 0.0088 = 0.7088
P(> 5000, forced to buy) = 0.1 - 0.5(0.0088) = 0.0956

18.37 (a) Construct the decision tree



(b) At D2, compute PW of cash flows and E(PW) using probability values.

<u>Expansion option</u>

(PW for D2, $120,000) = –100,000 + 120,000(P/F,15%,1) = $4352
(PW for D2, $140,000) = –100,000 + 140,000(P/F,15%,1) = $21,744
(PW for D2, $175,000) = $52,180

E(PW) = 0.3(4352 + 21,744) + 0.4(52,180) = $28,700

<u>No expansion option</u>

(PW for D2, $100,000 = $100,000(P/F,15%,1) = $86,960

E(PW) = $86,960

Conclusion at D2: Select no expansion option

(c) Complete rollback to D1 considering 3 year cash flow estimates.

Produce option, D1

E(PW of cash flows) = [0.5(75,000) + 0.4(90,000) + 0.1(150,000](P/A,15%,3)
= $202,063

E(PW for produce) = cost + E(PW of cash flows)
= -250,000 + 202,063
= $-47,937

Buy option, D1

At D2, E(PW) = $86,960

E(PW for buy) = cost + E(PW of sales cash flows)
= -450,000 + 0.55(PW sales up) + 0.45(PW sales down)

PW Sales up = 100,000(P/A,15%,2) + 86,960(P/F,15%,2)
= $228,320

PW sales down = (25,000 + 200,000)(P/F,15%,1)
= $195,660

E(PW for buy) = -450,000 + 0.55 (228,320) + 0.45(195,660)
= $-236,377

Conclusion: E(PW for produce) is larger than E(PW for buy); select produce option.

Note: Both returns are less than 15%, but the return is larger for produce option.

(d) The return on the initial investment would increase, but it would increase faster for the produce option.

Real Options

18.38 In $ billion units,

$PW_{option} = 4.0 – 4.3(P/F,9\%,1)$
$= 4.0 – 4.3(0.9174)$
$= \$0.05518$ ($55.18 million)

18.39 In $ million units,

$PW_{Invest\ now} = -80 + [35(0.333) + 25(0.333) + 10(0.333)](P/A,12\%,5)$
$= -80 + [35(0.333) + 25(0.333) + 10(0.333)](3.6048)$
$= \$4.028$ ($4.028 million)

$PW_{Invest\ later} = -4 + 0.9(P/F,12\%,1) – 80(P/F,12\%,1) + [35(0.5) + 25(0.5)]$
$\times(P/A,12\%,4)(P/F,12\%,1)$
$= -4 + 0.9(0.8929) – 80(0.8929) + [35(0.5) + 25(0.5)](3.0373)(0.8929)$
$= \$6.732$ ($6.732 million)

Conclusion: Company should implement the pilot option and delay the full-scale decision for 1 year

18.40 $PW_{Now} = -1{,}800{,}000 + 1{,}000{,}000(0.75)(P/A,15\%,5)$
$= -1{,}800{,}000 + 1{,}000{,}000(0.75)(3.3522)$
$= \$714{,}150$

$PW_{1\ year} = -100{,}000 – 1{,}900{,}000(P/F,15\%,1) + 1{,}000{,}000(0.70)(P/A,15\%,5)(P/F,15\%,1)$
$= -100{,}000 – 1{,}900{,}000(0.8696) + 1{,}000{,}000(0.70)(3.3522)(0.8696)$
$= \$288{,}311$

Clearly, Dupont should purchase the license now

18.41 (a) Find E(PW) after determining $E(R_t)$, the expected repair costs for each year t

$E(R_2) = 1/3(-500 -1000 -0) = \$-500$

$E(R_3) = 1/3(-1200 -1400 -500) = \$-1033$

$E(R_4) = 1/3(-850 -400 -2000) = \$-1083$

$E(PW) = -500(P/F,8\%,2) -1033(P/F,8\%,3) -1083(P/F,8\%,4)$
$= -500(0.8573) -1033(0.7938) - 1083(0.7350)$
$= \$-2045$

Without considering noneconomic factors, the warranty is worth an expected \$2045, or \$455 less than the \$2500 extended warranty cost.

(b) PW_{base} = -500(P/F,8%,3) -2000(P/F,8%,4)
= -500(0.7938) -2000(0.7350)
= \$-1867

(c) i = 0%

(d) No, because all of the resulting E(PW) < \$2500

Additional Problems and FE Exam Review Questions

18.42 Answer is (c)

18.43 Answer is (b)

18.44 Answer is (a)

18.45 E(PW) = -10,000(0.25) + 40,000(0.4) + 50,000(0.35)
= \$31,000
Answer is (d)

18.46 Answer is (a)

18.47 Answer is (b)

18.48 E(Revenue) = 95(0.1) + 118(0.35) + 125(0.55)
= \$119.55 billion
Answer is (d)

18.49 83,000 = 45,000(0.2) + 72,000(0.5) + PW_{opt}(0.3)
PW_{opt} = [83,000 - 45,000(0.2) - 72,000(0.5)]/0.3
= \$126,667
Answer is (c)

18.50 Answer is (d)

18.51 Answer is (c)

18.52 Answer is (a)

18.53 Answer is (a)

Solution to Case Study 1, Chapter 18

Sometimes, there is not a definitive answer to a case study exercise. Here are example responses.

SENSITIVITY TO THE ECONOMIC ENVIRONMENT

1. Spreadsheet analysis used for changes in MARR. *PW is not very sensitive*; plan A is selected for all three MARR values.

	A	B	C	D
1	Plan A, NCF, \$	Plan B, NCF, \$		
2	-10,000	-35,000		
3	-500	-300		
4	-500	-300		
5	-500	-300		
22	-500	-5,500	Not all years shown	
23	-500	-300		
24	-500	-300		
40	-500	-300		
41	-500	-300		
42	500	4,500		
43	PW of A, \$	PW of B, \$	MARR	
44	-19,688	-42,311	4%	
45	-16,599	-40,023	7%	
46	-14,867	-38,601	10%	

2. Sensitivity to changes in life is performed by hand. *Not very sensitive*; plan A has the best PW for all life estimates.

Expanding economy

$$n_A = 40(0.80) = 32 \text{ years}$$
$$n_1 = 40(0.80) = 32 \text{ years}$$
$$n_2 = 20(0.80) = 16 \text{ years}$$

$$\begin{aligned}PW_A &= -10{,}000 + 1000(P/F,10\%,32) - 500(P/A,10\%,32)\\ &= -10{,}000 + 1{,}000(0.0474) - 500(9.5264)\\ &= \$-14{,}716\end{aligned}$$

$$\begin{aligned}PW_B &= -30{,}000 + 5000(P/F,10\%,32) - 100(P/A,10\%,32) - 5000\\ &\qquad -200(P/F,10\%,16) - 5000(P/F,10\%,16) - 200(P/F,10\%,32)\\ &\qquad -200(P/A,10\%,32)\\ &= -35{,}000 + 4800(P/F,10\%,32) - 300(P/A,10\%,32) - 5200(P/F,10\%,16)\\ &= -35{,}000 + 4800(0.0474) - 300(9.5264) - 5200(0.2176)\\ &= \$-38{,}762\end{aligned}$$

Expected economy

$$
\begin{aligned}
PW_A &= -10{,}000 + 1000(P/F,10\%,40) - 500(P/A,10\%,40) \\
&= -10{,}000 + 1000(0.0221) - 500(9.7791) \\
&= \$-14{,}867
\end{aligned}
$$

$$
\begin{aligned}
PW_B &= -30{,}000 + 5000(P/F,10\%,40) - 100(P/A,10\%,40) - 5000 \\
&\qquad - 200(P/F,10\%,20) - 5000(P/F,10\%,20) - 200(P/F,10\%,40) \\
&\qquad - 200(P/A,10\%,40) \\
&= -35{,}000 + 4800(P/F,10\%,40) - 300(P/A,10\%,40) - 5200(P/F,10\%,20) \\
&= -35{,}000 + 4800(0.0221) - 300(9.7791) - 5200(0.1486) \\
&= \$-38{,}600
\end{aligned}
$$

Receding economy

$$
\begin{aligned}
n_A &= 40(1.10) = 44 \text{ years} \\
n_1 &= 40(1.10) = 44 \text{ years} \\
n_2 &= 20(1.10) = 22 \text{ years}
\end{aligned}
$$

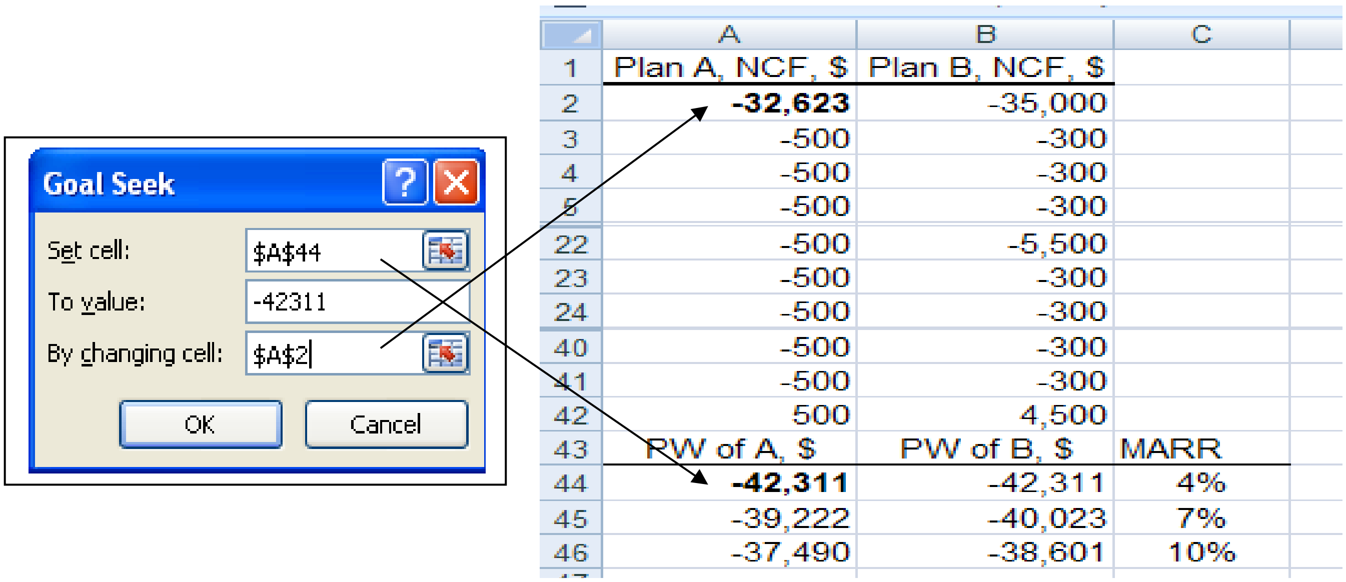
$$
\begin{aligned}
PW_A &= -10{,}000 + 1000(P/F,10\%,44) - 500(P/A,10\%,44) \\
&= -10{,}000 + 1000(0.0154) - 500(9.8461) \\
&= \$-14{,}908
\end{aligned}
$$

$$
\begin{aligned}
PW_B &= -30{,}000 + 5000(P/F,10\%,44) - 100(P/A,10\%,44) - 5000 \\
&\qquad - 200(P/F,10\%,22) - 5000(P/F,10\%,22) - 200(P/F,10\%,44) \\
&\qquad - 200(P/F,10\%,44) \\
&= -35{,}000 + 4800(P/F,10\%,44) - 300(P/A,10\%,44) - 5200(P/F,10\%,22) \\
&= -35{,}000 + 4800(0.0154) - 300(9.8461) - 5200(0.1228) \\
&= \$-38{,}519
\end{aligned}
$$

3. Use Goal Seek to find breakeven values of P_A for the three MARR values of 4%, 7%, and 10% per year. For MARR = 4%, the Goal Seek screen is below. Breakeven values are:

MARR, %	Breakeven P_A, $
4	–32,623
7	–33,424
10	–33,734

The P_A breakeven value is *not sensitive*, but all three outcomes are over 3X the $10,000 estimated first cost for plan A.



Solution to Case Study 2, Chapter 18

Sometimes, there is not a definitive answer to a case study exercise. Here are example responses.

SENSITIVITY ANALYSIS OF PUBLIC SECTOR PROJECTS -- WATER SUPPLY PLANS

1. Let x = weighting per factor

   Since there are 6 factors and one (environmental considerations) is to have a weighting that is double the others, its weighting is 2x. Thus,

$$2x + x + x + x + x + x = 100$$
$$7x = 100$$
$$x = 14.3\%$$

   Therefore, the environmental weighting is 2(14.3), or 28.6%

2.

Alt ID	Ability to Supply Area	Relative Cost	Engineering Feasibility	Institutional Issues	Environmental Considerations	Lead-Time Requirement	Total
1A	5(0.2)	4(0.2)	3(0.15)	4(0.15)	5(0.15)	3(0.15)	4.1
3	5(0.2)	4(0.2)	4(0.15)	3(0.15)	4(0.15)	3(0.15)	3.9
4	4(0.2)	4(0.2)	3(0.15)	3(0.15)	4(0.15)	3(0.15)	3.6
8	1(0.2)	2(0.2)	1(0.15)	1(0.15)	3(0.15)	4(0.15)	2.0
12	5(0.2)	5(0.2)	4(0.15)	1(0.15)	3(0.15)	1(0.15)	3.4

   The top three are the same as before: 1A, 3, and 4

3. For alternative 4 to be as economically attractive as alternative 3, its total annual cost would have to be the same as that of alternative 3, which is \$3,881,879. Thus, if P_4 is the capital investment,

$$3{,}881{,}879 = P_4(A/P, 8\%, 20) + 1{,}063{,}449$$
$$3{,}881{,}879 = P_4(0.10185) + 1{,}063{,}449$$
$$P_4 = \$27{,}672{,}361$$

$$\text{Decrease} = 29{,}000{,}000 - 27{,}672{,}361$$
$$= \$1{,}327{,}639 \text{ or } 4.58\%$$

4. Household cost at 100% = 3,952,959(1/12)(1/4980)(1/1)
= \$66.15

Decrease = 69.63 – 66.15
= \$3.48 or 5%

5. (a) Sensitivity analysis of M&O and number of households.

Alternative	Estimate	M&O, \$/year	Number of households	Total annual cost, \$/year	Household cost, \$/month
1A	Pessimistic	1,071,023	4980	3,963,563	69.82
	Most likely	1,060,419	5080	3,952,959	68.25
	Optimistic	1,049,815	5230	3,942,355	66.12
3	Pessimistic	910,475	4980	3,925,235	69.40
	Most likely	867,119	5080	3,881,879	67.03
	Optimistic	867,119	5230	3,881,879	**65.10**
4	Pessimistic	1,084,718	4980	4,038,368	71.13
	Most likely	1,063,449	5080	4,017,099	69.37
	Optimistic	957,104	5230	3,910,754	65.59

Conclusion: Alternative 3 - optimistic is the best.

(b) Let x be the number of households. Set alternative 4 - optimistic cost equal to \$65.10.

(3,910,754)/12(0.95)(x) = \$65.10
x = 5270

This is an increase of only 40 households.

Chapter 19
More on Variation and Decision Making under Risk

Certainty, Risk, and Uncertainty

19.1 (a) Discrete
(b) Discrete
(c) Continuous
(d) Continuous
(e) Discrete

19.2 (a) Continuous (assumed) and uncertain
(b) Discrete with risk
(c) Two variables: first is discrete and certain at \$800; second is continuous for ≥ \$800, but uncertain (at this point)
(d) Discrete with risk
(e) Discrete and certain

19.3 Needed or assumed information to calculate an expected value:
1. Treat output as discrete or continuous variable
2. If discrete, center points on cells, e.g., 8000, 15,000, and 18,000 units per week
3. Probability estimates for < 10,000 and /or > 20,000 units per week

Probability and Distributions

19.4 In \$ million units,

$$\begin{aligned} E(\text{damage}) &= 19(0.35) + 41(0.36) + 97(0.20) + 210(0.09) \\ &= \$59.71 \text{ million} \end{aligned}$$

19.5 Determine the probability values for C

C	0	1	2	3	4	≥ 5
P(C)	0.12	0.56	0.26	0.032	0 .022	0.006

(a) $P(C = 0 \text{ or } 1) = P(C=0) + P(C=1) = 0.12 + 0.56 = 0.68$ (68%)

(b) $P(C = 1 \text{ or } 2) = 0.56 + 0.26 = 0.82$ (82%)

(c) $P(C > 3) = P(C=4) + P(C\geq5) = 0.022 + 0.006= 0.028$ (2.8%)

19.6 (a) Discrete as shown

(b) $E(RI) = 6000(0.10) + 8500(0.21) + 9500(0.32) + 10{,}500(0.24) + 12{,}500(0.09) + 15{,}500(0.04)$

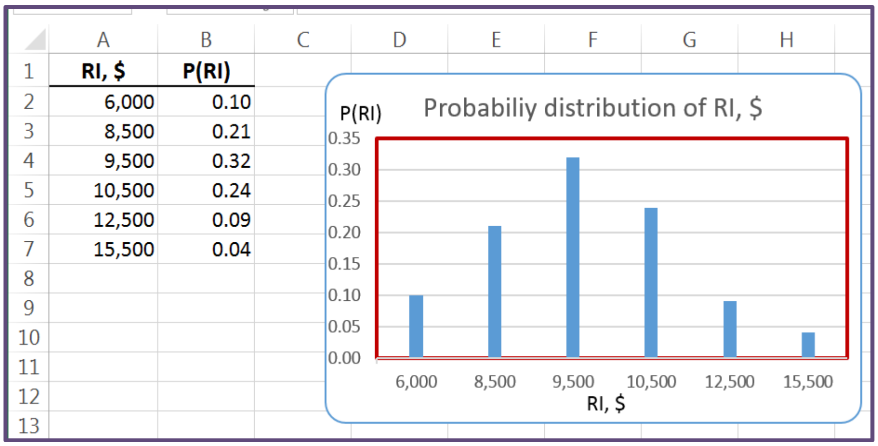
$= \$9690$

(c) $P(RI \geq 10{,}500) = P(RI = 10{,}500) + P(RI = 15{,}500)$

$= 0.24 + 0.09 + 0.04$

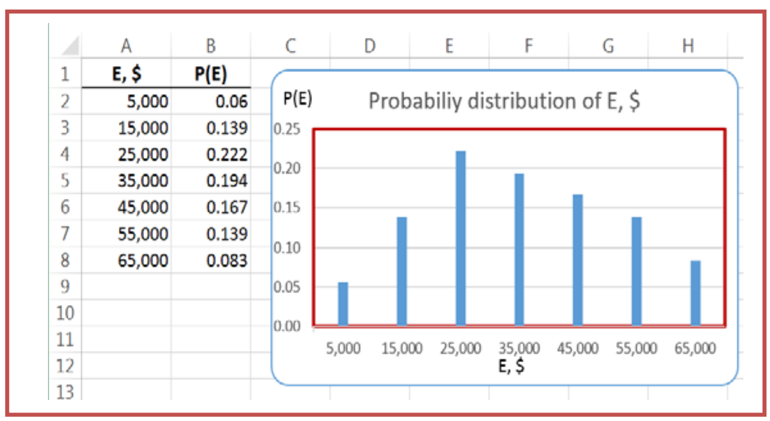
$= 0.37$ (37%)

(d) Plot shown for observed values of Royalty Income, RI



19.7 (a) Calculate probabilities and plot the distribution. Using a spreadsheet, the result is:

Expense range midpoint, E, \$1000	Number of months	Probability, P(E)
5	2	2/36 = 0.056
15	5	5/36 = 0.139
25	8	8/36 = 0.222
35	7	7/36 = 0.194
45	6	6/36 = 0.167
55	5	5/36 = 0.139
65	3	3/36 = 0.083
Total	36	1.000



(b) Can use months or probabilities; using probabilities

Let E_i = midpoint range i = 1, 2, …, 9. The \$45,000 midpoint includes \$40,000

$P(E > \$40{,}000) = 0.167 + 0.139 + 0.083 = 0.389$ (38.9%)

Using months, $P(E > \$40{,}000) = (6 + 5 + 3)/36 = 0.389$ (38.9%)

(c) From above table, $P(E = \$35{,}000) = 7/36 = 0.194$ (19.4%)

(d) Most frequently observed expenses is E = \$25,000

19.8 Use Equation [18.2] or [19.8] to find E(C)

Cell C_i, \$	$P(C_i)$	$C_i \times P(C_i)$, \$
600	0.06	36
800	0.10	80
1000	0.09	90
1200	0.15	180
1400	0.28	392
1600	0.15	240
1800	0.07	126
2000	0.10	200
	1.00	1344

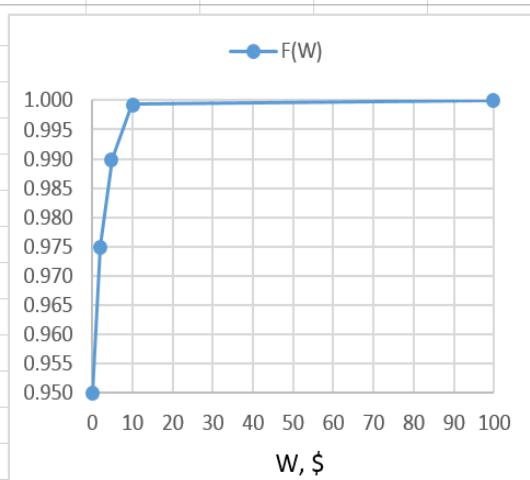
Sample expected value: E(C) = \$1344

19.9 (a) W is discrete; plot W vs. F(W)

W, \$	0	2	5	10	100
F(W)	0.950	0.975	0.990	0.9993	1.000

Spreadsheet plot of F(W) is below. Hand plot of F(W) will look like Figure 19-3(b)

W, \$	P(W)	F(W)
0	0.950	0.950
2	0.025	0.975
5	0.015	0.990
10	0.0093	0.999
100	0.0007	1.000
	1.000	



(b) $E(W) = 0.95(0) + ... + 0.0007(100)$
$= 0 + 0.05 + 0.075 + 0.093 + 0.07$
$= \$0.288$ per ticket

(c) $2.000 - 0.288 = \$1.712$
Long-term income for the state is \$1.71 per ticket

19.10 (a) $P(N) = (0.5)^N$ $N = 1, 2, 3, ...$

N	1	2	3	4	5	etc.
P(N)	0.5	0.25	0.125	0.0625	0.03125	
F(N)	0.5	0.75	0.875	0.9375	0.96875	

Plot P(N) and F(N); N is discrete.

P(L) is triangular like the distribution in Figure 19-5 with the mode at 5.

$$f(\text{mode}) = f(M) = \frac{2}{5-2} = \frac{2}{3}$$

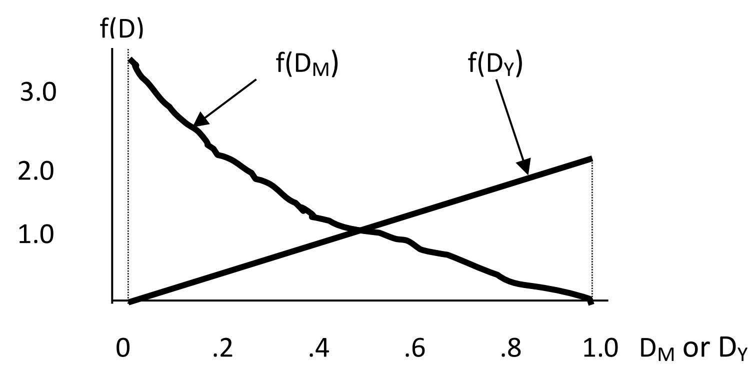
$$F(\text{mode}) = F(M) = \frac{5-2}{5-2} = 1$$

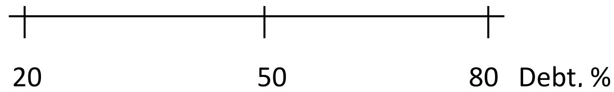
(b) $P(N = 1, 2 \text{ or } 3) = F(N \le 3) = 0.875$

19.11 (a) Determine several values of D_M and D_Y and plot.

D_M or D_Y	$f(D_M)$	$f(D_Y)$
0.0	3.00	0.0
0.2	1.92	0.4
0.4	1.08	0.8
0.6	0.48	1.2
0.8	0.12	1.6
1.0	0.00	2.0

$f(D_M)$ is a decreasing power curve and $f(D_Y)$ is increasing linear.





(b) Probability is larger that M (mature) companies have a lower debt percentage and that Y (young) companies have a higher debt percentage.

19.12 (a)

X_i	1	2	3	6	9	10
$F(X_i)$	0.2	0.4	0.6	0.7	0.9	1.0

(b) (1) $P(6 \le X \le 10) = F(10) – F(3) = 1.0 – 0.6 = 0.4$
or
$P(X = 6, 9 \text{ or } 10) = 0.1 + 0.2 + 0.1 = 0.4$

(2) $P(X = 4, 5 \text{ or } 6) = F(6) – F(3) = 0.7 – 0.6 = 0.1$
or
$P(X = 4, 5 \text{ or } 6) = P(X = 6) = 0.1$

(c) $P(X = 7 \text{ or } 8) = F(8) – F(6) = 0.7 – 0.7 = 0.0$

No sample values in the 50 have X = 7 or 8. A larger sample is needed to observe all values of X.

Random Samples

19.13 (a) Let p = probability such the 5p plus 1/2p equals 1.0

$$5p + 0.5p = 1$$
$$p = 1/5.5$$
$$= 0.18182$$

In $ million units for R, the probability statements are:

$P(R=2.6) = P(R=2.8) = \ldots\ldots = P(R=3.6) =0.18182$
$P(R=3.6) = 0.09091$

(b) $E(R) = 0.18182(2.6 + 2.8 + 3.0 + 3.2 + 3.4) + 0.09091(3.6)$
$= 2.72730 + 0.32728$
$= \$3.05458$ ($3.054 million)

19.14 The probability of occurrence of each situation is as follows:

All gas = 12/24 = 0.500
< 30% other/wind = 9/24 = 0.375

$\geq$ 30% other/wind = 3/24 = 0.125

E(R) = 5,270,000(0.50) + 7,850,000(0.375) + 12,130,000(0.125)
= $7,095,000

Difference = revenue – costs
= 7,095,000 - 6,800,000
= $295,000 greater than expenses

19.15 (a) Sample size is n = 40

Variable value	1	2	3	4	5
Assigned Numbers	0 -19	20 – 49	50 – 59	60 – 89	90 – 99
Times in sample	6	13	2	15	4
Sample probability	0.150	0.325	0.050	0.375	0.100

(b) P(T=2) = 0.325 Stated P(T = 2) = 0.30 (close)
P(T=5) = 0.100 Stated P(T = 5) = 0.10 (exactly the same)

19.16 (a) Function: = - PV(2%,5,-10000) – 800000 displays PW = $-847,135

(b) Expected value computations: E(P) = $886,000 and E(M&O) = $7,280 per year

Function: = - PV(2%,5,-7280) – 886000 displays $-920,314

PW is more costly by $73,179

	A	B	C	D	E	F	G
1	**First Cost Distribution**				**Maintenance Cost Distribution**		
2	**P, $1000**	**Frequency, Freq**	**P × Freq, $1000**		**M&O, $1000**	**Frequency, Freq**	**M&O × Freq, $1000**
3	550	1	550		2	3	6
4	650	6	3,900		6	15	90
5	750	3	2,250		10	3	30
6	850	4	3,400		14	4	56
7	1,050	8	8,400				182
8	1,150	2	2,300		**E(M&O), $1000**		**7.28**
9	1,350	1	1,350				
10		25	22,150				
11	**E(P), $1000**		**886.0**				
12	**PW at 2%**	**-$920,314**					

(c) Function: = - PV(2%,5,180000) – 800000 displays $48,423.

It is economically justified; but will likely be objected to by the driving public given the bridge's age, condition, and no significant improvements on visible parts of the bridge.

19.17 (a)

X	0	0.2	0.4	0.6	0.8	1.0
F(X)	0	0.04	0.16	0.36	0.64	1.00

Take X and p values from the graph. Some samples are:

RN	X	p, %
18	0.42	7.10
59	0.76	8.80
31	0.57	7.85
29	0.52	7.60

(b) Use the sample mean for the average p value. Our sample of 30 had p = 6.34%; yours will vary depending on the RNs from Table 19.2.

19.18 (a) When the RAND( ) function was used for 100 values in column A of a spreadsheet, the function = AVERAGE(A1:A100) resulted in 0.50750658; very close to 0.5.

(b) For the RAND results, count the number of values in each cell to determine how close it is to 10.

Sample Estimates – Average and Standard Deviation

19.19 (a) Mean = (452 + 364 + 415 +………+ 380)/11
= 411 mg/L

(b) Arrange values in increasing order and select middle value (i.e. 6th one)
Arranged values: 364,380,391,395,395,**404**,415,425,430,452,470

Median = 404 mg/L

(c) For mode, select value which occurs most frequently

Mode = 395 mg/L

(d) *By hand*:

COD	Mean, $\overline{X}$	$X_i - \overline{X}$	$(X_i - \overline{X})^2$
452	411	41	1681
364	411	-47	2209
415	411	4	16
395	411	-16	256
404	411	-7	49
470	411	59	3481
391	411	-20	400

395	411	-16	256
425	411	14	196
430	411	19	361
380	411	-31	961
4521		0	9866

$$s = \sqrt{9866/(11-1)}$$
$$= 31.4 \text{ mg/L}$$

Spreadsheet:

	A	B
1	**COD**	
2	452	
3	364	
4	415	
5	395	
6	404	
7	470	
8	391	
9	395	
10	425	
11	430	
12	380	
13	Mean	411.00
14	Std. Dev.	31.41

19.20 *By hand:*

(a) $\bar{X} = (108+99+84+93+80+90+83+83+96+85+89)/11 = 90$ ppb

(b)

Reading	Mean, $\bar{X}$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
108	90	18	324
99	90	9	81
84	90	-6	36
93	90	3	9
80	90	-10	100
90	90	0	0
83	90	-7	49
83	90	-7	49
96	90	6	36
85	90	-5	25
89	90	-1	1
990		0	710

$s = \sqrt{710/(11-1)} = 8.43$ ppb

(c) Range for $\pm 1s$ is $90 \pm 8.43 = 81.57 – 98.43$

Number of values in range = 8
% of values in range = 8/11 = 72.7%

<u>*Spreadsheet*</u>*:* Values entered into cells A1:A11

(a) Function: = AVERAGE(A1:A11) displays 90.0

(b) Function: = STDEV(A1:A11) displays 8.43

19.21 Use Equations [19.9] and [19.12].

Cell, X_i	f_i	X_i^2	f_iX_i	$f_iX_i^2$
200	4	40,000	800	160,000
400	8	160,000	3,200	1,280,000
600	9	360,000	5,400	3,240,000
800	14	640,000	11,200	8,960,000
1000	18	1,000,000	18,000	18,000,000
1200	25	1,440,000	30,000	36,000,000
1400	12	1,960,000	16,800	23,520,000
1600	10	2,560,000	16,000	25,600,000
	100		101,400	116,760,000

Sample mean: $\overline{X} = 101{,}400/100 = 1014.00$

Std. deviation: $s = \left[\frac{116{,}760{,}000}{99} - \frac{100}{99}(1014)^2 \right]^{1/2}$

$= (140{,}812.12)^{1/2}$
$= 375.25$

(b) $\overline{X} \pm 1s$ is $1014.00 \pm 375.25 = 638.75$ and 1,389.25

Number of values within $\pm$ 1s = 14+18+25 = 57
Percentage = (57/100)(100%) = 57%

19.22 (a) Convert P(Q) data into frequency values to determine s.

Q	P(Q)	QP(Q)	f	Q^2	fQ^2
1	0.2	0 .2	20	1	20
2	0.2	0 .4	20	4	80
3	0.2	0 .6	20	9	180
6	0.1	0 .6	10	36	360
9	0.2	1.8	20	81	1620
10	0.1	1.0	10	100	1000
		4.6			3260

Sample average: $\overline{Q} = 4.6$ units

Sample variance: $s^2 = \frac{3260}{99} - \frac{100}{99}(4.6)^2 = 11.56 \text{ units}^2$

Std. deviation $s = (11.56)^{0.5} = 3.40$ units

(b) Average and standard deviation values are shown.

	A	B	C	D	E	F	G	H
1	Q	P(Q)	P(Q)×Q	f(Q)	f(Q)×Q^2			
2	1	0.2	0.2	20	20			
3	2	0.2	0.4	20	80			
4	3	0.2	0.6	20	180			
5	4	0	0	0	0			
6	5	0	0	0	0			
7	6	0.1	0.6	10	360			
8	7	0	0	0	0			
9	8	0	0	0	0			
10	9	0.2	1.8	20	1620			
11	10	0.1	1	10	1000			
12	Average		4.6	100	3260			
13	Std. Dev.				3.40			
14								
15	Std. dev. function				=((E12/99)-(100/99)*(C12^2))^0.5			

(c) $\overline{Q} \pm 1s$ is $4.6 \pm 3.40 = 1.20$ and 8.00
50 values, or 50%, are in this range.

$\overline{Q} \pm 2s$ is $4.6 \pm 6.8 = -2.20$ and 11.40
All 100 values, or 100%, are in this range.

19.23 (a) Use Equations [19.15] and [19.16]. Substitute Y for D_Y.

$$f(Y) = 2Y$$

$$E(Y) = \int_0^1 (Y)2Ydy$$

$$= \left[\frac{2Y^3}{3}\right]_0^1$$

$$= 2/3 - 0 = 2/3$$

$$Var(Y) = \int_0^1 (Y^2)2Ydy - [E(Y)]^2$$

$$= \left[\frac{2Y^4}{4}\right]_0^1 - (2/3)^2$$

$$= \frac{2}{4} - 0 - \frac{4}{9}$$

$$= 1/18 = 0.05556$$

$$\sigma_Y = (0.05556)^{0.5} = 0.236$$

(b) $E(Y) \pm 2\sigma$ is $0.667 \pm 0.472 = 0.195$ and 1.139

Take the integral from 0.195 to 1.0 since the variable's upper limit is 1.0.

$$P(0.195 \le Y \le 1.0) = \int_{0.195}^{1} 2Ydy$$

$$= Y^2 \Big|_{0.195}^{1}$$

$$= 1 - 0.038 = 0.962 \qquad (96.2\%)$$

19.24 Use Equation [19.8] where $P(N) = (0.5)^N$

E(N) = 1(0.5) + 2(0.25) + 3(0.125) + 4(0.0625) + 5(.03125) + 6(.015625) + 7(.0078125)
+ 8(.003906) + 9(.001953) + 10(.0009766) + …
= 1.99+

E(N) can be calculated for as many N values as you wish. The limit to the series $N(0.5)^N$ is 2.0, the correct answer.

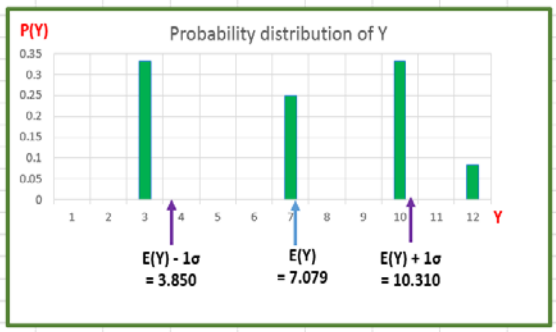
19.25 $E(Y) = 3(1/3) + 7(1/4) + 10(1/3) + 12(1/12)$
$= 1 + 1.75 + 3.333 + 1$
$= 7.083$

$Var(Y) = \sum Y^2P(Y) - [E(Y)]^2$
$= 3^2(1/3) + 7^2(1/4) + 10^2(1/3) + 12^2(1/12) - (7.083)^2$
$= 60.583 - 50.169$
$= 10.414$

$\sigma(Y) = [VAR(Y)]^{0.5} = 3.227$

$E(Y) \pm 1\sigma$ is $7.083 \pm 3.227 = 3.856$ and 10.310

	A	B	C	D
1	Y	P(Y)	Y × P(Y)	Y^2 × P(Y)
2	1	0	0	0
3	2	0	0	0
4	3	0.333	0.999	2.997
5	4	0	0	0
6	5	0	0	0
7	6	0	0	0
8	7	0.250	1.750	12.250
9	8	0	0	0
10	9	0	0	0
11	10	0.333	3.330	33.300
12	11	0	0	0
13	12	0.083	1.000	12.000
14	E(Y)		7.079	60.547
15				
16	VAR(Y)			10.435
17	σ(Y)			3.230



Simulation

19.26 Using a spreadsheet, the steps in Sec. 19.5 are applied.

1. CFAT given for years 0 through 6.
2. i varies between 6% and 10%.
   CFAT for years 7-10 varies between $1,600,000 and $2,400,000, written in $1000.
3. Uniform for both i and CFAT values.
4. Set up a spreadsheet. The example below has the following relations:

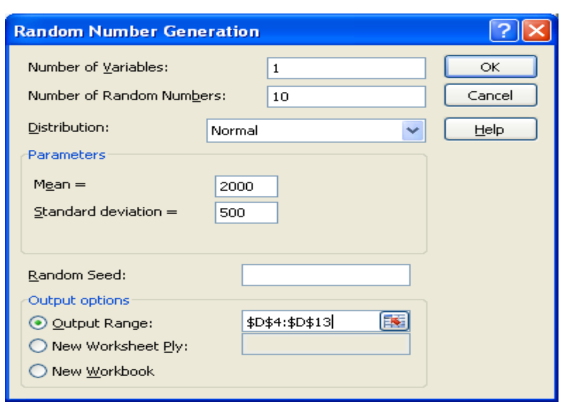
   Col A: = RAND ( )* 100 to generate random numbers from 0-100.
   Col B, cell B13: = INT((.04*A13+6) *100)/10000 converts the RN to i between 0.06 and 0.10. The % designation changes it to an interest rate between 6% and 10%.
   Col C: = RAND( )* 100 to generate random numbers from 0-100.
   Col D, cell D13: = INT (8*C13+1600) converts RN to a CFAT between $1600 and $2400.

Ten samples of i and CFAT for years 7-10 are below in columns B and D, respectively (highlighted).

	A	B	C	D	E	F	G	H
1						Annual CFAT	Annual CFAT	Annual CFAT
2			RN for	CFAT, $1000		using D4 for CFAT	using D5 for CFAT	using D6 for CFAT
3	RN for i	i	CFAT	years 7-10	Year	and B4 for MARR	and B5 for MARR	and B6 for MARR
4	9.752001	6.39%	4.617578	$ 1,636	0	-31,000	-31,000	-31,000
5	64.36685	8.57%	71.35531	$ 2,170	1	5,400	5,400	5,400
6	17.33263	6.69%	49.11906	$ 1,992	2	5,400	5,400	5,400
7	45.31801	7.81%	75.98175	$ 2,207	3	5,400	5,400	5,400
8	11.69914	6.46%	59.63702	$ 2,077	4	5,400	5,400	5,400
9	28.05337	7.12%	25.38174	$ 1,803	5	5,400	5,400	5,400
10	87.81461	9.51%	37.62056	$ 1,900	6	5,400	5,400	5,400
11	14.09394	6.56%	19.54264	$ 1,756	7	1,636	2,170	1,992
12	77.92364	9.11%	59.16107	$ 2,073	8	1,636	2,170	1,992
13	15.63792	6.62%	33.33903	$ 1,866	9	1,636	2,170	1,992
14					10	4,436	4,970	4,792
15	PW of CFAT, $1000					**613**	**-899**	**1,060**
16								
17	= INT((0.04*A13+6)*100)/10000		= INT(8*C13+1600)				= NPV($B$6,H5:H14)+H4	
18								

5. Columns F, G and H give 3 CFAT sequences, for example only, using rows 4, 5 and 6 RN generations. The entry for cells F11 through F13 is = D4 and cell F14 is = D4+2800, where S = $2800. The PW values are obtained using the NPV function.
6. Plot the PW values for as large a sample as desired. Or, following the logic of Figure 19-14, a spreadsheet relation can count the + and – PW values, with average and standard deviation calculated for the sample.
7. **Conclusion:**
   For certainty, accept the plan since PW = $767 exceeds zero at 7% per year.
   For risk, the result depends on the preponderance of positive PW values from the simulation, and the distribution of PW obtained in step 6.

19.27 Use the spreadsheet Random Number Generator (RNG) on the tools toolbar to generate CFAT values in column D from a normal distribution with μ = $2000 and σ = $500. The RNG screen image is shown below.



	A	B	C	D	E	F	G	H
1				RN from		Annual CFAT	Annual CFAT	Annual CFAT
2				Normal		using D4 for CFAT	using D5 for CFAT	using D6 for CFAT
3	RN for i	i		using RNG	Year	and B4 for MARR	and B5 for MARR	and B6 for MARR
4	16.0222366	6.64%		2376	0	-31,000	-31,000	-31,000
5	40.1214297	7.60%		1643	1	5,400	5,400	5,400
6	96.4793859	9.85%		2703	2	5,400	5,400	5,400
7	49.1846546	7.96%		2267	3	5,400	5,400	5,400
8	15.7990096	6.63%		1584	4	5,400	5,400	5,400
9	45.9941098	7.83%		2187	5	5,400	5,400	5,400
10	51.2164874	8.04%		2035	6	5,400	5,400	5,400
11	39.0480889	7.56%		2179	7	2,376	1,643	2,703
12	90.1333191	9.60%		1812	8	2,376	1,643	2,703
13	10.1544644	6.40%		2094	9	2,376	1,643	2,703
14					10	5,176	4,443	5,503
15	PW of CFAT, $1000					2,016	-846	-1,391

The spreadsheet above is the same form as that in Problem 19.26, except that CFAT values in column D for years 7 through 10 are generated using the RNG for the normal distribution as described above. The decision to accept the plan uses the same logic as that described in Problem 19.26.

Additional Problems and FE Exam Review Questions

19.28 Answer is (c)

19.29 Answer is (a)

19.30 $E(\$) = 0.22(34) + 0.31(38) + 0.47(55)$
$= 7.48 + 11.78 + 25.85$
$= \$45.11$
Answer is (d)

19.31 Answer is (d)

19.32

Reading	Mean, $\overline{X}$	$X_i - \overline{X}$	$(X_i - \overline{X})^2$
99	93	6	36
87	93	-6	36
93	93	0	0
90	93	-3	9
96	93	3	9
465		0	90

$s^2 = 90/(5 - 1)$
$= 22.5$
Answer is (b)

19.33 P(Income > \$8500) = 0.32 + 0.24 + 0.09 + 0.04
= 0.69
Answer is (c)

19.34 $s = \sqrt{4{,}680{,}000/(12} -1)$
= \$652.3
Answer is (b)

19.35 Four numbers (52, 67, 74, and 50) are in the range 50 through 74, which indicate type C.

P(Type C) = 4/12 = 0.33
Answer is (c)

19.36 Answer is (d)

Solution to Case Study, Chapter 19

USING SIMULATION AND THREE-ESTIMATE SENSITIVITY ANALYSIS

This simulation is left to the learner. The 7-step procedure from Section 19.5 can be applied here. Set up the RNG for the cash flow values of AOC, S, and n for each alternative. For each sample cash flow series, calculate the AW value for each alternative. To obtain a final answer of which alternative is the best, it is recommended that the number of positive and negative AW values be counted as they are generated. Then the alternative with the most positive AW values indicates which one to accept. Of course, due to the RNG generation of AOC, S and n values, this decision may vary from one simulation run to the next.