Surveying 1 / Dr. Najeh Tamim

CHAPTER 5

ANGLES, DIRECTIONS, AND ANGLE MEASURING EQUIPMENT

HORIZONTAL ANGLES

The horizontal angle between two lines intersecting in space is the angle measured between the projection of these two lines on a horizontal plane.
(In the figure below, the horizontal angle between AB & BC is angle A'B'C')

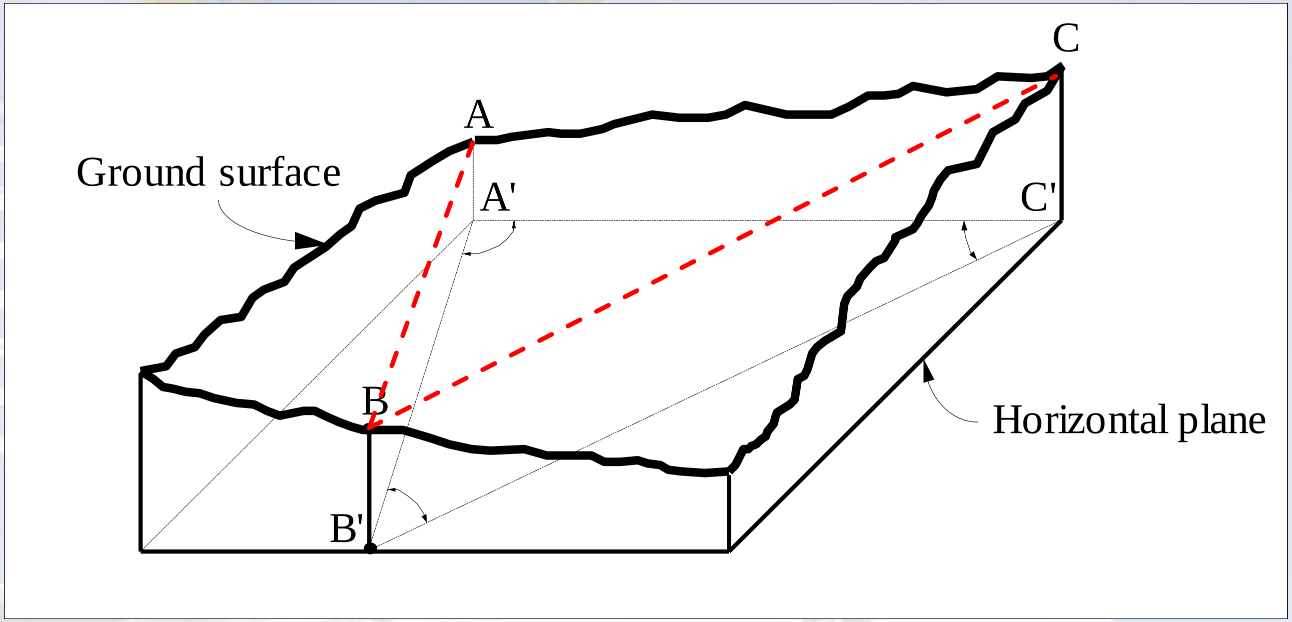


FIGURE 5.1: Horizontal angles.

VERTICAL AND ZENITH ANGLES

Vertical Angle of a line: The angle measured up (angle of rise) or down (angle of depression) from the horizontal line.

Zenith angle of a line: The angle measured from the zenith direction to the line (ranges from 0° to 180 °)

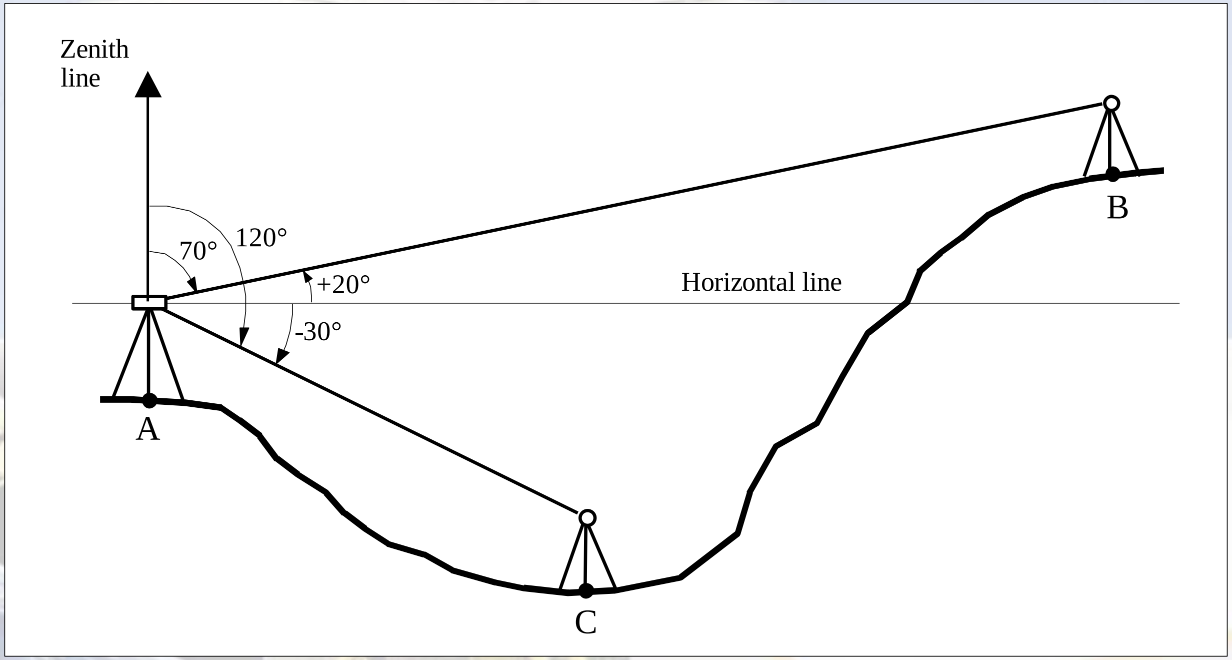


FIGURE 5.2: Vertical and zenith angles.

REFERENCE DIRECTION

1) TRUE OR GEOGRAPHIC NORTH: The direction towards the north pole. It lies on the meridian (great circle) passing through the point, the north and south poles).

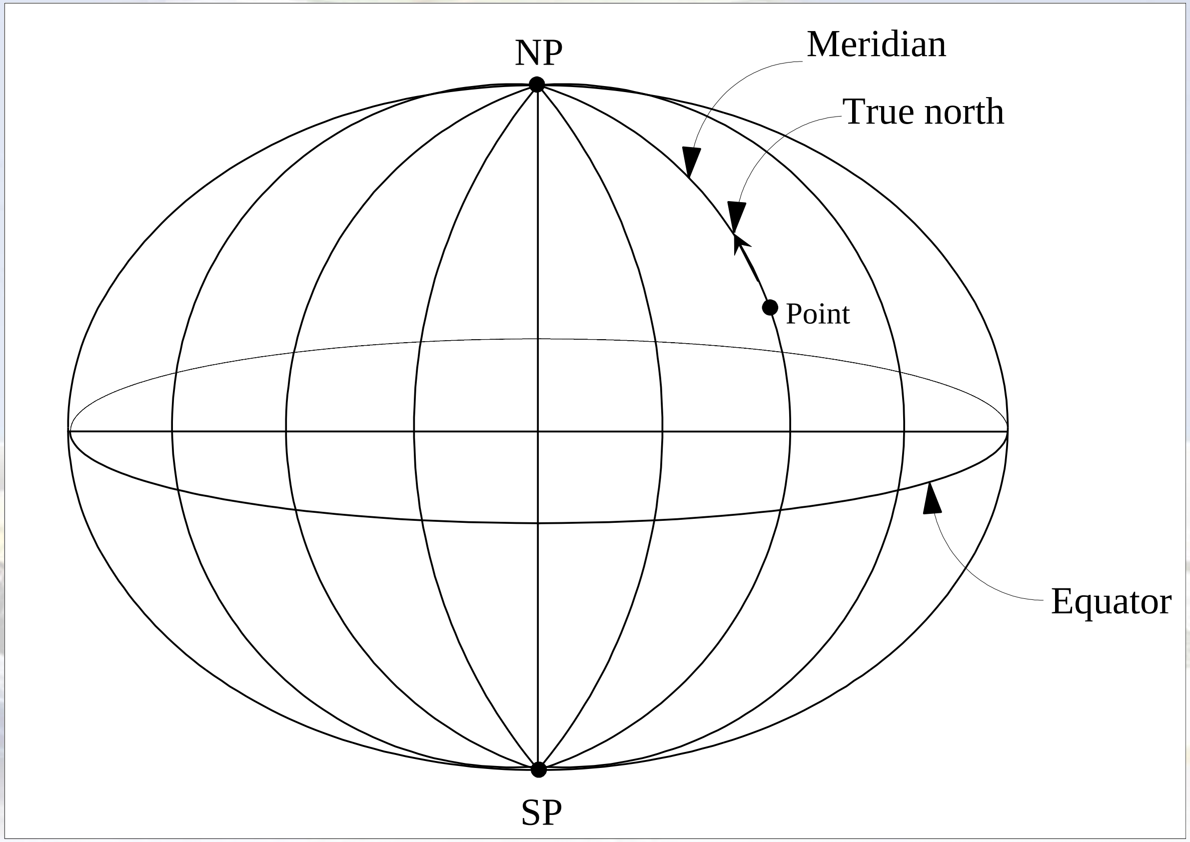


FIGURE 5.3: The true (geographic) north.

Approximate Methods for Locating the direction of True North:

a) The watch method: **Hold the watch horizontally in your hand with the short handle of the watch pointing towards the sun. Bisect the angle between the line pointing towards the sun and the line pointing towards the number 12. The direction of the bisecting line will be in the south direction. The opposite direction will be the north.**

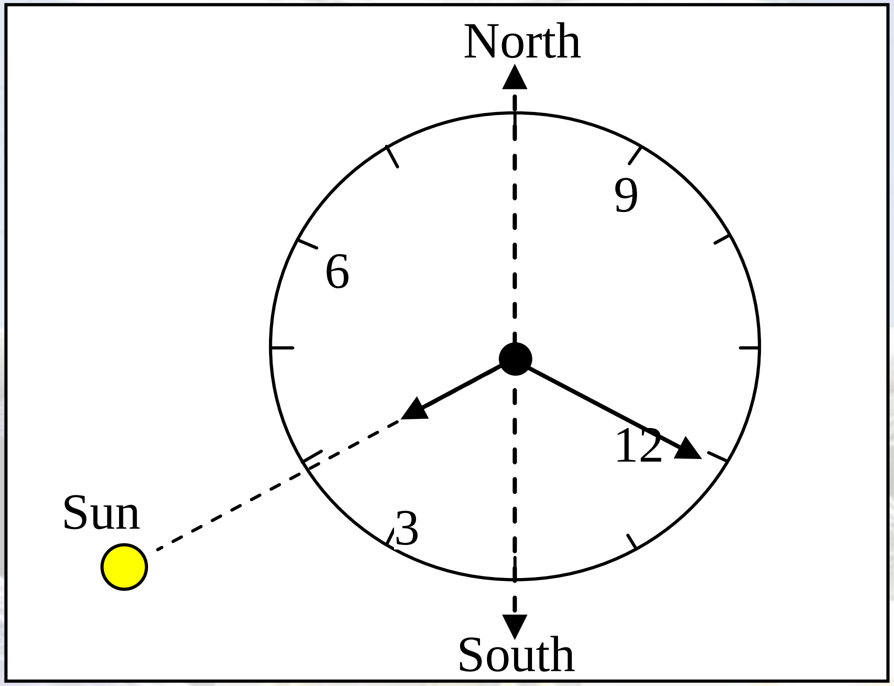


FIGURE 5.4: The watch method for determining the north direction.

b) The shadow method:

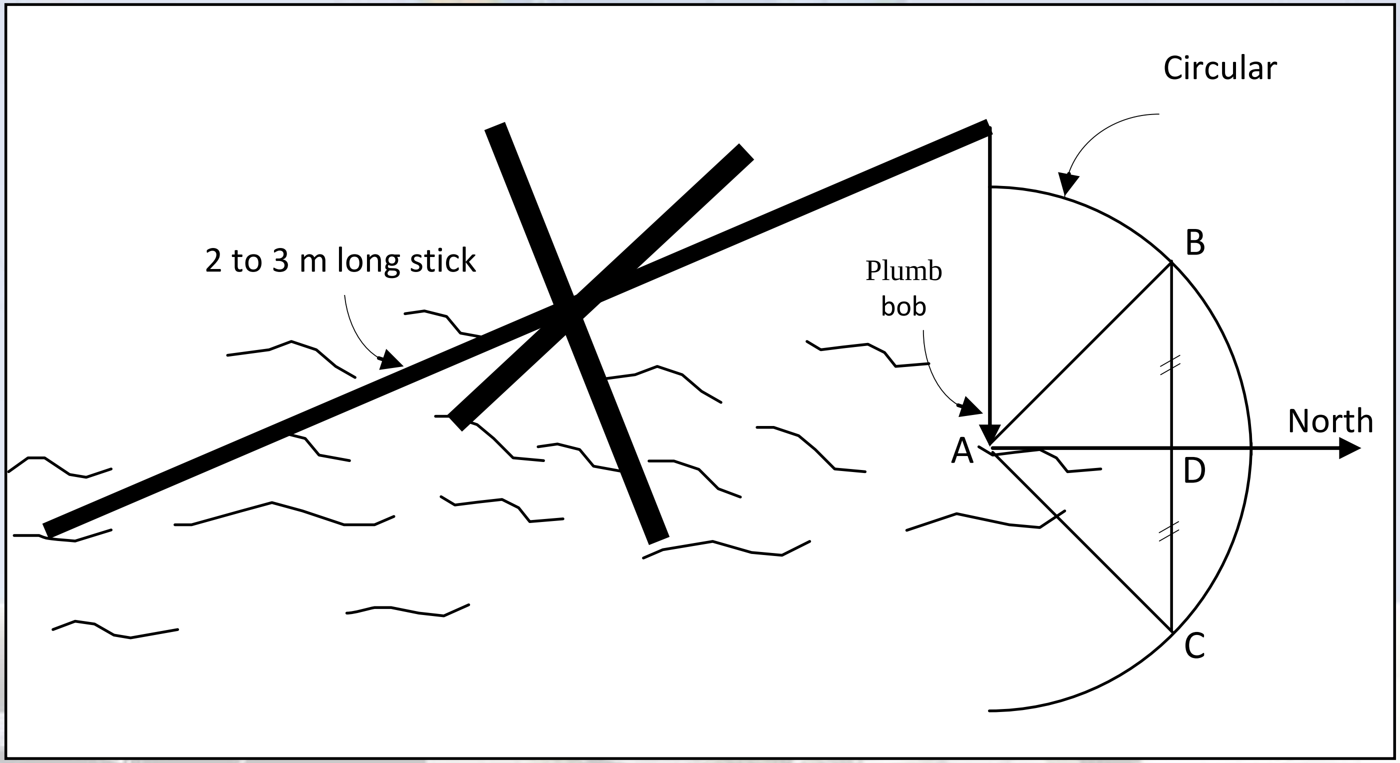


FIGURE 5.5: The shadow method for determining the north direction.

2) MAGNETIC NORTH: The direction towards the magnetic north pole. It lies on the meridian (great circle) passing through the point, the magnetic north and south poles). The angle between the true north and magnetic north is called magnetic declination.

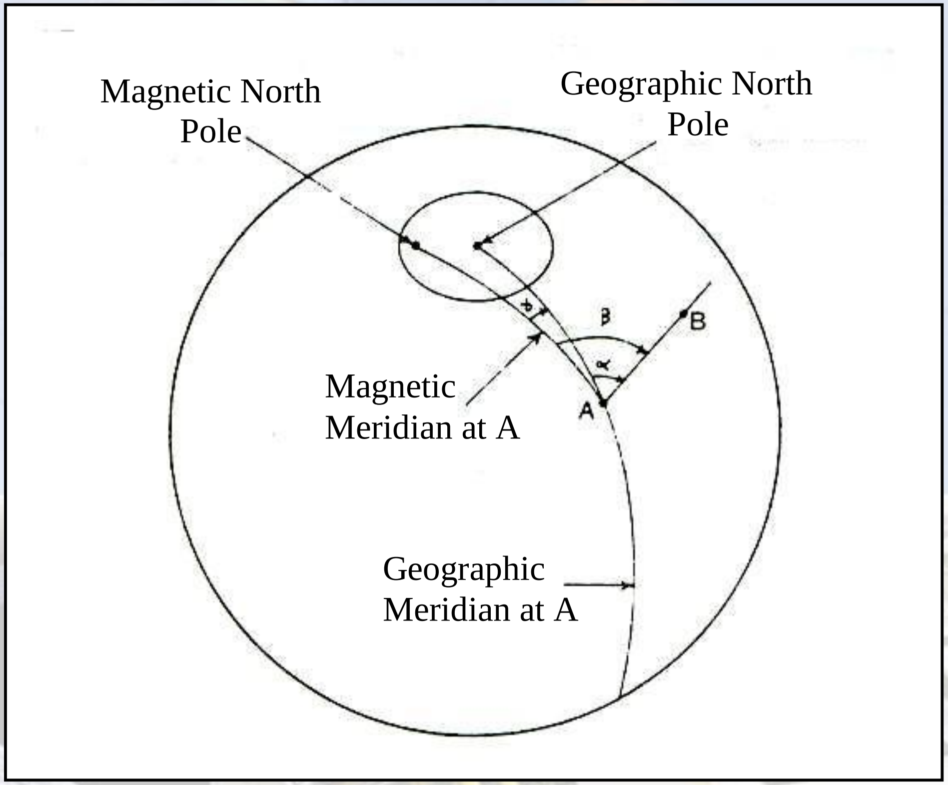


FIGURE 5.6: Relationship between true (geographic) and magnetic norths.

The compass is used to locate the direction of the magnetic north at a point. It is also used to measure the angle that a line makes with the magnetic north.

(a) Pocket compass

(b) Surveyor's compass

FIGURE 5.7: Magnetic compass.

3) ASSUMED NORTH: If the direction to the true or magnetic north is not known or cannot be located at the time of measurement, an assumed reference direction can be chosen. This is called the assumed north. It can be corrected later if the direction to the true or magnetic north at the place becomes known.

4) GRID NORTH: The direction parallel to the central meridian (true north) of the country.

REDUCED BEARING OF A LINE

(The acute angle that a line makes with the north or south direction, whichever is closer)

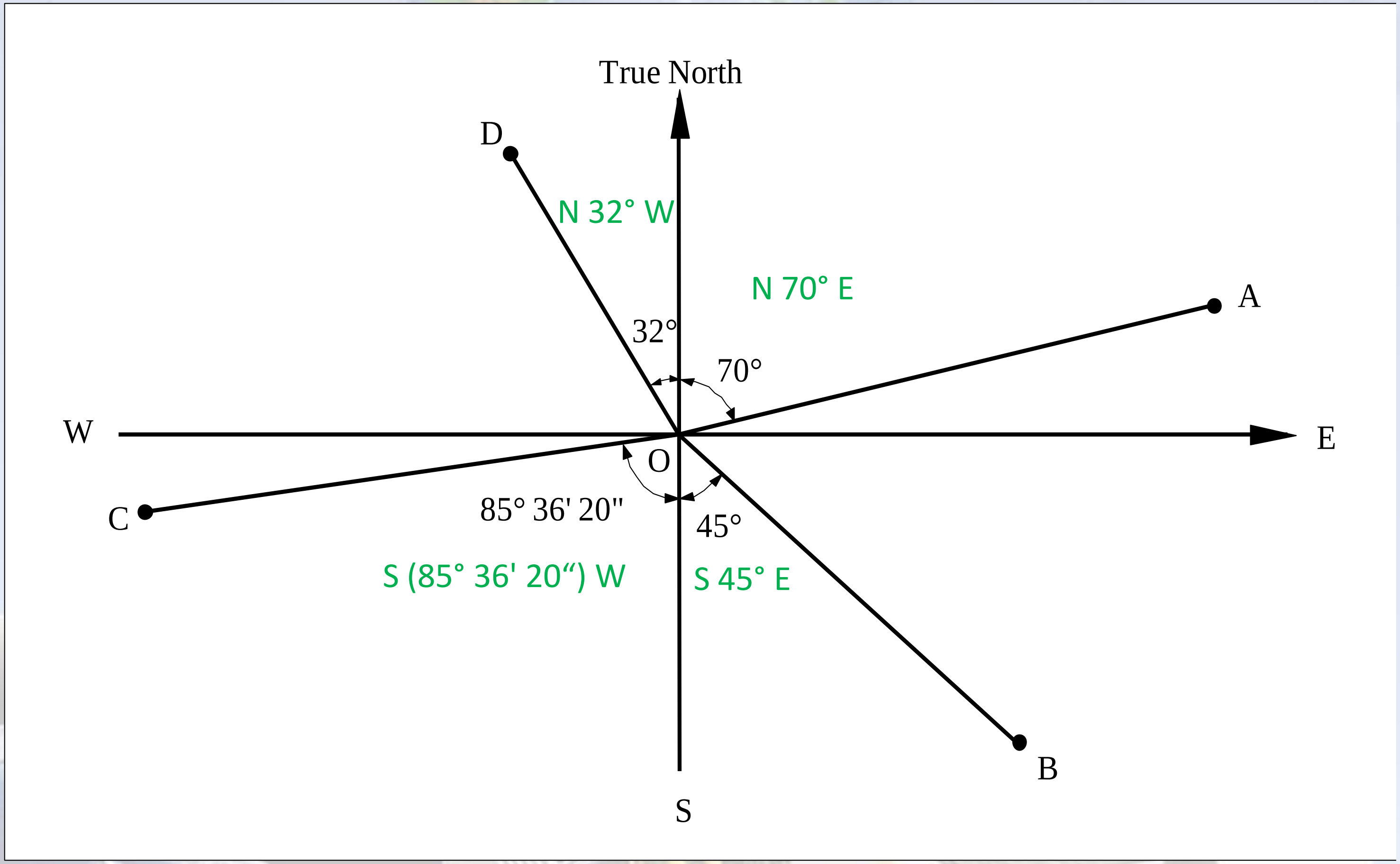


FIGURE 5.8: Reduced bearings.

Relationship between true and magnetic bearing

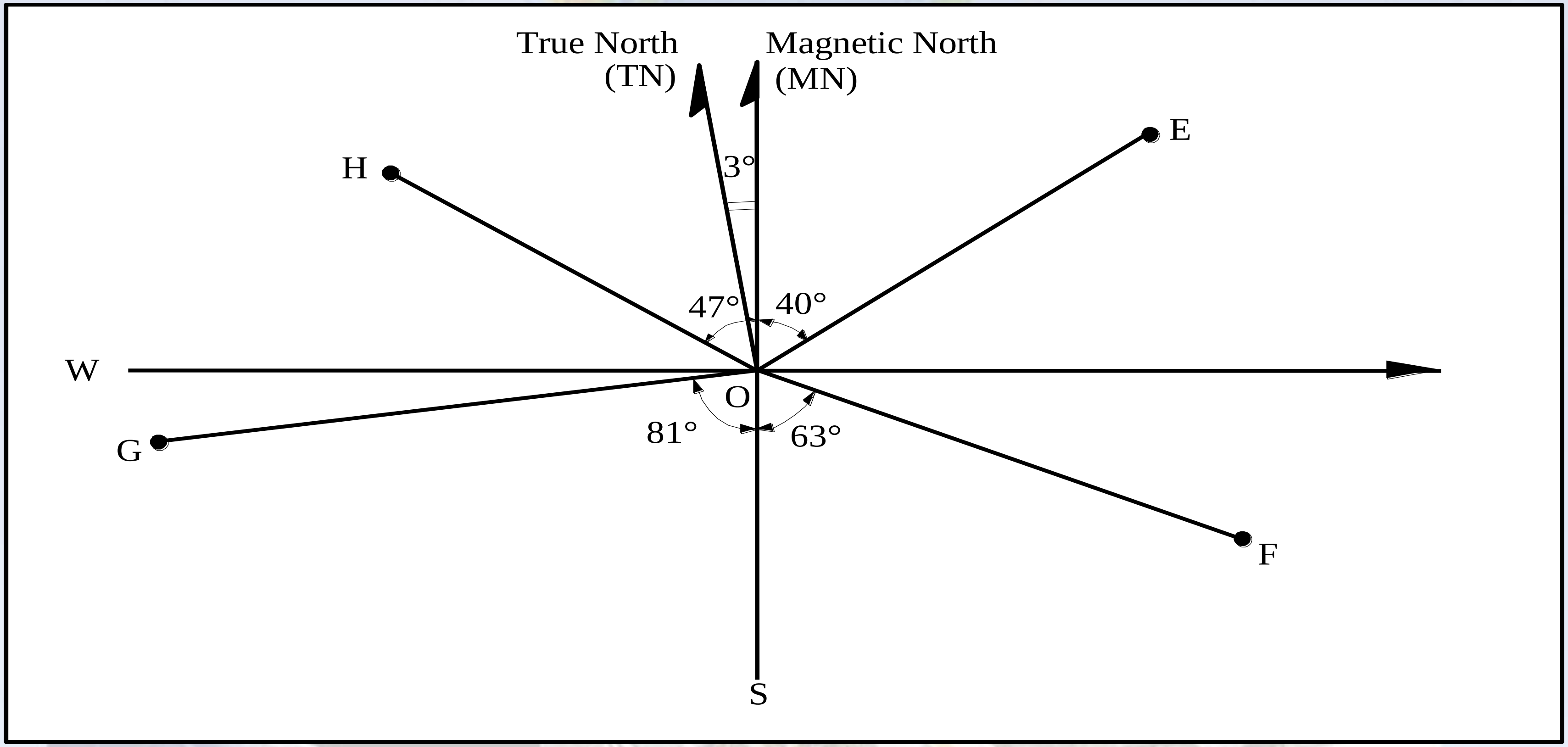


FIGURE 5.9: Magnetic bearings.

AZIMUTH OR WHOLE CIRCLE BEARING

(The azimuth of a line is the horizontal angle measured in a clockwise direction from the north direction to the line)

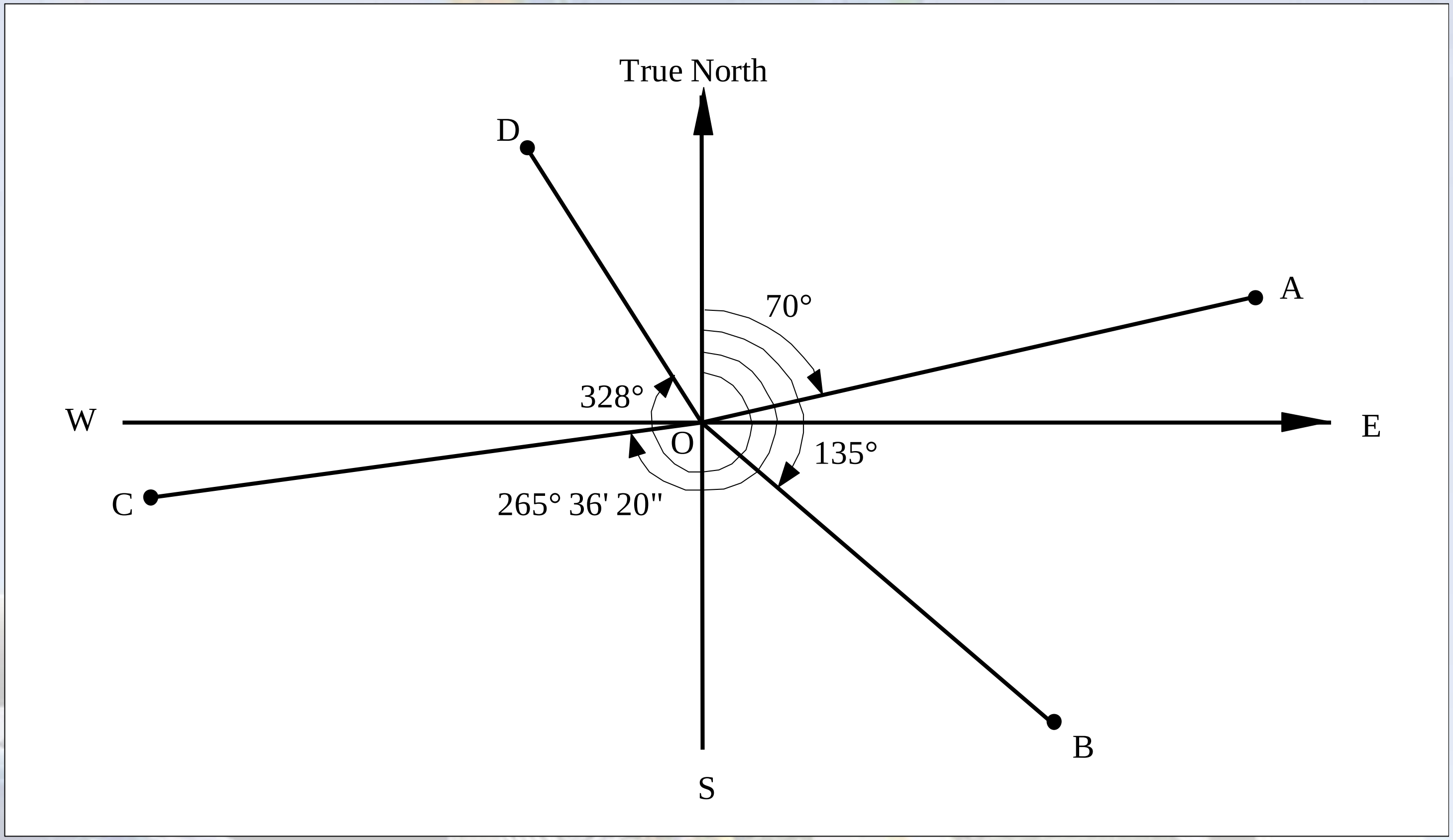


FIGURE 5.10: Azimuth of a line.

BACK REDUCED BEARING AND BACK AZIMUTH

- When measuring the forward azimuth of line AB, the north direction is set at A. For the back azimuth, the north direction is set at B (the end of the line).
- Back azimuth = forward azimuth ± 180°
- To calculate the back reduced bearing of a line, reverse the letters and keep the value of the angle.

Example:

The reduced bearing of a line = N 70° E

The back reduced bearing of the same line = S 70° W

PRINCIPAL ELEMENTS OF AN ANGLE-MEASURING INSTRUMENT

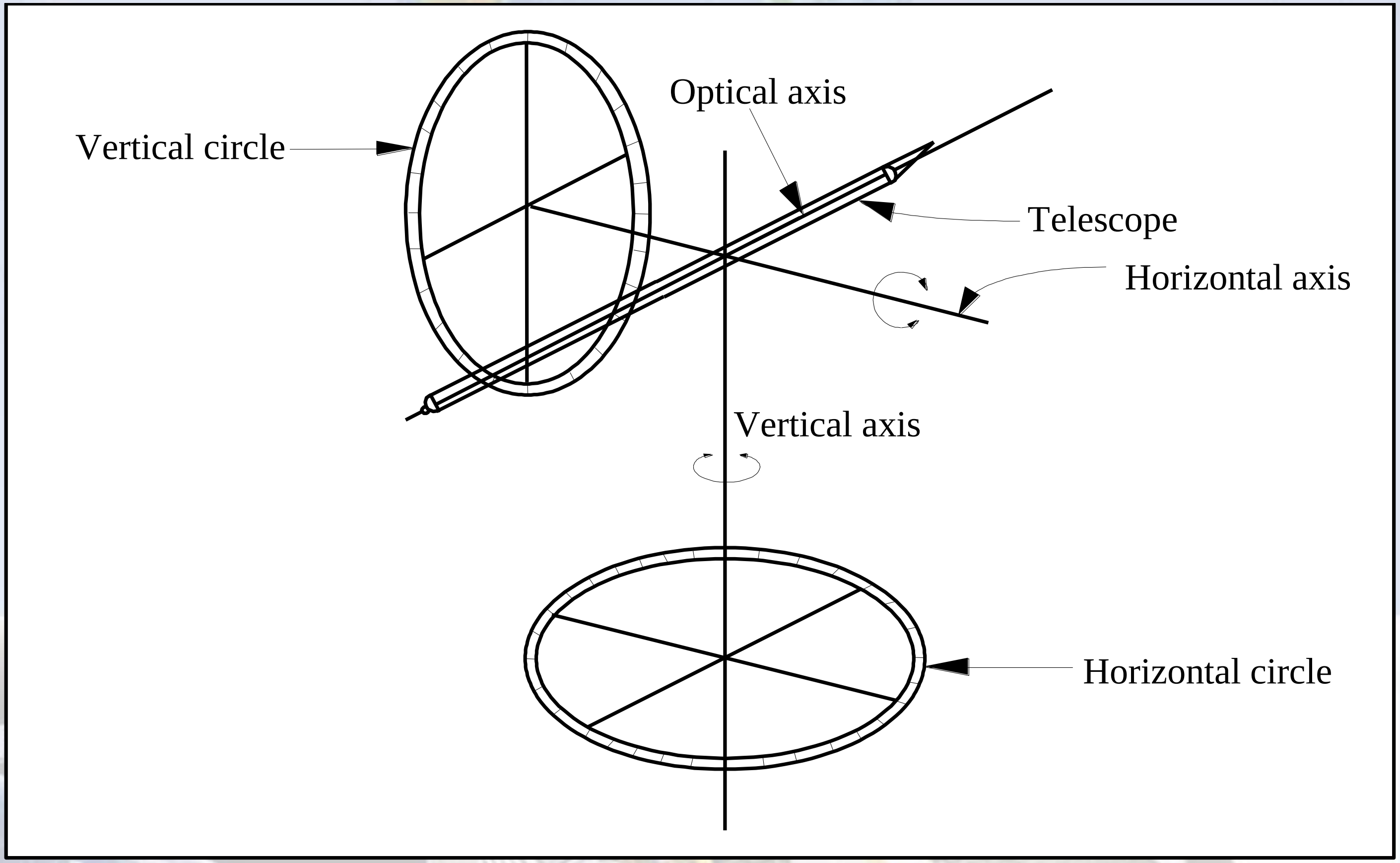


FIGURE 5.12: Principal elements of an angle measuring instrument.

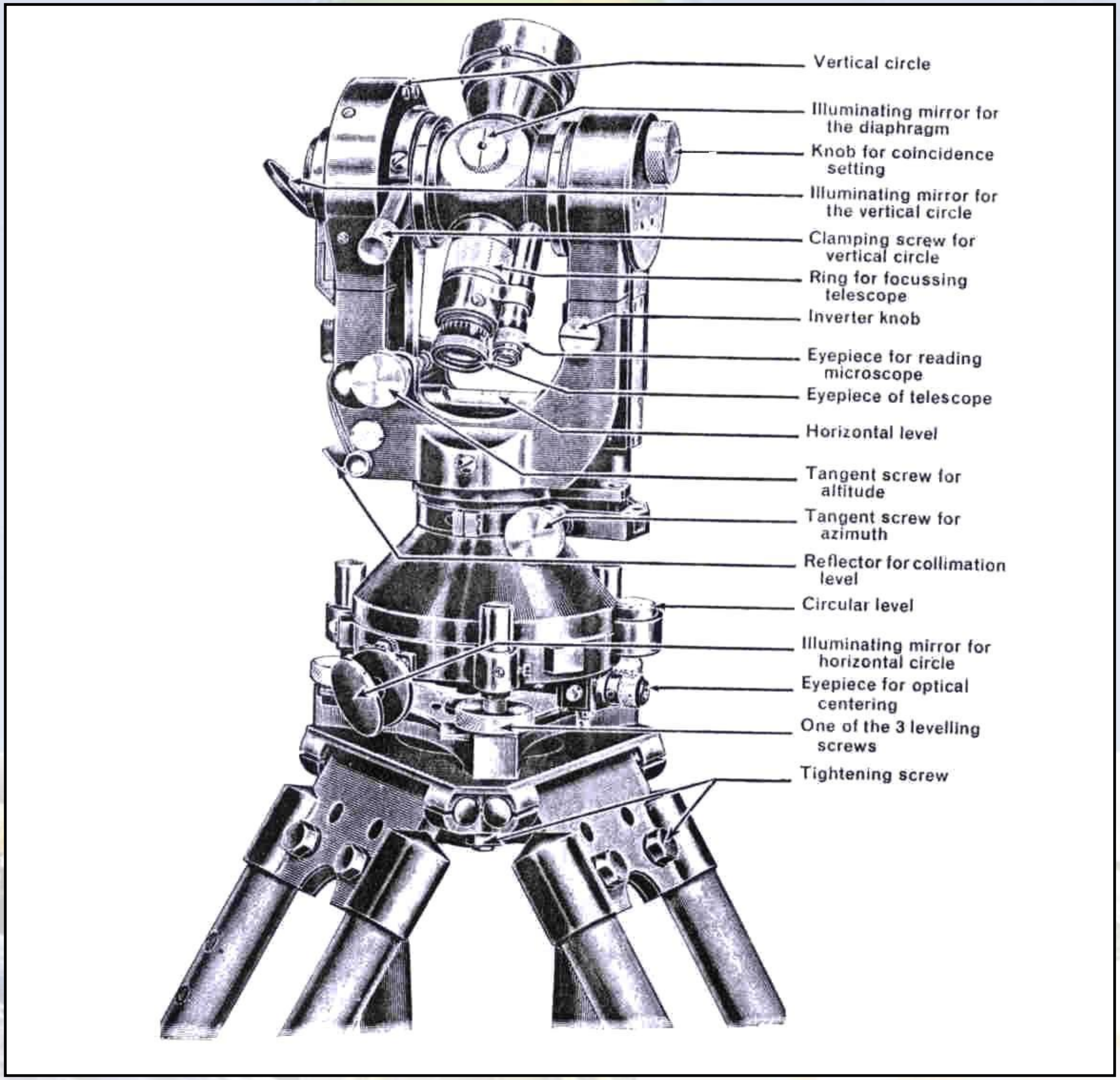


FIGURE 5.13: An example of a scale-reading (manual) Wild-T2 theodolite.

FIGURE 5.14: An example of a digital theodolite.

SETTING UP A THEODOLITE

To be covered in the lab.

MEASUREMENT OF A HORIZONTAL ANGLE

The theodolite should be exactly centered over B. Angle ABC is different from angle AB'C.

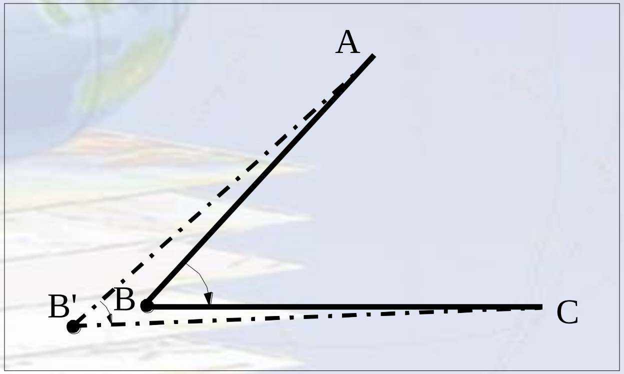


FIGURE 5.17: A wrong setup of the theodolite over station B.

Set up the theodolite over B. Direct the telescope towards point A and make the horizontal circle to read zero. Rotate the theodolite in a clockwise direction so that the telescope points towards C and read the value of the horizontal angle ABC.

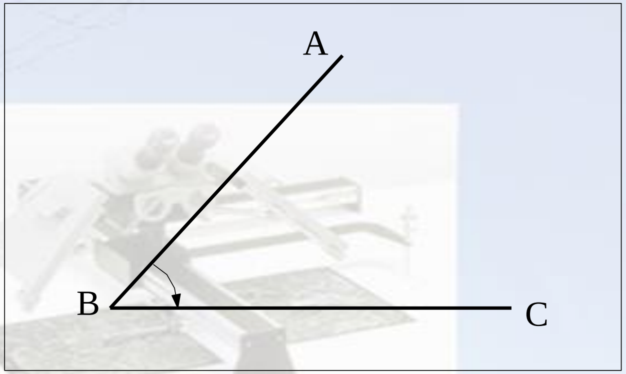
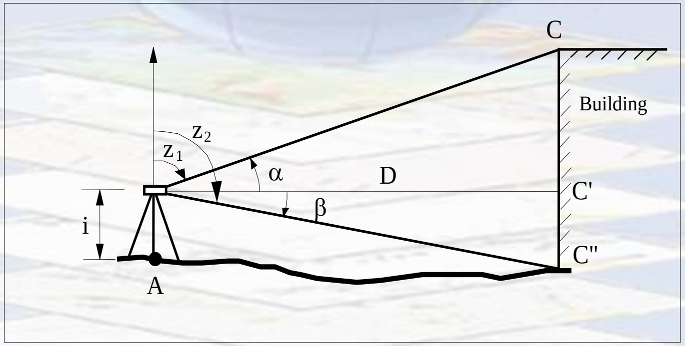


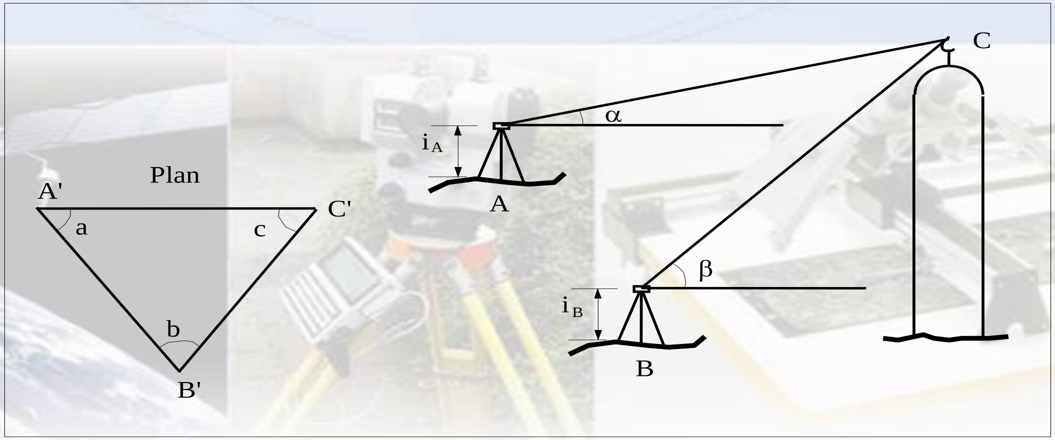
FIGURE 5.16: A horizontal angle.

MAIN APPLICATIONS OF THE THEODOLITE

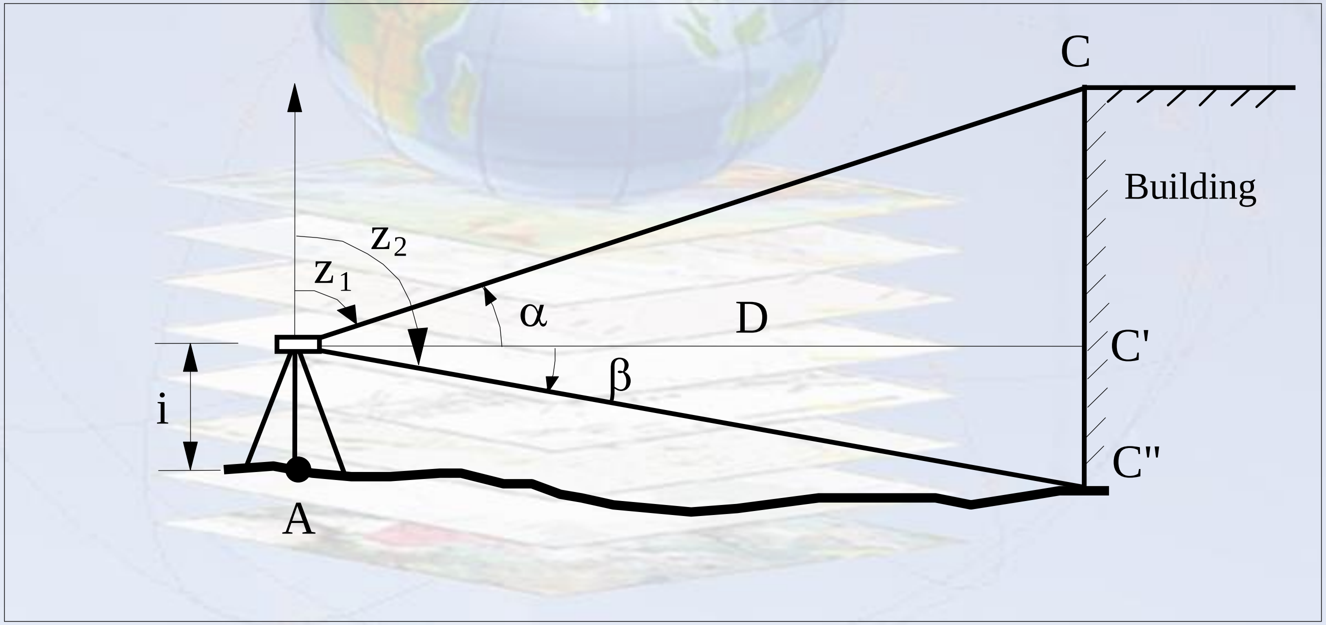
- **MEASUREMENT OF OBJECT HEIGHTS:**
- ***CASE (1):* Points whose horizontal distance from the theodolite is directly measured.**



- ***CASE (2):* Points whose horizontal distance from the theodolite is difficult to measure.**



CASE (1): ***Points whose horizontal distance from the theodolite is directly measured.***

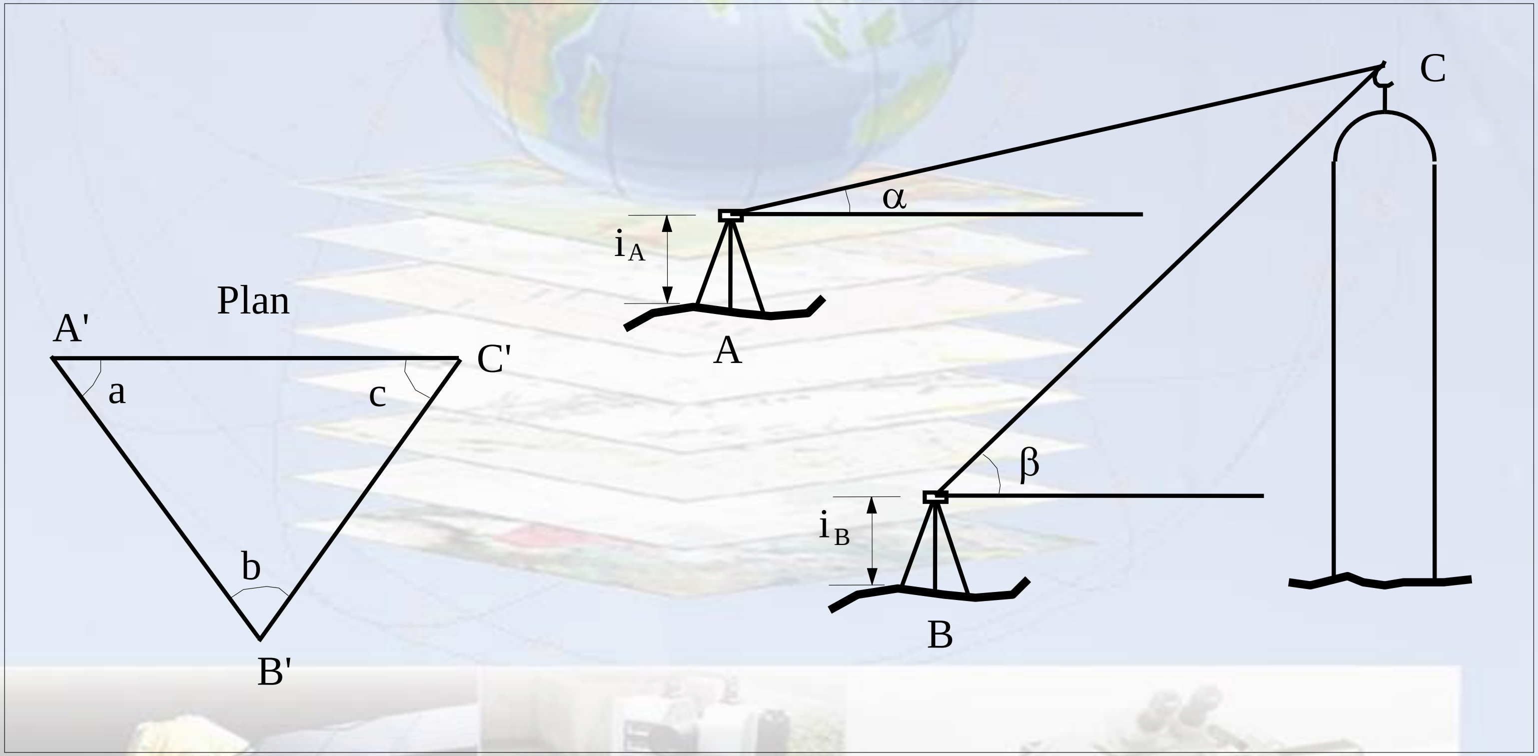


$$\Delta H = C\,C'' = D \cdot \tan\alpha - D \cdot \tan\beta$$

$$= D\left(\tan\alpha - \tan\beta\right)$$

$$= D\left(\frac{1}{\tan z_1} - \frac{1}{\tan z_2}\right)$$

CASE (2): *Points whose horizontal distance from the theodolite is difficult to measure.*



TACHEOMETRY

- distances and elevation differences are determined from instrumental readings alone, these usually being taken with a specially adapted theodolite.

- useful in broken terrain, e.g. river valleys, standing crops, etc., where direct linear measurements would be difficult and inaccurate.
  - Tangential method
  - Stadia method
  - Subtense bar method, and
  - Optical wedge method.

TANGENTIAL METHOD

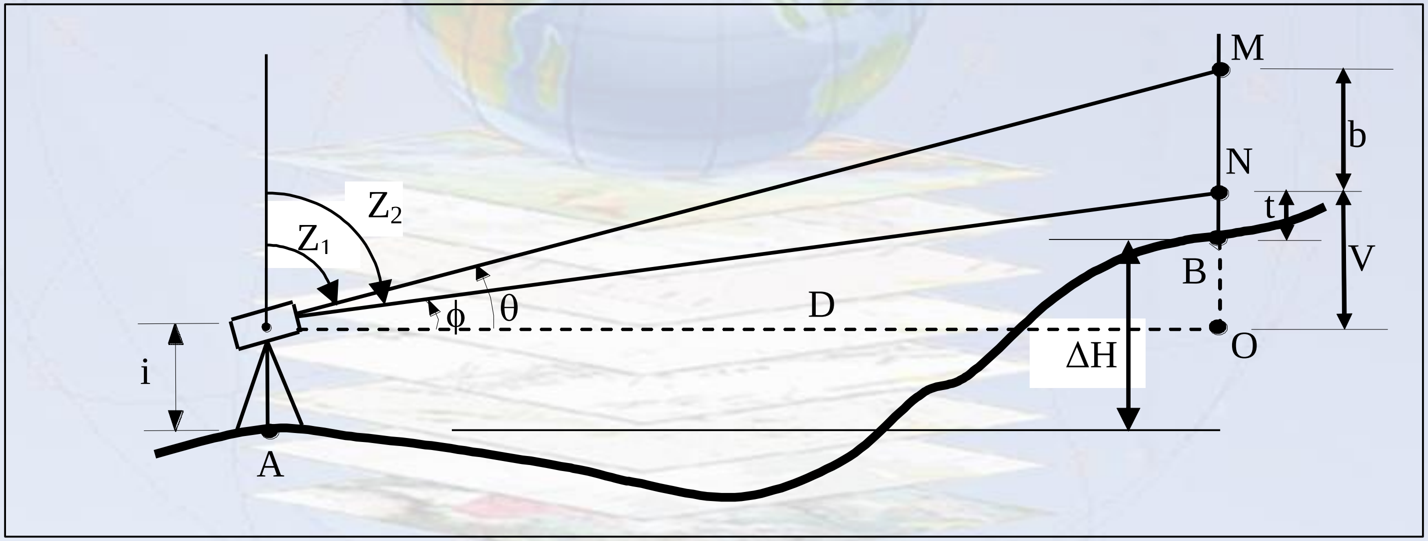


FIGURE 5.20: Tangential method.

$$\Rightarrow \quad D = \frac{b}{(\tan\theta - \tan\phi)} = \frac{b}{\left(\frac{1}{\tan z_1} - \frac{1}{\tan z_2}\right)}$$

$$\Delta H = i + V - BN = i + D \cdot \tan\phi - t$$

EXAMPLE:

The following readings were taken on a staff held vertically at point B.

Vertical Angle	Staff Reading
6° 15' 20" ± 5"	3.50 ± 0.005 m
5° 10' 45" ± 5"	1.00 ± 0.005 m

If you know that the theodolite is 1.65 m above A,

(a) Calculate the horizontal distance and elevation difference between points A and B, as well as, their standard errors.

(b) Do you recommend the tangential method for precise surveying, and why?

SOLUTION:

(a) $b = 3.50 - 1.00 = 2.50$ m

$$D = \frac{b}{(\tan\theta - \tan\phi)} = \frac{2.50}{\tan(6^\circ\ 15'\ 20'') - \tan(5^\circ\ 10'\ 45'')} = 131.75 \text{ m}$$

$$\Delta H = i + D \cdot \tan\phi - t$$

$$= 1.65 + 131.75 \times \tan(5^\circ\ 10'\ 45'') - 1.00 = 12.59 \text{ m}$$

From the Law of propagation of random errors:

$$\sigma_b^2 = (0.005)^2 + (0.005)^2 = 0.00005$$

For small angles θ and ϕ:

$$D = \frac{b}{(\tan\theta - \tan\phi)} \approx \frac{b}{\theta - \phi}$$, where θ and ϕ are in radian,

$$\sigma_D^2 = \left(\frac{\partial D}{\partial b}\right)^2 \cdot \sigma_b^2 + \left(\frac{\partial D}{\partial \theta}\right)^2 \cdot \sigma_\theta^2 + \left(\frac{\partial D}{\partial \phi}\right)^2 \cdot \sigma_\phi^2$$

$$= \left(\frac{1}{\theta - \phi}\right)^2 \cdot \sigma_b^2 + \left(\frac{-b}{(\theta - \phi)^2}\right)^2 \cdot \sigma_\theta^2 + \left(\frac{b}{(\theta - \phi)^2}\right)^2 \cdot \sigma_\phi^2$$

Substitute $\theta = 0.10918$ radian, $\phi = 0.09039$ radian, $b = 2.50$ m,

$\sigma_b^2 = 0.00005$ and $\sigma_\theta = \sigma_\phi = 2.424 \times 10^{-5}$ radian

$\Rightarrow \sigma_D^2 = 0.2005 \text{ m}^2$

$\Rightarrow \sigma_D = \pm\sqrt{0.2005} = \pm 0.45$ m

$$\sigma_{\Delta h}^2 = \sigma_i^2 + (\tan\phi)^2 \sigma_D^2 + (D \cdot \sec^2\phi)^2 \sigma_\phi^2 + \sigma_t^2$$

Consider $\sigma_i = 0.0$,

$$\Rightarrow \sigma_{\Delta h}^2 = \left(\tan(5° \; 10' \; 45'')\right)^2 \cdot (0.2005) + \left(131.75 \cdot \sec^2(5° \; 10' \; 45'')\right)^2 \cdot (2.424 \times 10^{-5})^2 + (0.005)^2$$

$\Rightarrow \sigma_{\Delta h} = \pm 0.04$ m

Final results:

Horizontal distance = D = 131.75 ± 0.45 m

Elevation difference = ΔH = 12.59 ± 0.04 m

(b) From part (a), we notice the high values of σ_D and $\sigma_{\Delta h}$ which makes the tangential method not suitable for precise surveying. In general, tacheometry gives rapid results and is easy to do, but does not give highly accurate results. It is generally used for topographic mapping.

STADIA METHOD

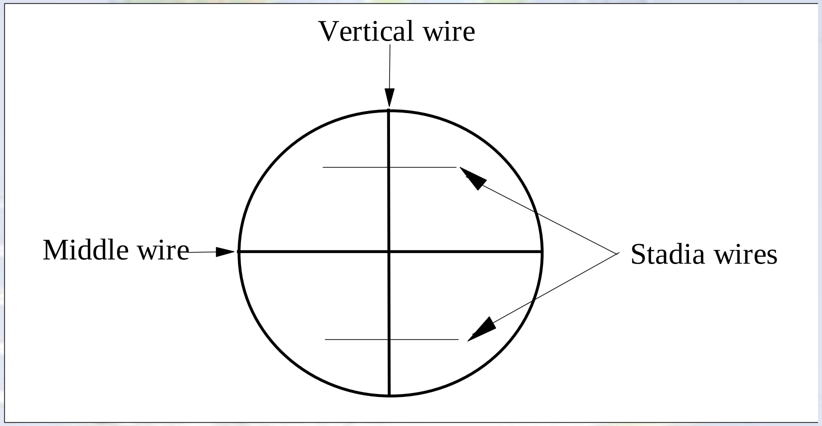


FIGURE 5.21: Stadia wires.

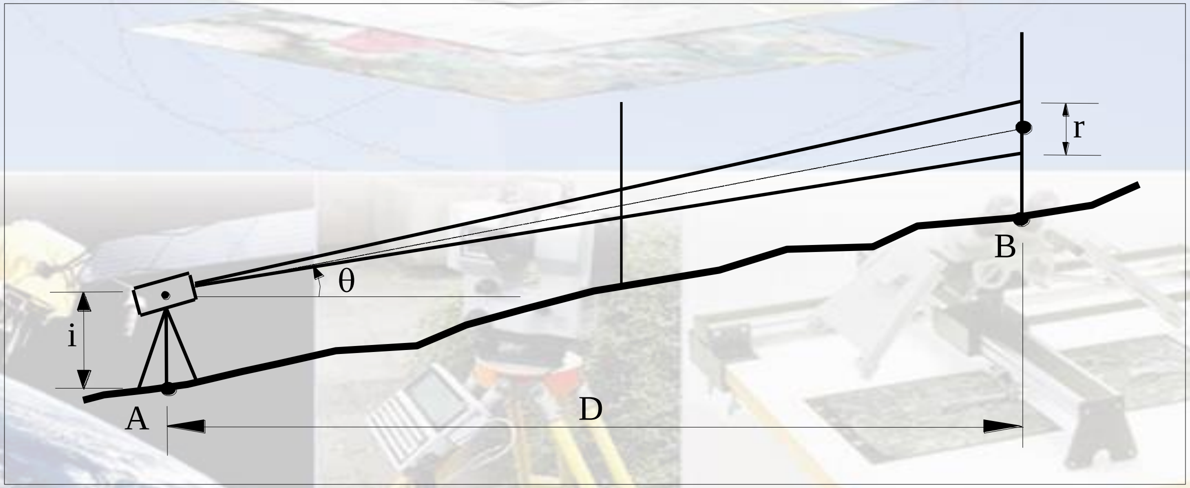


FIGURE 5.22: Stadia method.

Stadia Geometry for Horizontal Sight

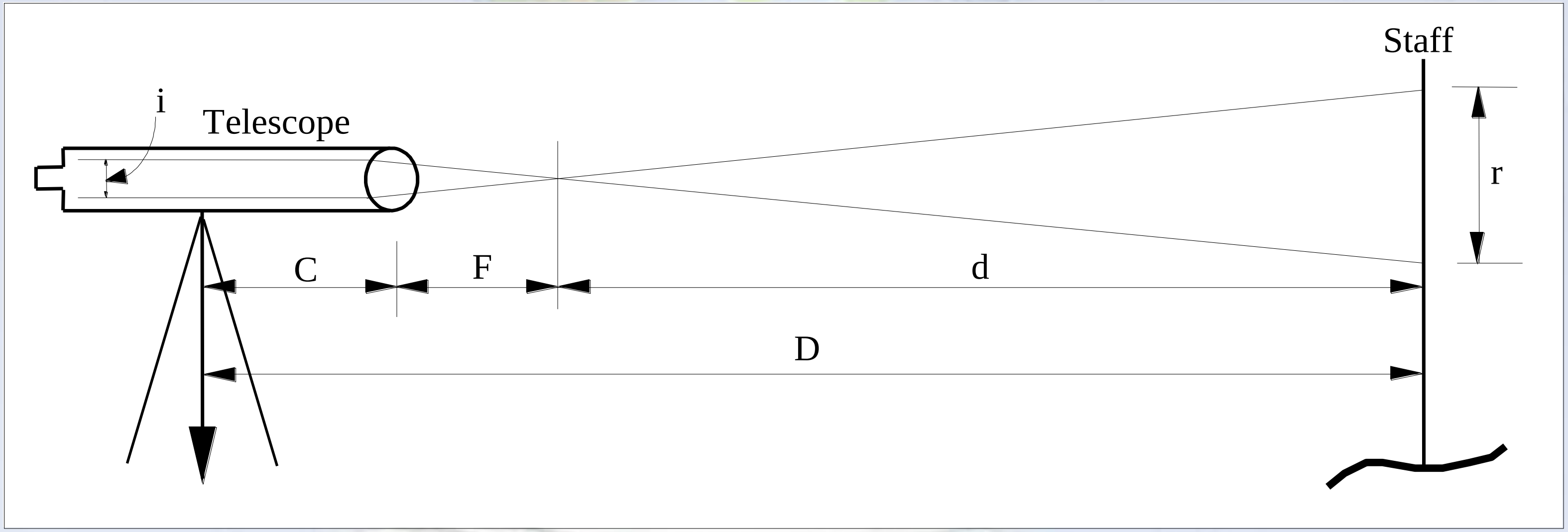


FIGURE 5.23: Stadia geometry for horizontal sight.

$$\frac{d}{r} = \frac{F}{i} \quad \Rightarrow \quad d = \frac{F}{i} \cdot r$$

From the above figure , the total distance from the staff to the plumb bob is:

$$D = \left(\frac{F}{i}\right) r + \left(F + C\right) = k.r + (F+C)$$

Stadia Geometry for Inclined Sight:

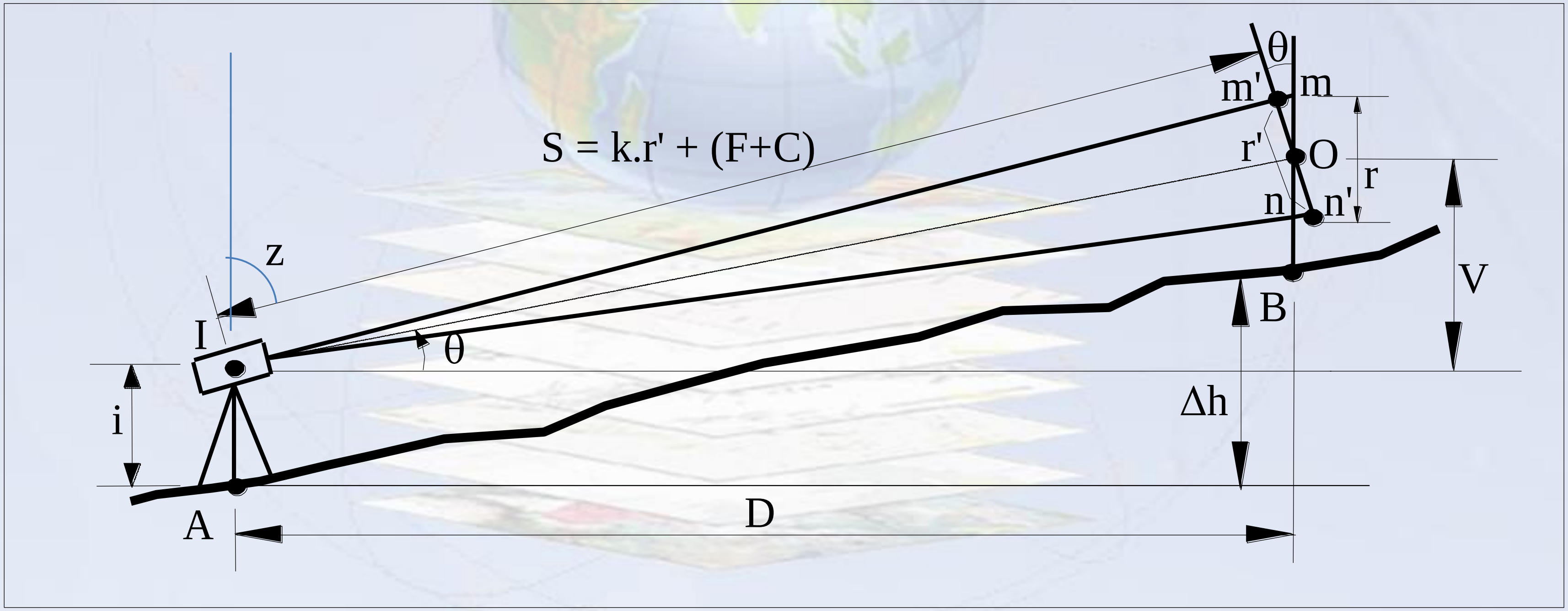


FIGURE 5.24: Stadia geometry for inclined sight.

With k = 100, and F+C = 0,

$D = kr\cos^2\theta = kr\sin^2 z$, $V = \frac{1}{2}kr\sin 2\theta = \frac{1}{2}kr\sin 2z$

$\Delta h = V + i - OB$

EXAMPLE:

The following readings were taken on a vertical staff with a theodolite having a constant k = 100 and F + C = 0.

Staff Station	Azimuth	Stadia Readings	Vertical Angle
A	27° 30'	1.000 1.515 2.025	+ 8° 00'
B	207° 30'	1.000 2.055 3.110	- 5° 00'

Calculate the mean slope between A and B.

SOLUTION:

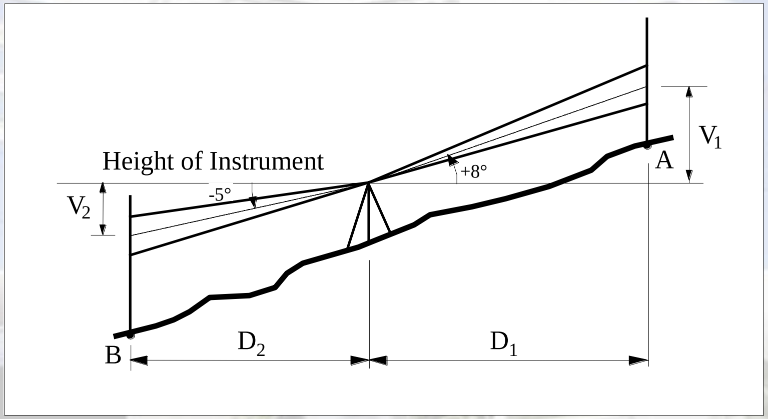


FIGURE 5.25

(1) Staff at Station A:

Staff intercept r = 2.025 - 1.000 = 1.025 m

Mid-reading = 1.515 m

$$D = kr\cos^2\theta$$

$$V = \frac{1}{2}kr\sin 2\theta$$

$$\Rightarrow D_1 = 100 \times 1.025 \times \cos^2(8°) = 100.515 \text{ m}$$

$$V_1 = \frac{1}{2} \times 100 \times 1.025 \times \sin(16°) = 14.126 \text{ m}$$

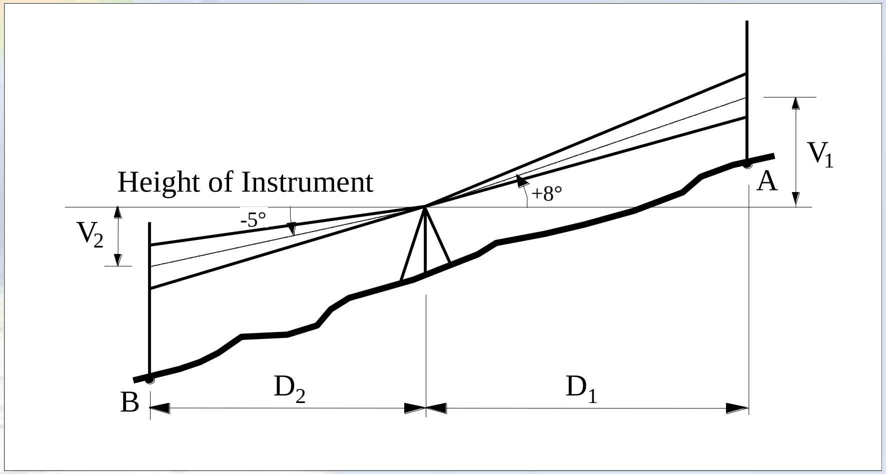


FIGURE 5.25

(2) Staff at Station B:

Staff intercept r = 3.110 - 1.000 = 2.110 m

Mid-reading = 2.055 m

$$\Rightarrow D_2 = 100 \times 2.110 \times \cos^2(-5°) = 209.397 \text{ m}$$

$$V_2 = \frac{1}{2} \times 100 \times 2.110 \times \sin(-10°) = -18.320 \text{ m}$$

Let h = height of instrument above datum, then

Elevation of point A = h + 14.126 - 1.515 = h + 12.611

Elevation of point B = h - 18.320 - 2.055 = h - 20.375

Elevation difference between B and A (ΔH_{BA}):

(ΔH_{BA}) = (h + 12.611) - (h - 20.375) = 32.986 m

From a consideration of azimuths, it will be seen that A, B and the instrument lie on a straight line (207° 30' - 27° 30' = 180°), so that the mean slope =

$$\frac{\text{Elevation difference}}{D_1 + D_2} = \frac{32.986}{100.515 + 209.397} = \frac{1}{9.4} = 1 \text{ in } 9.4 = 0.1064 = 10.64\%$$