Chapter 19

Formalized Sensitivity Analysis and Expected Value Decisions

LEARNING OBJECTIVES

1. Sensitivity to variation
2. Three estimates
3. Expected value
4. Expected value of cash flows
5. Decision trees

Sct 18.1 Determining Sensitivity to Parameter Variation

- A parameter is a variable or factor for which an estimate or stated value is required to conduct the analysis at hand.
- Examples:
  - P, F, A;
  - i, n;
  - Future costs, salvages, etc.
- Sensitivity analysis
  - Seeks to determine what parameters matter most in an economic analysis

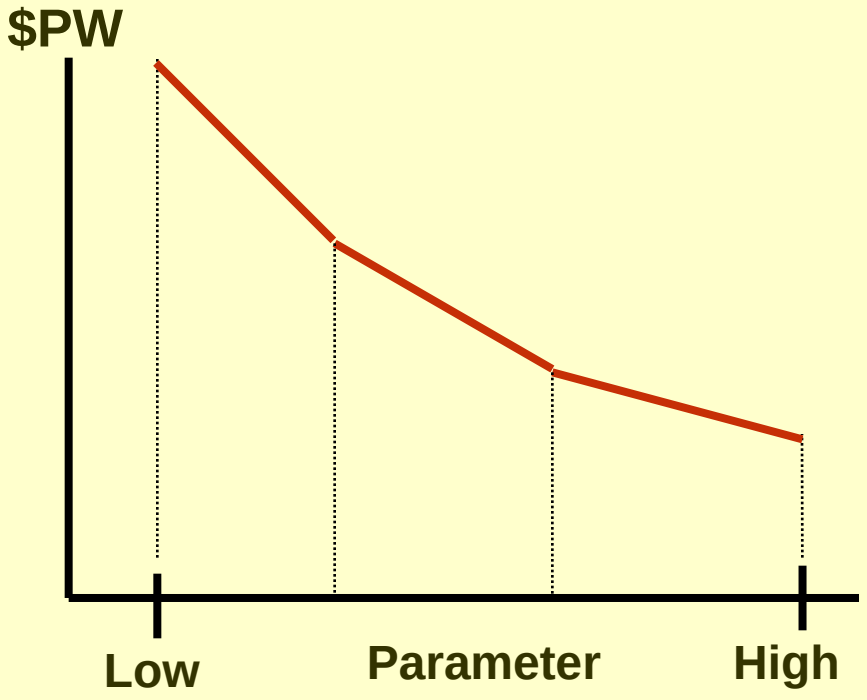
Sensitivity

- Sensitivity is concerned with variability
- Variance associated with input parameters impact the output variable the most
- The MARR as a parameter
  - Interest rates and other interest factors tend to be more stable from project to project
  - The analyst can limit the range over which these type of parameters vary

Visualizing the Impact of Parameters

- Plot the PW, AW, or ROR vs. input parameters
- Steps
  - Pre-select the desired input parameters
  - Select the probable range and increment of variation for each parameter
  - Select the measure of worth
  - Compute the results for each parameter
  - Graphically display the results by plotting the parameter vs. the measure of worth

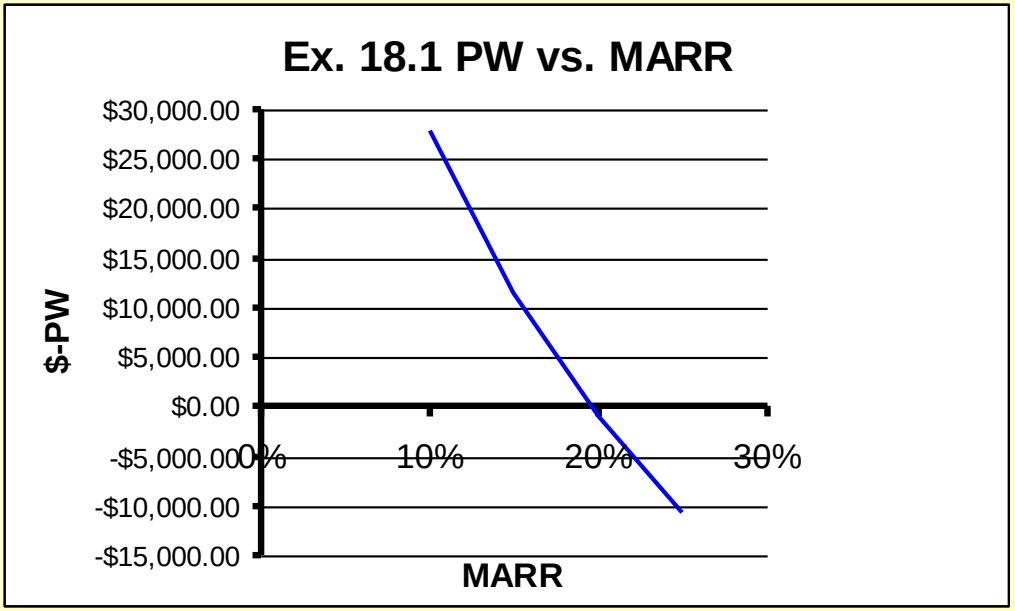
Example



Assume a parameter of interest (show on the X-axis). Vary that parameter from some low value to an assumed high value. Plot the resultant values on the Y-axis.

Sensitivity of PW: Example 18.1

Year	Cflow
0	-$80,000
1	$25,000
2	$23,000
3	$21,000
4	$19,000
5	$17,000
6	$15,000
7	$13,000
8	$11,000
9	$9,000
10	$7,000



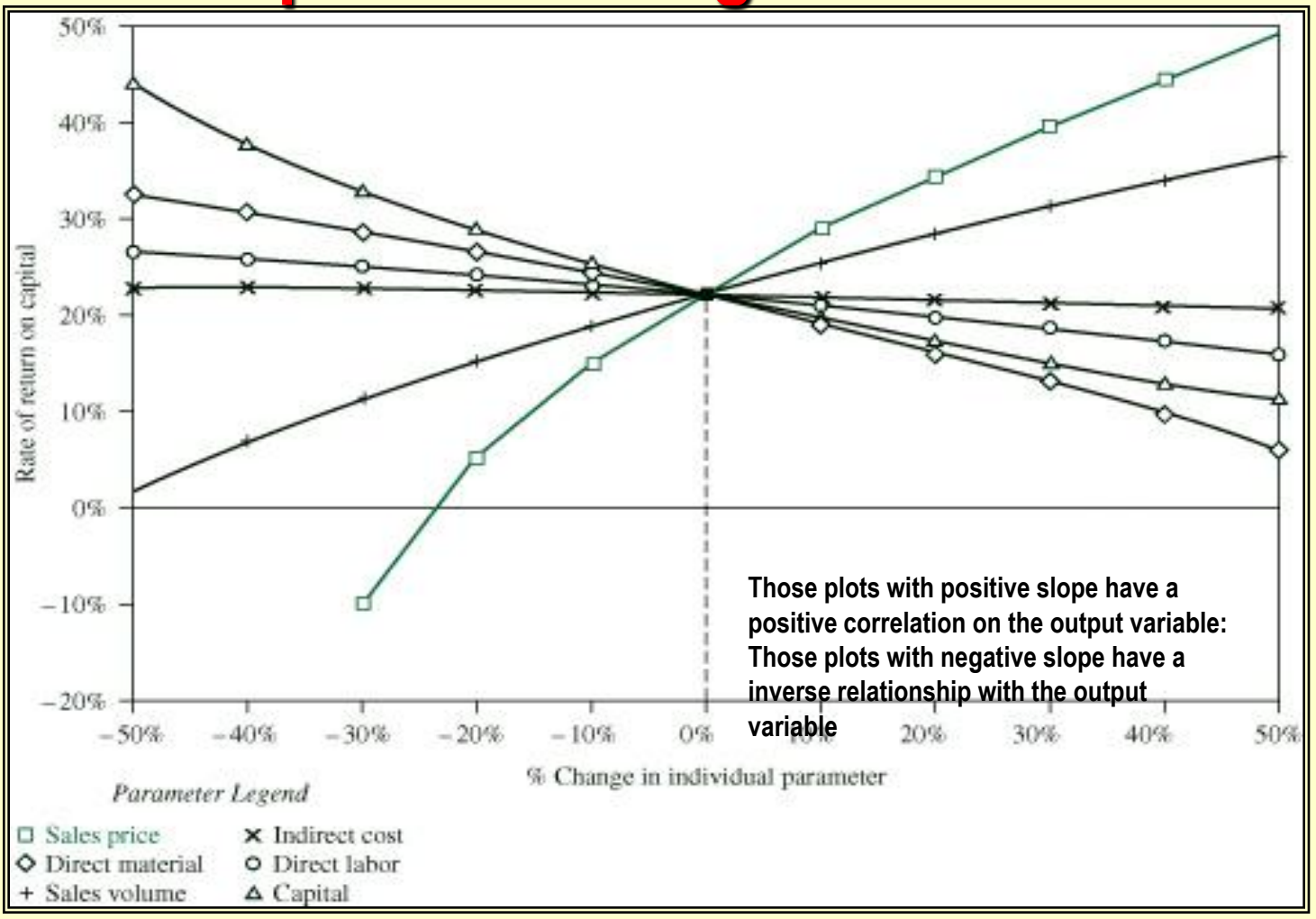
MARR	PW(i%)
10%	$27,831.49
15%	$11,510.26
20%	-$962.36
25%	-$10,711.51

One can better visualize the relationship on PW vs. a selected range of discount rates. As the discount rate increases from 10% to 25% the resultant PW is substantially lowered at a rather accelerated rate.

Sensitivity of Several Parameters

- For several parameters with one alternative
  - Graph the percentage change for each parameter vs. the measure of worth
  - One will plot the percent deviation from the most likely estimate on the x-axis
- This type of plot results in what is termed a spider plot

Spider Plot: Figure 18-3



Sct 18.2 Formalized Sensitivity Analysis Using Three Estimates

- Given an input parameter of interest
  - Provide three estimates for that parameter
    - A pessimistic estimate, P
    - A most likely estimate, ML
    - An optimistic estimate, O
- Note: This approach comes from PERT/CPM analysis and is based upon the *beta* distribution

Three Estimates: Example 18.3

- Three alternatives (A, B, C) with 4 Parameters
  - First cost, salvage value, AOC, and life
  - For *each parameter* we formulate

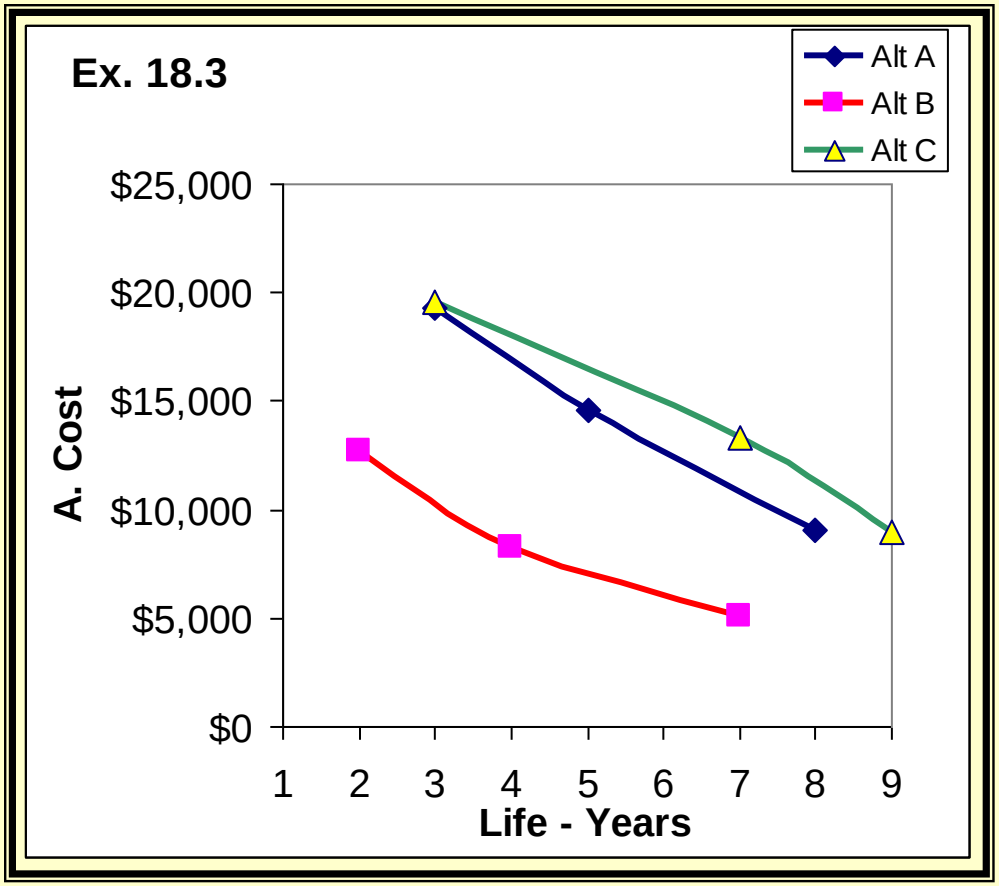
Parameter	
P	pessimistic estimate
ML	most likely estimate
O	optimistic estimate

- See Table 18.2 and observe the dominance by alternative B (cost problem so lower cost is preferred)

Example 18.3 -- Setup

Strategy	First Cost	SV	AOC	Life
Alt A.				
P	-20,000	0	-11,000	3
ML	-20,000	0	-9,000	5
O	-20,000	0	-5,000	8
Alt. B				
P	-15,000	500	-4,000	2
ML	-15,000	1,000	-3,500	4
O	-15,000	2,000	-2,000	7
Alt. C				
P	-30,000	3,000	-8,000	3
ML	-30,000	3,000	-7,000	7
O	-30,000	3,000	-3,500	9

Plot for Example 18.3



Observe the dominance by alternative B over A and C. A plot like this clearly shows the relationships.

Sct 18.3 Economic Variability and The Expected Value

- **Expected Value**
  - Long-run average based upon occurrence and probability of occurrence
- **Definition of Expected Value**

$$E(X) = \sum_{i=1}^{m} X_i P(X_i)$$

X_i = value of the variable *X* for i from 1 to *m* different values

$P(X_i)$ = probability that a specific value of X will occur

Subject to:

$$\sum_{i=1}^{m} P(X_i) = 1.0$$

See Example 18.4

Sct 18.4 Expected Value Computations for Alternatives

- Two applications for use of Expected Value (EV)
  1. Prepare information for a more complete analysis of an economic analysis
  2. To evaluate expected utility of a fully formulated alternative
- Examples 18.5 and 18.6 illustrate this concept

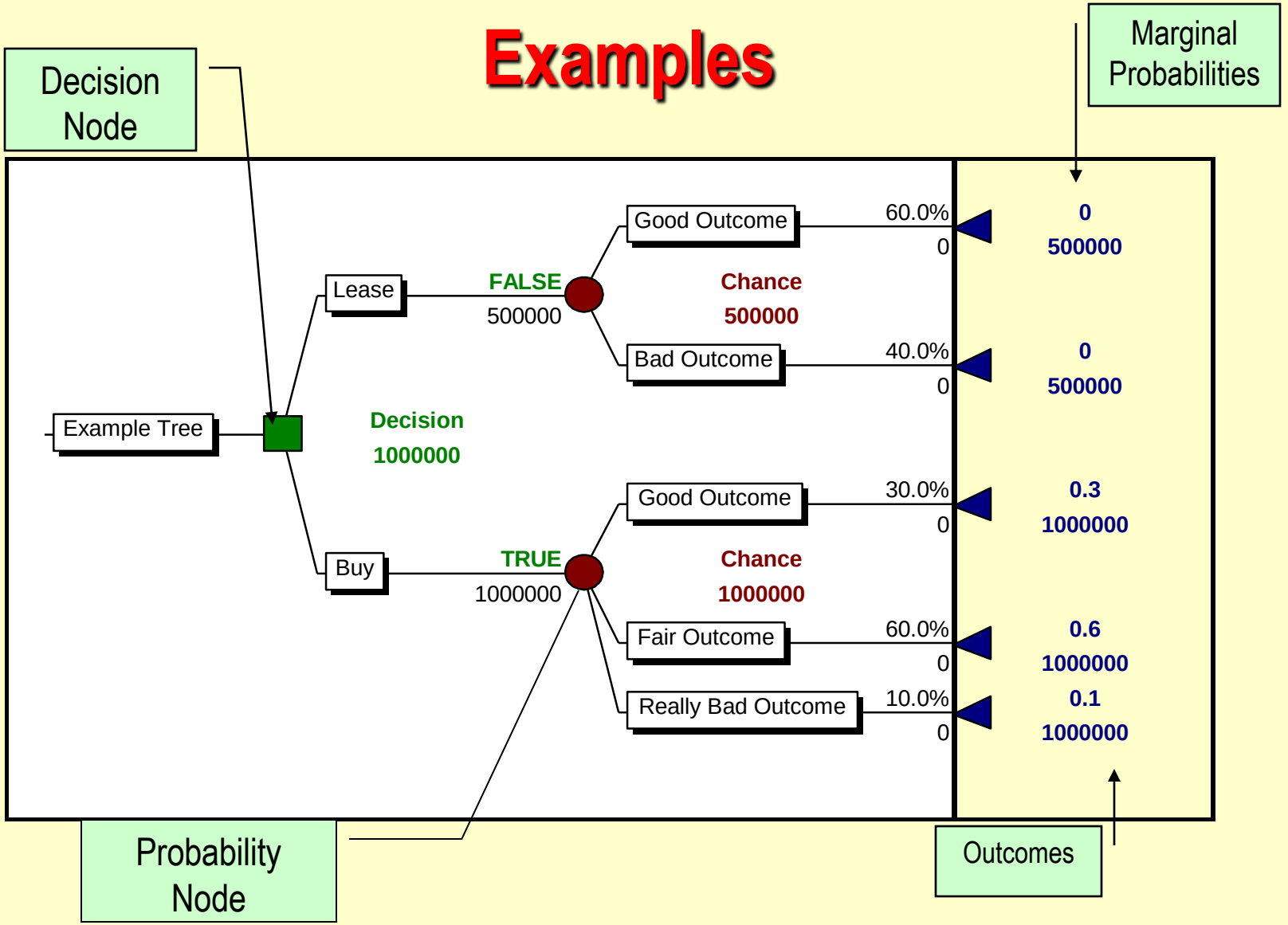
Sect 18.5 Staged Evaluation of Alternatives Using Decision Trees

- Some problems involve staged decisions that occur in sequence
- Define the staged decisions and assign the respective probabilities to the various defined outcomes
- Useful tool for modeling such a process involves Decision Trees
- The objective: ***Make Risk More Explicit***

Decision Tree Attributes

- More than one stage of alternative selection
- Selection of an alternative at one stage that leads to another stage
- Expected results from a decision at each stage
- Probability estimates for each outcome
- Estimates of economic value (cost or revenue) for each outcome
- Measure of worth as the selection criterion, e.g. E(PW)

Examples


Decision Node
Marginal Probabilities
Example Tree
Decision
1000000
Lease
FALSE
500000
Buy
TRUE
1000000
Good Outcome
60.0%
0
Chance
500000
Bad Outcome
40.0%
0
Good Outcome
30.0%
0
Chance
1000000
Fair Outcome
60.0%
0
Really Bad Outcome
10.0%
0
0
500000
0
500000
0.3
1000000
0.6
1000000
0.1
1000000
Probability Node
Outcomes

Solving Decision Trees

- Once designed, Decision Tree is solved by folding back the tree
- First, define all of the decision and the decision points
- Define the various outcomes given the decision
- Assign the probabilities to the mutually exclusive outcomes emanating from each decision node
- See Example 18-8 for a comprehensive analysis illustrating the decision tree approach

Important Points for Decision Trees

- Estimate the probabilities associated with each outcome
- These probabilities must sum to 1 for each set of outcomes (branches) that are possible from a given decision
- Required economic information for each decision alternative are investments and estimated cash flows

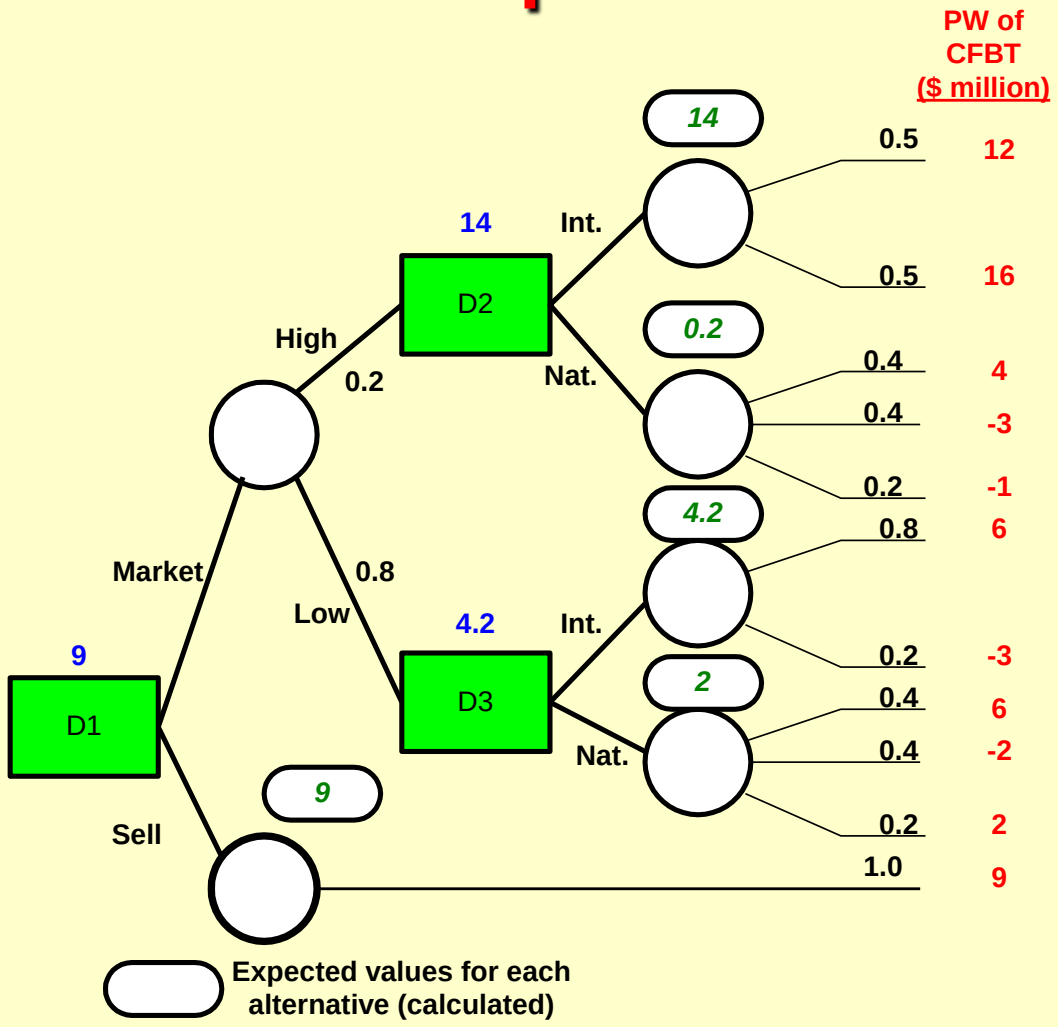
Starting Out

- Assuming the tree logic has been defined…
- Start at the top right of the tree
  - Determine the PW for each outcome branch applying the time value of money
  - Calculate the expected value for each decision alternative as:

$$E(decision) = \sum (outcome\ \text{estimate})\text{P}(\text{outcome})$$

  - At each decision node, select the best ***E*(decision) value**
  - Continue moving to the left of the tree back to the root in order to select the best alternative
  - Trace the best decision path back through the tree

Example 18.8


PW of CFBT ($ million)
14
0.5
12
0.5
16
14
Int.
D2
0.2
High
0.2
Nat.
0.4
4
0.4
-3
0.2
-1
4.2
0.8
6
Market
0.8
Low
4.2
Int.
9
D1
D3
0.2
-3
2
0.4
6
Nat.
0.4
-2
9
0.2
2
Sell
1.0
9
Expected values for each alternative (calculated)

Chapter Summary

- The emphasis is on sensitivity to variation in one or more parameters using a specific measure of worth.
- When two alternatives are compared compute and graph the measure of worth for different values of the parameter to determine when each alternative is better.
- When several parameters are expected to vary over a predictable range, the measure of worth is plotted and calculated using three estimates for a parameter:
  - **Most likely**
  - **Pessimistic**
  - **Optimistic**

Summary - continued

- The combination of parameter and probability estimates results in the expected value relations

$$E(X) = \sum(X)P(X)$$

- This expression is also used to calculate E(revenue), E(cost), E(cash flow) E(PW), and E(i) for the entire cash flow sequence of an alternative.
- E9X0 is a measure of central tendency

Summary - continued

- Decision trees are used to make a series of alternative selections.
- This is a way to explicitly take risk into account.
- It is necessary to make several types of estimates for a decision tree:
  - **Outcomes for each possible decision, cash flows, and probabilities.**
  - **Expected value computations are coupled with those for the measure of worth to "solve" the tree structure.**
  - **Assist is identifying the best alternative stage-by-stage.**

Chapter 18
End of Set