

## Rate of Return Evaluation for Multiple Alternatives

### **Tabulation of Net cash flow:**

It is necessary to prepare a cash-flow tabulation for each of two alternatives as well as the net cash flow that results when the annual cash flows of the two alternatives are compared.

Table\* Format for cash-flow tabulation:

	(1) cash flow	(2)	(3) = (2)-(1)
Year	Alternative A	Alternative B	Net Cash flow
0			
1			
2			
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If the alternatives have equal lives, the years column will go from 0 to n, the life of the alternatives. If the alternatives have unequal lives, the years column will go from 0 to the least common multiple of the two lives. The use of the least – common multiple rule is necessary because rate-of-return analysis on the net cash flow values must always be done over the same number of years for each alternative.

The alternative with the higher initial investment will always be regarded as alternative B, That is

$$\text{Net cash flow} = \text{cash flow}_B - \text{cash flow}_A$$

Incremental Rate-of-Return Evaluation using the present worth method:

The procedure is as follows:

- 1- Order the alternatives and all the one with the smaller initial investment alternative A.
- 2- Prepare the cash-flow and net cash-flow tabulation using the least common multiple of years.
- 3- Draw a net cash-flow diagram.
- 4- Set up and find the incremental return  $i^*_{B-A}$  using the present worth method, eq.\* or a computer program.
- 5- If  $i^*_{B-A} < \text{MARR}$  select alternative A.

If  $i^*_{B-A} > \text{MARR}$ , select alternative B.

### **Ex:**

	<b>Semiautomatic</b>	<b>Fully Automatic</b>
First cost	8000\$	13000\$
Annual disbursement	3500	1600
Salvage value	0	2000
Life, years	10	5

Determine which machine should be selected if the MARR is 15%.

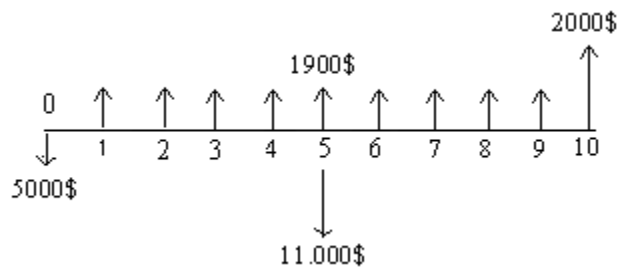
Solution:

- 1- Alternative A is the semiautomatic (s) and alternatives B is the fully automatic (f) machine.
- 2- The cash flows for 10 years are tabulated in table ↓
- 3- The net cash-flow diagram is given in fig. ↓
- 4- The incremental rate of return equation for net cash flow is  

$$0 = -5000 + 1900 (P/A, i\%, 10) - 11.000 (P/F, i\%, 5) + 2000 (P/F, i\%, 10)$$
 Solution shows that  $i^*_{f-s}$  is between 12 and 15%, by interpolation  $i^*_{f-s} = 12.65\%$ .
- 5- Since the rate of return on the extra investment is less than the 15% minimum attractive rate, the lower-cost or semiautomatic machine should be purchased.  
 If  $i^*_{f-s} \geq 15\%$  had been determined, the fully automatic machine would have been selected.

### Cash flow

<u>Year</u>	<u>(1)semiautomatic</u>	<u>(2)Fully automatic</u>	<u>(3) = (2)-(1) Difference</u>
0	-8000	-13000	-5000
1-5	-3500	-1600	+1900
5	...	(+2000-13000)	-11000
6-10	-3500	-1600	+1900
10	...	<u>+2000</u>	<u>+2000</u>
	-43.000	-38000	+5000



The incremental rate of return = 12.65%

Where as if MARR = 15%, we can select semiautomatic, because the break-even  $i$  value is less than 15%.

### **Rate of Return Evaluation using an AW equation:**

The AW equation on the incremental cash flow must be written over the least common multiple of lives, just as for a PW equation.

For equal lives, the AW- based ROR equation for incremental cash flow takes the convenient, general form:

$$O = \pm \Delta P(A/P, i, n) \pm \Delta SV (A/F, i, n) \pm \Delta A$$

Where the  $\Delta$  (delta) symbol represents the difference in P, SV and A in the incremental cash-flow tabulation (net cash flow).

We emphasize once again that the incremental cash flow can be used for the AW method, but to do so correctly, the LCM (least common multiple of lives must be used, just as in the PW method.

If the lives are unequal, we can tabulate the incremental cash flows for the LCM and setup the AW-based relation, to perform the analysis using actual cash flow. The Aw for one cycle of each alternative is developed and  $i_{B-A}^k$  is determining using:

$$O = AW_B - AW_A$$

#### **Example:**

	<b><u>Steam Station</u></b>	<b><u>Nuclear Station</u></b>
First cost, \$	8000	13000
Annual cost \$	3500	1600
Salvage value, \$	0	2000
Life, Years	10	5

Determine which machine should be selected using an AW based ROR method and an MARR of 15% per year?

**Solution:**

$$AW_{\text{Nuclear}} = 13.000 (A/P, i, 5) + 2000 (A/F, i, 5) - 1600$$

$$AW_{\text{year}} = 8000 (A/P, i, 10) - 3500$$

From the form of eq.

$$0 = AW_{\text{nuclear}} - AW_{\text{steam}}$$

$$0 = -13.000 (A/P, I, 5) + 2000 (A/F, I, 5) + 8000 (A/P, I, 10) + 1900$$

The results of manual solution yields an interpolated value of

$$i_{N-S}^* = 12.65\%$$

Since  $i^* = 12.65\%$  is less than a MARR of 15%, the steam station better than Nuclear.

**Selection from Multiple Alternatives using ROR Analysis:**

Multiple alternatives, that is, more two. When the ROR method is applied, the entire investment must return at least the minimum attractive rate of return. When the returns on several alternatives equal or exceed the MARR, at least one of their will be justified in that its

$$ROR > MARR$$

If the return on the extra investment equals or exceeds the MARR, then the extra investment should be made in order to maximize the total return on the money available. Thus, for ROR analysis of multiple alternatives the following criteria are used. Select the one alternative that:

- 1- Requires the largest investment, and
- 2- Indicates that the extra investment over another acceptable alternative is justified.

**The ROR procedure is:**

- 1- Order the alternatives by increasing initial investment (smallest to largest).
- 2- Determine the nature of the cash-flow series: some positive or all negatives.

- a) Some positive cash flows, i.e., incomes. Consider the do-nothing alternative as the defender and compute the incremental cash flows between the do-nothing alternative and the lowest initial-investment alternative. Go to step 3.
  - b) All negative cash flows, i.e., costs only, consider the lowest initial investment alternative as the defender and the next-higher investment as the challenger.
- 3- Set up the ROR relation and determine  $i^*$  for the defender. If  $i^* < \text{MARR}$ , remove the lowest investment alternative from further consideration and compute the overall ROR for the next-higher-investment alternative. Repeat this step until  $i^* \geq \text{MARR}$  for one of the alternatives, then this alternative becomes the defender and the next higher – investment alternative is labeled the challenger.
  - 4- Determine the annual incremental (net) cash flow between the challenger and defender using the relation:  
Incremental cash flow = challenger – defender cash flow
  - 5- Calculate the  $i^*$  for the incremental cash flow series using a PW based or AW-based equation.
  - 6- If  $i^* \geq \text{MARR}$ , the challenger becomes the defender and the previous defender is removed from further consideration. Conversely, if the  $i^* < \text{MARR}$ , the challenger is removed from further consideration and the defender remains the defender against the next challenger.
  - 7- Repeat steps 4 to 6 until only one alternative remains. It is the selected one.

Note that in steps 4 to 6 only two alternatives are compared at any one time. It is very important, therefore, that the correct alternatives be compared.

### **Example:**

If the MARR is 10%, use ROR analysis to select the one economically best location.

<b>Location</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
Building cost \$	-200.000	-275.000	-190.000	-350.000
Annual cash flow\$	22.000	+35.000	+19500	+42.000
Life, years	30	30	30	30

### **Solution:**

- 1- The alternatives are ordered by increasing building cost in table 4, first time.

2- Some positive cash flows are present, use step 2, part (a) to compare location C with the do nothing alternative.

3- The ROR relation is

$$O = -190.000 + 19500 (P/A, i^*, 30)$$

Table , column 1 presents the calculated  $(P/A, i^*, 30)$  factor value of 9.7436 and  $i_c^* = 9.63\%$ , since  $9.63\% < 10\%$  location C is eliminated.

Now the comparison is A to do-nothing, and column 2 shows that  $i_A^* = 10.49\%$ . This eliminates the do-nothing alternative, the defender is now A and the challenger is B.

4- The incremental cash-flow series, column 3, and  $i^*$  for B-to-A comparison is determined from

$$O = -275.000 - (-200.000) + (35.000 - 22.000) (P/A, i^*, 30)$$

$$= -75.000 + 13.000 (P/A, i^*, 30) \quad *$$

5- From the tables, look up the P/A factor at the MARR, which is  $(P/A, 10\%, 30) = 9.4269$ . Now, any P/A value from eq.\* greater than 9.4269 indicates that the  $i^*$  will be less than 10% and is therefore unacceptable. The B/A factor from eq.\* is 5.7692. For reference purposes,  $i^* = 17.28\%$ .

6- Alternative B is justified incrementally (new defender), thereby eliminating A.

7- Comparing D to B (steps 4 and 5) results in the PW relation  $0 = -75.000 + 7000 (P/A, i^*, 30)$  and a P/A value of 10.7143 ( $i_{D-B}^* = 8.55\%$ ). Location D is thereby eliminated and only alternative B remains, it is selected.

Table ↓

Location	C	A	B	D
Building cost, \$	-190.000	-200.000	-275.000	-350.000
Cash flow, \$	+19500	+22000	+35000	+42.000
Projects compared	C to do no	A to do no	B to A	D to B
Incremental cost, \$	-190.000	-200.000	-75.000	-75.000
Incremental cash flow, \$	+19500	+22.000	+13.000	+7000
$(P/A, i^*, 30)$	9.7436	9.0909	5.7692	10.7143
$i^* \%$	9.63	10.49	17.28	8.55
Increment Justified?	No	Yes	Yes	No
Project selected	Do-no	A	B	B