Chapter 7

Rate of Return One Project

LEARNING OUTCOMES

1. **Understand meaning of ROR**
2. **Calculate ROR for cash flow series**
3. **Understand difficulties of ROR**
4. **Determine multiple ROR values**
5. **Calculate External ROR (EROR)**
6. **Calculate *r* and *i* for bonds**

Interpretation of ROR

Rate paid on *unrecovered balance* of borrowed money such that final payment brings balance to exactly zero with interest considered

ROR equation can be written in terms of PW, AW, or FW

Use trial and error solution by *factor* or *spreadsheet*

Numerical value can range from *-100% to infinity*

ROR Calculation and Project Evaluation

- To determine ROR, find the *i** value in the relation

$$PW = 0 \quad \text{or} \quad AW = 0 \quad \text{or} \quad FW = 0$$

- Alternatively, a relation like the following finds i*

$$PW_{outflow} = PW_{inflow}$$

- For evaluation, a project is economically viable if

$$i^* \geq MARR$$

Finding ROR by Spreadsheet Function

Using the RATE function

= RATE(n,A,P,F)

P = $-200,000   A = $-15,000

n = 12   F = $435,000

Function is

= RATE(12,-15000,-200000,450000)

Display is i* = 1.9%

Using the IRR function

= IRR(first_cell, last_cell)

	A	B
1	Year	CF,$
2	0	-200,000
3	1	-15,000
4	2	-15,000
5	3	-15,000
6	4	-15,000
7	5	-15,000
8	6	-15,000
9	7	-15,000
10	8	-15,000
11	9	-15,000
12	10	-15,000
13	11	-15,000
14	12	435,000
15	**IRR function**	**1.9%**

= IRR(B2:B14)

ROR Calculation Using PW, FW or AW Relation

ROR is the unique i* rate at which a PW, FW, or AW relation equals exactly 0

Example: An investment of $20,000 in new equipment will generate income of $7000 per year for 3 years, at which time the machine can be sold for an estimated $8000. If the company's MARR is 15% per year, should it buy the machine?

Solution:: The ROR equation, based on a PW relation, is:

$$0 = -20,000 + 7000(P/A,i^*,3) + 8000(P/F,i^*,3)$$

Solve for i* by trial and error or spreadsheet: i* = 18.2% per year

Since i* > MARR = 15%, *the company should buy the machine*

Special Considerations for ROR

- *Incremental analysis* necessary for multiple alternative evaluations (discussed later)

- May get *multiple i* values* (discussed later)

- i* assumes *reinvestment* of positive cash flows earn *at i* rate* (may be unrealistic)

Multiple ROR Values

Multiple i* values may exist when there is more than one sign change in net cash flow (CF) series.
Such CF series are called non-conventional

Two tests for multiple i* values:

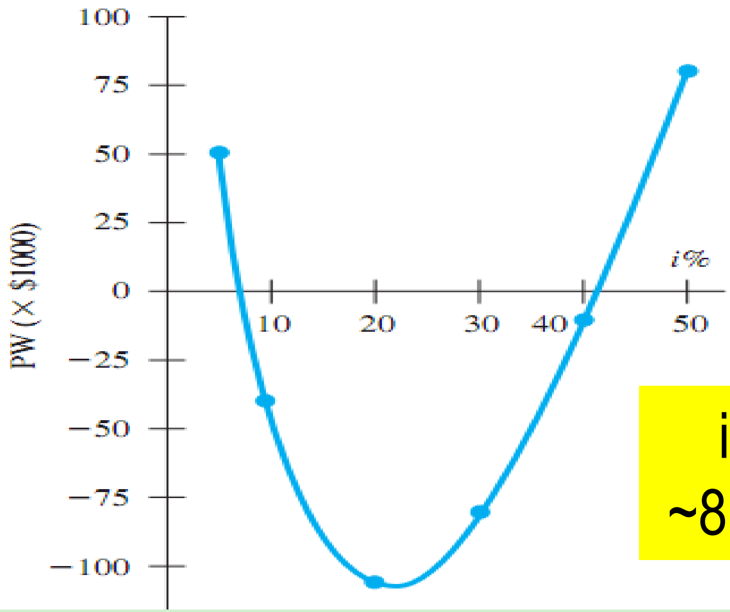
Descarte's rule of signs: total number of real i* values is ≤ the number of sign changes in *net cash flow series.*

Norstrom's criterion: if the *cumulative cash flow* starts off negatively and has only *one sign change*, there is only one positive root .

Plot of PW for CF Series with Multiple ROR Values

Year	Cash Flow ($1000)	Sequence Number	Cumulative Cash Flow ($1000)
0	+2000	S_0	+2000
1	−500	S_1	+1500
2	−8100	S_2	−6600
3	+6800	S_3	+200

i%	5	10	20	30	40	50
PW ($1000)	+51.44	−39.55	−106.13	−82.01	−11.83	+81.85



i* values at ~8% and ~41%

Example: Multiple i* Values

Determine the maximum number of i* values for the cash flow shown below

Year	Expense	Income	**Net cash flow**	**Cumulative CF**
0	-12,000	-	-12,000	-12,000
1	-5,000	+ 3,000	-2,000	-14,000
2	-6,000	+9,000	+3,000	-11,000
3	-7,000	+15,000	+8,000	-3,000
4	-8,000	+16,000	+8,000	+5,000
5	-9,000	+8,000	-1,000	+4,000

Solution:

The sign on the net cash flow changes twice, indicating two possible i* values

The cumulative cash flow begins negatively with *one sign change*

Therefore, there is only one i* value (i* = 8.7%)

Removing Multiple i* Values

Two new interest rates to consider:

- ***Investment rate i_i*** **– rate at which extra funds are *invested* *external* to the project**

- ***Borrowing rate i_b*** **– rate at which funds are borrowed *from an external source* to provide funds to the project**

Two approaches to determine External ROR (EROR)

- (1) Modified ROR (MIRR)
- (2) Return on Invested Capital (ROIC)

Modified ROR Approach (MIRR)

Four step Procedure:

- Determine PW in *year 0* of all negative CF at i_b
- Determine FW in *year n* of all positive CF at i_i
- Calculate EROR = i' by FW = PW(F/P,i',n)
- If i' ≥ MARR, project is economically justified

Example: EROR Using MIRR Method

For the NCF shown below, find the EROR by the MIRR method if MARR = 9%, i_b = 8.5%, and i_i = 12%

Year	0	1	2	3
NCF	+2000	-500	-8100	+6800

Solution:

PW_0 = -500(P/F,8.5%,1) - 8100(P/F,8.5%,2)
= $-7342

FW_3 = 2000(F/P,12%,3) + 6800
= $9610

PW_0(F/P,i',3) + FW_3 = 0

$-7342(1 + i')^3 + 9610 = 0$

i' = 0.939 (9.39%)

Since i' > MARR of 9%, project is justified

Return on Invested Capital Approach

- Measure of how effectively project uses funds that ***remain internal to project***
- ROIC rate, i'', is determined using ***net-investment procedure***

Three step Procedure

(1) Develop series of FW relations for each year t using:

$$F_t = F_{t-1}(1 + k) + NCF_t$$

where: $k = i_i$ if $F_{t-1} > 0$ and $k = i''$ if $F_{t-1} < 0$

(2) Set future worth relation for last year n equal to 0 (i.e., $F_n = 0$); solve for i''

(3) If i'' ≥ MARR, *project is justified*; otherwise, *reject*

ROIC Example

For the NCF shown below, find the EROR by the ROIC method if MARR = 9% and $i_i = 12\%$

Year	0	1	2	3
NCF	+2000	-500	-8100	+6800

Solution:

Year 0: $F_0 = \$+2000$ — $F_0 > 0$; invest in year 1 at $i_i = 12\%$

Year 1: $F_1 = 2000(1.12) - 500 = \$+1740$ — $F_1 > 0$; invest in year 2 at $i_i = 12\%$

Year 2: $F_2 = 1740(1.12) - 8100 = \$-6151$ — $F_2 < 0$; use i'' for year 3

Year 3: $F_3 = -6151(1 + i'') + 6800$ — Set $F_3 = 0$ and solve for i''

$$-6151(1 + i'') + 6800 = 0$$

$$i'' = 10.55\%$$

Since i'' > MARR of 9%, project is justified

Important Points to Remember

About the computation of an EROR value

- EROR values are dependent upon the selected investment and/or borrowing rates
- Commonly, multiple i* rates, i' from MIRR and i'' from ROIC have different values

About the method used to decide

- For a definitive economic decision, set the MARR value and *use the PW or AW method* to determine economic viability of the project

ROR of Bond Investment

Bond is ***IOU*** with face value **(V)**, coupon rate **(b)**, no. of payment periods/year **(c)**, dividend **(I)**, and maturity date **(n)**. Amount paid for the bond is **P.**

$$I = Vb/c$$

General equation for i*: $0 = - P + I(P/A,i^*,n_{x}c) + V(P/F,i^*,n_{x}c)$

A $10,000 bond with 6% interest payable quarterly is purchased for $8000. If the bond matures in 5 years, what is the ROR (a) per quarter, (b) per year?

Solution: **(a)** I = 10,000(0.06)/4 = $150 per quarter

ROR equation is: **0 = -8000 + 150(P/A,i*,20) + 10,000(P/F,i*,20)**

By trial and error or spreadsheet: i* = 2.8% per quarter

(b) Nominal i* per year = 2.8(4) = 11.2% per year

Effective i* per year = $(1 + 0.028)^4 – 1$ = 11.7% per year

Summary of Important Points

- ROR equations can be written in terms of PW, FW, or AW and usually require *trial and error solution*
- i* assumes *reinvestment* of positive cash flows *at i* rate*
- More than 1 sign change in NCF may cause *multiple i* values*
- Descarte's rule of signs and Norstrom's criterion useful when *multiple i* values* are suspected
- EROR can be calculated using MIRR or ROIC approach. Assumptions about investment and borrowing rates is required.
- General ROR equation for bonds is

$$0 = - P + I(P/A,i^*,n_xc) + V(P/F,i^*,n_xc)$$