Chapter 6

Annual Worth Analysis

Chapter 5 – Annual Worth Analysis

PURPOSE

Compare alternatives using an annual worth basis

TOPICS

- **AW calculations**
- **Alternative evaluation using AW**
- **AW of permanent investments**
- **Spreadsheet usage**

Sec 5.1 – AW Advantage

- AW is also called
  - AE – annual equivalent
  - EAC – equivalent annual cost
  - EUAC (or EUAW) – equivalent uniform annual cost (or worth)
- **Compare alternatives over only one life cycle – no LCM to meet equal service assumption**
- Same AW amount assumed for future cycles, and estimates change with inflation rate
- If this assumption not correct; use a study period and specific estimates for cash flows

Sec 5.1 – AW and Multiple Life Cycles

AW of cycle 1 with i = 22%

AW = -20,000(A/P,22%,3) -8000

= $-17,793 per year

AW over 2 cycles

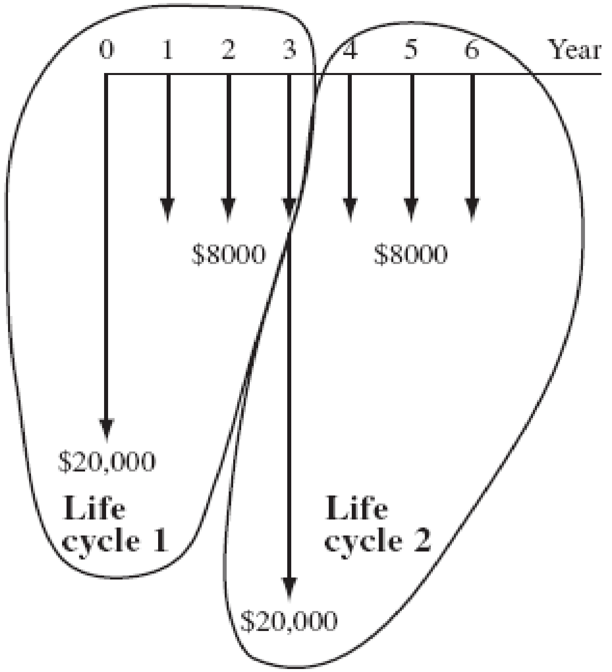
AW = -20,000(A/P,22%,3)

- 20,000(A/P,22%,6) -8000

= $-17,793 per year

Demonstrates that AW will be the same for any number of cycles

Estimated costs over two life cycles



Sec 5.1 – Calculating Project AW

- ✓ Use project PW or FW to determine AW with n = LCM for equal service or length of study period

$$AW = PW(A/P,i\%,n) = FW(A/F,i\%,n)$$

- ✓ AW is the sum of 2 separate components:
  - Capital recovery (CR)
  - Equivalent annual A of operating costs (A of AOC)

AW = CR + A of AOC

Sec 5.1 – Capital Recovery (CR)

What does CR mean?

CR is the annual equivalent cost A incurred by initially spending an amount P on an asset (project) and using it for n years plus the return on the investment P at i% per year

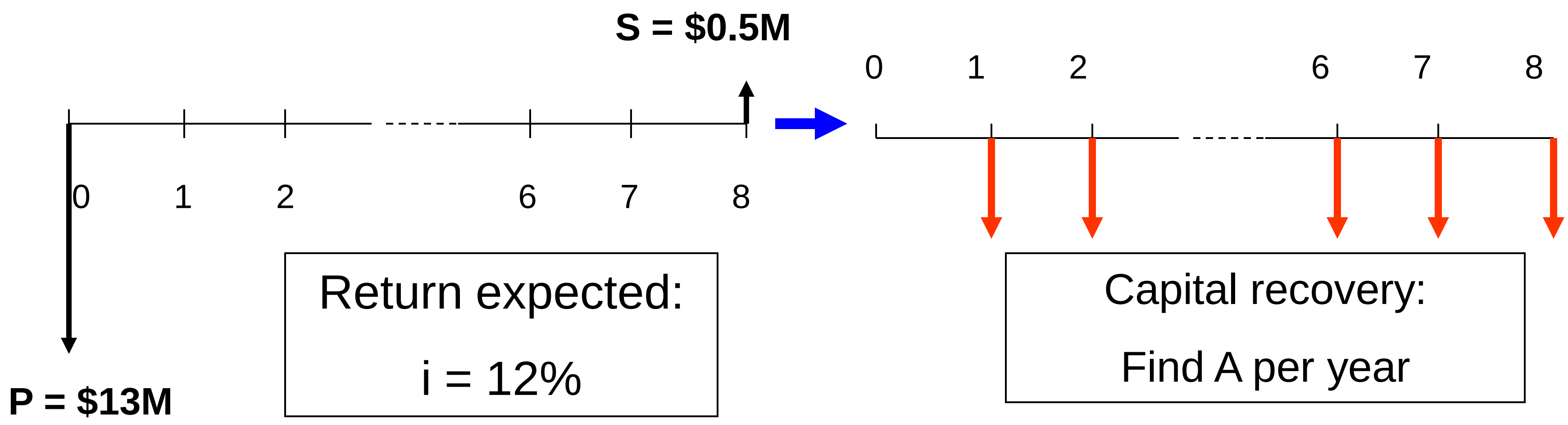
Example: **Project costs P = $-13 million**

Estimated salvage S = $0.5 million

Estimated life = 8 years

Expected return i = 12% per year

Sec 5.1 – Capital Recovery



Capital recovery is the equivalent annual amount A to recover $13M at 12% per year if the salvage is $0.5M after 8 years

$$CR = -13M(A/P,12\%,8) + 0.5M(A/F,12\%,8)$$

$$= \$-2{,}576M \text{ per year}$$

Conclusion: Project must develop revenue of at least $2.576M per year to recover P and make 12% on the investment

Sec 5.1 – Capital Recovery Formula

- **General formula for CR**

$$CR = -P(A/P,i,n) + S(A/F,i,n)$$

- Alternative formula to calculate CR

$$CR = -(P-S)(A/P,i,n) + Si$$

- For previous estimates, using alternate

$$CR = -(13 – 0.5)(A/P,12\%,8) + 0.5(0.12)$$
$$= \$-2,576M \text{ per year}$$

Sec 5.1 – Calculating AW

AW: sum of CR plus A value of annual costs

AW = CR + A of AOC

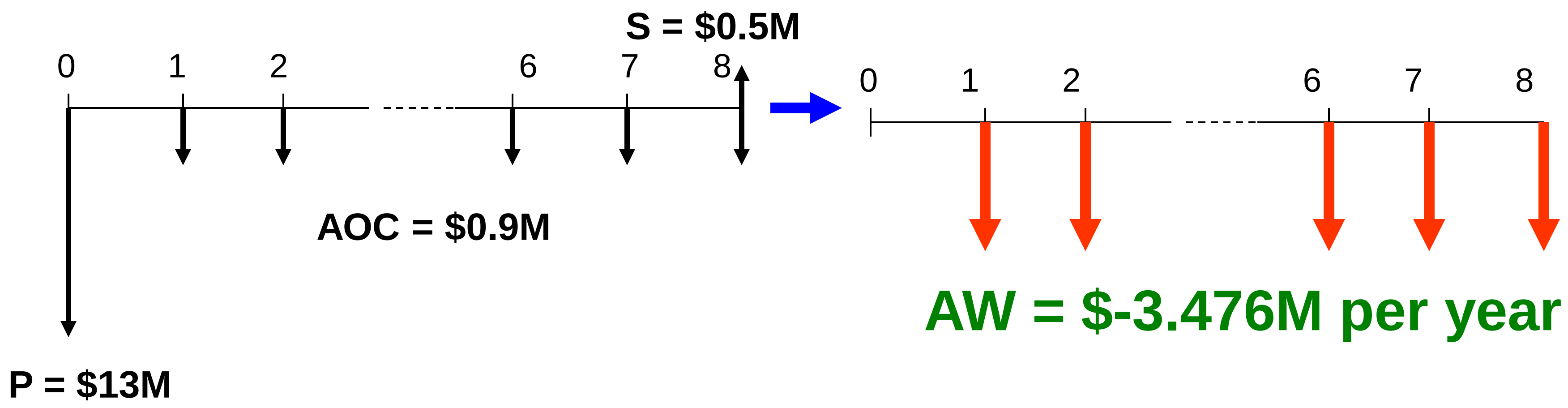
- **If AOC estimate is same each year, add A to CR**
- **If AOC varies, find A value first, then add to CR**

Example: For previous project, estimate AOC at $0.9M each year.

AW = -2.576M – 0.9M = $-3.476M per year

Conclusion: Project must develop revenue of at least $3.476M per year to recover P, A and return 12% per year

Sec 5.1 – Example Cash Flows for AW



This is the AW for all future life cycles of 8 years each, provided costs estimates change at the inflation or deflation rate

Sec 5.2 – AW-based Evaluation

Single project analysis

- **Calculate AW at stated MARR**
- **Acceptance criterion:**
  If AW ≥ 0, project is economically justified

Multiple alternatives

- **Calculate AW of each alternative at MARR over respective life or study period**
- **Selection criterion:**
  Select alternative with most favorable AW value, that is,

  numerically largest AW value

Sec 5.2 – AW Evaluation – Example 1

Equipment	X	Y
First cost, $	40,000	75,000
AOC, $ per year	25,000	15,000
Life, years	4	6
Salvage value, $	10,000	7,000

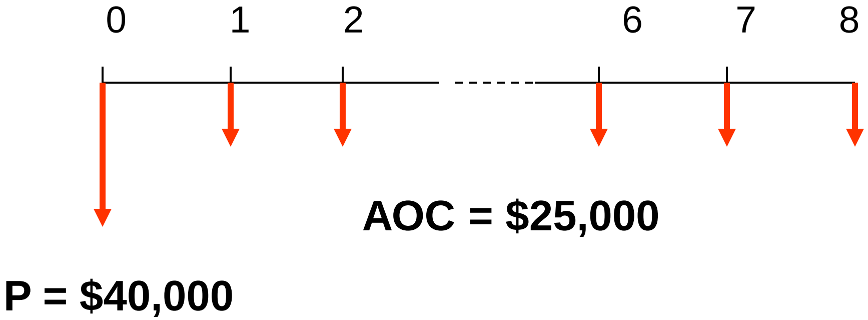
To select the more economic alternative at i = 12%, compare AW_X over 4 years with AW_Y over 6 years

AW_X = -40,000(A/P,12%,4) + 10,000(A/F,12%,4) – 25,000
= $-36,077

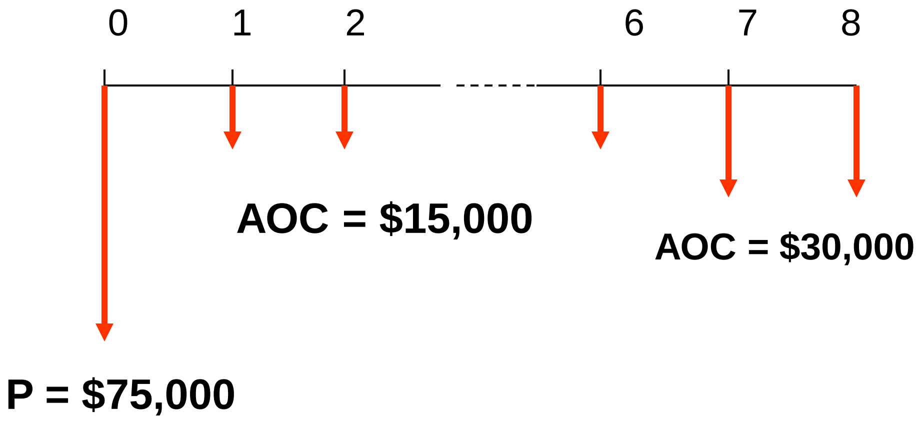
AW_Y = -75,000(A/P,12%,6) + 7,000(A/F,12%,6) – 15,000
= $-32,380 **Select Y**

Sec 5.2 – AW Evaluation – Example 2

In the previous example, assume the selected equipment will be retained for a total of 8 years. Additionally, assume after 8 years S = 0 for X and Y; and AOC_X continues at $25,000, but AOC_Y doubles starting in year 7



Alternative X



Alternative Y

Sec 5.2 – AW Evaluation – Example 2

Study period is n = 8 years

AW_X = -40,000(A/P,12%,8) – 25,000
= $-33,052

AW_Y = -75,000(A/P,12%,8) – 15,000
- 15,000(F/A,12%,2)(A/F,12%,8)
= $-32,683

Still select Y as the cheaper choice;

however the advantage of Y over X is now only 1/10 of the previous AW values

Sec 5.3 – AW of Permanent Investment

- AW of alternative that will last 'forever'
- This is the annual worth equivalent of capitalized cost (CC)
- Solve for AW in relation PW = AW(1/i) from chapter 4

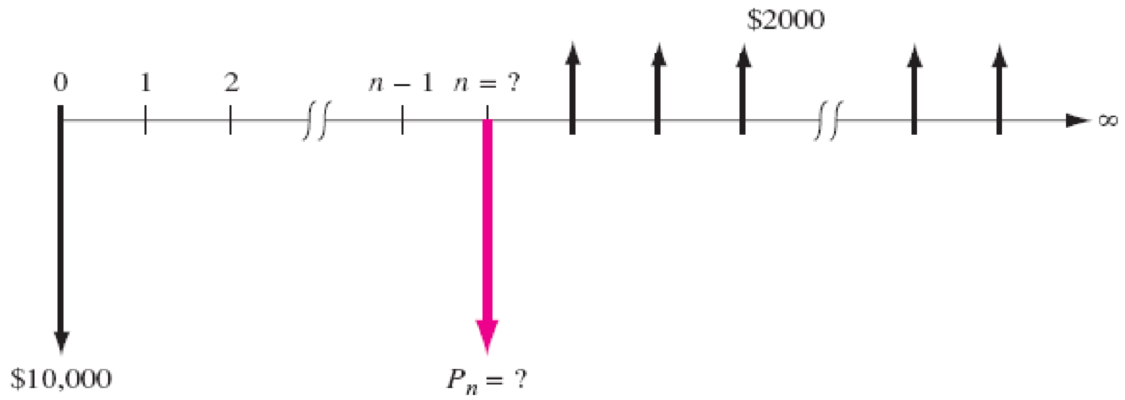
$$AW = PW(i) = CC(i)$$

Procedure: Regular interval cash flows – find AW over one cycle

Non-regular intervals – find P, then calculate AW = P(i) for long-term AW value

Sec 5.3 – AW of Permanent Investment

Example: How long must $10,000 remain invested at 5% per year so that $2000 per year can be withdrawn forever?



P = A/i determines total in year n to generate $2000 forever

P = 2000/0.05 = $40,000

F/P factor determines n if money grows at 5%, with no withdrawals

40,000 = 10,000(F/P,5%,n)

n = 28.4 years

Sec 5.3 – AW of Permanent Investment

Example 5.6 demonstrates comparison of short-lived and long-lived (forever) alternatives at i = 5%

For each proposal, determine CR and AW values

Prop	P and S	AOC	Life	CR and AW
A	P = $650,000 S = $17,000	A = $170,000	10	CR over 10; add AOC
B	P = $4 million	A = $5,000 ------------------ $30,000 every 5 years	'forever'	CR over ∞; add AOC; add periodic repair over 5 years
C	P = $6 million	A = $3,000	50	CR over 50; add AOC

Sec 5.3 – Example 5.6 (cont)

AW_A = - 650,000(A/P,5%,10) + 17,000(A/F,5%,10) - 170,000

= **$-252,824**

AW_B = - 4,000,000(0.05) - 5,000 - 30,000(A/F,5%,5)

= **$-210,429**

AW_C = - 6,000,000(A/P,5%,50) - 3,000

= **$-331,680**

Select proposal B

Sec 5.4 – Spreadsheet Evaluation Using AW Analysis

Use PMT function with n = life of alternative or study period

= PMT(i%,n,P,-S) – A

- This provides sum of CR and A of AOC, if AOC is uniform
- Note signs on P, S and A to obtain correct sign on result
- n values are one life cycle for each alternative, since LCM is not necessary for AW-based evaluation

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- If AOC is not uniform, enter annual estimates, find their P value, then use PMT. Or, use embedded NPV function

= PMT(i%,n, P+NPV(i%,year_1_cell,year_n_cell))

Sec 5.4 – Spreadsheet Evaluation Using AW

Spreadsheet-based AW analysis in Example 5.6 –
3 alternatives with different lives

	A	B	C	D	E	F	G	H	I	J
1										
2	Recap of Estimates				Calculation of AW values					
3	Proposal	A	B	C	Proposal	A	B	C		
4	First cost, $	650,000	4,000,000	6,000,000	Capital recovery, CR	-82,826	-200,000	-328,660		
5	Life, years	10	permanent	50						
6	Annual costs, $/year	170,000	5000	3000	Annual costs, A	-170,000	-5,000	-3,000		
7	Periodic cost, $		30,000 every 5 years				-5,429			
8	Salvage, $	17,000			AW value, CR + A	-252,826	-210,429	-331,660		
9										
10										
11										
12										

= -4000000*(0.05)

= PMT(5%,50,6000000)

= PMT(5%,5,,30000)

= PMT(5%,10,650000,-17000)

AW$_A$; n = 10: PMT(5%,10,650000,-17000) - 170,000 = $-252,826

AW$_B$; n = ∞: -4,000,000*0.05 - 5,000 + PMT(5%,5,,30000) = $-210,429

AW$_C$; n = 50: PMT(5%,50,6000000) - 3,000 = $-331,660

Sec 5.4 – AW Spreadsheet Evaluation – Sign Usage

- On the spreadsheet, note the careful use of minus signs to ensure a correct PMT function response.

➢ First costs and expenses have positive signs in PMT statement

Alternatives A and C: = PMT(i%,n,P,-S)

Alternative B periodic expense: = PMT(i%,n,,30000)

➢ **PMT function is preceded by + sign**