

Chapter 5

Present Worth and Capitalized cost Evaluation

Chapter 4 – Present Worth Analysis

PURPOSE

Identify types of alternatives; and compare alternatives using a present worth basis

TOPICS

- **Formulating alternatives**
- **Single and equal-life alternatives**
- **Different-life alternatives**
- **Capitalized cost alternative evaluation**
- **Independent alternatives**
- **Spreadsheet usage**

Sec 4.1 – Formulating Alternatives

Types of alternatives

- ✓ **Mutually exclusive (ME)** - only one viable project can be accepted. Do-nothing (DN) alternative is selected if none are justified economically
- ✓ **Independent** - more than one project can be selected. DN is one of the projects
- ✓ **Do-nothing** – maintain status quo/current approach

Types of cash flow estimates for an alternative

- **Revenue** – estimates include costs, revenues and (possibly) savings
- **Cost** – only cost estimates included; revenues assumed equal for all alternatives

Sec 4.1 – Formulating Alternatives

Much of the emphasis in professional engineering practice is on ME, cost alternatives. However, all tools in Eng Econ can be used to evaluate ME and independent alternatives that are revenue- or cost-based. Examples of both are included later.

Notes: P value of cash flows is now called PW, or present worth

P now represents first cost of an alternative

Sec 4.2 – PW of a Single Alternative

Single project analysis

- Calculate PW at stated MARR
- **Criterion:** If $PW \geq 0$, project is economically justified

Example: MARR = 10%

First cost, $P = \$-2500$

Annual revenue, $R = \$2000$

Annual cost, $AOC = \$-900$

Salvage value, $S = \$200$

Life, $n = 5$ years

$$\begin{aligned} PW &= P + S(P/F, 10\%, 5) \\ &\quad + (R - AOC)(P/A, 10\%, 5) \\ &= -2500 + 200(P/F, 10\%, 5) \\ &\quad + (2000 - 900)(P/A, 10\%, 5) \\ &= \$1794 \end{aligned}$$

PW > 0; project is economically justified

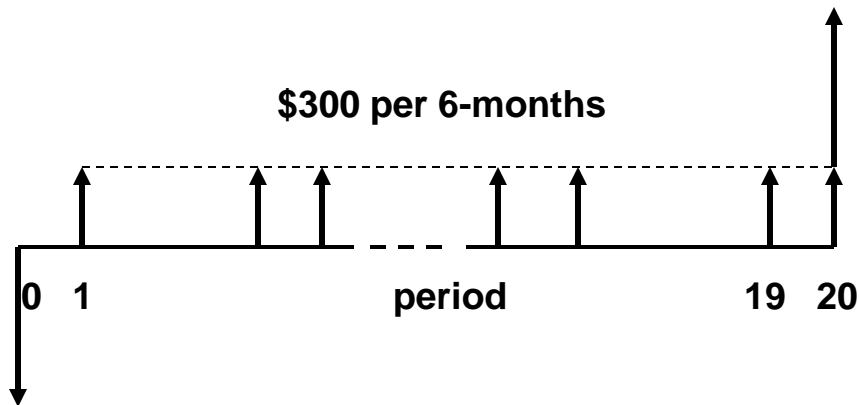
Sec 4.2 – Single Alternative Example – Bond Investment

- **Bond** – like an IOU; issued by corporations and all levels of government to raise capital
- **Face value, V** – Value of bond; this amount is returned at end of bond's life. Purchase price may be discounted
- **Life, n** – years to bond maturity, e. g., 5, 10, 20+
- **Dividend, I** – periodic interest payments to purchaser based on coupon rate, b

$$I = \frac{(\text{bond face value})(\text{bond coupon rate})}{\text{number of payments per year}} = \frac{Vb}{c}$$

Sec 4.2 – Single Alternative Example – Bond Investment

Example: A 10-year \$10,000 6% coupon rate bond is purchased at 5% discount. Bond dividend is paid semi-annually. Will the investor make 7% per year compounded semiannually?



Calculate PW at $i = 3.5\%$ per 6-month period for $n = 20$ periods

$$I = (10,000)(0.06)/2 = \$300$$

$$\begin{aligned} \text{PW} &= -10,000(0.95) \\ &\quad + 10,000(P/F, 3.5\%, 20) \\ &\quad + 300(P/A, 3.5\%, 20) \\ &= \$-210.62 \end{aligned}$$

Bond will not make the required return

Sec 4.2 – Equal-life ME Alternatives

- Calculate PW of each alternative at MARR
- Equal-service of alternatives is assumed
- **Selection criterion:** Select alternative with most favorable PW value, that is,

numerically largest PW value

PW_1	PW_2	Select
\$-1,500	\$-500	2
-2,500	500	2
2,500	1,500	1

Note : Not
the absolute
value

Sec 4.2 – Equal-life ME Alternatives

Example: Two ME cost alternatives for traffic analysis. Revenues are equal. MARR is 10% per year. Select one. (cont→)

Estimate	Electric-powered	Solar-powered
P, \$/unit	-2,500	-6,000
AOC, \$/year	-900	-50
S, \$	200	100
n, years	5	5

Sec 4.2 – Equal-life ME Alternatives

Determine PW_E and PW_S ; select larger PW

$$\begin{aligned}PW_E &= -2500 - 900(P/A, 10\%, 5) + 200(P/F, 10\%, 5) \\ &= \$-5788\end{aligned}$$

$$\begin{aligned}PW_S &= -6000 - 50(P/A, 10\%, 5) + 100(P/F, 10\%, 5) \\ &= \$-6127\end{aligned}$$

Conclusion: $PW_E > PW_S$; select electric-powered

Sec 4.3 – Different-life Alternatives

- PW evaluation always requires *equal-service* between all alternatives
- Two methods available:
 - Study period (same period for all alternatives)
 - Least common multiple (LCM) of lives for alternatives
- Study period method is recommended
- **Evaluation approach:** Determine each PW at stated MARR; select alternative with numerically largest PW

Sec 4.3 – Different-life Alternatives

Study Period of length n years (periods)

- n is same for each alternative
- If life $> n$, use market value estimate in year n for salvage value
- If life $< n$, estimate costs for remaining years

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Estimates outside time frame
of the study period are ignored

Sec 4.3 – Different-life Alternatives

LCM Method

- ❖ **Assumptions** (may be unrealistic at times)
 - ✓ Same service needed for LCM years (e.g., LCM of 5 and 9 is 45 years!)
 - ✓ Alternatives available for multiple life cycles
 - ✓ Estimates are correct over all life cycles (true only if cash flow estimate changes match inflation/deflation rate)



Evaluation approach: obtain LCM, repeat purchase and life cycle for LCM years; calculate PW over LCM; select alternative with most favorable PW

Sec 4.3 - Different-life Analysis - Example

	Location A	Location B
First cost, \$	−15,000	−18,000
Annual lease cost, \$ per year	−3,500	−3,100
Deposit return, \$	1,000	2,000
Lease term, years	6	9

Use PW to select lower-cost alternative:

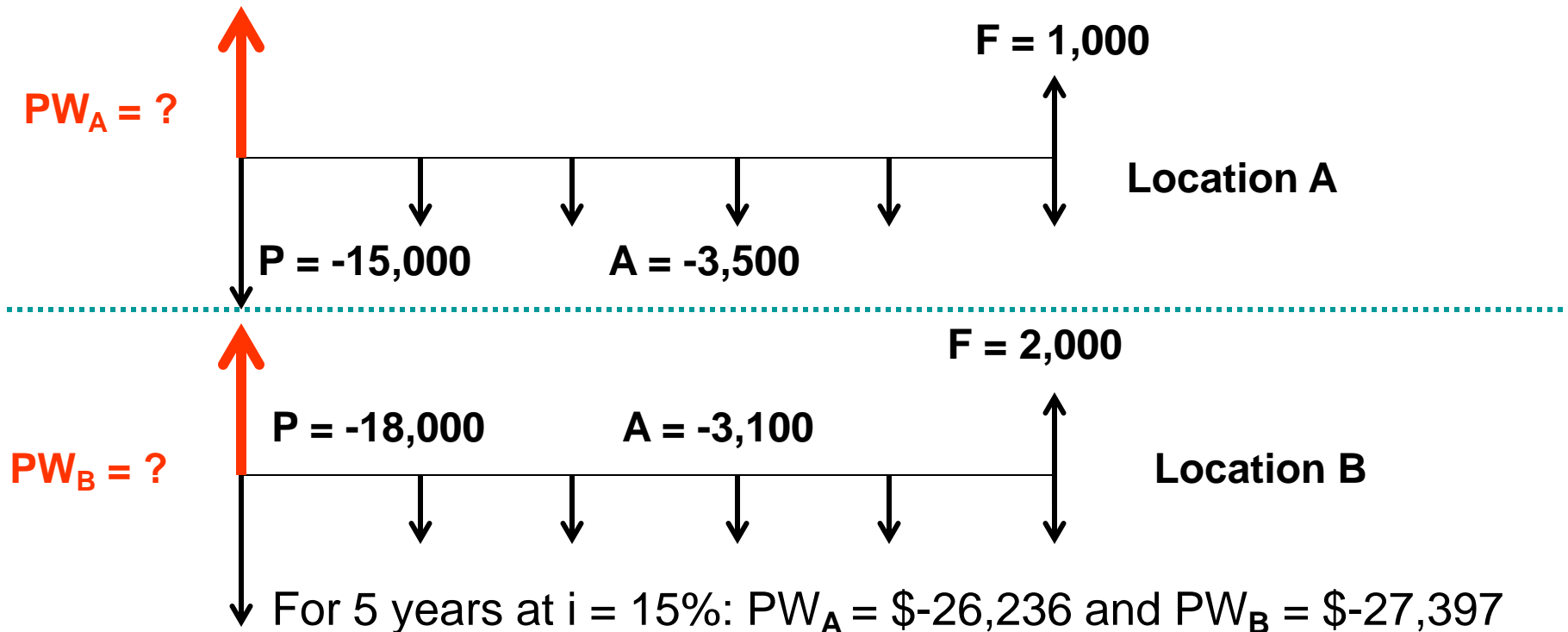
- For 5-year study period
- Using LCM of alternatives' lives

Assume MARR = 15% per year (cont →)

Sec 4.3 - Different-life Analysis - Example

Study period of 5 years

Assume deposit returns are good estimates after 5 years



Select Location A with lower PW of costs

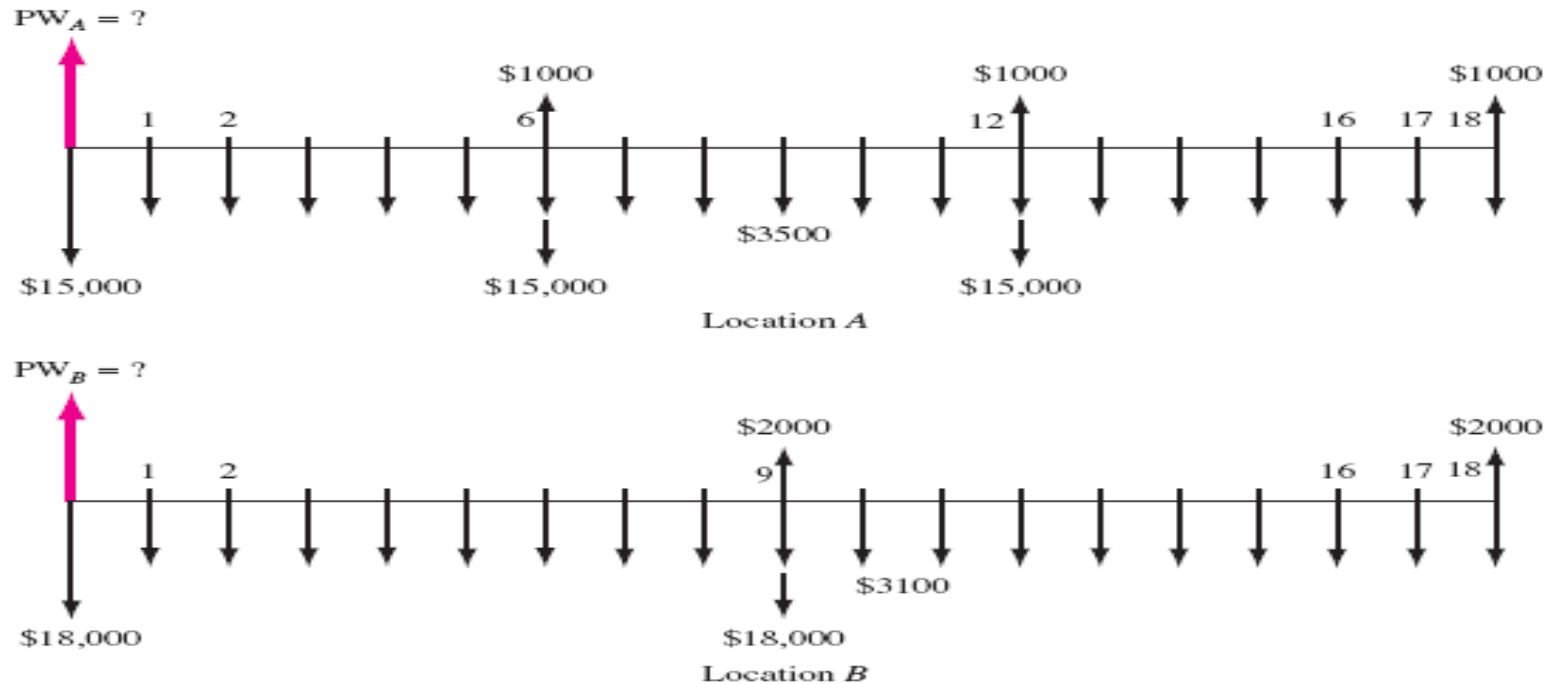
Sec 4.3 - Different-life Analysis - Example

LCM evaluation

- LCM is 18 years
- Repurchase A twice (years 6 and 12)
- Repurchase B once (year 9)
- Assume all cash flow estimates (including first cost end-of-lease 'deposit return') are correct for repeated life cycles to total 18 years

(cont →)

Sec 4.3 - Different-life Analysis - Example



For 18 years at MARR = 15%: $PW_A = \$-45,036$

For 18 years at MARR = 15%: $PW_B = \$-41,384$

Select location B

Note: Selection changed from 5-year study period

Sec 4.3 – Future Worth Evaluation

- FW evaluation of alternatives is especially applicable for LARGE capital investment situations when maximizing the future worth of a corporation is important
 - e.g., buildings, power generation, acquisitions
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- **Evaluation approach:** Determine FW value from cash flows or PW with an n value in F/P factor
 - ❖ equal to study period, or
 - ❖ equal to LCM of alternatives' lives

Sec 4.3 – Life Cycle Costing (LCC)

- Another application of PW analysis
- Useful when entire life cycle of a system is under evaluation
- e.g., new car model or aircraft model; introducing new technology
- PW evaluation must include cost estimates for *all stages* of the product or service:
 - Design (initial and detail)
 - Development
 - Production cost
 - Marketing cost
 - Operating costs
 - Warranty commitments
 - Phase-out costs
 - etc.

Sec 4.4 – Capitalized Cost (CC)

- PW of alternative that will last ‘forever’
- Especially applicable to public project evaluation (dams, bridges, irrigation, hospitals, police, etc.)
- CC relation is derived using the limit as $n \rightarrow \infty$ for the P/A factor

$$PW = A(P/A, i\%, n) = A \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right]$$

$$PW = A[1/i]$$

Sec 4.4 – Capitalized Cost

- Refer to PW as CC when n is large (can be considered infinite). Then

$$CC = \frac{A}{i} = \frac{AW}{i}$$

and

$$AW = CC \times i$$

Example: If \$10,000 earns 10% per year, \$1,000 is interest earned annually for eternity. Principal remains in tact

- Cash flows for CC computations are of two types -- recurring and nonrecurring

Sec 4.4 – Capitalized Cost

Procedure to find CC

1. Draw diagram for 2 cycles of recurring cash flows and any nonrecurring amounts
2. Calculate PW (CC) for all nonrecurring amounts
3. Find AW for 1 cycle of recurring amounts; then add these to all A series applicable for all years 1 to ∞ (or long life)
4. Find CC for amount above using $CC = AW/i$
5. Add all CC values (steps 2 and 4)

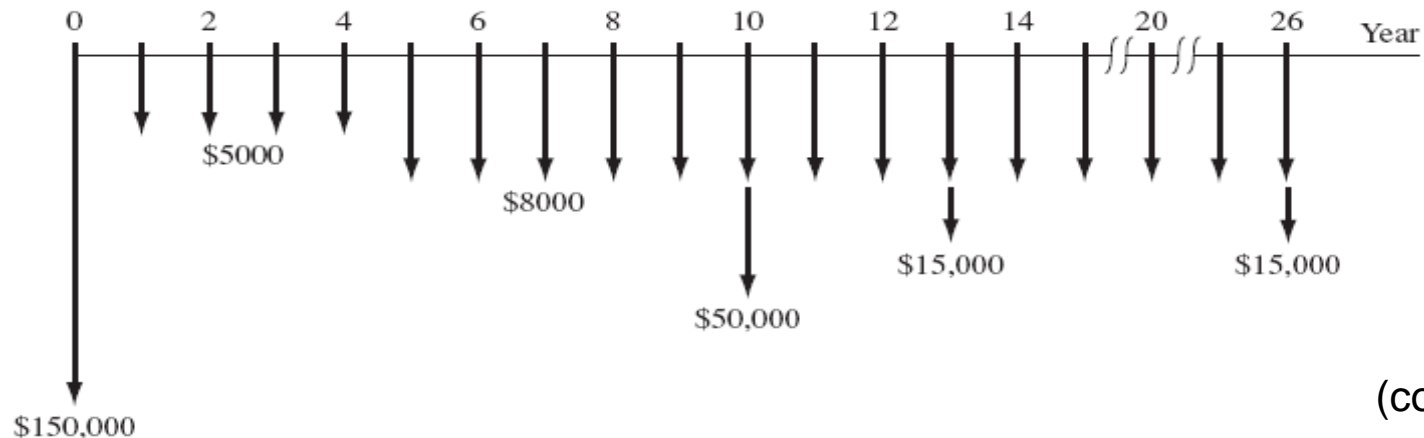
Sec 4.4 – CC Computation - Example

Find CC and A values at $i = 5\%$ of long-term public project with cash flows below. Cycle time is 13 years.

Nonrecurring costs: first \$150,000; one-time of \$50,000 in year 10

Recurring costs: annual maintenance of \$5000 (years 1-4) and \$8000 thereafter; upgrade costs \$15,000 each 13 years

Step 1



(cont →)

Sec 4.4 – CC Computation - Example

2. CC of nonrecurring costs:

$$CC_1 = -150,000 - 50,000(P/F, 5\%, 10) = \text{\$-180,695}$$

3. AW of recurring \$15,000 upgrade:

$$AW = -15,000(A/F, 5\%, 13) = \text{\$-847 per year}$$

AW of recurring maintenance costs years 1 to ∞ :

$$AW = \text{\$-5000 per year forever}$$

4. CC of extra \$3000 maintenance for years 5 to ∞ :

$$CC_2 = -3000(P/F, 5\%, 4)/0.05 = \text{\$-49,362}$$

CC for recurring upgrade and maintenance costs:

$$CC_3 = (-847 - 5000)/0.05 = \text{\$-116,940}$$

5. Total CC obtained by adding all three CC components

$$CC_T = -180,695 - 49,362 - 116,940 = \text{\$-346,997}$$

The AW value is the annual cost forever:

$$AW = CC \times i = -346,997(0.05) = \text{\$-17,350}$$

Sec 4.4 – CC Evaluation of Alternatives

- For two long-life or infinite-life alternatives:
SELECT ALTERNATIVE WITH LOWER CC OF COSTS

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- For one infinite life and one finite life:
Determine CC for finite life alternative using
AW of 1 life cycle and relation $CC = AW/i$
SELECT ALTERNATIVE WITH LOWER CC OF COSTS

Sec 4.4 – CC Evaluation of Alternatives - Example

1 long-term (assumed infinite); 1 finite life

Long-term alternative (LT): \$8 million now; \$25,000 renewal annual contract

Short-term alternative (ST): \$2.75 million now; \$120,000 AOC; life is $n = 5$ years

Select better at MARR = 15% per year

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$$CC_{LT} = -8,000,000 - 25,000/0.15 = \$-8.17 \text{ million}$$

$$CC_{ST} = AW/0.15$$

$$= [-2,750,000(A/P, 15\%, 5) - 120,000]/0.15$$

$$= \$-6.27 \text{ million}$$

Conclusion: Select ST with lower CC of costs

Sec 4.5 – Independent Projects

Situation: Select from several (m) projects.

Revenue and costs are estimated for each

Solution approach: Basically different from that for ME alternatives

- ❖ One-time projects; no equal-service evaluation necessary; LCM not necessary
- ❖ Two types of budget situations are possible -- **no limit** or **stated limit**
- ❖ **No limit:** select from none (DN alternative) to all m projects using criterion

SELECT ALL PROJECTS WITH $PW \geq 0$ AT MARR

Sec 4.5 – Independent Projects

Procedure for *stated budget limited* evaluation

- ❖ No more than specified amount **(*b*)** can be invested **and** each project must demonstrate $PW \geq 0$ at MARR
- ❖ Form ME bundles of projects which do not exceed limit. Include DN alternative. There are 2^m ME bundles
- ❖ Procedure:
 1. Determine all bundles with total investment $\leq b$
 2. Calculate PW of all projects included in bundles. (Note: any bundle with a $PW < 0$ project can be eliminated now)
 3. Add project PW values to get total PW for each viable bundle
 4. Select bundle with largest PW value. These are the projects to accept

Sec 4.5 – Independent Projects - Example

Project	Initial Investment	Annual Net Cash Flow	Life, Years
F	\$ -8,000	\$3870	6
G	-15,000	2930	9
H	-6,000	2080	5
J	-10,000	5060	3

(cont →)

Select from 4 independent projects at MARR of 15% per year; $b = \$15,000$

PROCEDURE:

1. Total of $2^4 = 16$ bundles. Only 6 require \$15,000 or less:

F, G, H, J, FH, DN

2. $PW = \text{investment} + NCF(P/A, 15\%, n)$

Project	Life, n	PW at 15%
F	6	\$6646
G	9	-1019 (out)
H	5	973
J	3	1553

Sec 4.5 – Independent Projects - Example

3. PW of viable bundles (after G is removed)

$$PW_F = \$6,646$$

$$PW_H = \$973$$

$$PW_J = \$1,553$$

$$PW_{FH} = 6,646 + 973 = \$7,619$$

$$PW_{DN} = \$0$$

4. Bundle with largest PW is **FH**. Select these two projects

Sec 4.6 – Spreadsheet Evaluation of ME Alternatives

- For one project evaluation, equal-life and study period comparisons with same annual amounts A, use the single-cell PV function

$$= P - PV(i\%, n, A, F)$$

(Note minus sign on PV function)

- For different-life alternatives or when annual amounts vary, enter cash flow (CF) series and use the NPV function

$$= P + NPV(i\%, \text{year}_1_CF_cell, \text{last_year_CF_cell})$$

(Note that initial cost P is not included in NPV function)

Sec 4.6 – Spreadsheet Usage

	Purchase generator	Lease generator
P, \$	-120,000	Extra -40,000 now and extra -20,000 in year 6
S, \$	40,000	None
n, years	3	6
AOC, \$/year	-8,000	-20,000

Determine which alternative is cheaper at
MARR = 12% per year

(cont →)

Sec 4.6 – Spreadsheet Usage

	A	B	C	D	E	F	G	H
1								
2	Year	Purchase Generator	Lease Generator					
3	0	-120,000	-40,000					
4	1	-8,000	-20,000					
5	2	-8,000	-20,000					
6	3	-88,000	-20,000					
7	4	-8,000	-20,000					
8	5	-8,000	-20,000					
9	6	32,000	-40,000					
10	PW value	-\$189,568	-\$132,361					
11								
12								
13	Repurchase at end of year 3: = 40000-8000-120000			PW value over 2 cycles: = -120000 + NPV(12%,B4:B9)				
14								
15								
16								
17								

Select lease option; its PW of costs is lower