# Chapter 3

# Use of Multiple Factors

# LEARNING OBJECTIVES

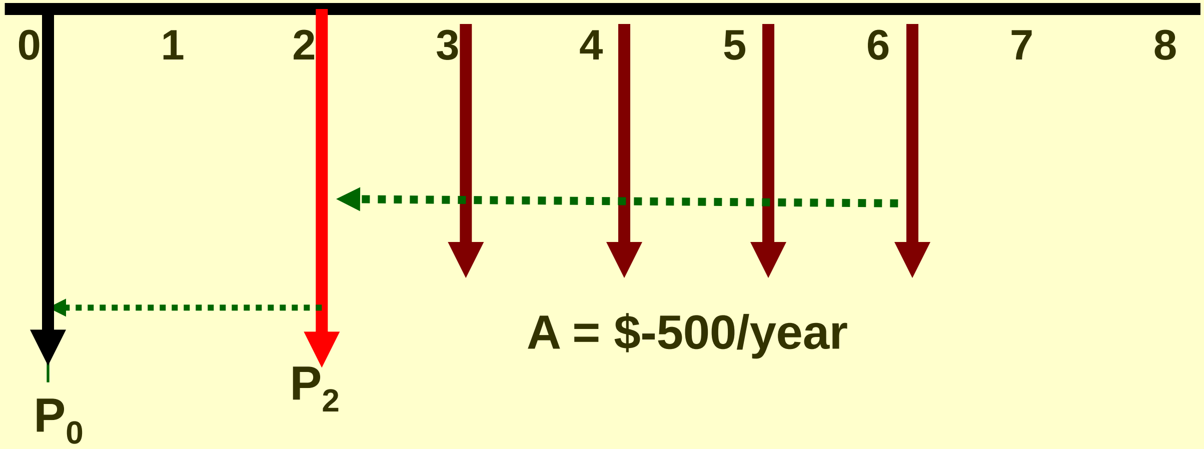
1. Dealing with shifted series
2. Shifted series and single amounts
3. Shifted gradients
4. Decreasing gradients
5. Spreadsheet applications

# Sct 3.1 Calculations for Uniform Series that are Shifted

- For a *shifted* series the present worth point in time is NOT $t = 0$.
- It is shifted either to the left of "0" or to the right of "0".
- Remember, when dealing with a uniform series:
  - The PW point is always one period to the left of the first series value, no matter where the series falls on the time line.

# Shifted Uniform Series

**Consider:**

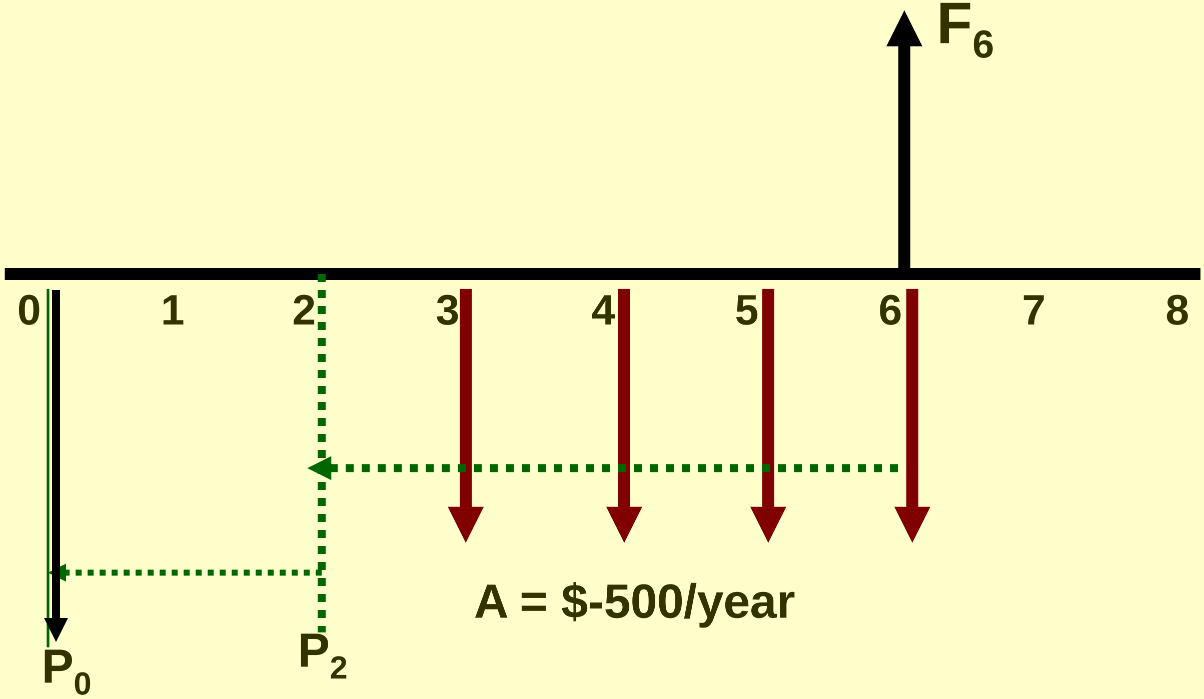



**P of this series is at t = 2 ($P_2$)** or $F_2$

$P_2 = -500(P/A,i\%,4)$ or could refer to as $F_2$

$P_0 = P_2(P/F,i\%,2)$ or could be $F_2(P/F,i\%,2)$

# Example of Shifted Series P and F




- F for this series is at t = 6; $F_6$ = A(F/A,i%,4)
- $P_0$ for this series at t = 0 is

$$P_0 = -500(P/A,i\%,4)(P/F,i\%,2)$$

# Using Spreadsheet Functions

- Net Present Value for a shifted series without a base amount. Excel function is:

  =NPV(i%,second_cell:last_cell) + first_cell

- To determine an equivalent A over all n years for a shifted series, use

  =PMT(i%,n,cell_with_P)

# Embedding Financial Functions

- Often, an Excel function can be embedded within another function. In case on previous slide, embed NPV function to find cell_with_P in PMT function.

=PMT(i%,n,NPV(i%,second_cell:last_cell) + first_cell)

- See Example 3.2.

# Sct 3.2 Calculations Involving Uniform-Series and Randomly-Placed Single Amounts

- Draw and correctly label the cash flow diagram that defines the problem
- Locate the present and future worth points for each series
- Write the time value of money equivalence relationships
- Substitute the correct factor values and solve

# Series with additional single cash flows

It is common to find cash flows that are combinations of series and single, randomly-placed cash flows

## For present worth, P

- Solve for the series present worth values then move to t = 0
- Then solve for the P at t = 0 for the single cash flows using the P/F factor for each cash flow
- Add the equivalent P values at t = 0

## For future worth, F

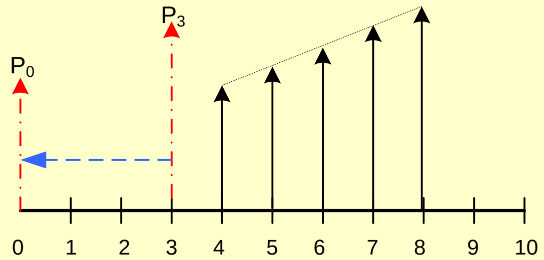
- Convert all cash flows to an equivalent F using the F/A and F/P factors in year t = n
- Add the equivalent F values at t = n

# Sct 3.3 Calculation for Shifted Gradients

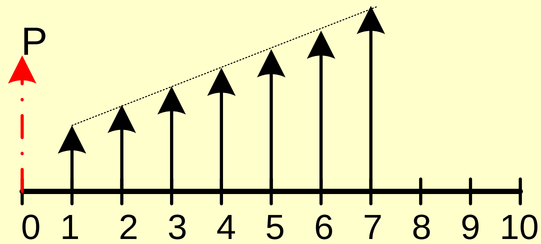
- The Present Worth of an arithmetic gradient (linear gradient) is always located:
  - One period to the left of the first cash flow in the series ( “0” gradient cash flow) or,
  - Two periods to the left of the “1G” cash flow

# Shifted Gradient

- A ***Shifted Gradient*** has its present value point <u>removed from time</u> t = 0

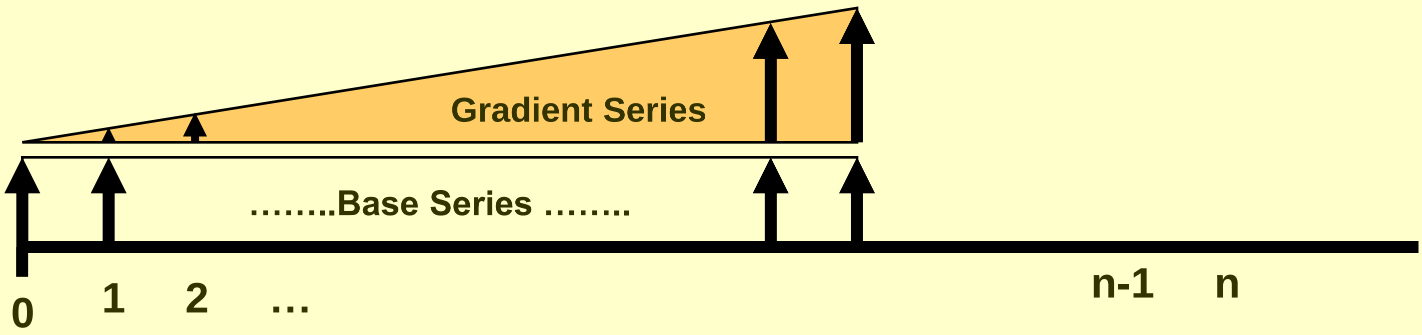



- A ***Conventional Gradient*** has its present worth point at t = 0



# Example of a Conventional Gradient

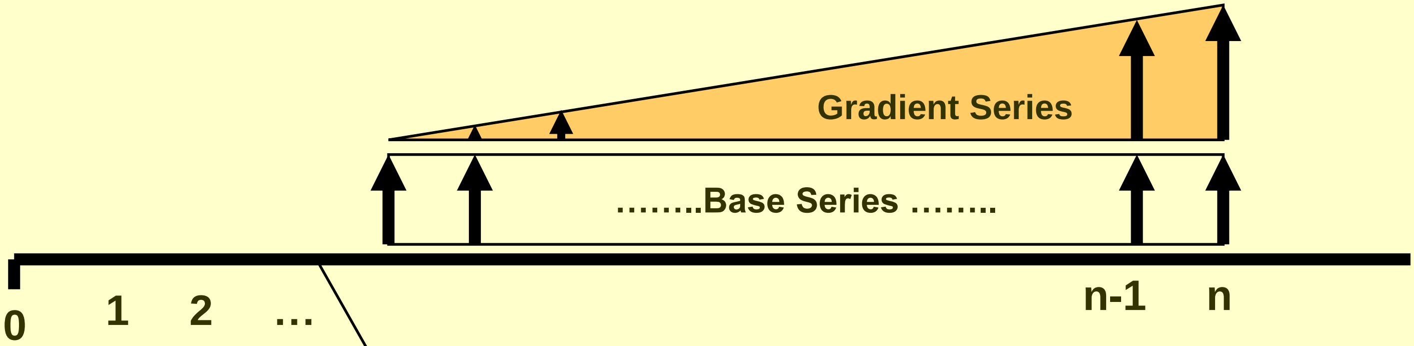
- **Consider:**




**This represents a conventional gradient**

**The present worth point is t = 0**

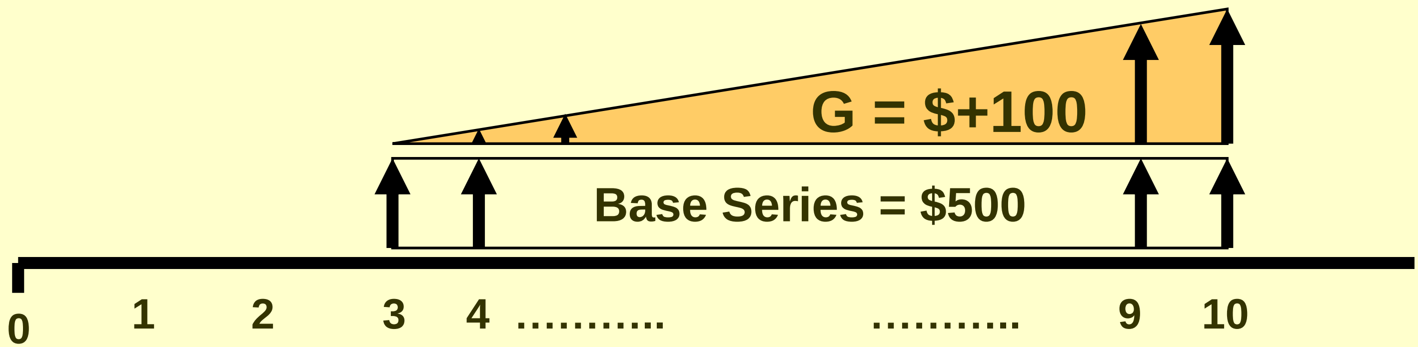
# Example of a Shifted Gradient




The present worth point for the base series and the gradient is here!

**This represents a shifted gradient**

# Shifted Gradient: Numerical Example



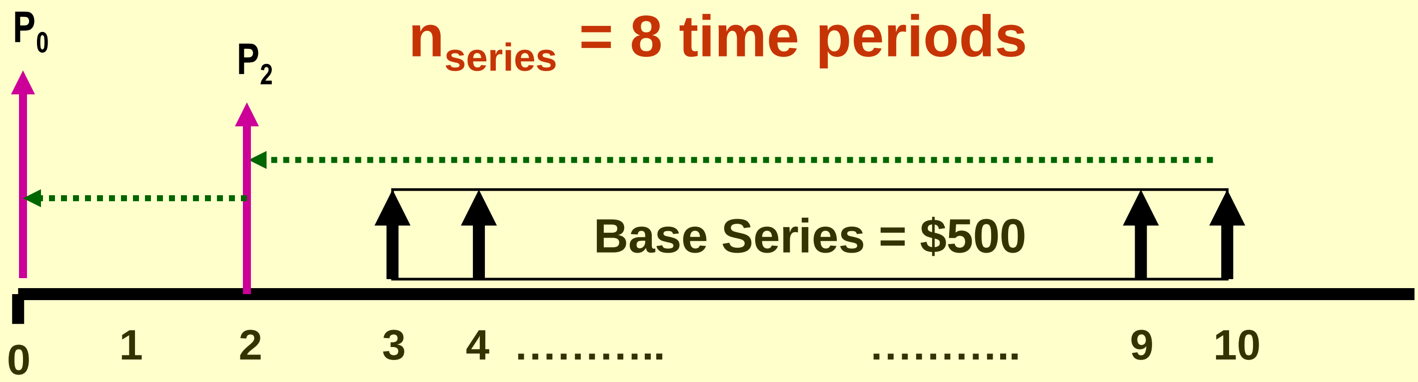

**Cash flows start at t = 3**

**$500/year increasing by $100/year through year 10; i = 10%; <u>Find P at t = 0</u>**

# Shifted Gradient: Numerical Example

## PW of the base series

$n_{series}$ = 8 time periods

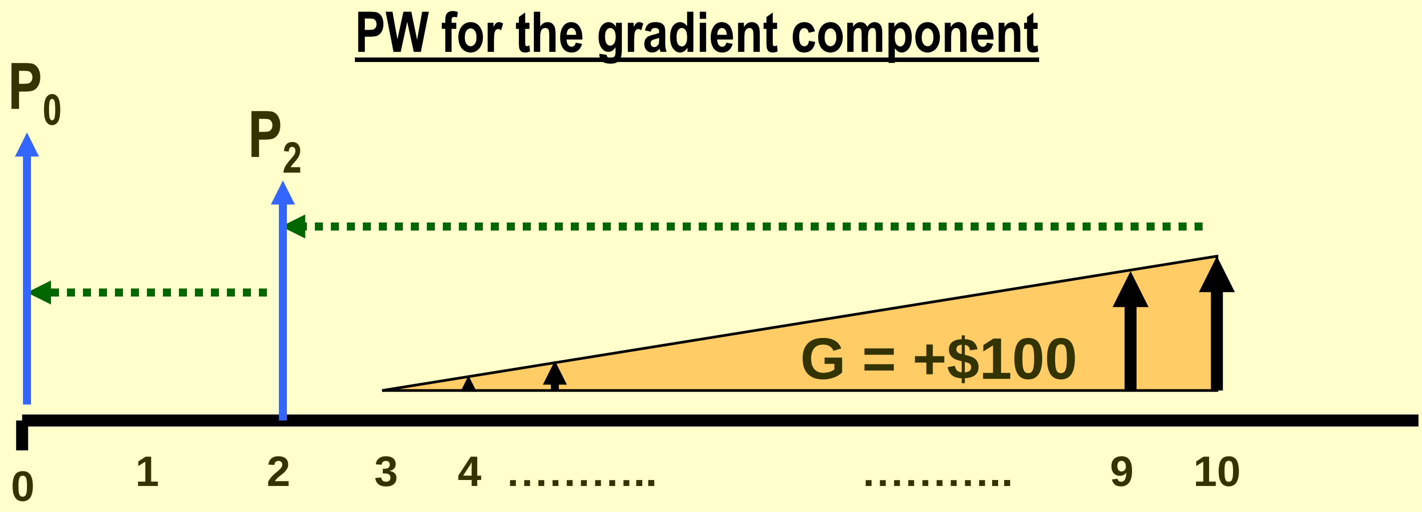



$P_2 = 500(P/A,10\%,8) = 500(5.3349) = \$2667.45$

$P_0 = 2667.45(P/F,10\%,2) = 2667.45(0.8264)$

$= \underline{\$2204.38}$

# Shifted Gradient: Numerical Example




$P_2$ = \$100(P/G,10%,8) = \$100(16.0287) = \$1,602.87

$P_0$ = \$1,602.87(P/F,10%,2) = \$1,602.87(0.8264)

= **\$1,324.61**

# Example: Total Present Worth Value

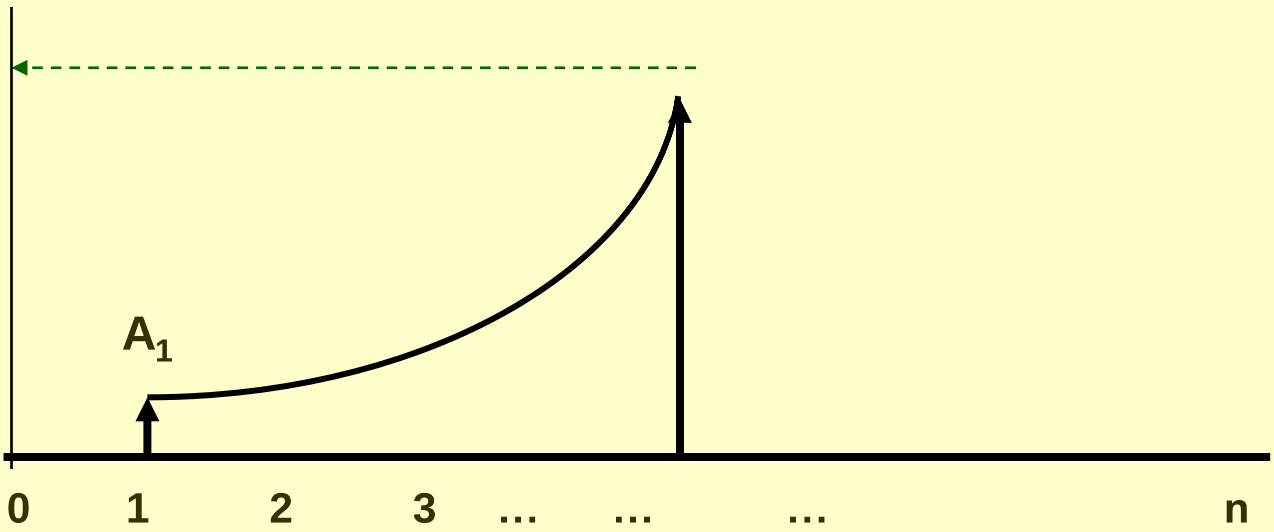
- **For the base series**
  - $P_0$ = $2204.38
- **For the arithmetic gradient**
  - $P_0$ = $1,324.61
- **Total present worth**
  - P = $2204.38 + $1,324.61 = **$3528.99**

# To Find A for a Shifted Gradient

1) Find the present worth of the gradient at actual time 0
2) Then apply the (A/P,$i$,$n$) factor to convert the present worth to an equivalent annuity (series)

# Shifted Geometric Gradient

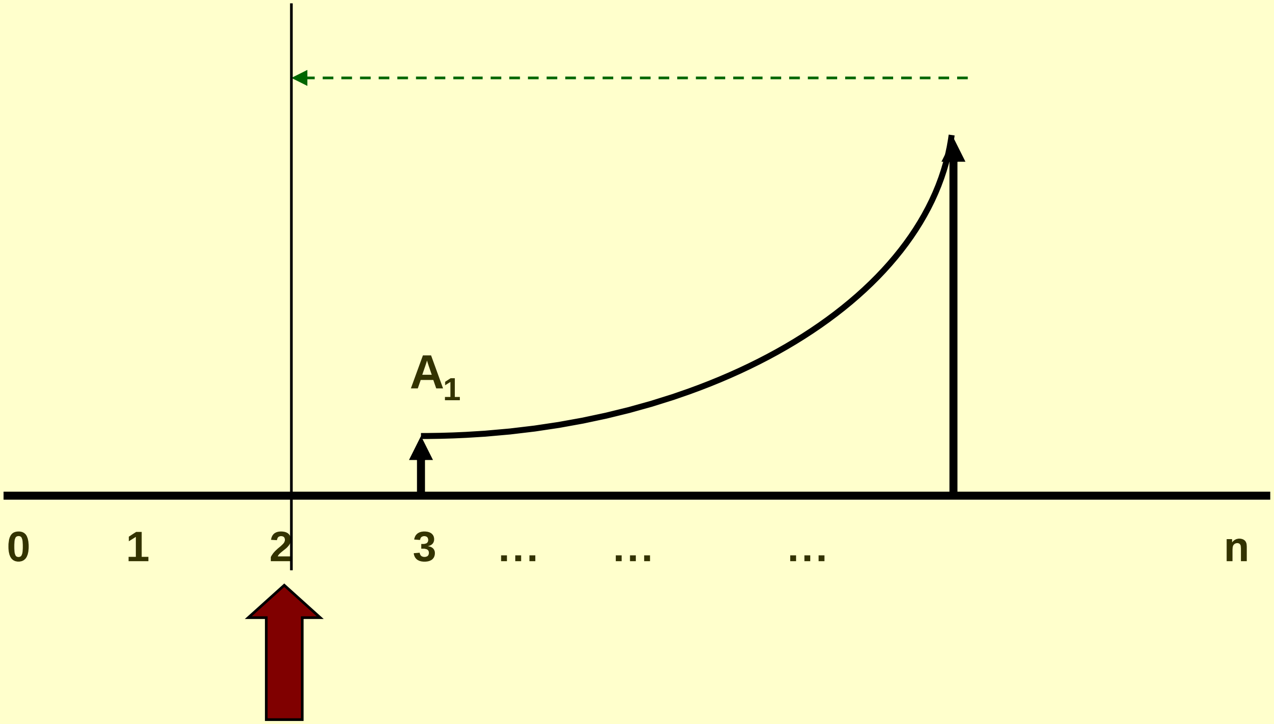
## Conventional Geometric Gradient




**Present worth point is at t = 0 for a conventional geometric gradient**

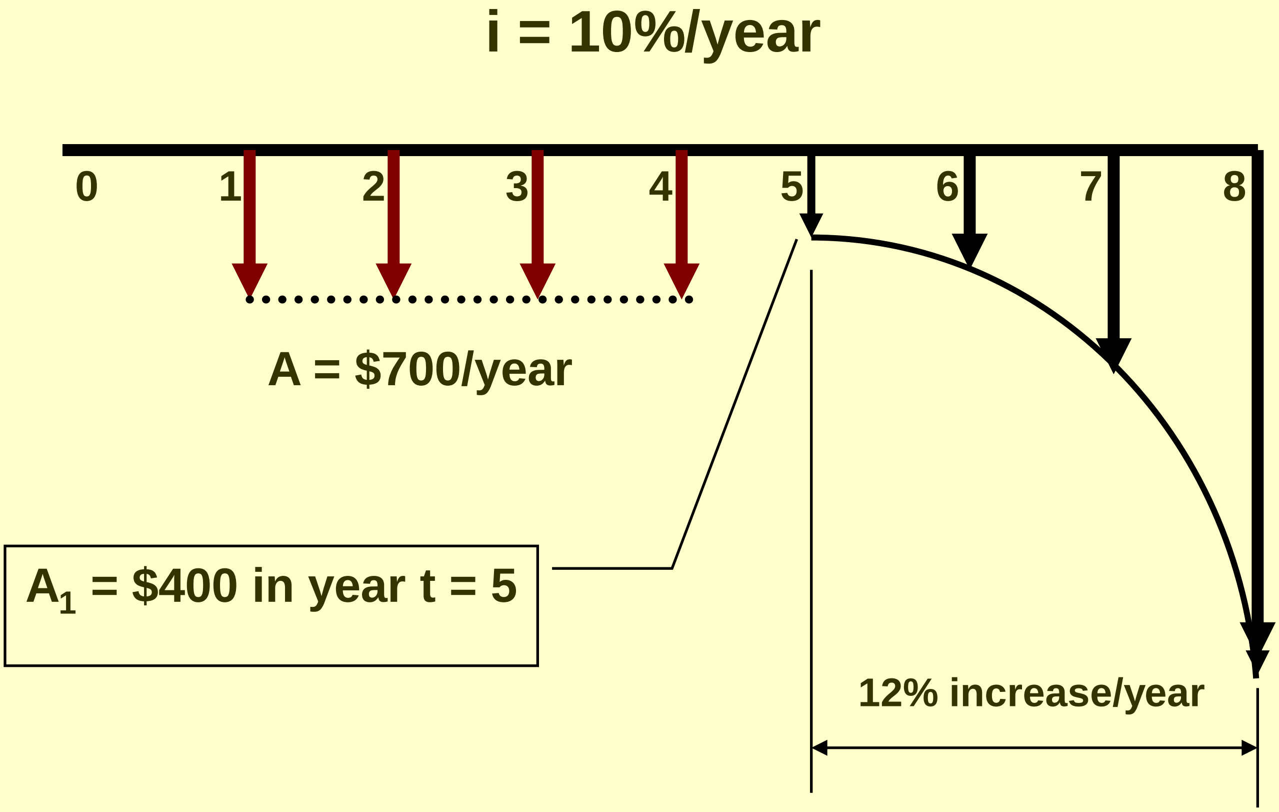
# Shifted Geometric Gradient

## Shifted Geometric Gradient



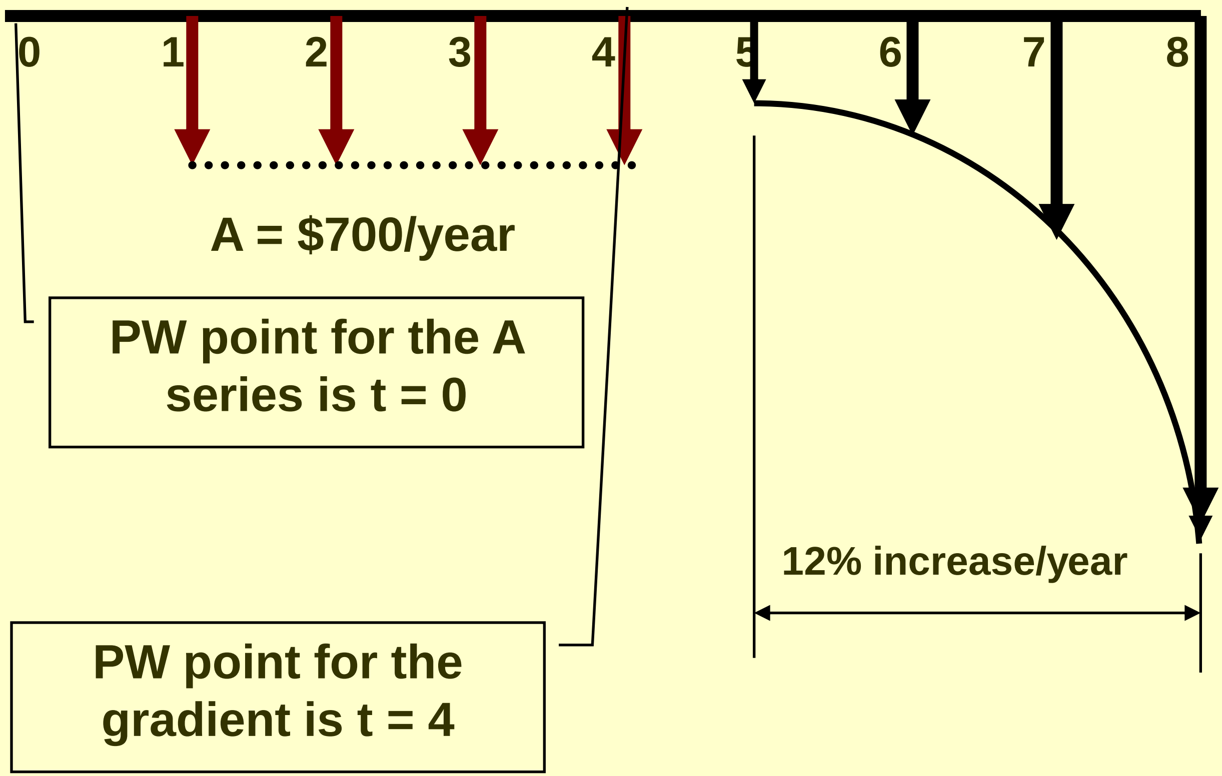

**Present worth point is at t = 2 for this example**

# Shifted Geometric Gradient Example



# Geometric Gradient Example

i = 10%/year

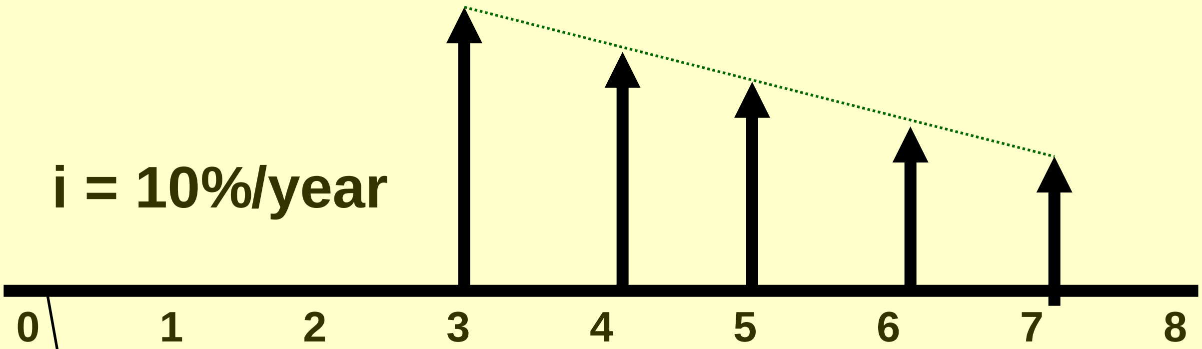


# Sct 3.4 Shifted Decreasing Arithmetic Gradients

<u>Given the following shifted, decreasing gradient</u>

$F_3$ = \$1,000; G = \$-100

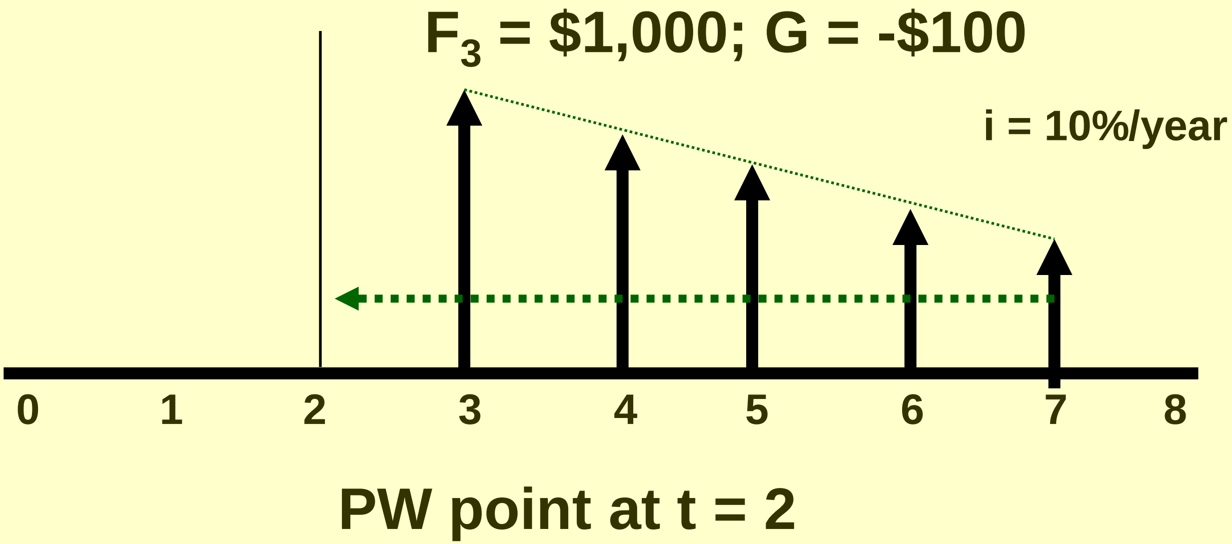
i = 10%/year




Find the present worth at t = 0

# PW for Shifted Decreasing Gradient

First, find PW at t = 2

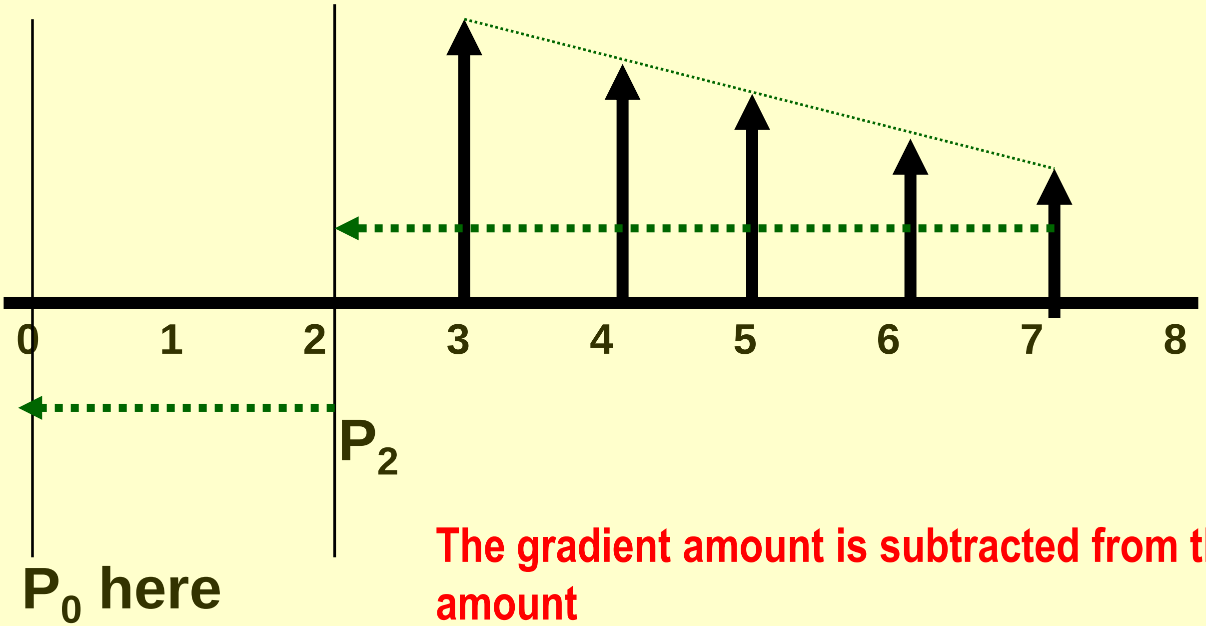


# Shifted Decreasing Gradient Example

## Second, find the PW at t = 0

i = 10%/year

$F_3$ = \$1,000; G = \$-100



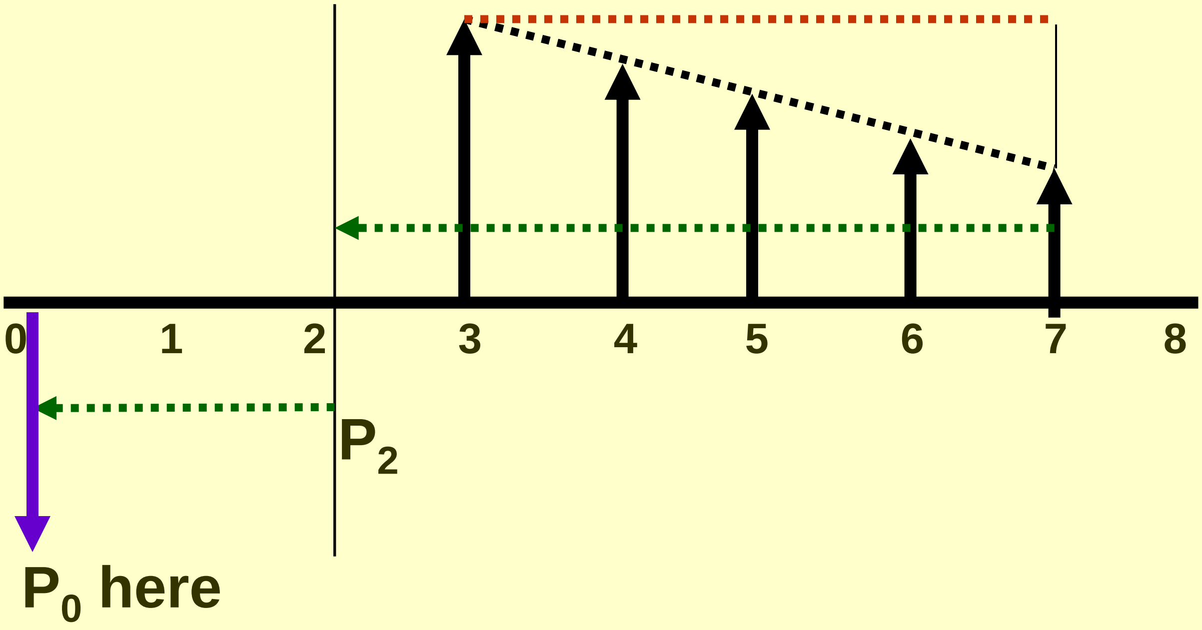

The gradient amount is subtracted from the base amount

# Shifted Decreasing Gradient Example

$F_3$ = \$1,000; G = -\$100

i = 10%/year

Base amount = \$1,000



# Sct 3.5 Spreadsheet Applications

## NPV Function in Excel

- **NPV** function is a basic financial function
- Requires that all cells in the defined time range have an entry
- The entry can be $0…but not blank! A "0" value must be entered
- Incorrect results can be generated if one or more cells in the defined range is left blank

# Summary

- Chapter summarizes cash flow patterns that are shifted away from time t = 0
- Illustrations of using multiple factors to perform PW or FW analysis for shifted cash flows
- Illustrations of shifted arithmetic and geometric gradients
- Illustrations of the power of Excel financial functions

# CHAPTER 3

## End of Slide Set