Chapter 3

Use of Multiple Factors

LEARNING OBJECTIVES

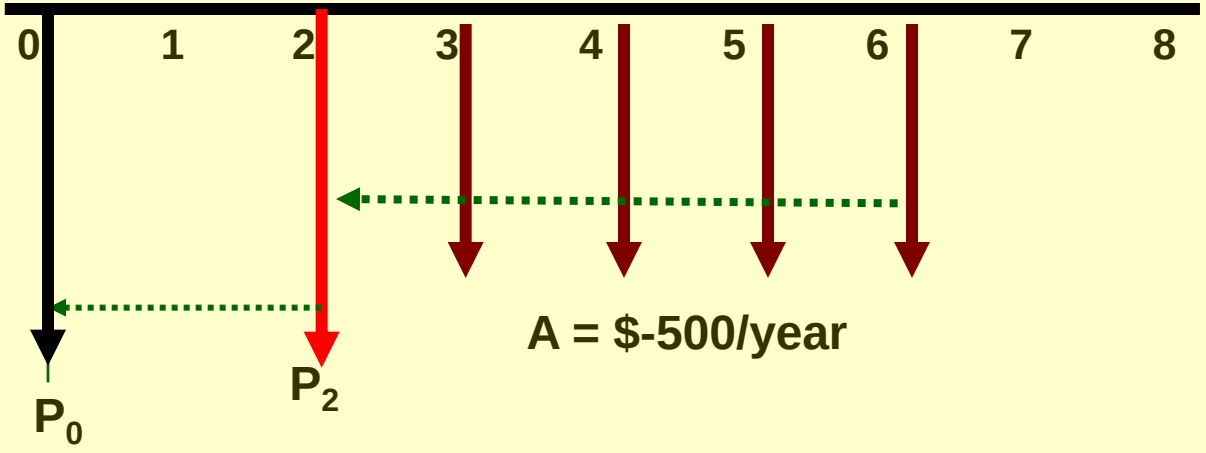
1. Dealing with shifted series
2. Shifted series and single amounts
3. Shifted gradients
4. Decreasing gradients
5. Spreadsheet applications

Sct 3.1 Calculations for Uniform Series that are Shifted

- For a *shifted* series the present worth point in time is NOT $t = 0$.
- It is shifted either to the left of "0" or to the right of "0".
- Remember, when dealing with a uniform series:
  - The PW point is always one period to the left of the first series value, no matter where the series falls on the time line.

Shifted Uniform Series

Consider:

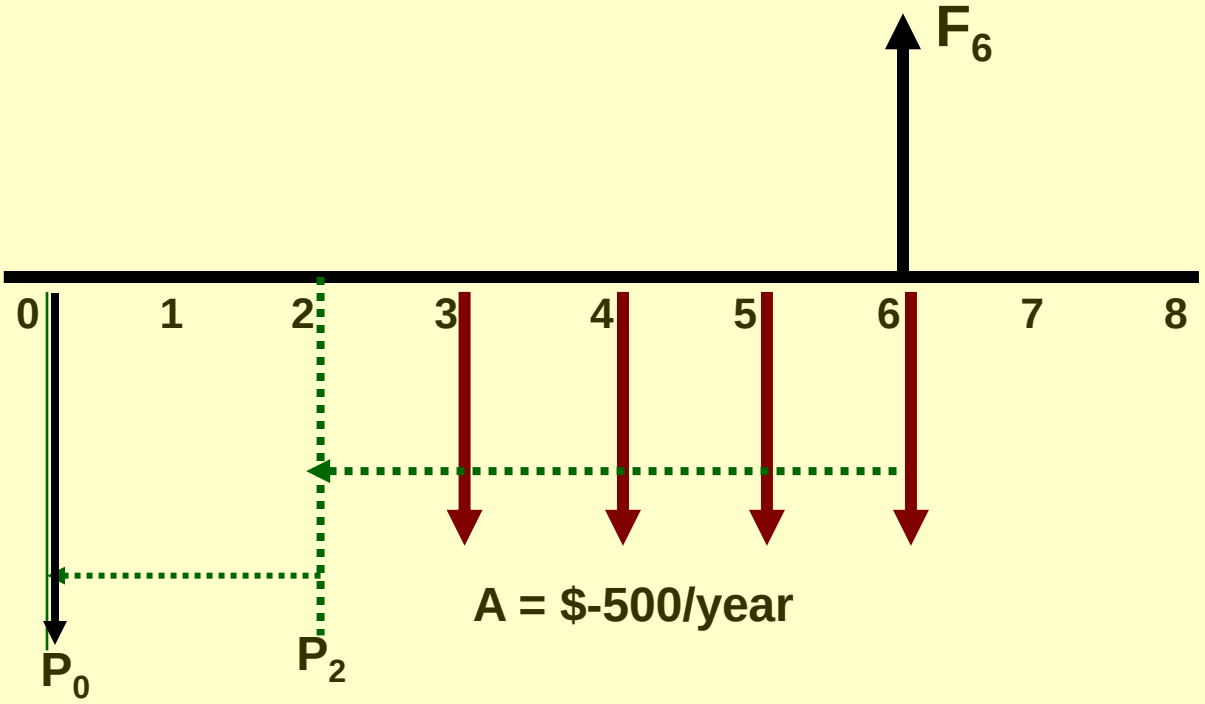


P of this series is at t = 2 (P_2) or F_2

$P_2 = -500(P/A,i\%,4)$ or could refer to as F_2

$P_0 = P_2(P/F,i\%,2)$ or could be $F_2(P/F,i\%,2)$

Example of Shifted Series P and F



- F for this series is at t = 6; $F_6 = A(F/A,i\%,4)$
- P_0 for this series at t = 0 is

$$P_0 = -500(P/A,i\%,4)(P/F,i\%,2)$$

Using Spreadsheet Functions

- Net Present Value for a shifted series without a base amount. Excel function is:

  =NPV(i%,second_cell:last_cell) + first_cell

- To determine an equivalent A over all n years for a shifted series, use

  =PMT(i%,n,cell_with_P)

Embedding Financial Functions

- Often, an Excel function can be embedded within another function. In case on previous slide, embed NPV function to find cell_with_P in PMT function.

=PMT(i%,n,NPV(i%,second_cell:last_cell) + first_cell)

- See Example 3.2.

Sct 3.2 Calculations Involving Uniform-Series and Randomly-Placed Single Amounts

- Draw and correctly label the cash flow diagram that defines the problem
- Locate the present and future worth points for each series
- Write the time value of money equivalence relationships
- Substitute the correct factor values and solve

Series with additional single cash flows

It is common to find cash flows that are combinations of series and single, randomly-placed cash flows

For present worth, P

- Solve for the series present worth values then move to t = 0
- Then solve for the P at t = 0 for the single cash flows using the P/F factor for each cash flow
- Add the equivalent P values at t = 0

For future worth, F

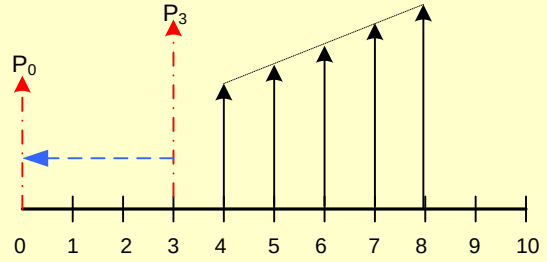
- Convert all cash flows to an equivalent F using the F/A and F/P factors in year t = n
- Add the equivalent F values at t = n

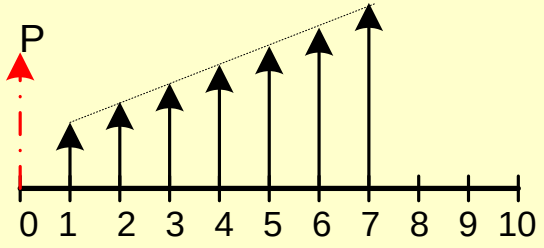
Sct 3.3 Calculation for Shifted Gradients

- The Present Worth of an arithmetic gradient (linear gradient) is always located:
  - One period to the left of the first cash flow in the series ("0" gradient cash flow) or,
  - Two periods to the left of the "1G" cash flow

Shifted Gradient

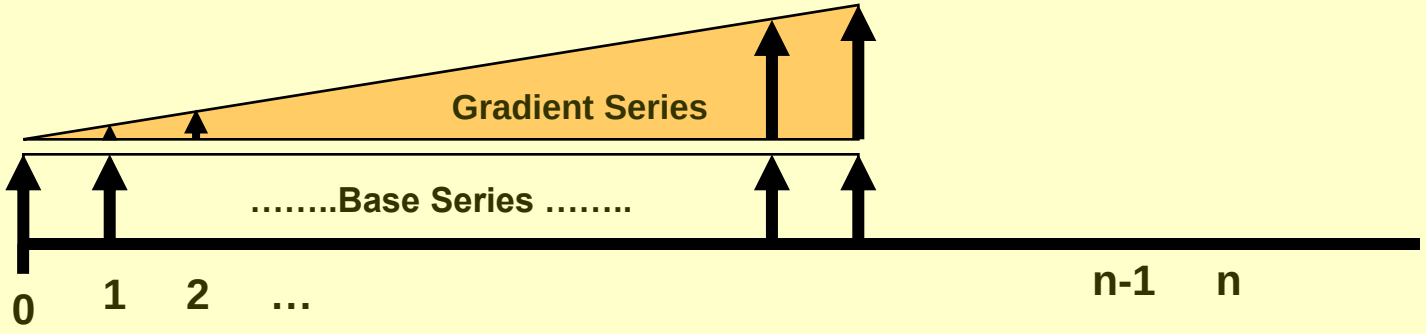
- A *Shifted Gradient* has its present value point removed from time t = 0
- A *Conventional Gradient* has its present worth point at t = 0





Example of a Conventional Gradient

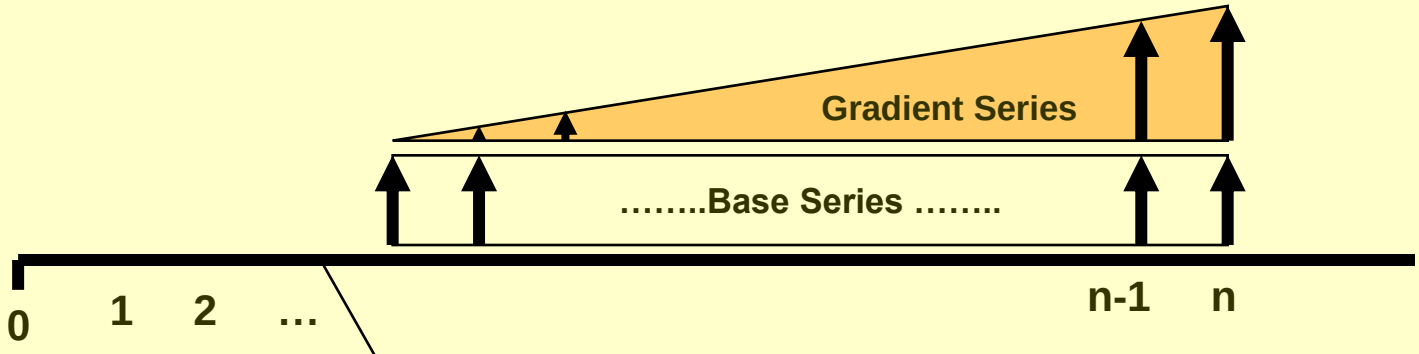
- **Consider:**



This represents a conventional gradient

The present worth point is t = 0

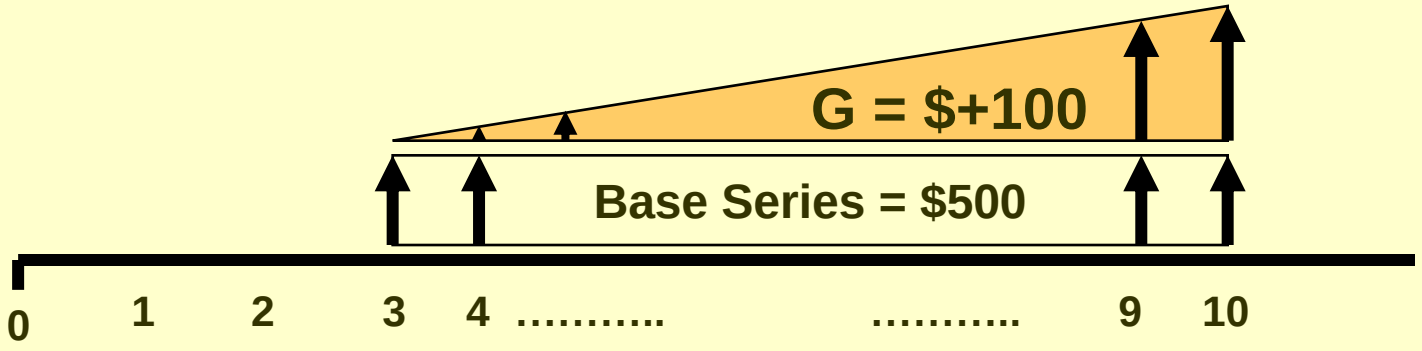
Example of a Shifted Gradient



The present worth point for the base series and the gradient is here!

This represents a shifted gradient

Shifted Gradient: Numerical Example

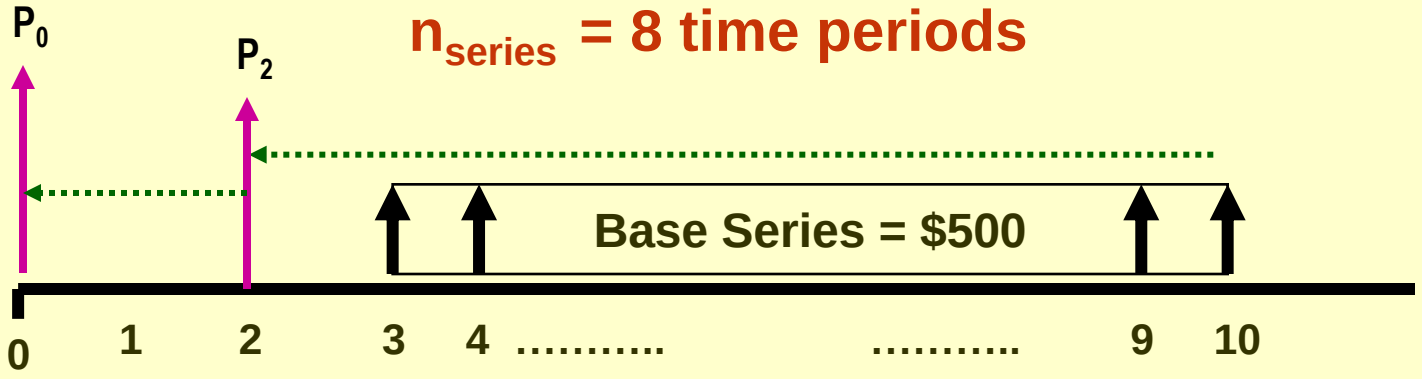


Cash flows start at t = 3

$500/year increasing by $100/year through year 10; i = 10%; Find P at t = 0

Shifted Gradient: Numerical Example

PW of the base series



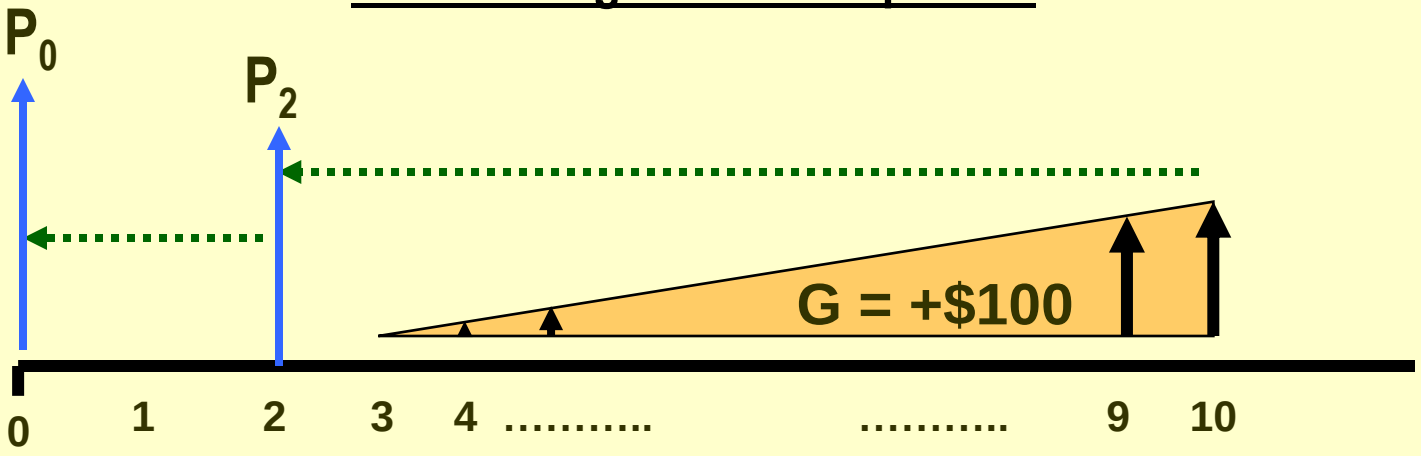
$P_2 = 500(P/A,10\%,8) = 500(5.3349) = \$2667.45$

$P_0 = 2667.45(P/F,10\%,2) = 2667.45(0.8264)$

$= \underline{\$2204.38}$

Shifted Gradient: Numerical Example

PW for the gradient component



$P_2 = \$100(P/G,10\%,8) = \$100(16.0287) = \$1,602.87$

$P_0 = \$1,602.87(P/F,10\%,2) = \$1,602.87(0.8264)$

$= \underline{\$1,324.61}$

Example: Total Present Worth Value

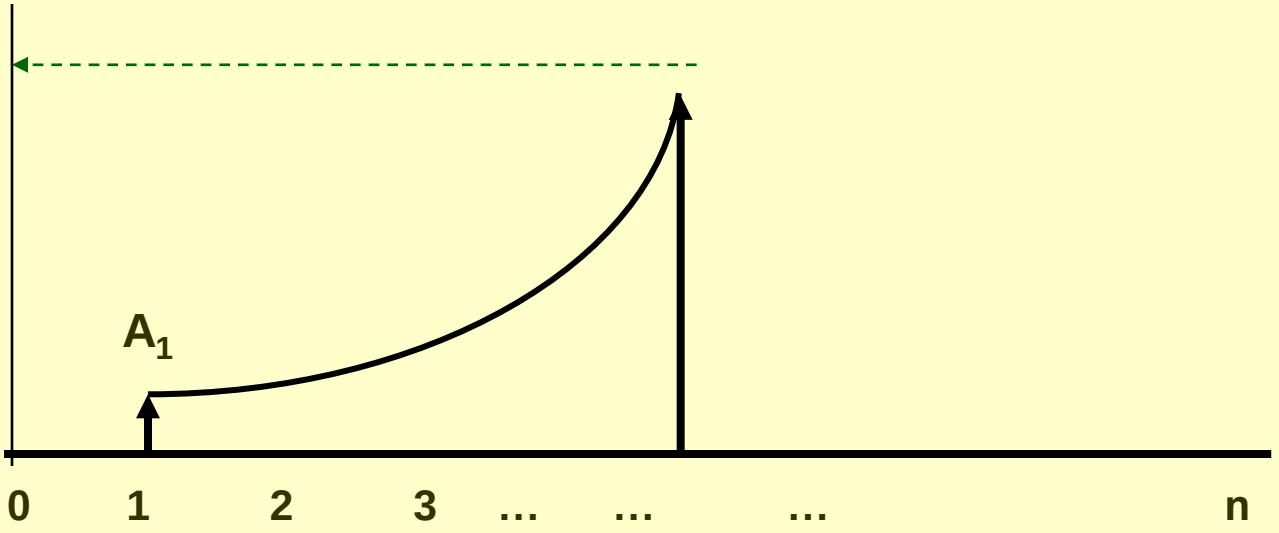
- **For the base series**
  - P_0 = $2204.38
- **For the arithmetic gradient**
  - P_0 = $1,324.61
- **Total present worth**
  - P = $2204.38 + $1,324.61 = **$3528.99**

To Find A for a Shifted Gradient

1) Find the present worth of the gradient at actual time 0
2) Then apply the (A/P,*i*,*n*) factor to convert the present worth to an equivalent annuity (series)

Shifted Geometric Gradient

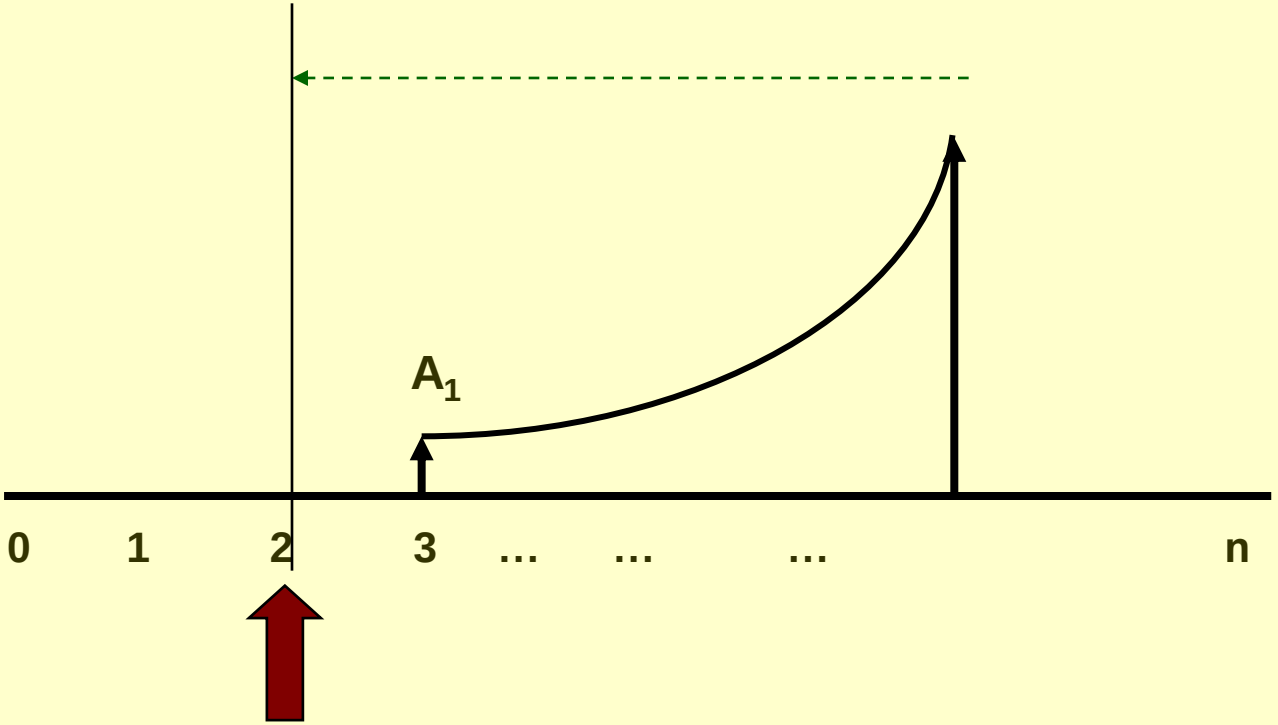
Conventional Geometric Gradient



Present worth point is at t = 0 for a conventional geometric gradient

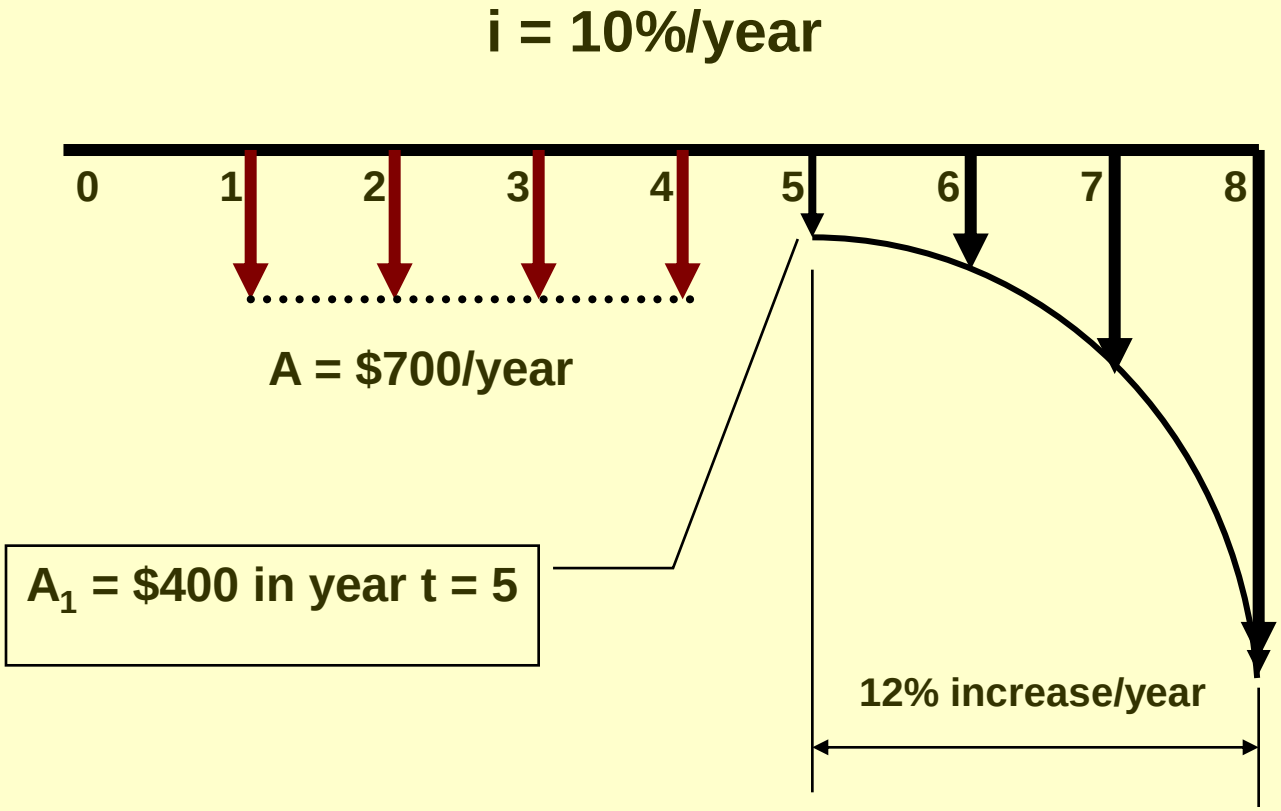
Shifted Geometric Gradient

Shifted Geometric Gradient



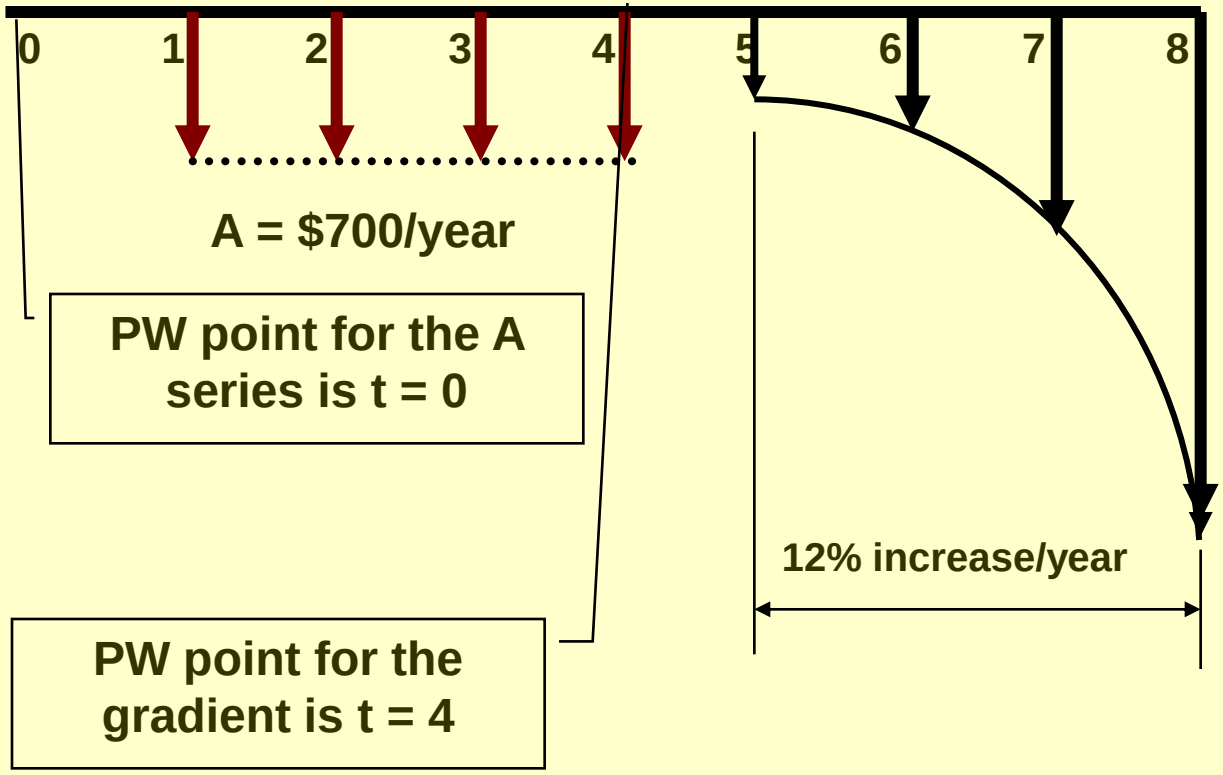
Present worth point is at t = 2 for this example

Shifted Geometric Gradient Example



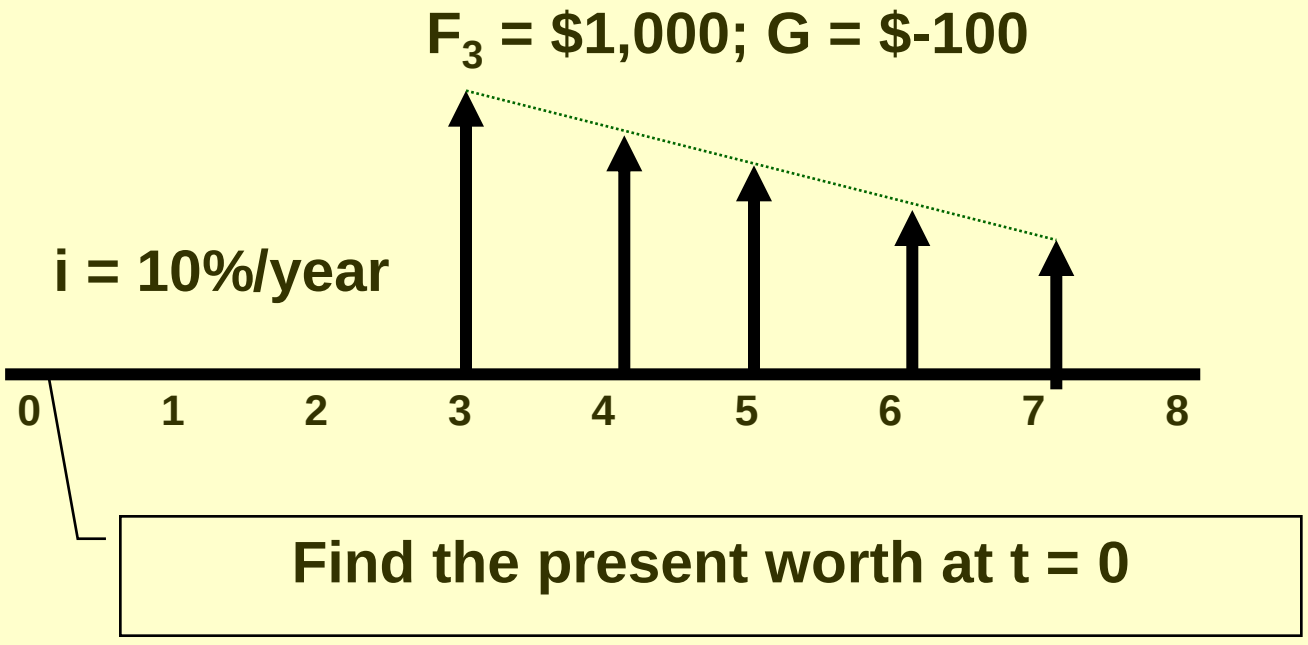
Geometric Gradient Example

i = 10%/year



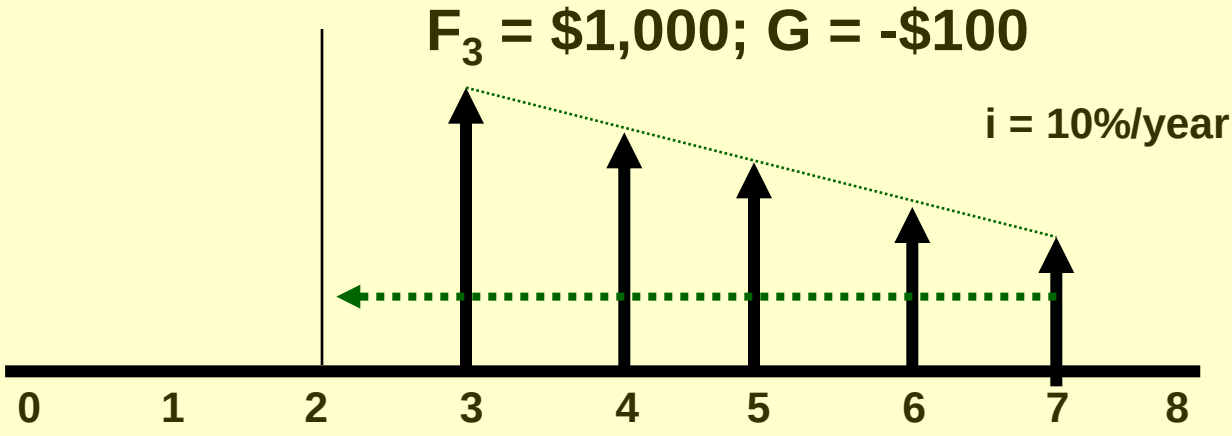
Sct 3.4 Shifted Decreasing Arithmetic Gradients

Given the following shifted, decreasing gradient



PW for Shifted Decreasing Gradient

First, find PW at t = 2

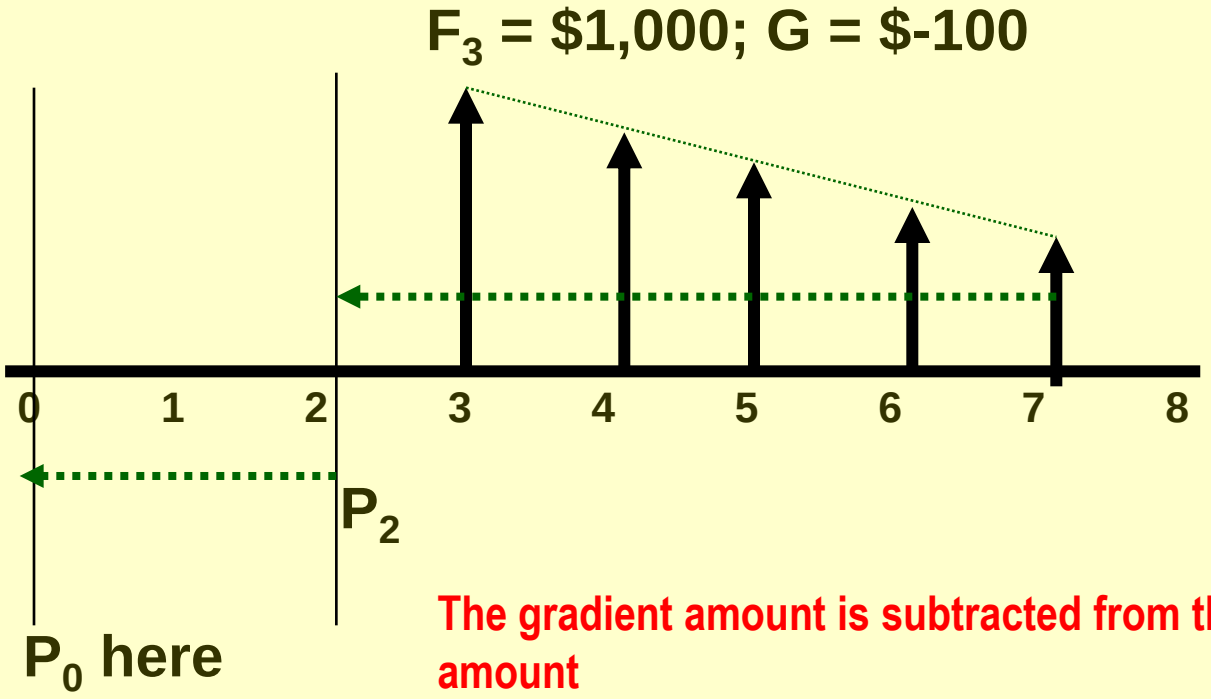


PW point at t = 2

Shifted Decreasing Gradient Example

Second, find the PW at t = 0

i = 10%/year



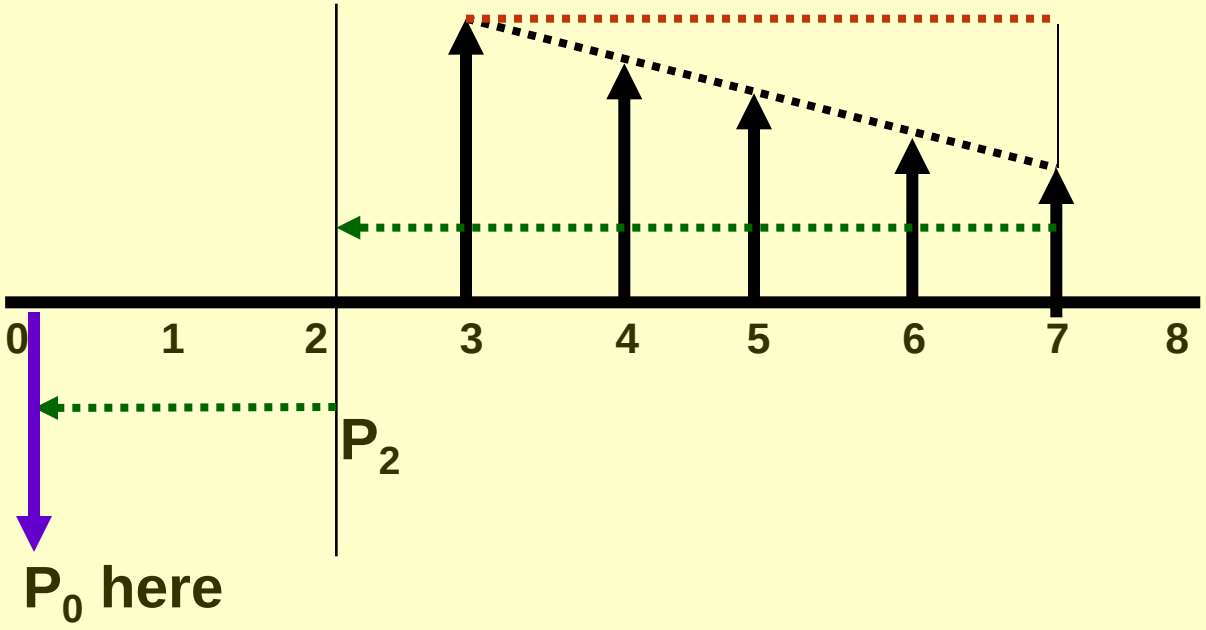
The gradient amount is subtracted from the base amount

Shifted Decreasing Gradient Example

$F_3 = \$1{,}000;\ G = -\$100$

i = 10%/year

Base amount = $1,000



Sct 3.5 Spreadsheet Applications

NPV Function in Excel

- NPV function is a basic financial function
- Requires that all cells in the defined time range have an entry
- The entry can be $0…but not blank! A "0" value must be entered
- Incorrect results can be generated if one or more cells in the defined range is left blank

Summary

- Chapter summarizes cash flow patterns that are shifted away from time t = 0
- Illustrations of using multiple factors to perform PW or FW analysis for shifted cash flows
- Illustrations of shifted arithmetic and geometric gradients
- Illustrations of the power of Excel financial functions

CHAPTER 3

End of Slide Set