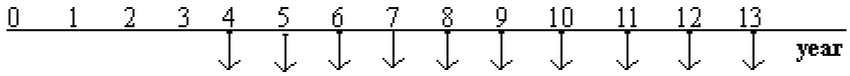
Chapter 4
Use of Multiple Factors

Locating the present worth and future worth:

When a uniform series of payments begins at a time other than at the end of interest period 1, several methods can be used to find the present worth. For example, the present worth of the uniform series of fig. * could be determined by only of the following methods:

- Use the P/F factor to find the present worth of each disbursement at year 0 and add them.
- Use the F/P factor to find the future worth of each disbursement in year 13, add them, and then find the present worth of the total using:
  P = F (P/F,i, 13)
- Use the F/A factor to find the future worth by F = A/F/A, i, 10), and then find the present worth of the total using P = F (P/F, i, 13).
- Use the P/A factor to compute the "present worth" (which will be located in year 3, not year 0), and then find the present worth in year 0 by using the (P/F, i, 3) factor.



A = 50$

Note that P is located 1 year prior to the beginning of the first annual amount.
Remember the following rule.

1) The present worth is always located one interest period prior to the first uniform-series amount when using the P/A factor.
2) The future worth is always located in the same period as the last uniform payment when using the F/A factor.
3) The number of period n that should be used with the P/A or F/A factors is equal to the number of payments of fig.* n=10.

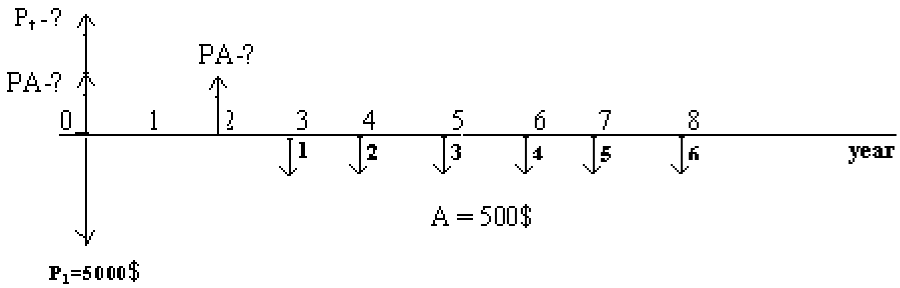
Calculations for a uniform series that begins after period 1:

Steps:

1) Draw a cash-flow diagram of the receipts and disbursements.
2) Locate the present worth or future worth of each series on the cash-flow diagram.
3) Determine n by renumbering the cash-flow diagram.
4) Draw the cash flow diagram representing desired equivalent cash flow.
5) Set up and solve the equations.

Example 1:

A person buys a small piece of land for 5000$ and deferred annual payments of 500$ a year for 6 years starting 3 years from now. What is the present worth of the investment if the interest rate is 8% per year?



$$PA = 500\ (P/A, 8\%, 6)$$

Since PA is located in year 2, it is necessary to find PA in year 0:

$$P_A = P_A\ (P/F, 8\%, 2)$$

The total present worth is determined by adding PA and the initial investment P1>

$$P_t = P_1 + P_A =$$

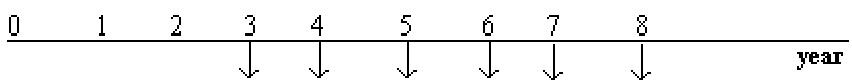
$$= 5000+500\ (P/A, 8\%, 6)\ (P/F1\ 8\%, 2) =$$

$$= 5000+500\ (4.6229)\ (0.8573) =$$

$$= 6981.6\$$$

Example 2:

Calculate the 8 year equivalent uniform annual worth amount at 16% per year interest for the uniform disbursements shown in fig.*



$$A = 800\$$$

1. Present worth method:

$P_A = 8..\ (P/A, 16\%, 6)$

$P_T = P_A\ (P/F, 16\%, 2) = 800\ (P/A, 16\%, 6)\ (P/F, 16\%, 2)$

$= 800\ (3.6847)\ (0.7432) = 2190.78\$$

Where P+ is the total present worth of the cash flow. The equivalent series A for 8 years can now be determined via the A/P factor

$$A = P_t\ (A/P, 16\%, 8) = 504.36\$$$

2- Future – worth method

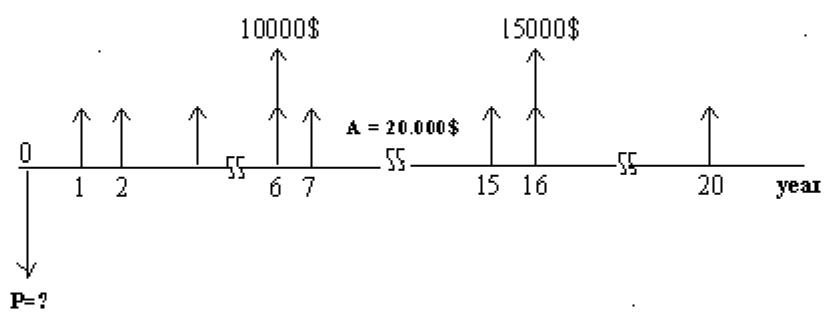
$$F = 800\ (F/A, 16\%, 6) = 7184\$$$

The A/F factor is now used to obtain A.

$$A^1 = F\ (A/F, 16\%, 8) = 504.46\$$$

➪ Calculations involving uniform-series and Randomly placed amounts:

Ex.:



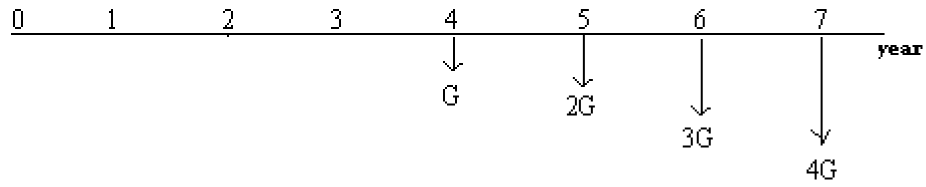
$$P = 20.000\ (P/A, 16\%,20) + 10.000\ (P/F, 16\%, 6) + 15000\ (P/F, 16\%, 16)$$
$$= 124.075\$$$

➪ Present worth and equivalent uniform series of shifted gradients:

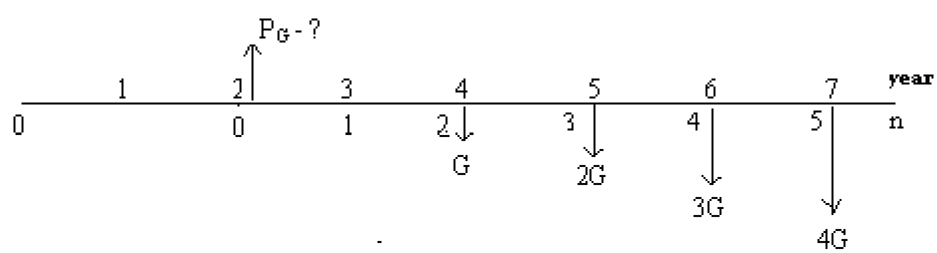
The present worth of a uniform gradient will always be located 2 periods before the gradient starts.

Ex:

For the cash flows of fig. 2, locate the gradient present worth.

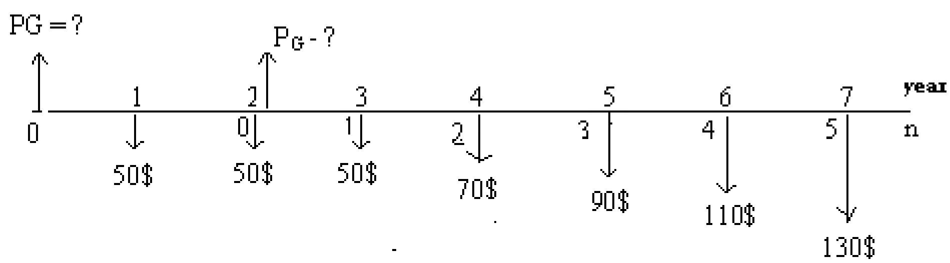


The present worth of a gradient, P_G



Ex.

Find the equivalent annual worth for the cash-flow amounts in fig.*



<u>Solution:</u>

The solution steps to compute the equivalent amount A are :

1- Consider the 50$ base amount as on annual amount for all 7 years.

2- Find the present worth P_G of the 20$ gradient that starts in actual year 4 as shown on the gradient year time scale where n = 5.

$$P_G = 20\ (P/G, i, 5)$$

3- Bring the gradient present worth back to actual year O.

$$P_o = P_G\ (P/F, i, 2) = 20\ (P/G, i, 5)\ (P/F, i, 2)$$

4- Annualize the gradient present worth from year 0 through year + to obtain A_G.

$$A_G = Po\ (A/P, i, 7)$$

5- Finally, add the base amount to the gradient annual worth to determine A.

$$A = 20\ (P/G, i, 5)\ (P/F, i, 2)\ (A/P, i, 7) + 50$$

Decreasing Gradients:

The use of the gradient factors is the same for increasing and decreasing gradients, except that in the case of decreasing gradients the following are true:

1- The base amount is equal to the largest amount attained in the gradient series.

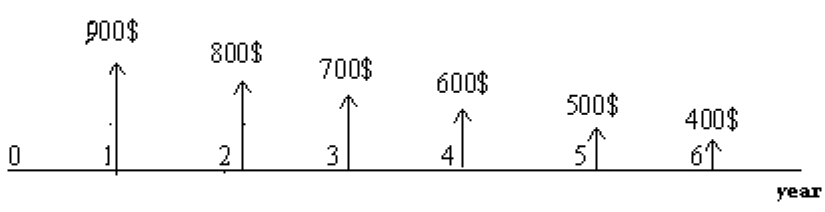
2- The gradient term is subtracted from the base amount instead of added, thus, the term – G/A/G, I, n) or – G (P/G, I, n) is used in the computations.

The present worth of the gradient will still take place 2 periods before the gradient starts and the A value will start at period 1 and continue through period n.

Ex:

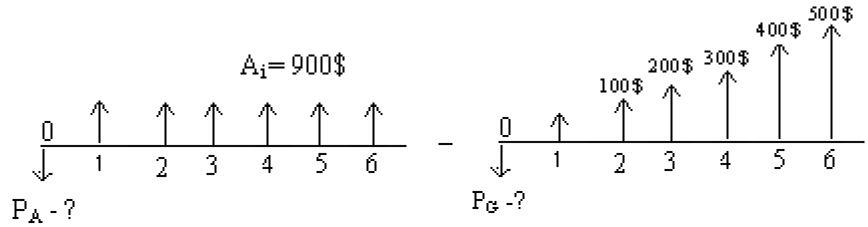
Find the (a) present worth

(b) annual worth of the receipts shown in fig.** for i = 7% per year



(a)

(a) The total present worth is:



(b) (c)

(a) = (b) – (c)

$P_T = P_A - P_G$ = 900 (P/A, 7%, 5) – 100 (P/G, 7%, 6)

= 900 (4.7665) – 100 (10, 9784) =

= 3192.01$

(b) The annual-worth series is comprised of two components: the base amount and the gradients equivalent uniform amount. The annual-receipt series (A_1=900$) is the base amount, and the gradient uniform series A_G is subtracted from A_1

$A = A_1 - A_G$ = 900 – 100 (A/G, 7%, 6)

= 900 – 100 (2-3032)

= 669.68 per year for year 1 to 6