Chapter 2

Factors: How Time and Interest Affect Money

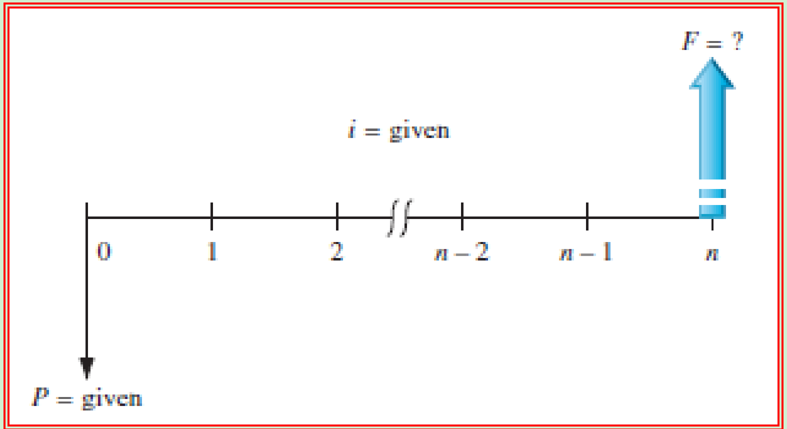
LEARNING OUTCOMES

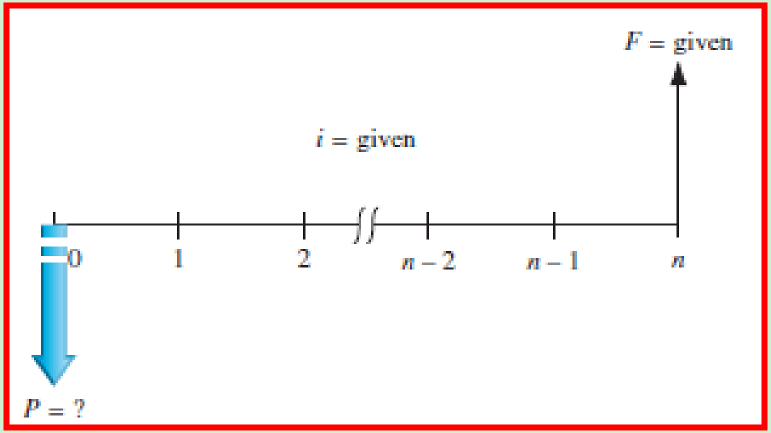
1. F/P and P/F Factors
2. P/A and A/P Factors
3. F/A and A/F Factors
4. Factor Values
5. Arithmetic Gradient
6. Geometric Gradient
7. Find i or n

Single Payment Factors (F/P and P/F)

Single payment factors involve only P and F.

Cash flow diagrams are as follows:





Formulas are as follows:

$F = P(1 + i)^n$

$P = F[1 / (1 + i)^n]$

Terms in parentheses or brackets are called *factors.* Values are in tables for i and n values

Factors are represented in *standard factor notation such as (F/P,i,n),*

where letter to left of slash is what is sought; letter to right represents what is given

F/P and P/F for Spreadsheets

Future value F is calculated using FV function:

= FV(i%,n,,P)

Present value P is calculated using PV function:

= PV(i%,n,,F)

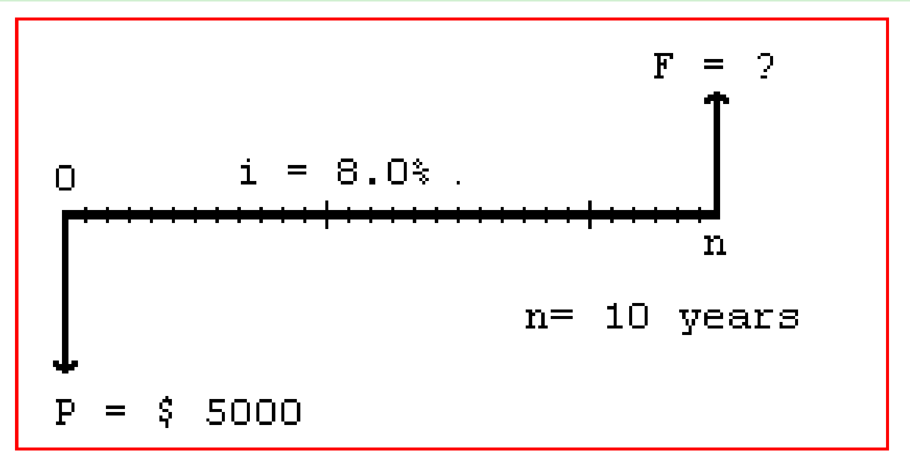
Note the use of double commas in each function

Example: Finding Future Value

A person deposits $5000 into an account which pays interest at a rate of 8% per year. The amount in the account after 10 years is closest to:

(A) $2,792 (B) $9,000 (C) $10,795 (D) $12,165

The cash flow diagram is:



Solution:

$$F = P(F/P,i,n)$$
$$= 5000(F/P,8\%,10)$$
$$= 5000(2.1589)$$
$$= \$10,794.50$$

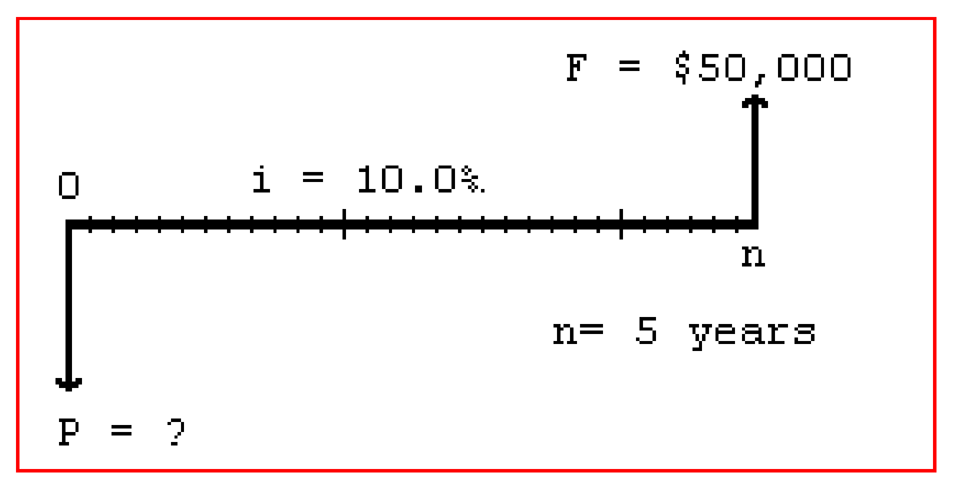
Answer is (C)

Example: Finding Present Value

A small company wants to make a single deposit now so it will have enough money to purchase a backhoe costing $50,000 five years from now. If the account will earn interest of 10% per year, the amount that must be deposited now is nearest to:

(A) $10,000 (B) $ 31,050 (C) $ 33,250 (D) $319,160

The cash flow diagram is:



Solution:

$P = F(P/F,i,n)$

$= 50,000(P/F,10\%,5)$

$= 50,000(0.6209)$

$= \$31,045$

Answer is (B)

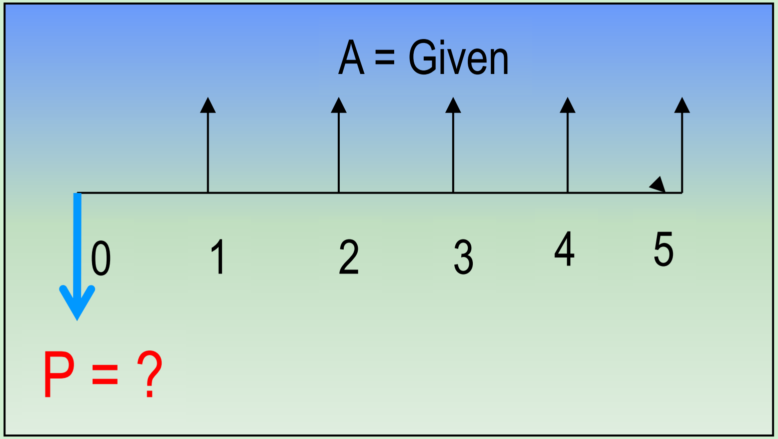
Uniform Series Involving P/A and A/P

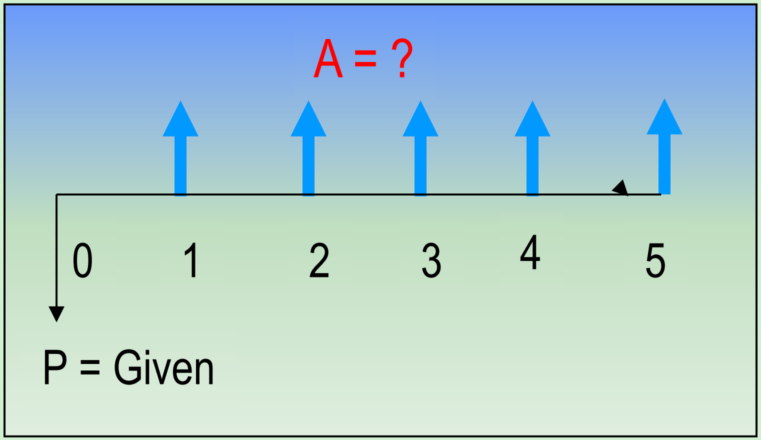
The uniform series factors that involve **P and A** are derived as follows:

(1) Cash flow occurs in ***consecutive*** interest periods

(2) Cash flow amount is ***same*** in each interest period

The cash flow diagrams are:





P = A(P/A,i,n) ⟵ Standard Factor Notation ⟶ A = P(A/P,i,n)

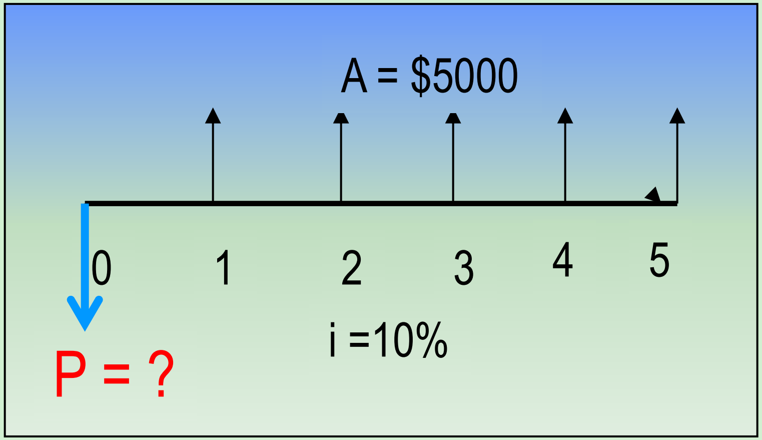
Note: P is one period *Ahead* of first A value

Example: Uniform Series Involving P/A

A chemical engineer believes that by modifying the structure of a certain water treatment polymer, his company would earn an extra \$5000 per year. At an interest rate of 10% per year, how much could the company afford to spend now to just break even over a 5 year project period?

(A) \$11,170 (B) 13,640 (C) \$15,300 (D) \$18,950

The cash flow diagram is as follows:



Solution:

$P = 5000(P/A,10\%,5)$

$= 5000(3.7908)$

$= \$18,954$

Answer is (D)

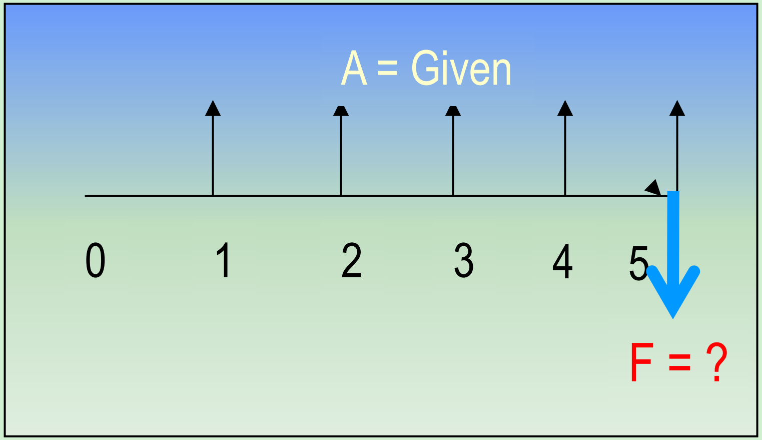
Uniform Series Involving F/A and A/F

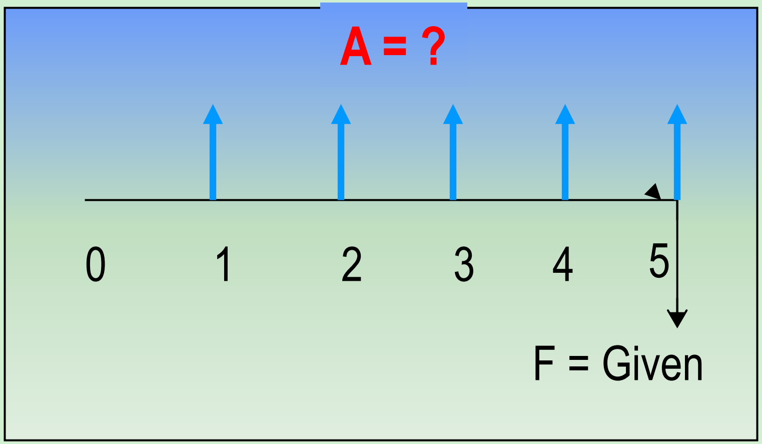
The uniform series factors that involve **F and A** are derived as follows:

(1) Cash flow occurs in ***consecutive*** interest periods

(2) Last cash flow occurs in ***same*** period as F

Cash flow diagrams are:





F = A(F/A,i,n) ⟵ Standard Factor Notation ⟶ A = F(A/F,i,n)

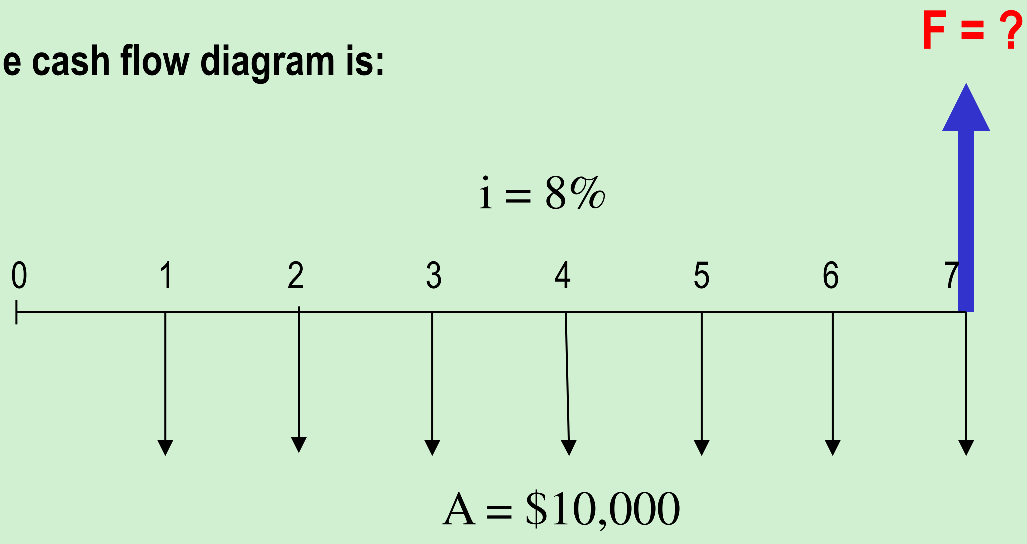
Note: F takes place in the *same* period as last A

Example: Uniform Series Involving F/A

An industrial engineer made a modification to a chip manufacturing process that will save her company $10,000 per year. At an interest rate of 8% per year, how much will the savings amount to in 7 years?

(A) $45,300 (B) $68,500 (C) $89,228 (D) $151,500

The cash flow diagram is:



Solution:

$$F = 10{,}000(F/A,8\%,7)$$

$$= 10{,}000(8.9228)$$

$$= \$89{,}228$$

Answer is (C)

Factor Values for Untabulated i or n

3 ways to find factor values for untabulated i or n values

- **Use formula**
- **Use spreadsheet function with corresponding P, F, or A value set to 1**
- **Linearly interpolate in interest tables**

Formula or spreadsheet function is fast and accurate
Interpolation is only approximate

Example: Untabulated i

Determine the value for (F/P, 8.3%,10)

Formula: $F = (1 + 0.083)^{10} = 2.2197$ ⟵ **OK**

Spreadsheet: = FV(8.3%,10,,1) = 2.2197 ⟵ **OK**

Interpolation: 8% ------ 2.1589
8.3% ------ x
9% ------ 2.3674

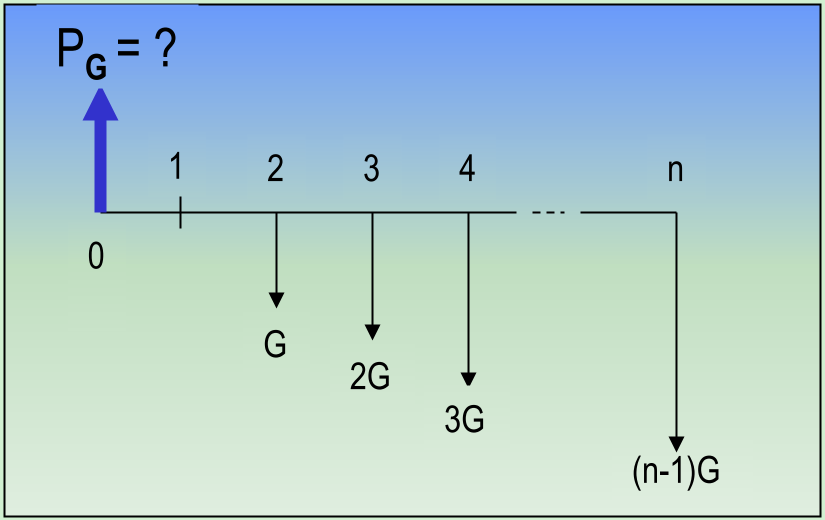
$x = 2.1589 + [(8.3 - 8.0)/(9.0 - 8.0)][2.3674 – 2.1589]$
$= 2.2215$ ⟵ **(Too high)**

Absolute Error = 2.2215 – 2.2197 = 0.0018

Arithmetic Gradients

Arithmetic gradients *change* by the *same amount* each period

The cash flow diagram for the P_G of an arithmetic gradient is:



Standard factor notation is

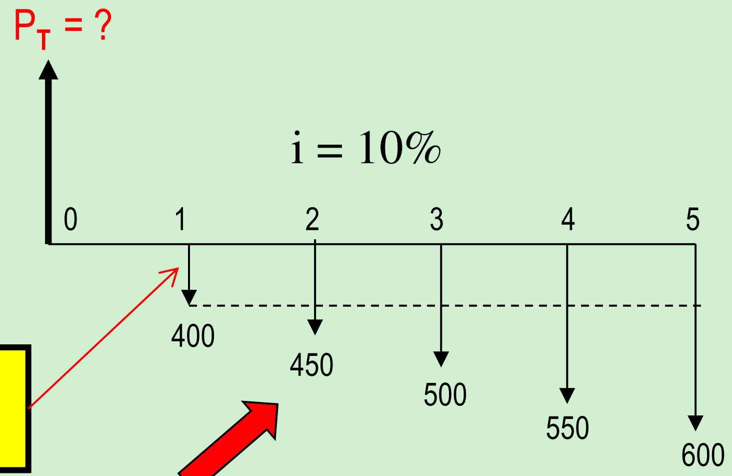
$$P_G = G(P/G,i,n)$$

G starts between periods 1 and 2
(not between 0 and 1)

This is because cash flow in year 1 is usually not equal to G and is handled separately as a *base amount* (shown on next slide)

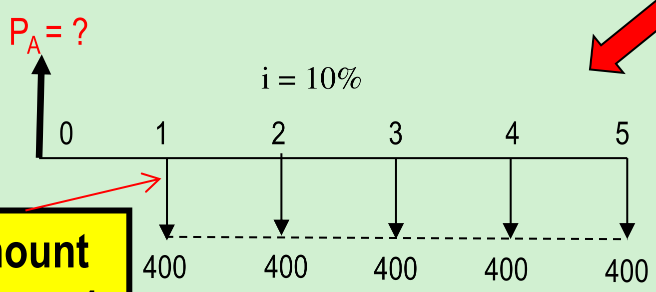
Note that P_G is located Two Periods Ahead of the first change that is equal to G

Typical Arithmetic Gradient Cash Flow



Amount in year 1 is base amount

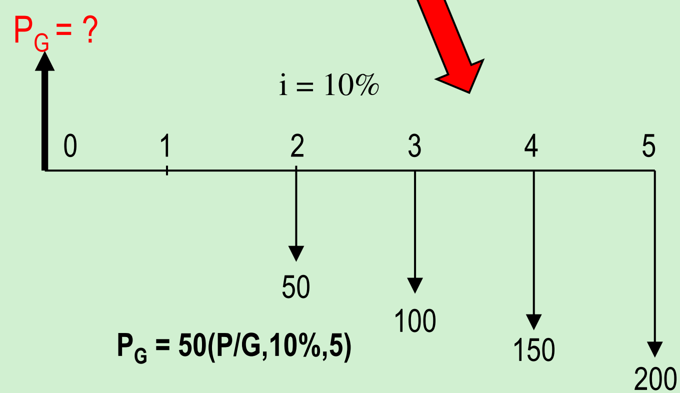
This diagram = this *base amount* plus this gradient



Amount in year 1 is base amount

$P_A = 400(P/A,10\%,5)$

+

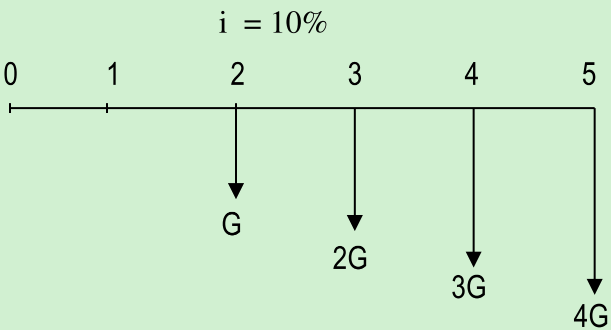


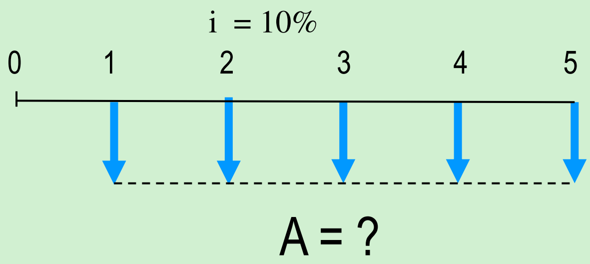
$P_G = 50(P/G,10\%,5)$

$P_T = P_A + P_G = 400(P/A,10\%,5) + 50(P/G,10\%,5)$

Converting Arithmetic Gradient to A

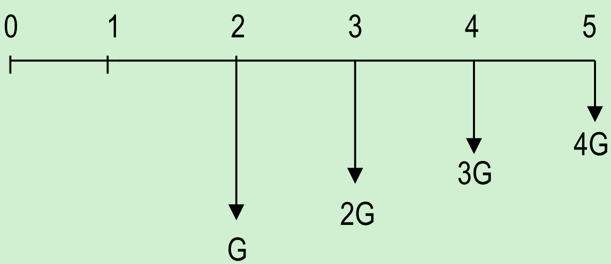
Arithmetic gradient can be converted into equivalent A value using G(A/G,i,n)





General equation when base amount is involved is

A = base amount + G(A/G,i,n)



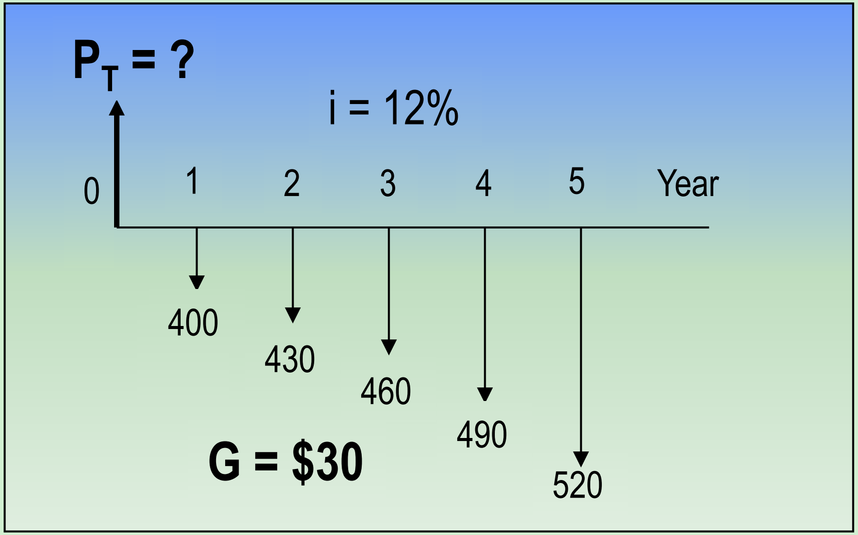
For decreasing gradients, change plus sign to minus

A = base amount - G(A/G,i,n)

Example: Arithmetic Gradient

The present worth of \$400 in year 1 and amounts increasing by \$30 per year through year 5 at an interest rate of 12% per year is closest to:

(A) \$1532 **(B) \$1,634** **(C) \$1,744** **(D) \$1,829**



Solution:

P_T = 400(P/A,12%,5) + 30(P/G,12%,5)
= 400(3.6048) + 30(6.3970)
= \$1,633.83

Answer is (B)

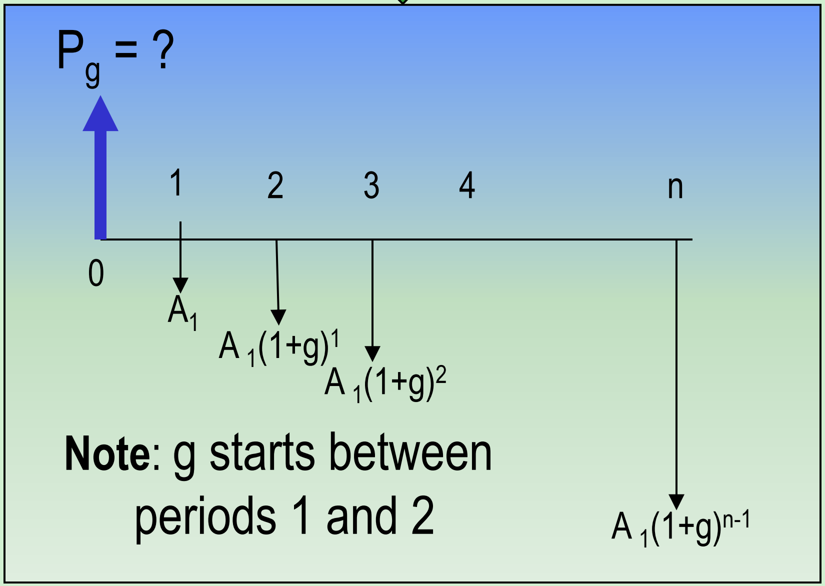
The cash flow could also be converted into an **A** value as follows:

A = 400 + 30(A/G,12%,5)
= 400 + 30(1.7746)
= \$453.24

Geometric Gradients

Geometric gradients **change by the** ***same percentage*** **each period**

Cash flow diagram for present worth of geometric gradient



There are ***no tables*** for geometric factors

Use following equation for g ≠ i:

$$P_g = A_1\{1- [(1+g)/(1+i)]^n\}/(i-g)$$

where: A_1 = cash flow in period 1

g = rate of increase

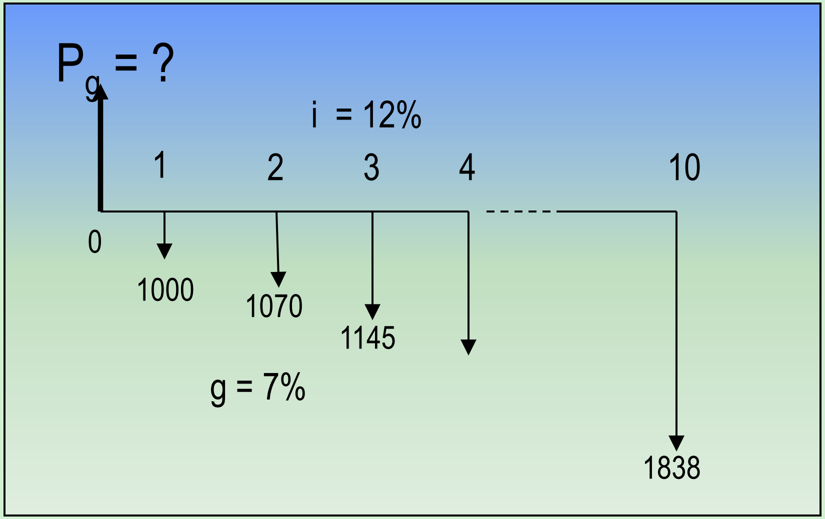
If g = i, $P_g = A_1 n/(1+i)$

Note: If g is negative, change signs in front of both g values

Example: Geometric Gradient

Find the present worth of $1,000 in year 1 and amounts increasing by 7% per year through year 10. Use an interest rate of 12% per year.

(a) $5,670 (b) $7,333 (c) $12,670 (d) $13,550



Solution:

$P_g = 1000[1-(1+0.07/1+0.12)^{10}]/(0.12-0.07)$

$= \$7,333$

Answer is (b)

To find A, multiply P_g by (A/P,12%,10)

Unknown Interest Rate i

Unknown interest rate problems involve solving for i, given n and 2 other values (P, F, or A)

(Usually requires a trial and error solution or interpolation in interest tables)

Procedure: Set up equation with all symbols involved and solve for i

A contractor purchased equipment for $60,000 which provided income of $16,000 per year for 10 years. The annual rate of return of the investment was closest to:

(a) 15% (b) 18% (c) 20% (d) 23%

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P,i\%,10) = 16{,}000$$

$$(A/P,i\%,10) = 0.26667$$

From A/P column at n = 10 in the interest tables, i is between 22% and 24%

Answer is (d)

Unknown Recovery Period n

Unknown recovery period problems involve solving for n, given i and 2 other values (P, F, or A)

(Like interest rate problems, they usually require a trial & error solution or interpolation in interest tables)

Procedure: Set up equation with all symbols involved and solve for n

A contractor purchased equipment for $60,000 that provided income of $8,000 per year. At an interest rate of 10% per year, the length of time required to recover the investment was closest to:

(a) 10 years **(b) 12 years** **(c) 15 years** **(d) 18 years**

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P,10\%,n) = 8{,}000$$

$$(A/P,10\%,n) = 0.13333$$

From A/P column in i = 10% interest tables, n is between 14 and 15 years **Answer is (c)**

Summary of Important Points

- In P/A and A/P factors, P is *one period ahead* of first A
- In F/A and A/F factors, F is in *same period as last* A
- To find untabulated factor values, best way is to use *formula or spreadsheet*
- For arithmetic gradients, gradient G starts between *periods 1 and 2*
- Arithmetic gradients have 2 parts, *base amount* (year 1) and *gradient amount*
- For geometric gradients, gradient g starts been *periods 1 and 2*
- In geometric gradient formula, A_1 is amount in *period 1*
- To find unknown i or n, *set up equation involving all terms* and solve for i or n