

Chapter 2

Factors: How Time and Interest Affect Money

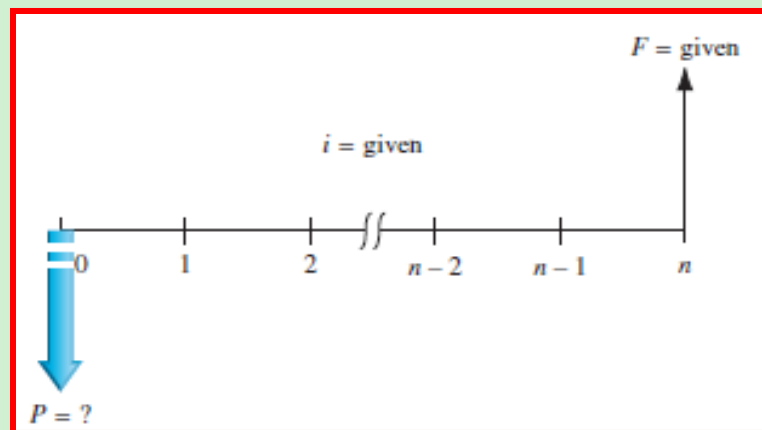
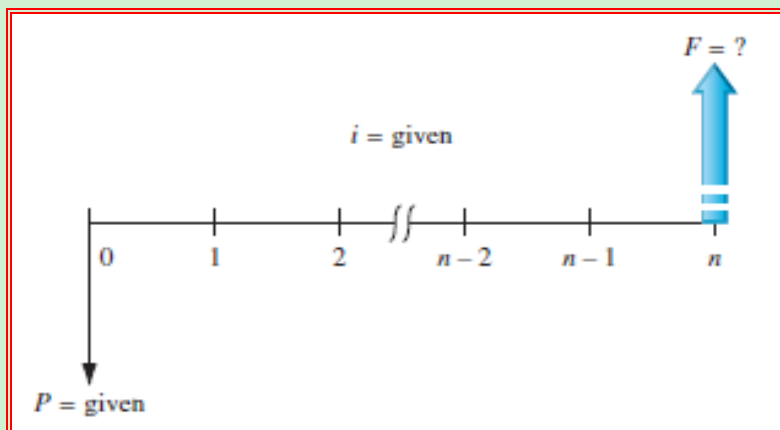
LEARNING OUTCOMES

- 1. F/P and P/F Factors**
- 2. P/A and A/P Factors**
- 3. F/A and A/F Factors**
- 4. Factor Values**
- 5. Arithmetic Gradient**
- 6. Geometric Gradient**
- 7. Find i or n**

Single Payment Factors (F/P and P/F)

Single payment factors involve only **P** and **F**.

Cash flow diagrams are as follows:



↑

$$F = P(1 + i)^n$$

Formulas are as follows:

↓

$$P = F[1 / (1 + i)^n]$$

Terms in parentheses or brackets are called *factors*. Values are in tables for i and n values

Factors are represented in **standard factor notation** such as **(F/P, i, n)**,
where letter to left of slash is what is sought; letter to right represents what is given

F/P and P/F for Spreadsheets

Future value F is calculated using FV function:

$$= \text{FV}(i\%, n, , P)$$

Present value P is calculated using PV function:

$$= \text{PV}(i\%, n, , F)$$

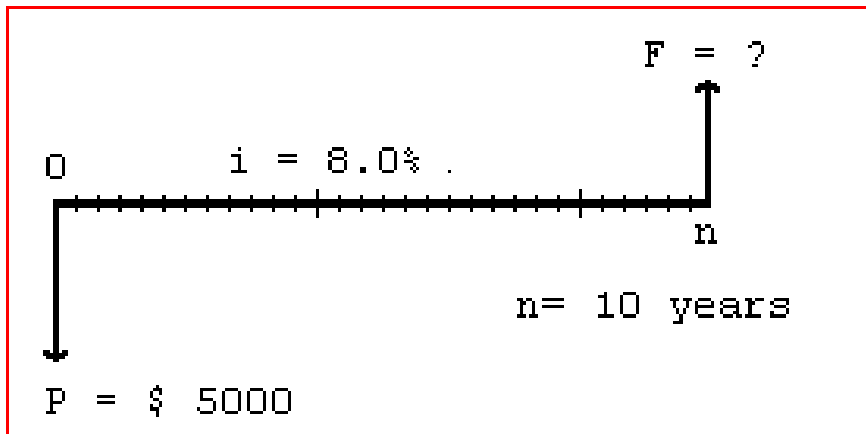
Note the use of double commas in each function

Example: Finding Future Value

A person deposits \$5000 into an account which pays interest at a rate of 8% per year. The amount in the account after 10 years is closest to:

- (A) \$2,792 (B) \$9,000 (C) \$10,795 (D) \$12,165

The cash flow diagram is:



Solution:

$$\begin{aligned} F &= P(F/P, i, n) \\ &= 5000(F/P, 8\%, 10) \\ &= 5000(2.1589) \\ &= \$10,794.50 \end{aligned}$$

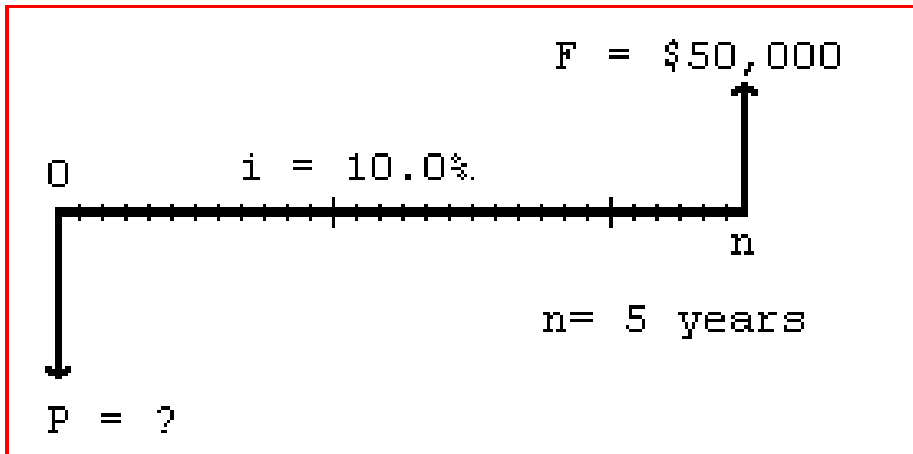
Answer is (C)

Example: Finding Present Value

A small company wants to make a single deposit now so it will have enough money to purchase a backhoe costing \$50,000 five years from now. If the account will earn interest of 10% per year, the amount that must be deposited now is nearest to:

- (A) \$10,000 (B) \$ 31,050 (C) \$ 33,250 (D) \$319,160

The cash flow diagram is:



Solution:

$$\begin{aligned} P &= F(P/F, i, n) \\ &= 50,000(P/F, 10\%, 5) \\ &= 50,000(0.6209) \\ &= \$31,045 \end{aligned}$$

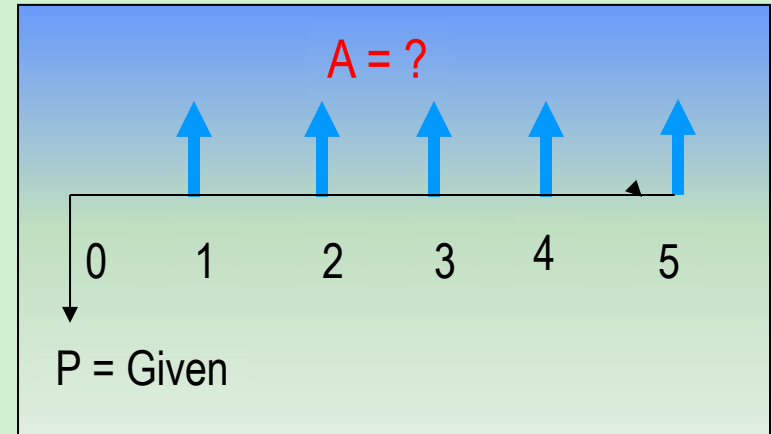
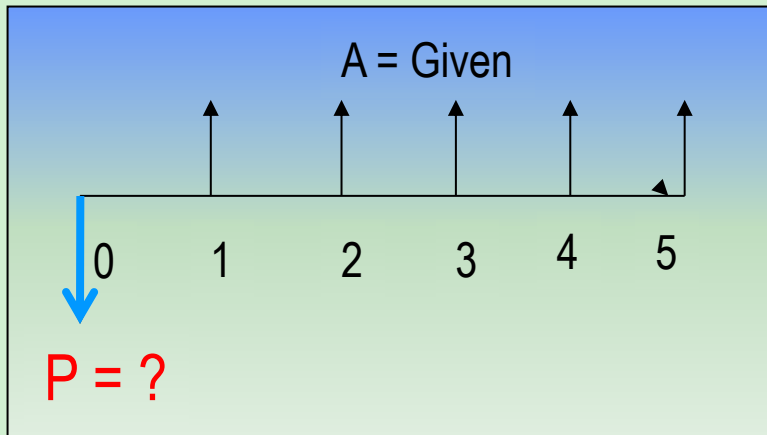
Answer is (B)

Uniform Series Involving P/A and A/P

The uniform series factors that involve **P** and **A** are derived as follows:

- (1) Cash flow occurs in **consecutive** interest periods
- (2) Cash flow amount is **same** in each interest period

The cash flow diagrams are:



$$P = A(P/A, i, n) \longleftarrow \text{Standard Factor Notation} \longrightarrow A = P(A/P, i, n)$$

Note: P is one period **Ahead** of first A value

Example: Uniform Series Involving P/A

A chemical engineer believes that by modifying the structure of a certain water treatment polymer, his company would earn an extra \$5000 per year. At an interest rate of 10% per year, how much could the company afford to spend now to just break even over a 5 year project period?

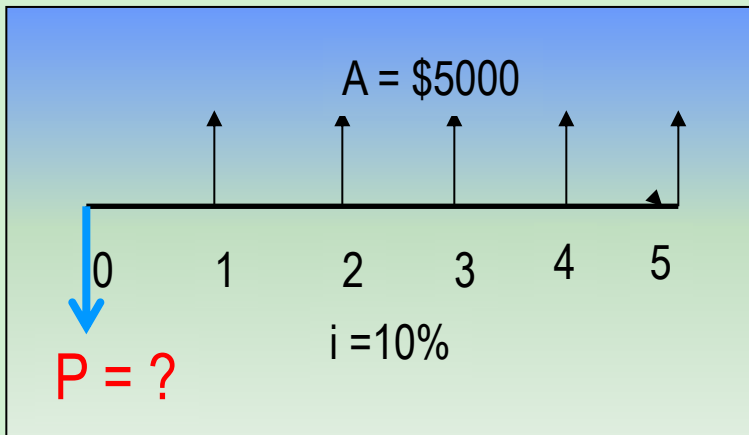
(A) \$11,170

(B) 13,640

(C) \$15,300

(D) \$18,950

The cash flow diagram is as follows:



Solution:

$$\begin{aligned} P &= 5000(P/A, 10\%, 5) \\ &= 5000(3.7908) \\ &= \$18,954 \end{aligned}$$

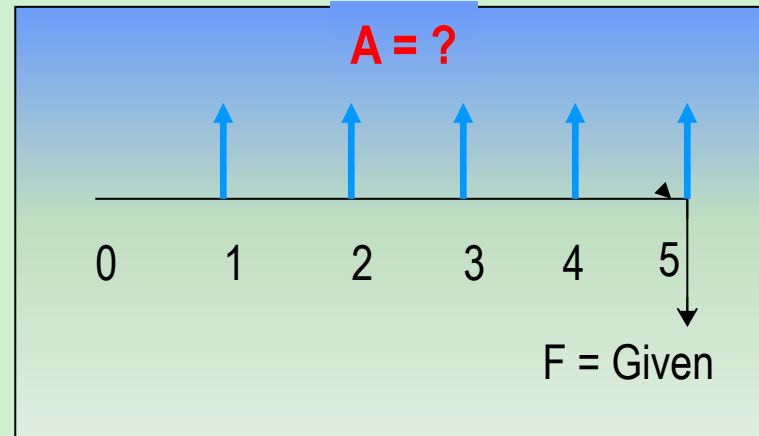
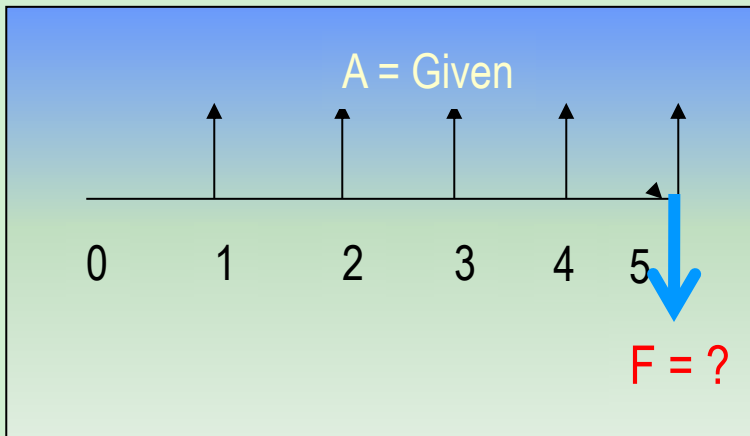
Answer is (D)

Uniform Series Involving F/A and A/F

The uniform series factors that involve **F** and **A** are derived as follows:

- (1) Cash flow occurs in **consecutive** interest periods
- (2) Last cash flow occurs in **same** period as F

Cash flow diagrams are:



$$F = A(F/A, i, n) \longleftarrow \text{Standard Factor Notation} \longrightarrow A = F(A/F, i, n)$$

Note: F takes place in the **same** period as last A

Example: Uniform Series Involving F/A

An industrial engineer made a modification to a chip manufacturing process that will save her company \$10,000 per year. At an interest rate of 8% per year, how much will the savings amount to in 7 years?

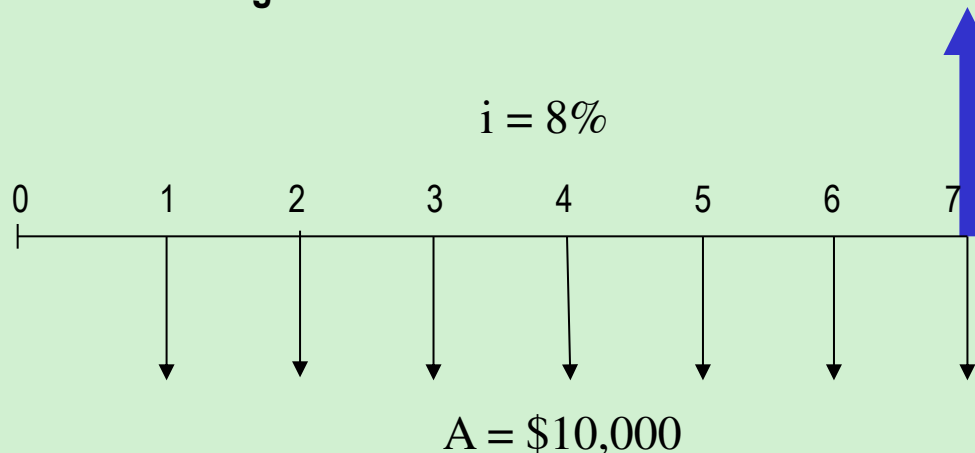
(A) \$45,300

(B) \$68,500

(C) \$89,228

(D) \$151,500

The cash flow diagram is:



Solution:

$$\begin{aligned} F &= 10,000(F/A, 8\%, 7) \\ &= 10,000(8.9228) \\ &= \$89,228 \end{aligned}$$

Answer is (C)

Factor Values for Untabulated i or n

3 ways to find factor values for untabulated i or n values

- ✳ Use formula
- ✳ Use spreadsheet function with corresponding P , F , or A value set to 1
- ✳ Linearly interpolate in interest tables

Formula or spreadsheet function is fast and accurate
Interpolation is only approximate

Example: Untabulated i

Determine the value for (F/P, 8.3%,10)

Formula: $F = (1 + 0.083)^{10} = 2.2197$ ← OK

Spreadsheet: $=FV(8.3\%,10,,1) = 2.2197$ ← OK

Interpolation:

8%	-----	2.1589
8.3%	-----	x
9%	-----	2.3674

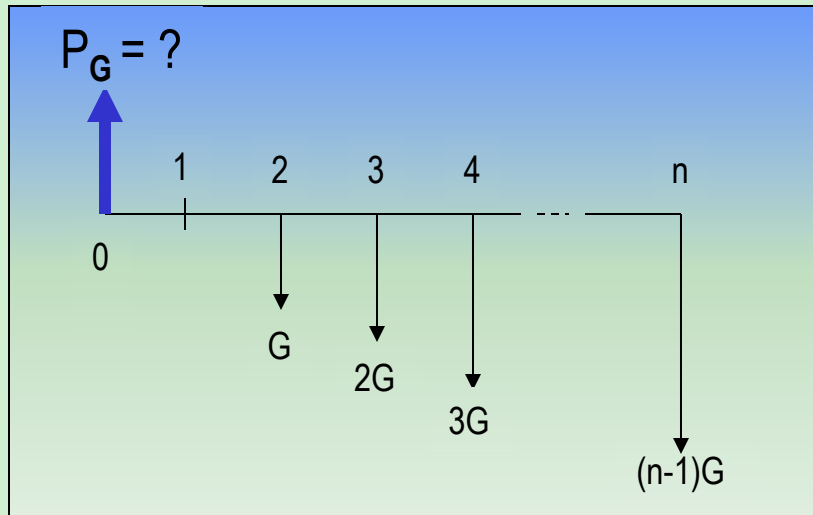
$x = 2.1589 + [(8.3 - 8.0)/(9.0 - 8.0)][2.3674 - 2.1589]$
 $= 2.2215$ ← (Too high)

Absolute Error = $2.2215 - 2.2197 = 0.0018$

Arithmetic Gradients

Arithmetic gradients *change* by the *same amount* each period

The cash flow diagram for the P_G of an arithmetic gradient is:



Standard factor notation is

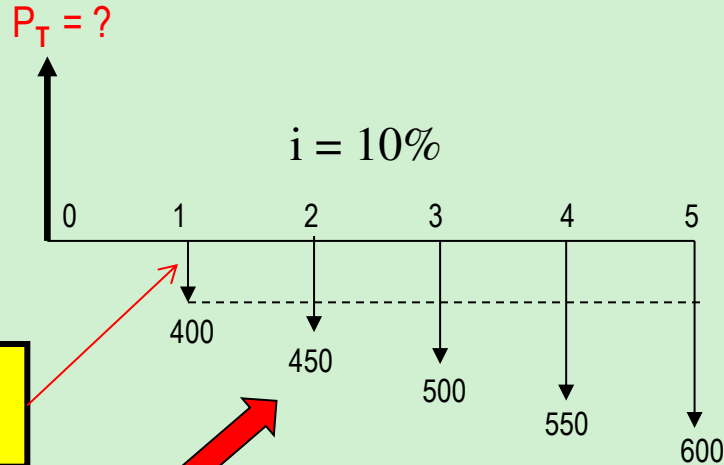
$$P_G = G(P/G, i, n)$$

G starts between periods 1 and 2
(not between 0 and 1)

This is because cash flow in year 1 is usually not equal to G and is handled separately as a *base amount* (shown on next slide)

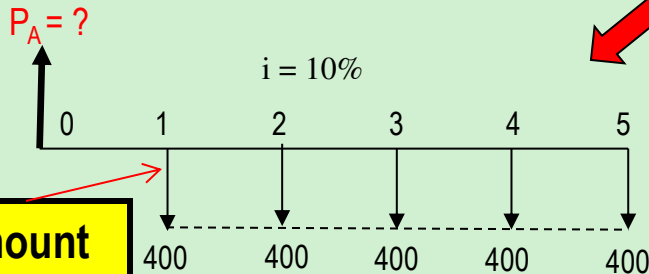
Note that P_G is located Two Periods Ahead of the first change that is equal to G

Typical Arithmetic Gradient Cash Flow



Amount in year 1
is base amount

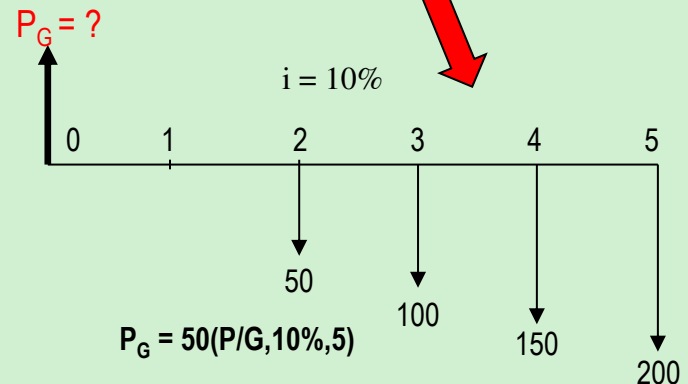
This diagram = this base amount plus this gradient



Amount
in year 1
is base
amount

$$P_A = 400(P/A, 10\%, 5)$$

+

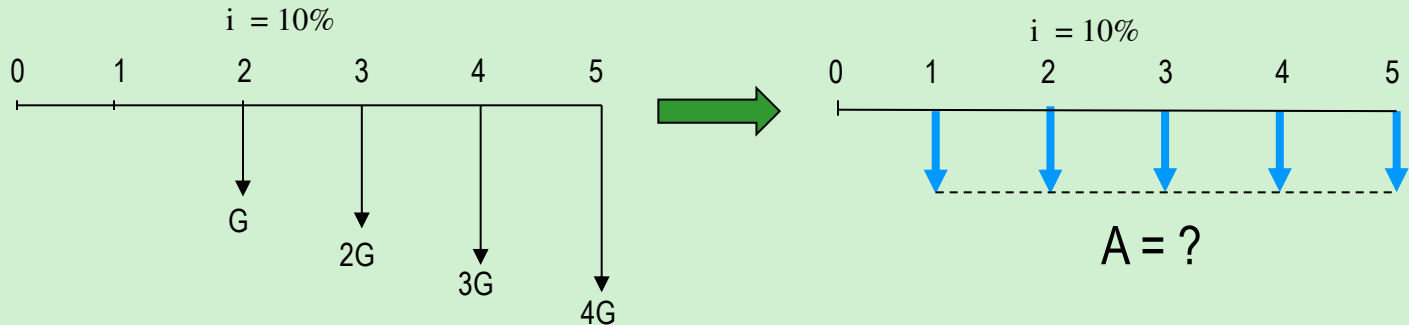


$$P_G = 50(P/G, 10\%, 5)$$

$$P_T = P_A + P_G = 400(P/A, 10\%, 5) + 50(P/G, 10\%, 5)$$

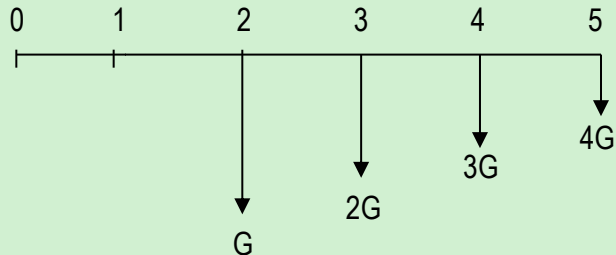
Converting Arithmetic Gradient to A

Arithmetic gradient can be converted into equivalent A value using $G(A/G, i, n)$



General equation when base amount is involved is

$$A = \text{base amount} + G(A/G, i, n)$$



For decreasing gradients,
change plus sign to minus

$$A = \text{base amount} - G(A/G, i, n)$$

Example: Arithmetic Gradient

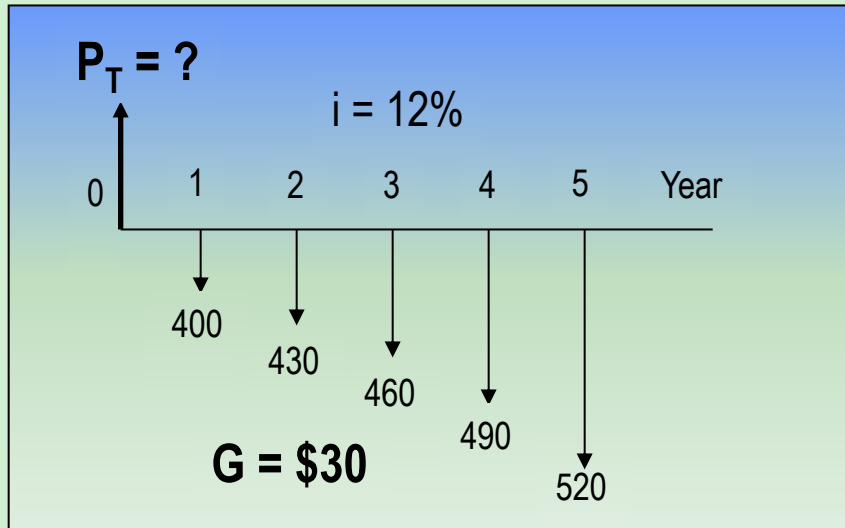
The present worth of \$400 in year 1 and amounts increasing by \$30 per year through year 5 at an interest rate of 12% per year is closest to:

(A) \$1532

(B) \$1,634

(C) \$1,744

(D) \$1,829



Solution:

$$\begin{aligned} P_T &= 400(P/A, 12\%, 5) + 30(P/G, 12\%, 5) \\ &= 400(3.6048) + 30(6.3970) \\ &= \$1,633.83 \end{aligned}$$

Answer is (B)

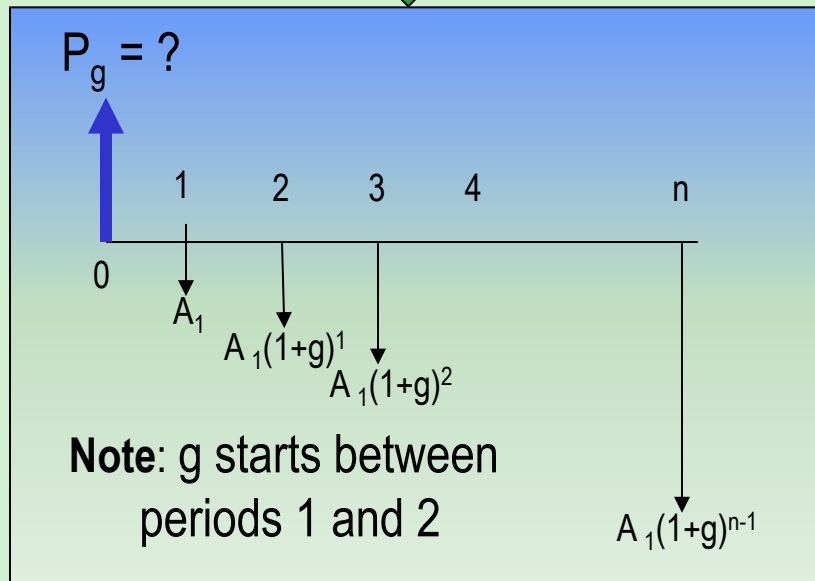
The cash flow could also be converted into an **A** value as follows:

$$\begin{aligned} A &= 400 + 30(A/G, 12\%, 5) \\ &= 400 + 30(1.7746) \\ &= \$453.24 \end{aligned}$$

Geometric Gradients

Geometric gradients change by the **same percentage** each period

Cash flow diagram for present worth
of geometric gradient



There are **no tables** for geometric factors

Use following equation for $g \neq i$:

$$P_g = A_1 \{1 - [(1+g)/(1+i)]^n\} / (i-g)$$

where: A_1 = cash flow in period 1
 g = rate of increase

$$\text{If } g = i, P_g = A_1 n / (1+i)$$

Note: If g is **negative**, change signs in front of both g values

Example: Geometric Gradient

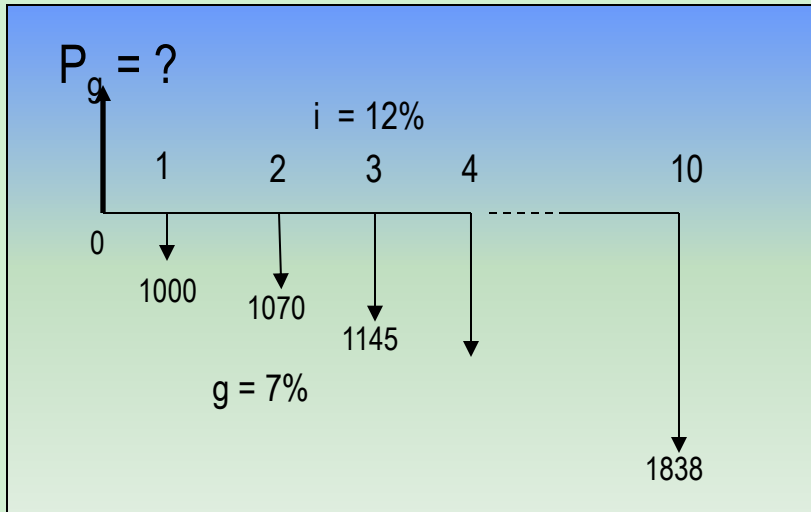
Find the present worth of \$1,000 in year 1 and amounts increasing by 7% per year through year 10. Use an interest rate of 12% per year.

(a) \$5,670

(b) \$7,333

(c) \$12,670

(d) \$13,550



Solution:

$$P_g = 1000[1 - (1 + 0.07/1 + 0.12)^{10}] / (0.12 - 0.07) \\ = \$7,333$$

Answer is (b)

To find A , multiply P_g by $(A/P, 12\%, 10)$

Unknown Interest Rate i

Unknown interest rate problems involve solving for i , given n and 2 other values (P , F , or A)

(Usually requires a trial and error solution or interpolation in interest tables)

Procedure: Set up equation with all symbols involved and solve for i

A contractor purchased equipment for \$60,000 which provided income of \$16,000 per year for 10 years. The annual rate of return of the investment was closest to:

(a) 15%

(b) 18%

(c) 20%

(d) 23%

Solution:

Can use either the P/A or A/P factor. Using A/P :

$$60,000(A/P, i\%, 10) = 16,000$$

$$(A/P, i\%, 10) = 0.26667$$

From A/P column at $n = 10$ in the interest tables, i is between 22% and 24% **Answer is (d)**

Unknown Recovery Period n

Unknown recovery period problems involve solving for n,
given i and 2 other values (P, F, or A)

(Like interest rate problems, they usually require a trial & error solution or interpolation in interest tables)

Procedure: Set up equation with all symbols involved and solve for n

A contractor purchased equipment for \$60,000 that provided income of \$8,000 per year. At an interest rate of 10% per year, the length of time required to recover the investment was closest to:

- (a) 10 years (b) 12 years (c) 15 years (d) 18 years

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60,000(A/P, 10\%, n) = 8,000$$

$$(A/P, 10\%, n) = 0.13333$$

From A/P column in $i = 10\%$ interest tables, n is between 14 and 15 years **Answer is (c)**

Summary of Important Points

- ✦ In P/A and A/P factors, P is *one period ahead* of first A
- ✦ In F/A and A/F factors, F is in *same period* as last A
- ✦ To find untabulated factor values, best way is to use *formula or spreadsheet*
- ✦ For arithmetic gradients, gradient G starts between *periods 1 and 2*
- ✦ Arithmetic gradients have 2 parts, *base amount* (year 1) and *gradient amount*
- ✦ For geometric gradients, gradient g starts been *periods 1 and 2*
- ✦ In geometric gradient formula, A_1 is amount in *period 1*
- ✦ To find unknown i or n, *set up equation involving all terms* and solve for i or n