Example 2:

Samples of plastic are analyzed for scratch and shock Resistance. The Results from 50 samples are summarized as follows:

		Shock Resistance: high	Low
Scratch Resistance	high	40	4
	low	1	5

let A denote the event that a sample has high shock Resistance and B denote the event that a sample space has high scratch Resistance

determine The Number of samples in

1→ $A \cap B$ $\boxed{40}$ ⇒ A ⇒ 44 samples

B ⇒ 44

2→ A' $\boxed{9}$

3→ $A \cup B$ $\boxed{45}$

example: Measurements of the time Needed to complete a chemical reaction might be modeled with the Sample space $S = R^+$ [The set of the real Number]

let: $E_1 = \{x \mid 1 \le x < 10\}$

$E_2 = \{x \mid 3 < x < 118\}$

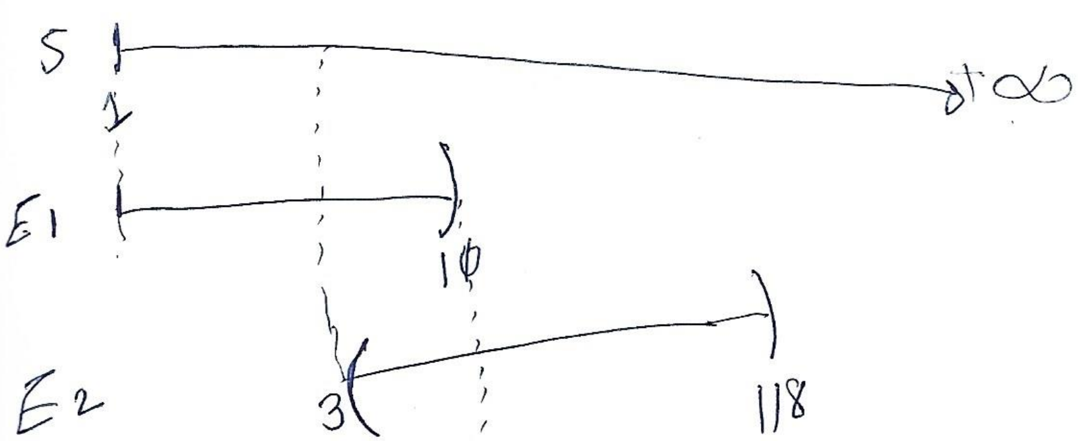
Then

$E_1 \cup E_2 = \{x \mid 1 \le x < 118\}$

$E_1 \cap E_2 = \{x \mid 3 < x < 10\}$

$E_1' = \{x \mid x \ge 10\} + \{x \mid 0 < x < 1\}$

$E_1' \cap E_2 = \{x \mid 10 \le x < 118\}$



2-1 Sample Spaces and Events

2-1.4 Counting Techniques

Permutations ⇒ linear permutation n position

⇒ select 1 from n ⇒ select 1 from (n-1) ⇒ ... so on.

The number of **permutations** of n different elements is $n!$ where

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 \qquad (2\text{-}1)$$

consider ~~the~~ ~~set~~ a set of elements $S = \{a, b, c\}$

A permutation of the elements is an ordered sequence of the elements.

for example abc, acb, bac, bca

cab, cba are all the permutation of S

$3! = 3 \times 2 \times 1 = 6$

Counting techniques.   مراجعة : (1)

→ Multiplication Rule. <slide 21>

If the Number of Ways of completing step 1 is n_1
" " " " " " 2 " n_2
3 " n_3
k " n_k

and so on..

the total Number of Ways of completing the operation is $n_1 \times n_2 \times n_3 \times \dots \times n_k$

→ Permutations. <22>

the Number of permutations of n different elements is $n!$

→ permutation of subsets <24>

the Number of permutation of subsets of r elements selected from a set of n different elements is

$$P_r^n = \frac{n!}{(n-r)!}$$

printed circuit with 8-location # of designs using different components

Permutations of similar objects

② مراجعة

the number of permutations of $n = n_1 + n_2 + \cdots + n_r$ objects of which n_1 are of ~~type~~ one type

n_2 " " second "

n_r rth "

$$\Rightarrow \frac{n!}{n_1!\, n_2!\, n_3! \cdots n_r!}$$

~~of~~ operational machine has 4 operation 2-drills & 2 Notches مثال

Now Combinations

the Number of combinations, subsets of size r that can be selected from a set of n elements is denoted as

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

printed circuit board with 8-location select 4 identical ~~Parts~~ components to be placed on the board how many different design is possible

A bin of 50 parts, contains 3 defective parts and 47 non-defective parts. (1A)

a sample of 6 parts is selected from the 50 parts.

that is, each part can only be selected once and the sample is a subset of the 50 parts.

⇒ how many different samples are there of size 6 that contains exactly 2 defective parts.

solution:

a subset of 2-defective parts can be formed by first choosing 2-deffective parts from 3 different parts

$$\binom{3}{2} = \frac{3!}{2!\,(3-1)!} = \frac{3\times 2\times 1}{\cancel{2}\times 1\ \times 1} = 3 \text{ different ways.}$$

then the second step is to select the remaing four parts from 47 acceptable parts in the bin

$$\Rightarrow \binom{47}{4} = \frac{47!}{4!\,(47-4)!} = 178\,365 \text{ different ways.}$$

therefore, from the multiplication rule, the number of sub sets of size 6 that contains exactly 2 defective parts and 4-non-defective parts.

$$3 * 178\,365 = 535\,095$$

additional ~~computational~~,

the total Number of different subsets of size 6 is found to be.

$$\binom{50}{6} = \frac{50!}{6!\,(44)!} = 15,890,700$$

different subsets.

the probability that a ~~the~~ sample contains exactly 2 defective parts is

$$\frac{535095}{15,890,700} = 0.034 = 3.4\%$$

exercise 2-34 page 30

(1)

⇒ a wireless garage door openere has a code determined by the up or down setting of 12 switches, how many outcomes are in the sample space of possible codes?

From counting techniques

⇒ Two possible ways for each switch

the sequence of swisch is important

that means ~~of~~ we have steps

then, like saying ~~of~~ we have

12 steps & each step consists of 2 ways

then

Number of possible ways (codes) $= \underbrace{2 \times 2 \times 2 \times \cdots \times 2}_{12 \text{ times}}$

$= 2^{12} = 4096$ code.

⇒ Similar to binary code.

in C++ ~~byte~~ (char) is of size

& 1 byte ⇒ 8 bit

possible decimal

2-35 < ~~slide~~ page 30 >. ②

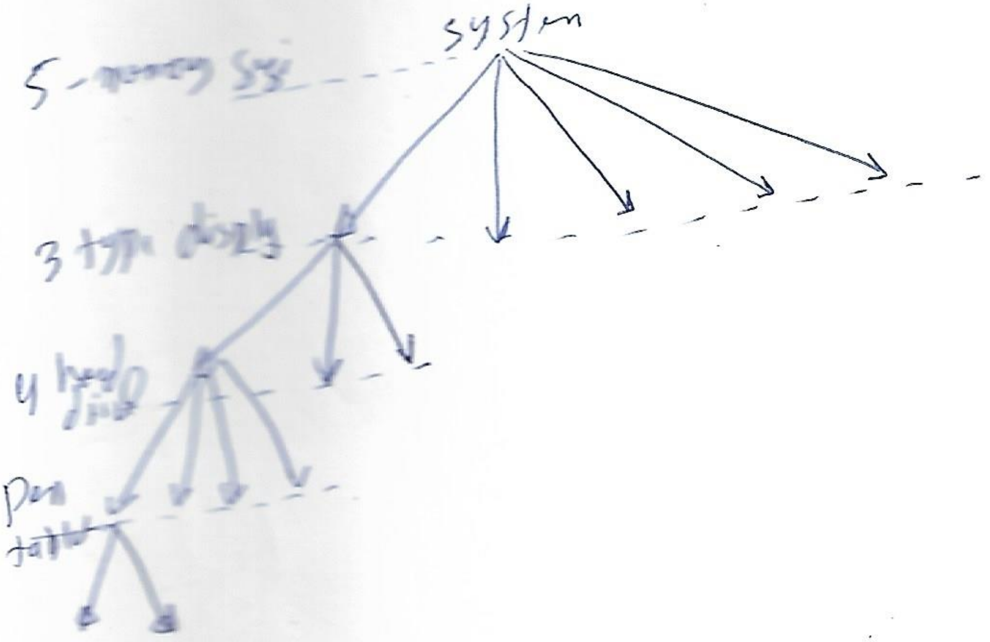
An order for a PDA < personal digital assistant > can specify any one of

→ 5 memory sizes
→ 3 types of display
→ 4 sizes of hard disks

and can either include or Not a pen tablet

how many different system can be ordered ?

from multiplication ~~counting~~ techniques. the answer is $5 \times 3 \times 4 \times 2 = 120$ different systems.



<2-38> page 30

A manufacturing operation consists of 10 operations ~~h~~ that can be completed in any order.

How many different production sequence are possible.

$\Rightarrow$ $10! = 3\,628\,800$ different sequence

<2-39> page 30

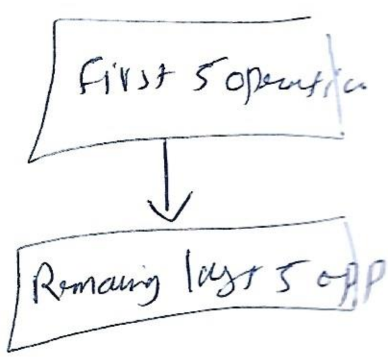
A manufacturing operations consists of 10 operations.

5 operations must be completed before any of the Remaining 5 operations can begin.

Within each set of 5, operations can be completed in any order.

how many different production sequences are possible?

$\Rightarrow$ Notic We have steps



Now within each ~~operat~~ set of 5 operation can be completed in any order

of possible ways for First set = 5! -

(2-40) page 30

in a sheet metal operation.

3 Notches
4 bends

are required;

if the operations can be done in any order, how many different ways of completing the manufacturing are possible.

→

the Number of permutation of $n = n_1 + n_2 + \cdots + n_r$ objects of which n_1 are of type 1

& n_2 are of type 2,

then $\frac{7!}{3!4!} = 35$ sequences are possible

(2-41) page 30 (5)

A lot of 140 semiconductor chips is inspected by choosing a sample of 5 chips

Assume 10 of the chips do NOT conform to customer requirements.

(a) how many different samples are possible?

→ the Number of samples of size 5 is

$$\binom{140}{5} = \frac{140!}{5!\,(140-5)!} = 416\,965\,528$$

(b) how many samples of 5 contains exactly one non-conforming chip

→ # of ways of selecting 1 non-conf sample

$$= \binom{10}{1} = \frac{10!}{1!\,(10-1)!} = \frac{10!}{9!} = 10$$

→ # of ways of selecting 4 - conforming samples

$$\binom{130}{4} = \frac{130!}{4!\,(130-4)!} = 11\,358\,880$$

→ then, the Number of samples that contains

(c) how many samples of five contain ~~at least one~~ non-conforming chip

→ the Number of samples that contain at least one non-conforming chip is the ~~#~~

the total Number of samples $\binom{140}{5}$

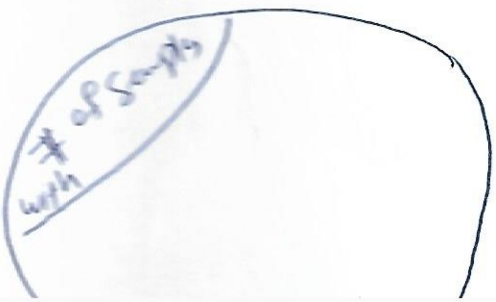
minus the Number of samples that contain No - Non conforming chips

$\binom{130}{5}$

that is

$$\binom{140}{5} - \binom{130}{5} = \frac{140!}{5!\,(135)!} - \frac{130!}{5!\,(125)!}$$

$$= 130\ 721\ 725$$



total # of samples = # of samples with all comforms + # of samples at least one non-conforming

Exercise 1 < 2-43 > page 31

7

(a) process has 5 samples

control has 2 samples

the # of possible sequences $= \frac{7!}{2!\,5!} = 21$

(b)

Now we consider the 5 processes are different,

that means like we have 5 different types

& we have 2 identical control samples of ~~control~~

$n_1 = 1$

$n_2 = 1$

$n_3 = 1$

$n_4 = 1$

$n_5 = 1$

$n_6 = 2$

possible sequences $= \frac{7!}{1!\,1!\,1!\,1!\,1!\,2!}$

consider The ~~TWO~~ identical labels to be labeled as ~~X~~ & ~~X~~

consider The 5 different process samples to be labeled as A, B, C, D, E

then the possible sequences ~~should~~ must start by X

then the possible sequences will be permutation between the elements

A B C D E & X

~~go~~ sequences = 6! = 720

EX2 < 2-45 > page 31 (7)

(a) From multiplication Rule $10^3 = 1000$ prefixs are possible

$\boxed{\text{First digit}}$ 10 way
2nd digit 10 way
3rd digit, 10 way,

(b) $\boxed{\text{First digit}}$ $\Rightarrow$ 8 ways,

$\boxed{\text{middle digit}}$ $\Rightarrow$ 2 way.

$\boxed{\text{last digit}}$ $\Rightarrow$ 10 way,

possible prefixes $= 8 \times 2 \times 10$

(c) like saying selecting subset of size r from set of n - different digits

$$P_3^{10} = \frac{10!}{7!} = 720$$

$\Rightarrow$ or other way
No. of ways for selecting first digit = 10

EXERCISES:

2-58:

A credit card contains 16 digits between 0 & 9 only 100 millon numbers are valid, if a number is entered randomly, what is the probability that it is a valid number ??

$\Rightarrow$ digits 0, 1, 2, ---, 9

if the number hold one digit. then it has 10 possibilities $(10)^1$

if the number holds 2 digits, then it has 100 possibilities $(10)^2$

in our case we have 16 digits

# of possibilities $= 10^{16}$

but only 100 millon are valid $(10)^8$ valid

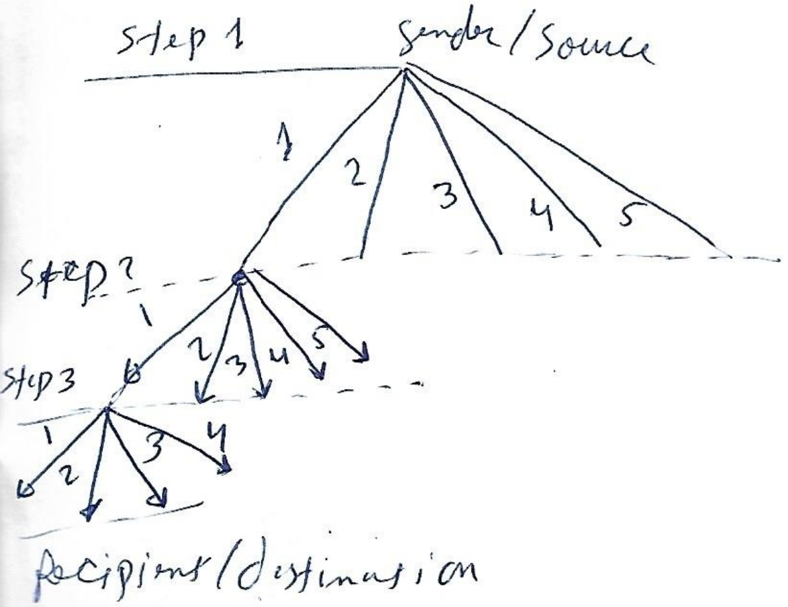
$$P(\text{valid}) = \frac{10^8}{10^{16}} = \frac{1}{10^8}$$

EXERCISES: 2-60

A message can follow different paths through servers on a Network. the sender's message can go to one of five servers for the first stage. step.
each of them can send to five servers at the second step, each of which can send to four servers at the third step.
and then message goes to the Recipient

a) How many paths are possible?

b) If all paths are equally likely, what is the prob. that a message passes through the first of four servers at the third step.



# of ways/paths that message passes through first of four servers at the step

$= 5 * 5 = 25$

then prob is $\frac{25}{100} = \frac{1}{4}$

paths $= 5 * 5 * 4 = \cancel{25}$ 100

Example: 2-74 page 41

A computer system uses passwords that are 6-characters and each character is one of 26 letters (a–z) or 10 integers (0–9)

// Uppercase letters are Not used.

let $\underline{A}$ denote the event that a password begin with a vowel either (a, e, i, o, u)

let B denote the event that a password ends with an even number (either (0, 2, 4, 6, 8)

Suppose a hacker selects a password at Random. determine the following Probabilities.

(a) $P(A)$ (b) $P(B)$

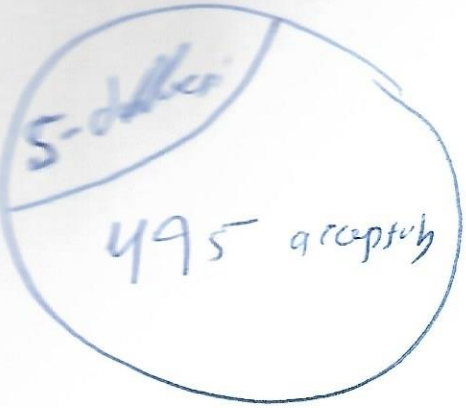
(c) $P(A \cap B)$ (d) $P(A \cup B)$

$P(A) = 5/36$ $P(A \cap B)$ = if both are independent
$= P(A) \cdot P(B) = 0.01929$

$P(B) = 5/$

Exercis <1> 2-84 page 46

1



total = 500

(a) let A : event That Sample is deffective

B : " " " acceptable

the prob that The First is deff $= \frac{5}{500}$

but The prob that The second is deffective given that The First is deffective

$= \frac{4}{499} = 0.0080$

(b) $p(dd) = \frac{5}{500} \cdot \frac{4}{499} = 0.000080$

deffective deffective

(c) $p(aa) = \frac{495}{} * \frac{494}{} = 0.98$

(d) P(First defective) = $\frac{5}{500}$

P(~~First~~ Second defective | ~~Second~~ First is defective) = $\frac{4}{499}$

P(third defective | First & Second are defective) = $\frac{3}{498}$

(e) P(~~First~~ third defective | First defective & Second okay)

~~P(d | d o)~~

~~498~~

$\frac{4}{498}$

⇒ then # of Remaining defective Samples are 4

⇒ # of Remaining all Samples are 498

then P(...) = $\frac{4}{498}$

(F) $\left(\frac{5}{500}\right)\left(\frac{4}{499}\right)\left(\frac{3}{498}\right)$ =

Exercise 2-86 page 46 (3)

2

(a) $\frac{1}{36^7}$

(b) $\underbrace{\text{First char}}\ \underbrace{\text{Remaining 6-char}}$

we have 5 possibility

we have 6 char each has 36 possibilities

$\Rightarrow$ # of ways $= 36^6$

From Counting Techniques total Number of possible sequences

$= 5 * 36^6$

$\Rightarrow$ the the prob $= \frac{1}{5 * 36^6}$

(c) $\underbrace{\text{First char}}_{\text{start with vowels}}\ \underbrace{\text{5 char}}_{\text{any 36}}\ \underbrace{\text{last char}}_{\text{is [illegible]}}$

4

From counting => then

total # of ways = $5 * 36 * 5$

The prob = $\dfrac{1}{5 * (36)^5 * 5}$

Example 2-23

Consider the 400 pairs in the following table.

		Surface flaw		
		Yes (F)	No	total
defective	yes (D)	10	18	28
	No	30	342	372
	total	40	360	400

$$P(D|F) = \frac{P(D \cap F)}{P(F)} = \frac{10/400}{40/400} = \frac{10}{40}$$

also Notice

$P(F) = 40/400$

$P(D) = 28/400$

$$P(F|D) = \frac{P(F \cap D)}{P(D)} = \frac{10/400}{28/400} = \frac{10}{28}$$

$$P(D|F) = \frac{P(F \cap D)}{P(F)} = \frac{10/400}{40/400} = \frac{10}{40}$$

$$P(D \mid F) = 10/40$$

$$P(\bar{D} \mid F) = \frac{P(\bar{D} \cap F)}{P(F)} = \frac{30/400}{40/400} = \frac{30}{40}$$

$$P(D \mid \bar{F}) = \frac{P(D \cap \bar{F})}{P(\bar{F})} = \frac{18/400}{360/400} = \frac{18}{360}$$

$$P(\bar{D} \mid \bar{F}) = \frac{P(\bar{D} \cap \bar{F})}{P(\bar{F})} = \frac{342/400}{360/400}$$

$$= 342/360$$