2-1 Sample Spaces and Events

2-1.4 Counting Techniques

Multiplication Rule

If an operation can be described as a sequence of k steps, and

if the number of ways of completing step 1 is n_1, and

if the number of ways of completing step 2 is n_2 for each way of completing step 1, and

if the number of ways of completing step 3 is n_3 for each way of completing step 2, and so forth,

the total number of ways of completing the operation is

$$n_1 \times n_2 \times \cdots \times n_k$$

2-1 Sample Spaces and Events

2-1.4 Counting Techniques

Permutations

The number of **permutations** of n different elements is $n!$ where

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 \qquad (2\text{-}1)$$

2-1 Sample Spaces and Events

2-1.4 Counting Techniques

Permutations : Example 2-10

A printed circuit board has eight different locations in which a component can be placed. If four different components are to be placed on the board, how many different designs are possible?

Each design consists of selecting a location from the eight locations for the first component, a location from the remaining seven for the second component, a location from the remaining six for the third component, and a location from the remaining five for the fourth component. Therefore,

$$P_4^8 = 8 \times 7 \times 6 \times 5 = \frac{8!}{4!} = 1680 \text{ different designs are possible.}$$

2-1 Sample Spaces and Events

2-1.4 Counting Techniques

Permutations of Subsets

The number of permutations of subsets of r elements selected from a set of n different elements is

$$P_r^n = n \times (n-1) \times (n-2) \times \cdots \times (n-r+1) = \frac{n!}{(n-r)!} \qquad (2\text{-}2)$$

2-1 Sample Spaces and Events

2-1.4 Counting Techniques

Permutations of Subsets: Example 2-11

Consider a machining operation in which a piece of sheet metal needs two identical diameter holes drilled and two identical size notches cut. We denote a drilling operation as d and a notching operation as n. In determining a schedule for a machine shop, we might be interested in the number of different possible sequences of the four operations. The number of possible sequences for two drilling operations and two notching operations is

$$\frac{4!}{2!\,2!} = 6$$

The six sequences are easily summarized: *ddnn*, *dndn*, *dnnd*, *nddn*, *ndnd*, *nndd*.

2-1 Sample Spaces and Events

2-1.4 Counting Techniques

Permutations of Similar Objects

The number of permutations of $n = n_1 + n_2 + \cdots + n_r$ objects of which n_1 are of one type, n_2 are of a second type, … , and n_r are of an rth type is

$$\frac{n!}{n_1!\, n_2!\, n_3! \ldots n_r!} \qquad (2\text{-}3)$$

2-1 Sample Spaces and Events

2-1.4 Counting Techniques

Permutations of Similar Objects: Example 2-12

A part is labeled by printing with four thick lines, three medium lines, and two thin lines. If each ordering of the nine lines represents a different label, how many different labels can be generated by using this scheme?

From Equation 2-3, the number of possible part labels is

$$\frac{9!}{4!\,3!\,2!} = 2520$$

2-1 Sample Spaces and Events

2-1.4 Counting Techniques

Combinations

The number of combinations, subsets of size r that can be selected from a set of n elements, is denoted as $\binom{n}{r}$ or C_r^n and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \qquad (2\text{-}4)$$

2-1 Sample Spaces and Events

2-1.4 Counting Techniques

Combinations: Example 2-13

A printed circuit board has eight different locations in which a component can be placed. If five identical components are to be placed on the board, how many different designs are possible?

Each design is a subset of the eight locations that are to contain the components. From Equation 2-4, the number of possible designs is

$$\frac{8!}{5!\,3!} = 56$$

2-2 Interpretations of Probability

2-2.1 Introduction

Probability

- Used to quantify likelihood or chance
- Used to represent risk or uncertainty in engineering applications
- Can be interpreted as our **degree of belief** or **relative frequency**

2-2 Interpretations of Probability

2-2.1 Introduction

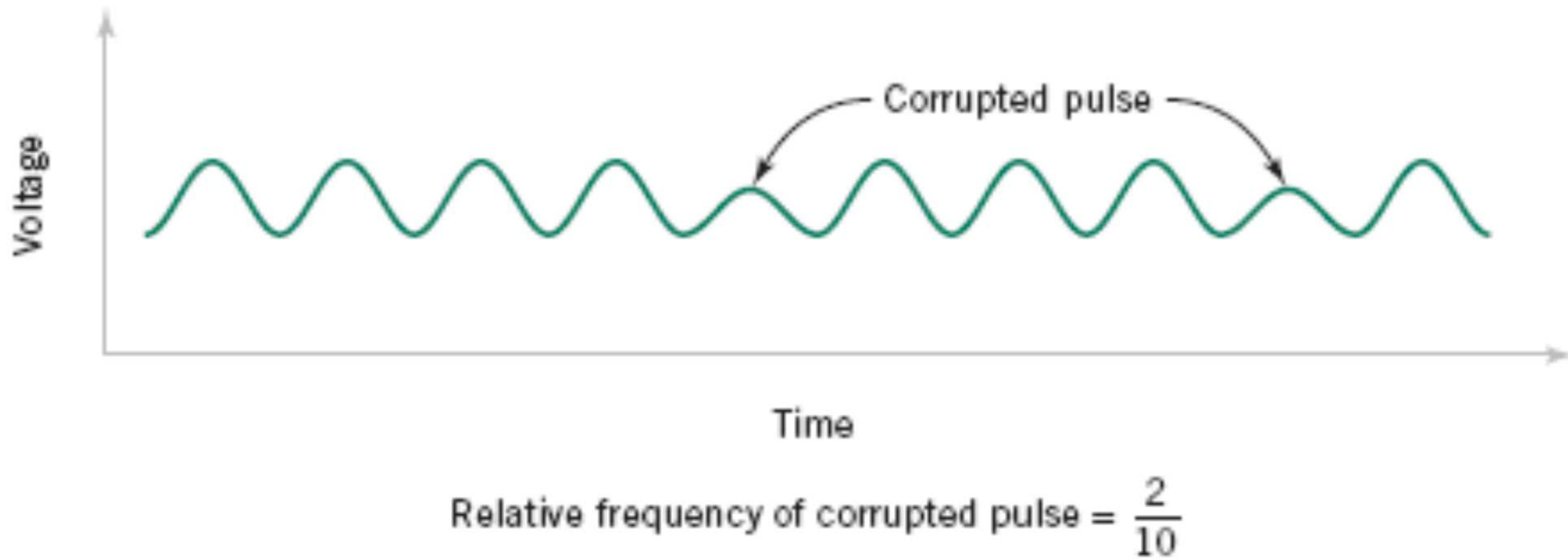

Figure 2-10 Relative frequency of corrupted pulses sent over a communications channel.

2-2 Interpretations of Probability

Equally Likely Outcomes

Whenever a sample space consists of N possible outcomes that are equally likely, the probability of each outcome is $1/N$.

2-2 Interpretations of Probability

Example 2-15

Assume that 30% of the laser diodes in a batch of 100 meet the minimum power requirements of a specific customer. If a laser diode is selected randomly, that is, each laser diode is equally likely to be selected, our intuitive feeling is that the probability of meeting the customer's requirements is 0.30.

Let E denote the subset of 30 diodes that meet the customer's requirements. Because E contains 30 outcomes and each outcome has probability 0.01, we conclude that the probability of E is 0.3. The conclusion matches our intuition. Figure 2-10 illustrates this example.

2-2 Interpretations of Probability

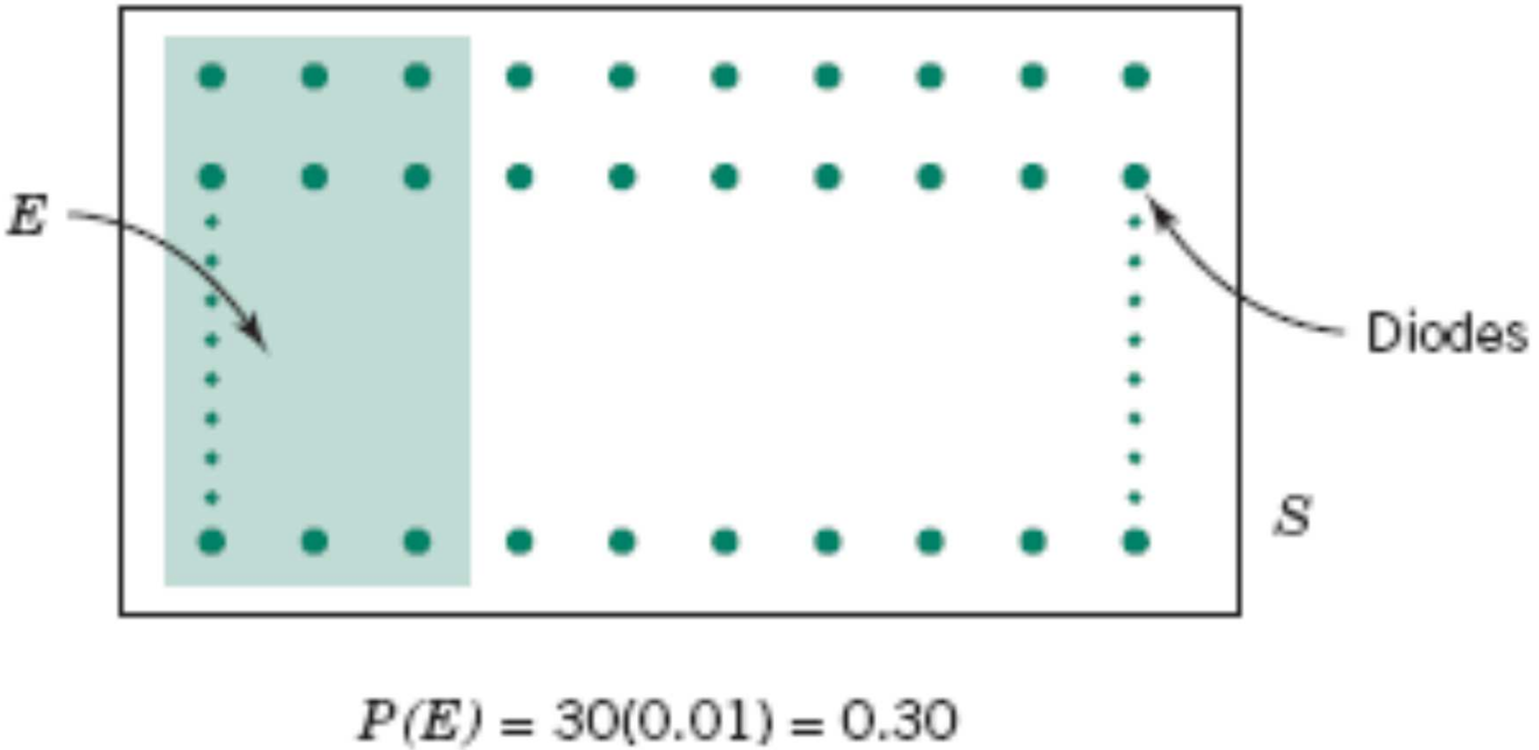

Figure 2-11 Probability of the event *E* is the sum of the probabilities of the outcomes in *E*

2-2 Interpretations of Probability

Definition

For a discrete sample space, the *probability of an event E,* denoted as $P(E)$, equals the sum of the probabilities of the outcomes in E.

2-2 Interpretations of Probability

Example 2-16

A random experiment can result in one of the outcomes $\{a, b, c, d\}$ with probabilities 0.1, 0.3, 0.5, and 0.1, respectively. Let A denote the event $\{a, b\}$, B the event $\{b, c, d\}$, and C the event $\{d\}$. Then,

$$P(A) = 0.1 + 0.3 = 0.4$$

$$P(B) = 0.3 + 0.5 + 0.1 = 0.9$$

$$P(C) = 0.1$$

Also, $P(A') = 0.6$, $P(B') = 0.1$, and $P(C') = 0.9$. Furthermore, because $A \cap B = \{b\}$, $P(A \cap B) = 0.3$. Because $A \cup B = \{a, b, c, d\}$, $P(A \cup B) = 0.1 + 0.3 + 0.5 + 0.1 = 1$. Because $A \cap C$ is the null set, $P(A \cap C) = 0$.

2-2 Interpretations of Probability

2-2.2 Axioms of Probability

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

If S is the sample space and E is any event in a random experiment,

(1) $P(S) = 1$

(2) $0 \leq P(E) \leq 1$

(3) For two events E_1 and E_2 with $E_1 \cap E_2 = \emptyset$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

2-3 Addition Rules

Probability of a Union

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \qquad (2\text{-}5)$$

2-3 Addition Rules

Mutually Exclusive Events

If A and B are mutually exclusive events,

$$P(A \cup B) = P(A) + P(B) \quad (2\text{-}6)$$

2-3 Addition Rules

Three Events

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \quad (2\text{-}7)$$

2-3 Addition Rules

A collection of events, $E_1, E_2, \ldots, E_k$, is said to be **mutually exclusive** if for all pairs,

$$E_i \cap E_j = \varnothing$$

For a collection of mutually exclusive events,

$$P(E_1 \cup E_2 \cup \ \ldots \ \cup E_k) = P(E_1) + P(E_2) + \ldots P(E_k) \qquad (2\text{-}8)$$

2-3 Addition Rules

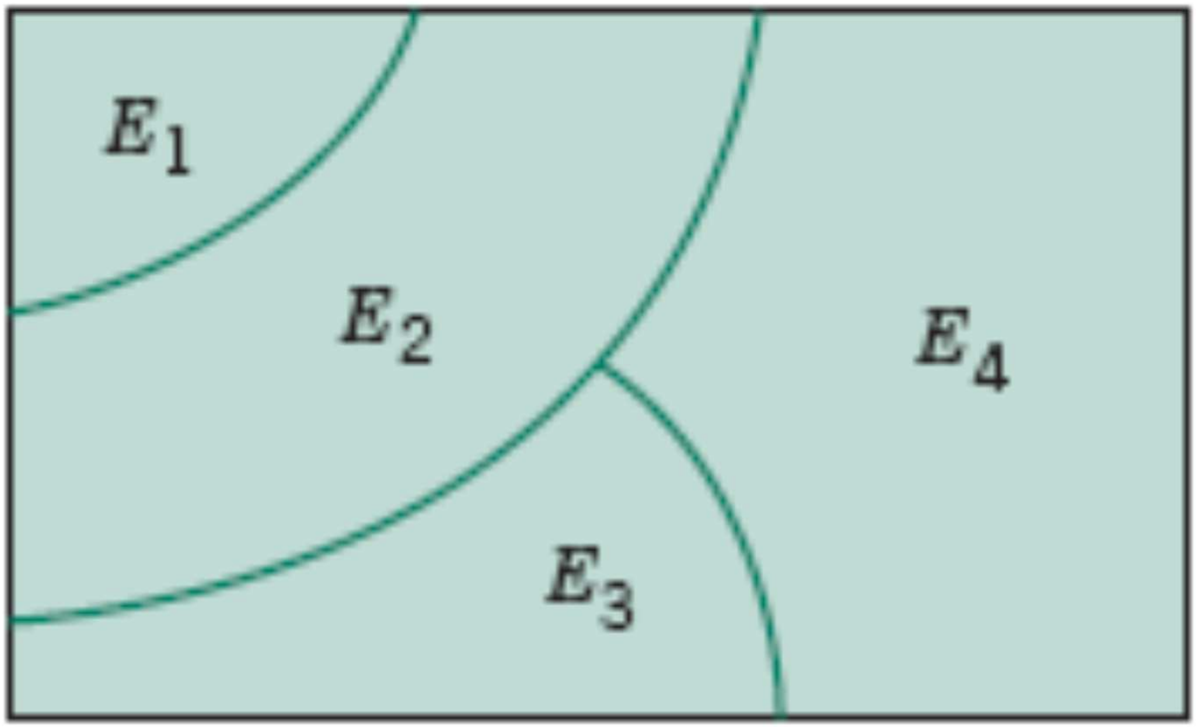

Figure 2-12 Venn diagram of four mutually exclusive events

2-3 Addition Rules

Example 2-21

A simple example of mutually exclusive events will be used quite frequently. Let X denote the pH of a sample. Consider the event that X is greater than 6.5 but less than or equal to 7.8. This probability is the sum of any collection of mutually exclusive events with union equal to the same range for X. One example is

$$P(6.5 < X \leq 7.8) = P(6.5 < X \leq 7.0) + P(7.0 < X \leq 7.5) + P(7.5 < X \leq 7.8)$$

Another example is

$$P(6.5 < X \leq 7.8) = P(6.5 < X \leq 6.6) + P(6.6 < X \leq 7.1) + P(7.1 < X \leq 7.4) + P(7.4 < X \leq 7.8)$$

The best choice depends on the particular probabilities available.