2 Probability

CHAPTER OUTLINE

LEARNING OBJECTIVES

After careful study of this chapter you should be able to do the following:

1. Understand and describe sample spaces and events for random experiments with graphs, tables, lists, or tree diagrams
2. Interpret probabilities and use probabilities of outcomes to calculate probabilities of events in discrete sample spaces
3. Use permutation and combinations to count the number of outcomes in both an event and the sample space.
4. Calculate the probabilities of joint events such as unions and intersections from the probabilities of individual events
5. Interpret and calculate conditional probabilities of events
6. Determine the independence of events and use independence to calculate probabilities
7. Use Bayes' theorem to calculate conditional probabilities
8. Understand random variables

2-1 Sample Spaces and Events

2-1.1 Random Experiments

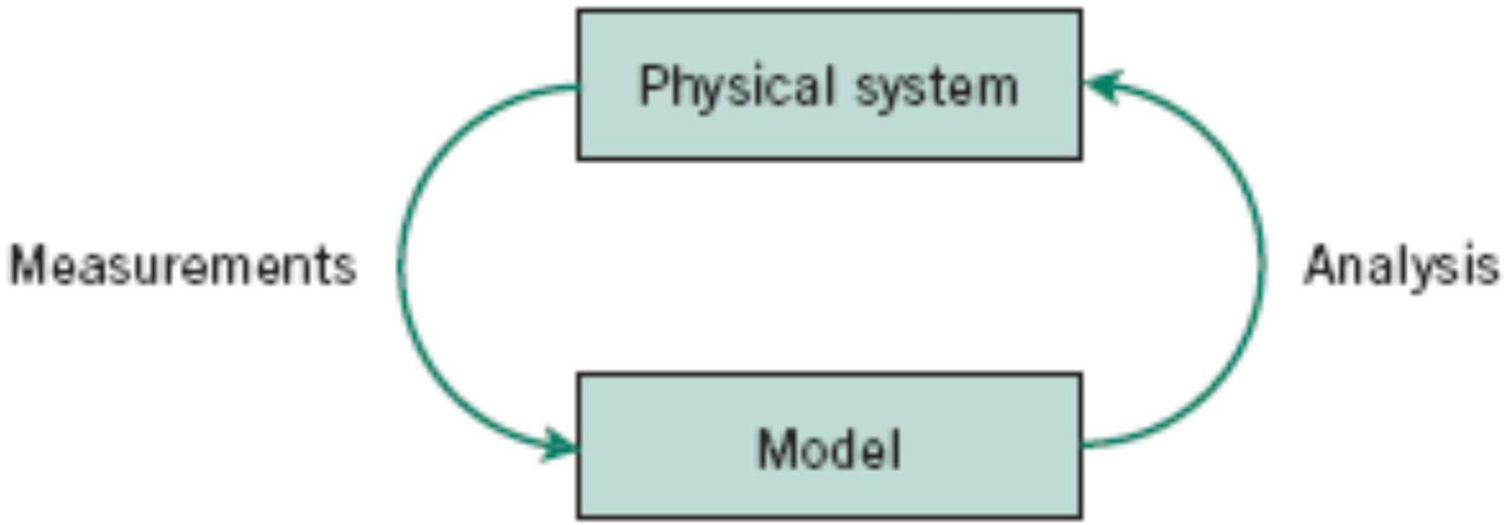

Figure 2-1 Continuous iteration between model and physical system.

2-1 Sample Spaces and Events

2-1.1 Random Experiments

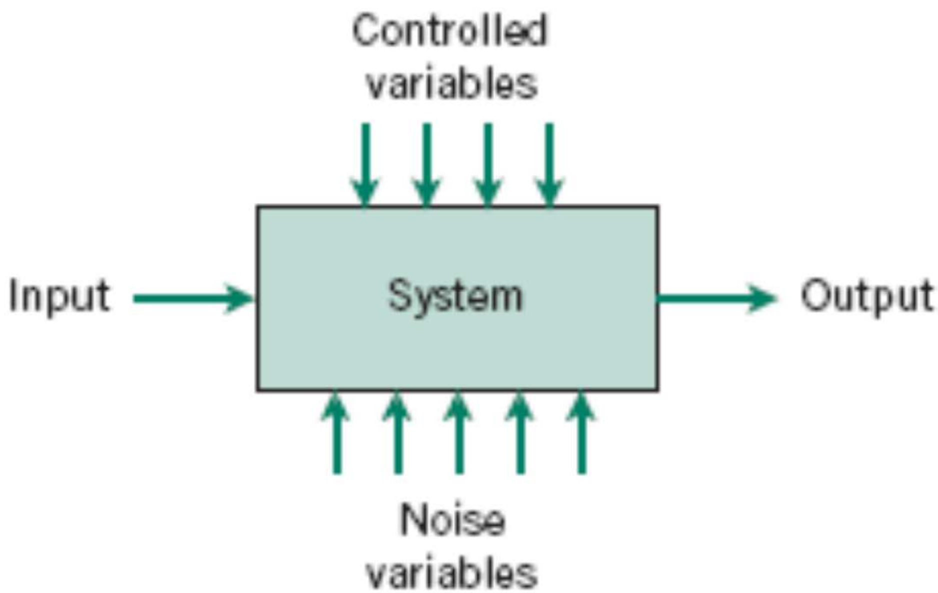

Figure 2-2 Noise variables affect the transformation of inputs to outputs.

2-1 Sample Spaces and Events

2-1.1 Random Experiments

Definition

An experiment that can result in different outcomes, even though it is repeated in the same manner every time, is called a **random experiment.**

2-1 Sample Spaces and Events

2-1.1 Random Experiments

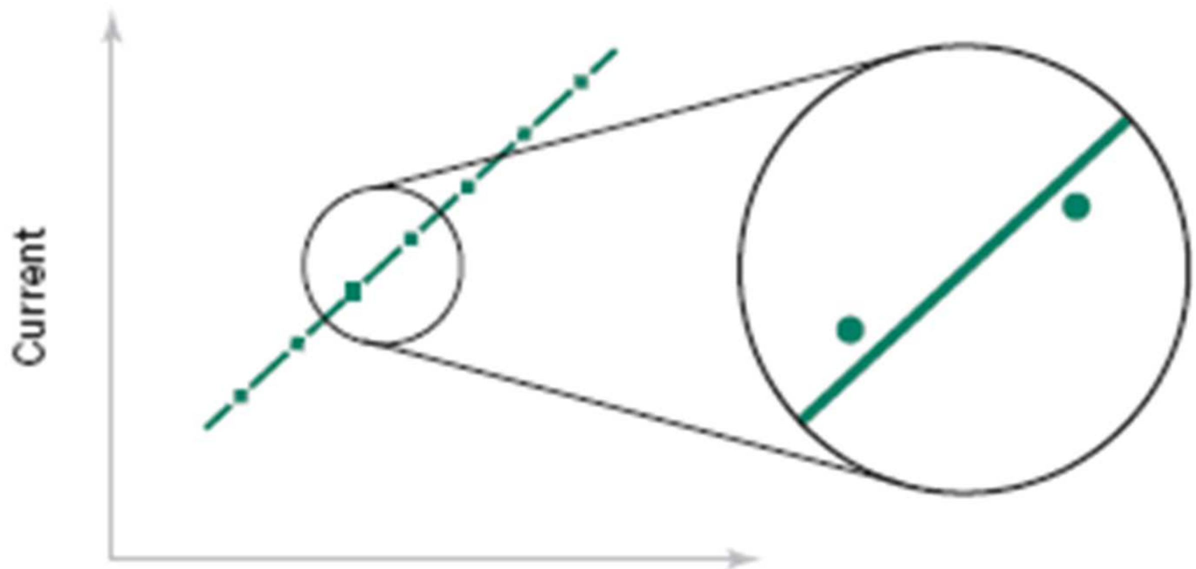

Figure 2-3 A closer examination of the system identifies deviations from the model.

2-1 Sample Spaces and Events

2-1.1 Random Experiments

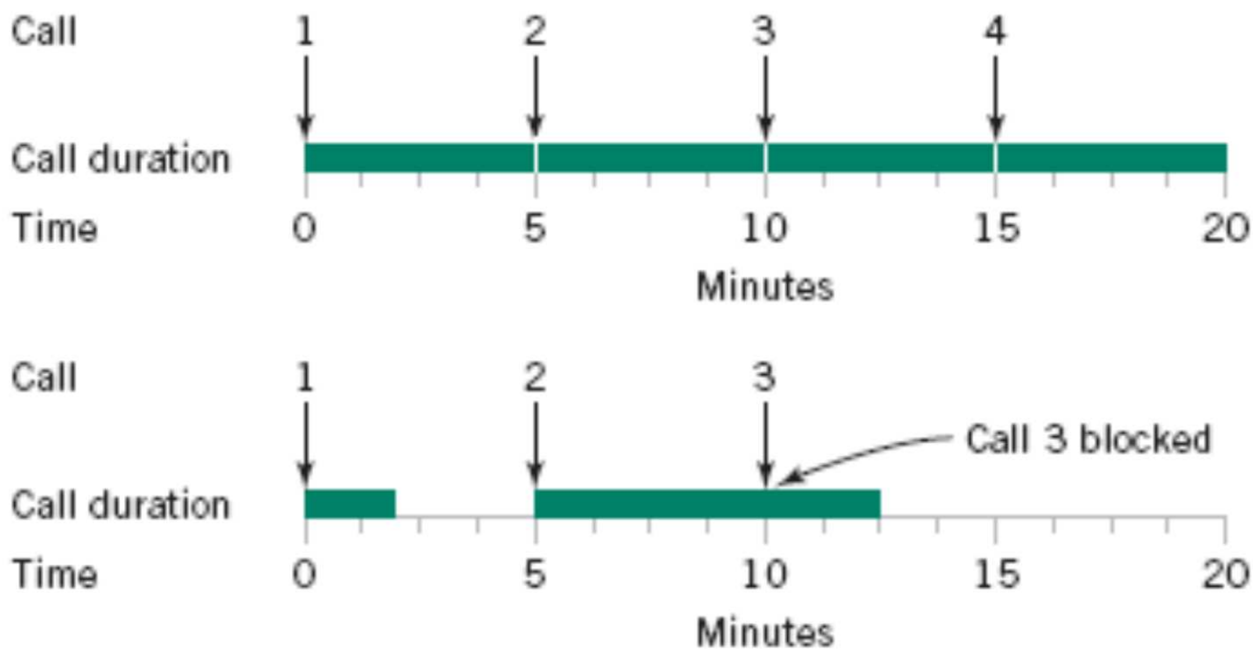

Figure 2-4 Variation causes disruptions in the system.

2-1 Sample Spaces and Events

2-1.2 Sample Spaces

Definition

The set of all possible outcomes of a random experiment is called the **sample space** of the experiment. The sample space is denoted as S.

2-1 Sample Spaces and Events

2-1.2 Sample Spaces

Example 2-1

Consider an experiment in which you select a molded plastic part, such as a connector, and measure its thickness. The possible values for thickness depend on the resolution of the measuring instrument, and they also depend on upper and lower bounds for thickness. However, it might be convenient to define the sample space as simply the positive real line

$$S = R^+ = \{x \mid x > 0\}$$

because a negative value for thickness cannot occur.

2-1 Sample Spaces and Events

Example 2-1 (continued)

If it is known that all connectors will be between 10 and 11 millimeters thick, the sample space could be

$$S = \{x \mid 10 < x < 11\}$$

If the objective of the analysis is to consider only whether a particular part is low, medium, or high for thickness, the sample space might be taken to be the set of three outcomes:

$$S = \{low, medium, high\}$$

If the objective of the analysis is to consider only whether or not a particular part conforms to the manufacturing specifications, the sample space might be simplified to the set of two outcomes

$$S = \{yes, no\}$$

that indicate whether or not the part conforms.

2-1 Sample Spaces and Events

Example 2-2

If two connectors are selected and measured, the extension of the positive real line R is to take the sample space to be the positive quadrant of the plane:

$$S = R^+ \times R^+$$

If the objective of the analysis is to consider only whether or not the parts conform to the manufacturing specifications, either part may or may not conform. We abbreviate *yes* and *no* as y and n. If the ordered pair yn indicates that the first connector conforms and the second does not, the sample space can be represented by the four outcomes:

$$S = \{yy, yn, ny, nn\}$$

2-1 Sample Spaces and Events

Example 2-2 (continued)

If we are only interested in the number of conforming parts in the sample, we might summarize the sample space as

$$S = \{0, 1, 2\}$$

As another example, consider an experiment in which the thickness is measured until a connector fails to meet the specifications. The sample space can be represented as

$$S = \{n, yn, yyn, yyyn, yyyyn, \text{and so forth}\}$$

2-1 Sample Spaces and Events

Tree Diagrams

- Sample spaces can also be described graphically with **tree diagrams.**
  - When a sample space can be constructed in several steps or stages, we can represent each of the n_1 ways of completing the first step as a branch of a tree.
  - Each of the ways of completing the second step can be represented as n_2 branches starting from the ends of the original branches, and so forth.

2-1 Sample Spaces and Events

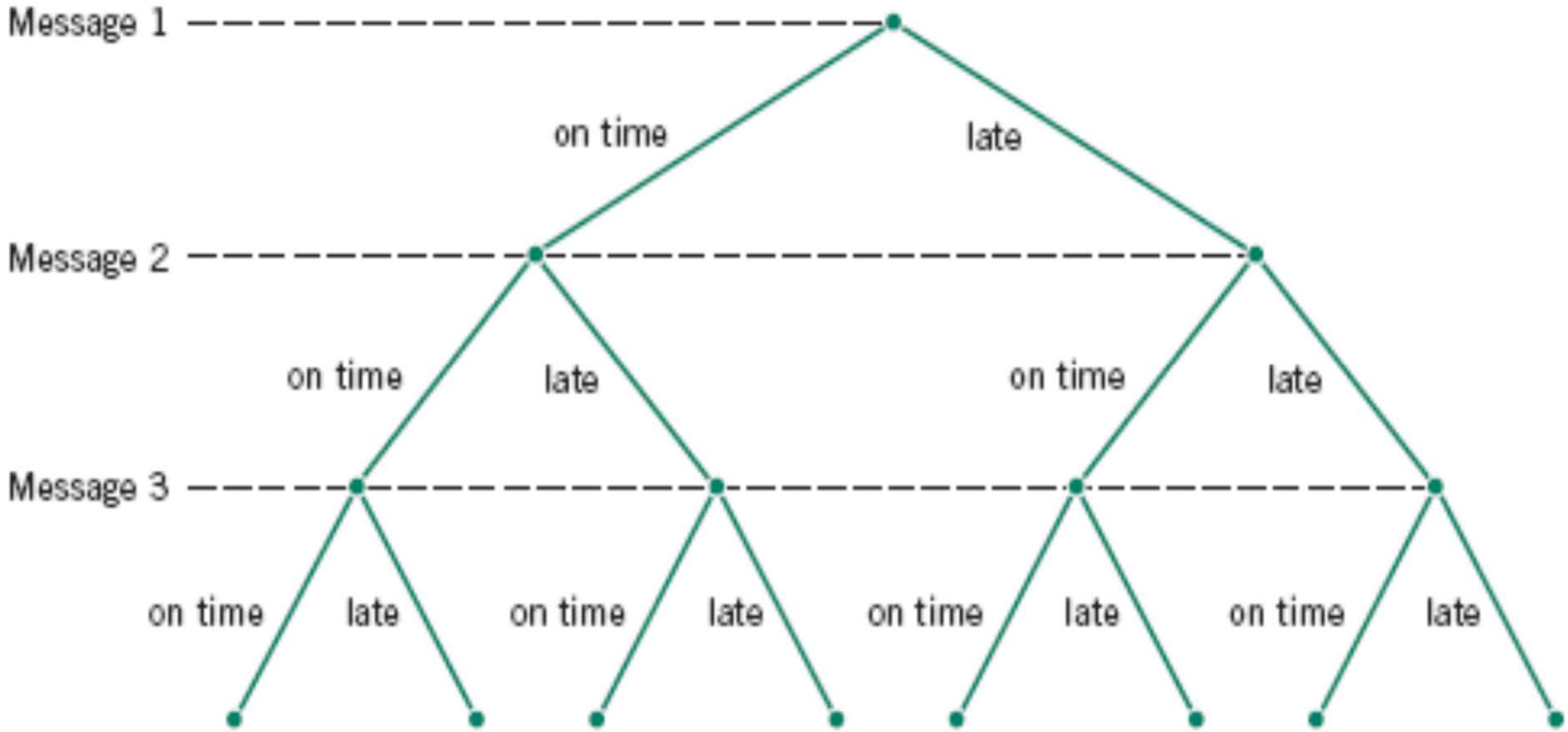

Figure 2-5 Tree diagram for three messages.

2-1 Sample Spaces and Events

Example 2-3

Each message in a digital communication system is classified as to whether it is received within the time specified by the system design. If three messages are classified, use a tree diagram to represent the sample space of possible outcomes.

Each message can either be received on time or late. The possible results for three messages can be displayed by eight branches in the tree diagram shown in Fig. 2-5.

2-1 Sample Spaces and Events

2-1.3 Events

Definition

An **event** is a subset of the sample space of a random experiment.

2-1 Sample Spaces and Events

2-1.3 Events

Basic Set Operations

- The **union** of two events is the event that consists of all outcomes that are contained in either of the two events. We denote the union as $E_1 \cup E_2$.
- The **intersection** of two events is the event that consists of all outcomes that are contained in both of the two events. We denote the intersection as $E_1 \cap E_2$.
- The **complement** of an event in a sample space is the set of outcomes in the sample space that are not in the event. We denote the component of the event E as E'.

2-1 Sample Spaces and Events

2-1.3 Events

Example 2-6

Consider the sample space $S = \{yy, yn, ny, nn\}$ in Example 2-2. Suppose that the set of all outcomes for which at least one part conforms is denoted as E_1. Then,

$$E_1 = \{yy, yn, ny\}$$

The event in which both parts do not conform, denoted as E_2, contains only the single outcome, $E_2 = \{nn\}$. Other examples of events are $E_3 = \varnothing$, the null set, and $E_4 = S$, the sample space. If $E_5 = \{yn, ny, nn\}$,

$$E_1 \cup E_5 = S \qquad E_1 \cap E_5 = \{yn, ny\} \qquad E_1' = \{nn\}$$

2-1 Sample Spaces and Events

Definition

Two events, denoted as E_1 and E_2, such that

$$E_1 \cap E_2 = \varnothing$$

are said to be **mutually exclusive.**

2-1 Sample Spaces and Events

Venn Diagrams

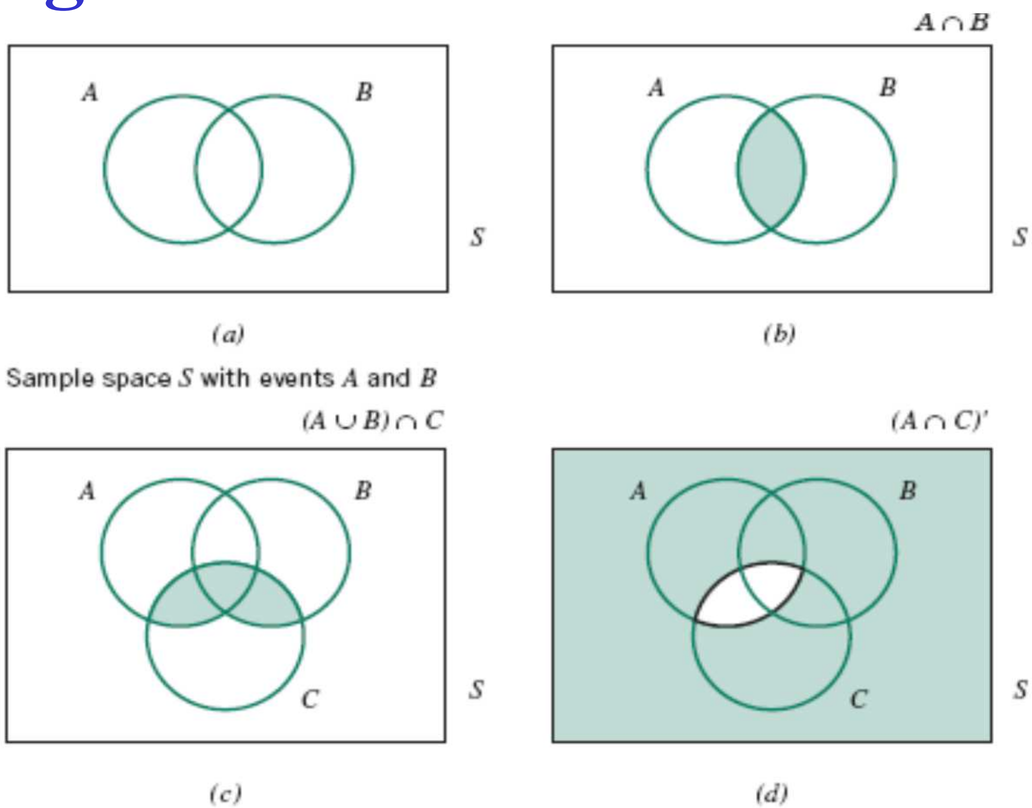

Figure 2-8 Venn diagrams.