# Example:

A city has a population of 3 million people that are evenly distributed over an area of 1000 $km^2$. We know that a percentage of the population is subscribed to a cellular system. Assume that the cellular system is an **Erlang B** system with a total band of 14 MHz and full duplex channel bandwidth of 40 kHz and covers the city using hexagonal cells with radius 2 km and a cluster size of 7 cells. Assume that each user makes 1 call each 2 hours with average call duration of 1 minute and the desired probability of call blocking is 0.005. Find:

a) The total number of cells in the system
b) The number of channels per cell
c) The total number of channels in the system
d) Traffic intensity per cell
e) Maximum carried traffic for the whole system
f) The total number of users who can use the system
g) Percentage of Population of the city who can subscribe to the cellular service
h) The theoretical maximum number of users who can be served at any time.

## Solution:

System is an Erlang B

$$\text{City Area} = 1000 \text{ km}^2$$

$$\text{Total System Bandwidth} = 14 \text{ MHz}$$

$$\text{Full Duplex Channel Bandwidth} = 40 \text{ kHz}$$

$$\text{Cells Shape} = \text{Hexagonal}$$

$$\text{Cell Radius } R = 2 \text{ km}$$

$$\text{Cluster Size } N = 7$$

$$\lambda = \frac{1 \text{ Calls}}{2 \text{ Hour}} = 0.5 \text{ Calls/Hour}$$

$$H = 1 \text{ Minute} = \frac{1}{60} \text{ Hours}$$

$$\Pr[\text{Blocking}] = 0.005$$

a) We can easily verify that the area of a hexagonal cell in terms of its radius is

$$\text{Hexagonal Cell Area} = 2.598\, R^2$$

So, the area of our cells is

$$\text{Cell Area} = 2.598(2)^2 = 10.392 \text{ km}^2$$

This gives a number of cell in the system equal to

$$\text{Number of Cells} = \frac{1000}{10.392} \approx 96 \text{ Cells}$$

b) We need to get the number of channels in the whole band first

$$\text{Number of Channels in Complete Band} = \frac{14 \text{ MHz}}{40 \text{ kHz}} = 350 \text{ Channels}$$

Dividing these channels equally among the cells of a cluster gives

$$\text{Number of Channels per Cell} = C = \frac{350}{7} = 50 \text{ Channels/Cell}$$

c) The system has 96 cells. We allocated 50 channels in each cell, so

$$\begin{aligned}\text{Total Number of Channels in the System} &= \left(50 \ \frac{\text{Channels}}{\text{Cell}}\right)(96 \text{ Cell}) \\ &= 4800 \text{ Channels}\end{aligned}$$

d) Given $C$ and the Probability of a call being blocked (GOS), and using the Erlang B chart, we see that each cell has a traffic intensity of

$$\text{Traffic Intensity per Cell} \approx 36 \text{ Erlangs}$$

e) Maximum carried traffic over the system assumes that all cells are experiencing the maximum traffic intensity to give

$$\begin{aligned}\text{Maximum Carried Traffic over the System} &= \ A) \ \text{Number of Cells}) \\ &= (36 \text{ Erlangs})(96 \text{ Cells}) \\ &= 3456 \text{ Erlangs}\end{aligned}$$

f) First, let us find maximum number of Users per Cell. We need to find traffic intensity per user which is

$$\text{Traffic Intensity per User} = A_U = \lambda \cdot H$$
$$= (0.5)\left(\frac{1}{60}\right) = 0.00833 \text{ Erlangs}$$

So,

$$\text{Users per cell} = U = \frac{A}{A_U} = \left(\frac{36}{0.00833}\right)$$
$$= 4322 \text{ Users/Cell}$$

and

$$\text{Total Users in the Whole system} = (4322 \text{ Users/Cell})(96 \text{ Cell})$$
$$= 414,912 \text{ Users}$$

g) This is given by

$$\text{Percentage of City Popuation who can Subscribe} = \frac{\text{Total Users}}{\text{City Population}}$$
$$= \frac{414{,}912}{3{,}000{,}000}$$
$$= 13.83\%$$