Investment Portfolio Management

Course Materials

Finance Department

Faculty of Economics and Social Sciences

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Faculty of Economic and Social Studies
Department of Finance- 2nd semester 2018/2019

Course Title and Course Code	**Investment Portfolio Management (10871420)**
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Course Units	3
Prerequisites	Investment Analysis and Management (10871320)

This course aims to provide you with an in-depth introduction to investment analysis and portfolio management. The overarching objectives are that students (1) gain a deep intuitive understanding of the concepts used in investment analysis, (2) learn the tools used in investment analysis, including excel modeling and regression analysis, and (3) gain confidence in applying the concepts and tools in managing a portfolio.

LEARNING OBJECTIVES

Upon finishing this course you will be able to

- Compute historical and expected returns, as well as risk measures and comprehend the importance of the risk-return relationship.
- Evaluate the risk-return characteristics of financial assets
- Design an optimal portfolio depending on the risk tolerance level and return required by the clients
- Evaluate and manage assets using one and multifactor models
- Use technical analysis
- Calculate portfolio performance measures and apply attribution analysis of the portfolio performance

METHOD OF TEACHING AND LEARNING:

- Class lectures, interactive learning (class discussions, group work) and practical problems solved in class.
- Project: Throughout the semester students practice on a trading simulation which includes several asset classes. At the end of the semester students present the portfolio they have constructed based on the risk-return profile of a hypothetical client. The students work in groups of 2-3.
- Office hours: students are encouraged to make full use of the office hours of their instructor, where they can ask questions, see their exam paper, and/or go over lecture material.
- Use of Moodle learning platform, where instructors post lecture notes, assignment instructions, timely announcements, as well as additional resources.
- Use of MS Excel application to apply the different concepts and techniques.

STUDENT EVALUATION

Your grade for the course will be based on two in-class exams (first and Second), a final exam, an empirical project and class participation. The course grade is determined as follows:

Grade = 0.15 Project + 0.20 ∗ First Exam + 0.20 ∗ Second Exam + 0.45 ∗ Final
All exams are cumulative, closed-books and closed-notes (if needed you will be provided with a formula sheet; no self-made "cheat sheets" are allowed). All you should bring is writing utensils and a calculator (no laptops or other electronic devices).

Tentative Exams schedule*:

Exam	Day	Date	Time
First	Sunday	03/03/2018	In Class
Second	Sunday	14/04/2018	In Class

* If your class is on Monday, then your exam date will be one day later

Grading Policies: If you have questions about the way I grade any of your work, talk to me immediately. If you disagree with the grade even after you have discussed it with me, you must submit your question in writing within one week of the day on which I return your work. If you appeal, I will re-grade your entire exam or assignment; your grade may increase, decrease, or remain the same.

Project Information: The project is an individual project. Each student is required to apply all the techniques given during the course to construct an investment portfolio. A presentation session will be held where each one will present his portfolio in 5 minutes and answer the questions in 5 minutes.

TEXTBOOK

1. Bodie, Kane and Marcus, "Essentials of Investments", McGraw Hill, 9th edition (international)
2. Reilly, Brown, "Investment Analysis and Portfolio Management" , 10th Edition

Suggested Materials

Financial magazines and financial pages of newspapers.

Web Resources

https://moodle.najah.edu
www.pex.ps
www.finance.yahoo.com
http://tools.tickerchart.com/

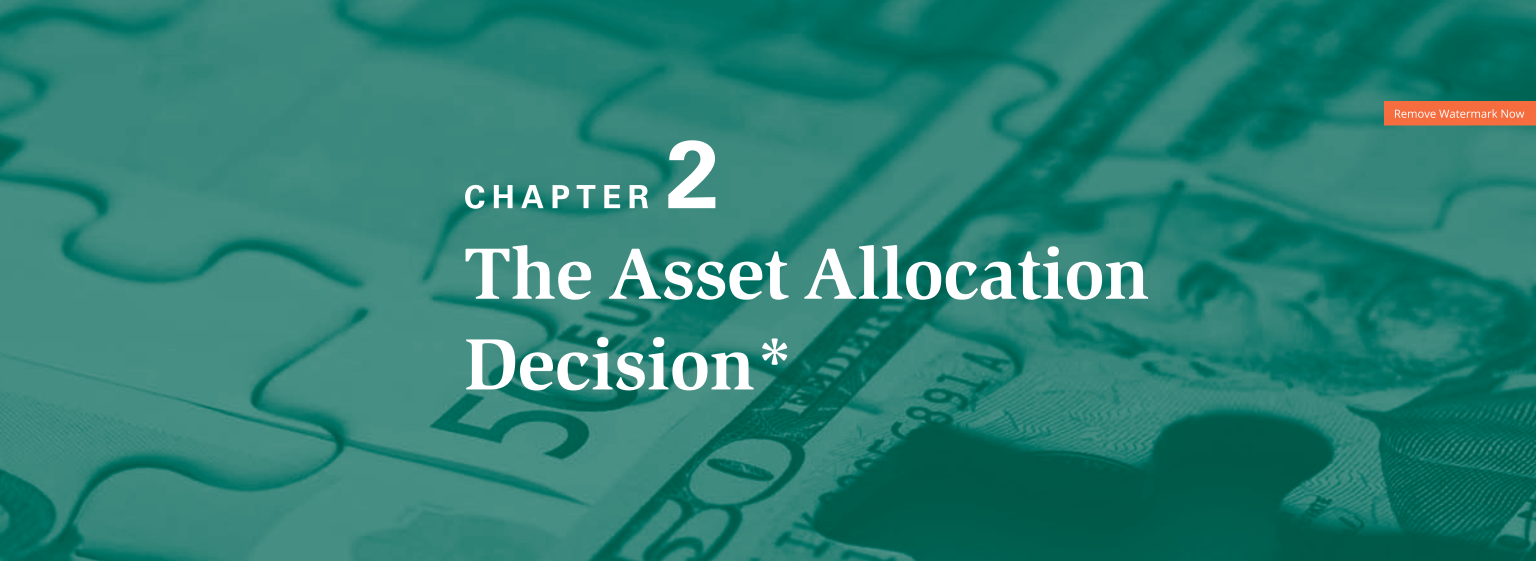
CLASS SYLLABUS

Learning Objectives	Lecture	Topics	Readings
Review the basic mathematics	1-2	Course Introduction • Syllabus	Handout

related to portfolio management		• Basic mathematics of Portfolio Management	
Define the objectives in constructing and managing a portfolio and learn to create an investment policy statement.	3-8	Portfolio Management Process and Asset Allocation Decision	RB, CH 2 PPT
Learn to compute historical and expected returns	9-13	Risk and Return	BKM, Ch5 Excel Application
Understand the principles of modern portfolio theory and the effect of diversification on investment portfolios. Design an optimal portfolio depending on the risk tolerance level and return required by the clients	14-20	Efficient Diversification	BKM, Ch 6 Excel Application
	21	**FIRST EXAM** and related discussions	
Evaluate and Manage assets using one and multifactor models	22-30	Capital Asset Pricing and Arbitrage Pricing	BKM, Ch 7 Excel Application
Understand the arguments of rational finance	31-35	Efficient Market Hypothesis (EMH)	BKM, Ch 8 Excel Application
Use technical Analysis	36-40	Behavioral Finance and Technical Analysis	BKM, Ch 9
	41	**SECOND EXAM** and related discussions	
Calculate portfolio performance measures and apply attribution analysis of the portfolio performance	42-46	Portfolio Performance Evaluation	BKM, Ch 18 Excel Application
	47-48	**FINAL EXAM**	

COURSE POLICY

- Attendance would be recorded every lecture. Unsatisfactory attendance (more than 6 hours absent) would result in a student being barred from sitting the final examination.
- All assignments **must** be submitted before/on the deadline. Late submission without prior permission from the lecturer will not be accepted.
- Tests **must** be taken on the date and time that it is given. No replacement tests would be given unless a valid medical certificate or official letter of permission is produced.
- Plagiarism and copying is a serious academic offence. Offenders would be awarded grade **F** either for the assignment/report concerned or the entire coursework and may be barred from sitting for the final examination.
- Please ensure your mobile phones are **silenced or switched off** during lectures.



CHAPTER 2

The Asset Allocation Decision*

After you read this chapter, you should be able to answer the following questions:

- What is involved in the asset allocation process?
- What are the four steps in the portfolio management process?
- What is the role of asset allocation in investment planning?
- Why is a policy statement important to the planning process?
- What objectives and constraints should be detailed in a policy statement?
- How and why do investment goals change over a person's lifetime?
- Why do asset allocation strategies differ across national boundaries?

The previous chapter informed us that *risk drives return*. Therefore, the practice of investing funds and managing portfolios should focus primarily on managing risk rather than on managing returns.

This chapter examines some of the practical implications of risk management in the context of asset allocation. **Asset allocation** is the process of deciding how to distribute an investor's wealth among different countries and asset classes for investment purposes. An **asset class** is comprised of securities that have similar characteristics, attributes, and risk/return relationships. A broad asset class, such as "bonds," can be divided into smaller asset classes, such as Treasury bonds, corporate bonds, and high-yield bonds. We will see that, in the long run, the highest compounded returns will most likely accrue to those investors with larger exposures to risky assets. We will also see that although there are no shortcuts or guarantees to investment success, maintaining a reasonable and disciplined approach to investing will increase the likelihood of investment success over time.

The asset allocation decision is not an isolated choice; rather, it is a component of a structured four-step portfolio management process that we present in this chapter. As we will see, the first step in the process is to develop an investment policy statement, or plan, that will guide all future decisions. Much of an asset allocation strategy depends on the investor's policy statement, which includes the investor's goals or objectives, constraints, and investment guidelines.

What we mean by an "investor" can range from an individual account to trustees overseeing a corporation's multibillion-dollar pension fund, a university endowment, or an insurance company portfolio. Regardless of who the investor is or how simple or complex the investment needs, he or she should develop a policy statement before

*The authors acknowledge the collaboration of Professor Edgar Norton of Illinois State University on this chapter.

making long-term investment decisions. Although most of our examples will be in the context of an individual investor, the concepts we introduce here—investment objectives, constraints, benchmarks, and so on—apply to any investor, individual or institution. We'll review historical data to show the importance of the asset allocation decision and discuss the need for investor education, an important issue for companies who offer retirement or savings plans to their employees. The chapter concludes by examining asset allocation strategies across national borders to show the effect of regulations, market environment, and culture on investing patterns; what is appropriate for a U.S.-based investor is not necessarily appropriate for a non-U.S.-based investor.

2.1 Individual Investor Life Cycle

Financial plans and investment needs are as different as each individual. Investment needs change over a person's life cycle. How individuals structure their financial plan should be related to their age, financial status, future plans, risk aversion characteristics, and needs.

2.1.1 The Preliminaries

Before embarking on an investment program, we need to make sure other needs are satisfied. No serious investment plan should be started until a potential investor has adequate income to cover living expenses and has a safety net should the unexpected occur.

Insurance Life insurance should be a component of any financial plan. Life insurance protects loved ones against financial hardship should death occur before our financial goals are met. The death benefit paid by the insurance company can help pay medical bills and funeral expenses and provide cash that family members can use to maintain their lifestyle, retire debt, or invest for future needs (for example, children's education, spouse retirement). Therefore, one of the first steps in developing a financial plan is to purchase adequate life insurance coverage.

Insurance can also serve more immediate purposes, including being a means to meet long-term goals, such as retirement planning. On reaching retirement age, you can receive the cash or surrender value of your life insurance policy and use the proceeds to supplement your retirement lifestyle or for estate planning purposes.

Insurance coverage also provides protection against other uncertainties. *Health* insurance helps to pay medical bills. *Disability* insurance provides continuing income should you become unable to work. *Automobile and home* (or rental) insurance provides protection against accidents and damage to cars or residences.

Although nobody ever expects to use his or her insurance coverage, a first step in a sound financial plan is to have adequate coverage "just in case." Lack of insurance coverage can ruin the best-planned investment program.

Cash Reserve Emergencies, job layoffs, and unforeseen expenses happen, and good investment opportunities emerge. It is important to have a cash reserve to help meet these occasions. In addition to providing a safety cushion, a cash reserve reduces the likelihood of being forced to sell investments at inopportune times to cover unexpected expenses. Most experts recommend a cash reserve equal to about six months' living expenses. Calling it a "cash" reserve does not mean the funds should be in cash; rather, the funds should be in investments you can easily convert to cash with little chance of a loss in value. Money market or short-term bond mutual funds and bank accounts are appropriate vehicles for the cash reserve.

Similar to the financial plan, an investor's insurance and cash reserve needs will change over his or her life. The need for disability insurance declines when a person retires. In contrast, other insurance, such as supplemental Medicare coverage or long-term-care insurance, may become more important.

2.1.2 Investment Strategies over an Investor's Lifetime

Assuming the basic insurance and cash reserve needs are met, individuals can start a serious investment program with their savings. Because of changes in their net worth and risk tolerance, individuals' investment strategies will change over their lifetime. In the following sections, we review various phases in the investment life cycle. Although each individual's needs and preferences are different, some general traits affect most investors over the life cycle.

The four life-cycle phases are shown in Exhibit 2.1 (the third and fourth phases—spending and gifting—are shown as concurrent) and described here.

Accumulation Phase Individuals in the early-to-middle years of their working careers are in the **accumulation phase**. As the name implies, these individuals are attempting to accumulate assets to satisfy fairly immediate needs (for example, a down payment for a house) or longer-term goals (children's college education, retirement). Typically, their net worth is small, and debt from car loans or their own past college loans may be heavy. As a result of their typically long investment time horizon and their future earning ability, individuals in the accumulation phase are willing to make relatively high-risk investments in the hopes of making above-average nominal returns over time.

Here we emphasize the wisdom of investing early and regularly in one's life. Funds invested in early life cycle phases, with returns compounding over time, will reap significant financial benefits during later phases. Exhibit 2.2 shows growth from an initial $10,000 investment over 20, 30, and 40 years at assumed annual returns of 7 and 8 percent. The middle-aged person who invests $10,000 "when he or she can afford it" will only reap the benefits of compounding for 20 years or so before retirement. In contrast, a person who begins saving at a younger age will reap the much higher benefits of funds invested for 30 or 40 years. Regularly investing as little as $2,000 a year reaps large benefits over time, as well. As shown in Exhibit 2.2, a person who has invested a total of $90,000—an initial $10,000 investment followed by $2,000 annual investments over 40 years—will have over half a million dollars accumulated assuming the 7 percent return. If the funds are invested more aggressively and earn the 8 percent return, the accumulation will be nearly three-quarters of a million dollars.

Exhibit 2.1 Rise and Fall of Personal Net Worth over a Lifetime

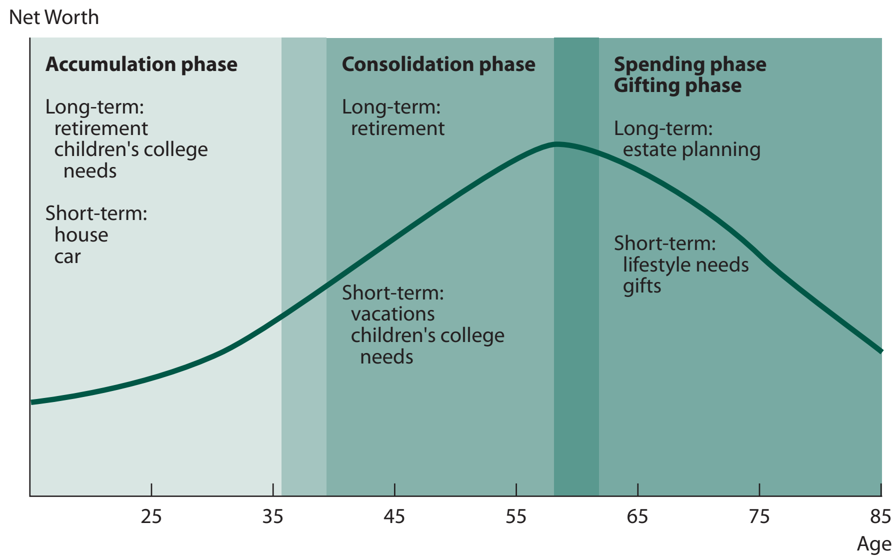


Exhibit 2.2 Benefits of Investing Early

		The Future Value of an Initial $10,000 Investment	The Future Value of Investing $2,000 Annually	The Future Value of the Initial Investment Plus the Annual Investment
Interest rate	7.0%			
20 years		$38,696.84	$81,990.98	$120,687.83
30 years		$76,122.55	$188,921.57	$265,044.12
40 years		$149,744.58	$399,270.22	$549,014.80
Interest rate	8.0%			
20 years		$46,609.57	$91,523.93	$138,133.50
30 years		$100,626.57	$226,566.42	$327,192.99
40 years		$217,245.21	$518,113.04	$735,358.25

Source: Calculations by authors.

Consolidation Phase Individuals in the **consolidation phase** are typically past the midpoint of their careers, have paid off much or all of their outstanding debts, and perhaps have paid, or have the assets to pay, their children's college bills. Earnings exceed expenses, so the excess can be invested to provide for future retirement or estate planning needs. The typical investment horizon for this phase is still long (20 to 30 years), so moderately high risk investments are attractive. At the same time, because individuals in this phase are concerned about capital preservation, they do not want to take abnormally high risks that may put their current nest egg in jeopardy.

Spending Phase The **spending phase** typically begins when individuals retire. Living expenses are covered by social security income and income from prior investments, including employer pension plans. Because their earning years have concluded (although some retirees take part-time positions or do consulting work), they are very conscious of protecting their capital. At the same time, they must balance their desire to preserve the nominal value of their savings with the need to protect themselves against a decline in the *real* value of their savings due to inflation. The average 65-year-old person in the United States has a life expectancy of about 20 years. Thus, although their overall portfolio may be less risky than in the consolidation phase, they still need some risky growth investments, such as common stocks, for inflation (purchasing power) protection.

The transition into the spending phase requires a sometimes difficult change in mindset; throughout our working life we are trying to save; suddenly we can spend. We tend to think that if we spend less, say 4 percent of our accumulated funds annually instead of 5, 6, or 7 percent, our wealth will last far longer. Although this is correct, a bear market early in our retirement can greatly reduce our accumulated funds. Fortunately, there are planning tools that can give a realistic view of what can happen to our retirement funds should markets fall early in our retirement years; this insight can assist in budgeting and planning to minimize the chance of spending (or losing) all the saved retirement funds. Annuities, which transfer risk from the individual to the annuity firm (most likely an insurance company), are another possibility. With an annuity, the recipient receives a guaranteed, lifelong stream of income. Options can allow for the annuity to continue until both a husband and wife die.

Gifting Phase The **gifting phase** is similar to, and may be concurrent with, the spending phase. In this stage, individuals may believe they have sufficient income and assets to cover their current and future expenses while maintaining a reserve for uncertainties. In such a case, excess assets can be used to provide financial assistance to relatives or friends, to establish charitable trusts, or to fund trusts as an estate planning tool to minimize estate taxes.

2.1.3 Life Cycle Investment Goals

During an individual's investment life cycle, he or she will have a variety of financial goals. **Near-term, high-priority goals** are shorter-term financial objectives that individuals set to fund purchases that are personally important to them, such as accumulating funds to make a house down payment, buy a new car, or take a trip. Parents with teenage children may have a near-term, high-priority goal to accumulate funds to help pay college expenses. Because of the emotional importance of these goals and their short time horizon, high-risk investments are not usually considered suitable for achieving them.

Long-term, high-priority goals typically include some form of financial independence, such as the ability to retire at a certain age. Because of their long-term nature, higher-risk investments can be used to help meet these objectives.

Lower-priority goals are just that—it might be nice to meet these objectives, but it is not critical. Examples include the ability to purchase a new car every few years, redecorate the home with expensive furnishings, or take a long, luxurious vacation. A well-developed policy statement considers these diverse goals over an investor's lifetime. The following sections detail the process for constructing an investment policy, creating a portfolio that is consistent with the policy and the environment, managing the portfolio, and monitoring its performance relative to its goals and objectives over time.

2.2 The Portfolio Management Process*

The process of managing an investment portfolio never stops. Once the funds are initially invested according to the plan, the real work begins in evaluating the portfolio's performance and updating the portfolio based on changes in the economic environment and the investor's needs.

The first step in the portfolio management process, as seen in Exhibit 2.3, is for the investor, either alone or with the assistance of an investment advisor, to construct a

Exhibit 2.3 The Portfolio Management Process

1. Policy Statement
   Focus: Investor's short-term and long-term needs, familiarity with capital market history, and expectations
2. Examine current and projected financial, economic, political, and social conditions
   Focus: Short-term and intermediate-term expected conditions to use in constructing a specific portfolio
3. Implement the plan by constructing the portfolio
   Focus: Meet the investor's needs at minimum risk levels
4. Feedback Loop: Monitor and update investor needs, environmental conditions, evaluate portfolio performance

*This section and the one that follows benefited from insights contained in Maginn, Tuttle, Pinto, and McLeavey (2007), especially Chapters 1 and 2.

policy statement. The policy statement is a road map; in it, investors specify the types of risks they are willing to take and their investment goals and constraints. All investment decisions are based on the policy statement to ensure that these decisions are appropriate for the investor. We examine the process of constructing a policy statement in the following section. Because investor needs, goals, and constraints change over time, the policy statement must be periodically reviewed and updated.

The process of investing involves assessing the future and deriving strategies that offer the best possibility of meeting the policy statement guidelines. In the second step of the portfolio management process, the portfolio manager studies current financial and economic conditions and forecasts future trends. The investor's needs, as reflected in the policy statement, and financial market expectations will jointly determine **investment strategy**. Economies are dynamic; they are affected by numerous industry struggles, politics, and changing demographics and social attitudes. Thus, the portfolio will require *constant* monitoring and updating to reflect changes in financial market expectations. We examine the process of evaluating and forecasting economic trends in Chapter 12.

The third step of the portfolio management process is to **construct the portfolio**. With the investor's policy statement and financial market forecasts as input, the advisors implement the investment strategy and determine how to allocate available funds across different countries, asset classes, and securities. This involves constructing a portfolio that will minimize the investor's risks while meeting the needs specified in the policy statement. Financial theory frequently assists portfolio construction, which is discussed in Part 2 of this book. Some of the practical aspects of selecting investments for inclusion in a portfolio are discussed in Part 4 and Part 5.

The fourth step in the portfolio management process is the **continual monitoring** of the investor's needs and capital market conditions and, when necessary, updating the policy statement. Based upon all of this, the investment strategy is modified accordingly. An important component of the monitoring process is to evaluate a portfolio's performance and compare the relative results to the expectations and the requirements listed in the policy statement. The evaluation of portfolio performance is discussed in Chapter 25. Once you have completed the four steps, it is important to recognize that this is a *continuous* process—it is essential to revisit all the steps to ensure that the policy statement is still valid, that the economic outlook has not changed, and so forth.

2.3 The Need for a Policy Statement

As noted in the previous section, a policy statement is a road map that guides the investment process. Constructing a policy statement is an invaluable planning tool that will help the investor understand his or her needs better as well as assist an advisor or portfolio manager in managing a client's funds. While it does not guarantee investment success, a policy statement will provide discipline for the investment process and reduce the possibility of making hasty, inappropriate decisions. There are two important reasons for constructing a policy statement: First, it helps the investor decide on realistic investment goals after learning about the financial markets and the risks of investing; second, it creates a standard by which to judge the performance of the portfolio manager.

2.3.1 Understand and Articulate Realistic Investor Goals

When asked about their investment goal, people often say, "to make a lot of money," or some similar response. Such a goal has two drawbacks: First, it may not be appropriate for the investor, and second, it is too open-ended to provide guidance for specific investments and time frames. Such an objective is well suited for someone going to the racetrack or buying lottery tickets, but it is inappropriate for someone investing funds in financial and real assets for the long term.

An important purpose of writing a policy statement is to help investors understand their own needs, objectives, and investment constraints. As part of this, investors need to learn about financial markets and the risks of investing. This background will help prevent them from making inappropriate investment decisions in the future based on unrealistic expectations and increase the possibility that they will satisfy their specific, measurable financial goals.

Thus, the policy statement helps the investor to specify realistic goals and become more informed about the risks and costs of investing. Market values of assets, whether they be stocks, bonds, or real estate, can fluctuate dramatically. For example, during the October 1987 crash, the Dow Jones Industrial Average (DJIA) fell more than 20 percent in one day; in October 1997, the Dow fell "only" 7 percent. A review of market history shows that it is not unusual for asset prices to decline by 10 percent to 20 percent over several months—for example, the months following the market peak in March 2000, and the major decline when the market reopened after September 11, 2001. The most recent "bloodbath" was the market decline of over 30 percent during 2008—and this decline was global. The problem is, investors typically focus on a single statistic, such as an 11 percent average annual rate of return on stocks, and expect the market to rise 11 percent every year. Such thinking ignores the risk of stock investing. Part of the process of developing a policy statement is for the investor to become familiar with the risks of investing, because we know that a strong positive relationship exists between risk and return.

One expert in the field recommends that investors should think about the following set of questions and explain their answers as part of the process of constructing a policy statement:

1. What are the real risks of an adverse financial outcome, especially in the short run?
2. What probable emotional reactions will I have to an adverse financial outcome?
3. How knowledgeable am I about investments and financial markets?
4. What other capital or income sources do I have? How important is this particular portfolio to my overall financial position?
5. What, if any, legal restrictions may affect my investment needs?
6. How would any unanticipated fluctuations in my portfolio value affect my investment policy?

Adapted from Charles D. Ellis, *Investment Policy: How to Win the Loser's Game* (Homewood, IL: Dow Jones-Irwin, 1985), 25–26. Reproduced with permission of the McGraw-Hill Companies.

In summary, constructing a policy statement is mainly the investor's responsibility. It is a process whereby investors articulate their realistic needs and goals and become familiar with financial markets and investing risks. Without this information, investors cannot adequately communicate their needs to the portfolio manager. Without this input from investors, the portfolio manager cannot construct a portfolio that will satisfy clients' needs. The result of bypassing this step will most likely be future aggravation, dissatisfaction, and disappointment.

2.3.2 Standards for Evaluating Portfolio Performance

The policy statement also assists in judging the performance of the portfolio manager. Performance cannot be judged without an objective standard; the policy statement provides that objective standard. The portfolio's performance should be compared to guidelines specified in the policy statement, not on the portfolio's overall return. For example, if an investor has a low tolerance for risky investments, the portfolio manager should not be fired simply because the portfolio does not perform as well as the risky S&P 500 stock index. The point is, because risk drives returns, the investor's lower-risk investments, as specified in the investor's policy statement, will probably earn lower returns than if all the investor's funds were placed in the aggregate stock market.

The policy statement will typically include a **benchmark portfolio**, or comparison standard. The risk of the benchmark, and the assets included in the benchmark, should agree with the client's risk preferences and investment needs. Notably, both the client and the portfolio manager must agree that the benchmark portfolio reflects the risk preferences and appropriate return requirements of the client. In turn, the investment performance of the portfolio manager should be compared to this benchmark portfolio. For example, an investor who specifies low-risk investments in the policy statement should compare the portfolio manager's performance against a low-risk benchmark portfolio. Likewise, an investor seeking high-risk, high-return investments should compare the portfolio's performance against a high-risk benchmark portfolio.

Because it sets an objective performance standard, the policy statement acts as a starting point for periodic portfolio review and client communication with managers. Questions concerning portfolio performance should be addressed in the context of the written policy guidelines. Managers should mainly be judged by whether *they consistently followed the client's policy guidelines*. The portfolio manager who makes unilateral deviations from policy is not working in the best interests of the client. Therefore, even significant deviations that result in higher portfolio returns can and should be grounds for the manager's dismissal.

Thus, we see the importance of constructing the policy statement: The client must first understand his or her own needs before communicating them to the portfolio manager who in turn, must implement the client's desires by following the investment guidelines. As long as policy is followed, shortfalls in performance should not be a major concern. Remember that the policy statement is designed to impose an investment discipline on the client and on the portfolio manager. The less knowledgeable they are, the more likely clients are to inappropriately judge the performance of the portfolio manager.

2.3.3 Other Benefits

A sound policy statement helps to protect the client against a portfolio manager's inappropriate investments or unethical behavior. Without clear, written guidance, some managers may consider investing in high-risk investments, hoping to earn a quick return. Such actions are probably counter to the investor's specified needs and risk preferences. Though legal recourse is a possibility against such action, writing a clear and unambiguous policy statement should reduce the possibility of such inappropriate manager behavior.

Just because a specific manager currently manages your account does not mean that person will always manage your funds. Because your portfolio manager may be promoted' dismissed or take a better job' your funds may come under the management of an individual you do not know and who does not know you. To prevent costly delays during this transition, you can ensure that the new manager "hits the ground running" with a clearly written policy statement. A policy statement should prevent delays in monitoring and rebalancing your portfolio and contribute to a seamless transition from one money manager to another.

To sum up, a clearly written policy statement helps avoid potential problems. When the client clearly specifies his or her needs and desires, the portfolio manager can more effectively construct an appropriate portfolio. The policy statement provides an objective measure for evaluating portfolio performance, helps guard against ethical lapses by the portfolio manager, and aids in the transition between money managers. Therefore, the first step before beginning any investment program is to construct a policy statement.

An appropriate policy statement should satisfactorily answer the following questions:

1. Is the policy carefully designed to meet the specific needs and objectives of this particular investor? (Cookie-cutter or one-size-fits-all policy statements are generally inappropriate.)
2. Is the policy written so clearly and explicitly that a competent stranger could use it to manage the portfolio in conformance with the client's needs? In case of a manager transition, could the new manager use this policy statement to handle your portfolio in accordance with your needs?
3. Would the client have been able to remain committed to the policies during the capital market experiences of the past 60 to 70 years? That is, does the client fully understand investment risks and the need for a disciplined approach to the investment process?
4. Would the portfolio manager have been able to maintain the policies specified over the same period? (Discipline is a two-way street; we do not want the portfolio manager to change strategies because of a disappointing market.)
5. Would the policy, if implemented, have achieved the client's objectives? (Bottom line: Would the policy have worked to meet the client's needs?)

Adapted from Charles D. Ellis, *Investment Policy: How to Win the Loser's Game* (Homewood, IL: Dow Jones-Irwin, 1985), 62. Reproduced with permission of the McGraw-Hill Companies.

2.4 Input to the Policy Statement

Before an investor and advisor can construct a policy statement, they need to have an open and frank exchange of information, ideas, fears, and goals. Specifically, the client and advisor need to discuss the client's investment objectives and constraints. To illustrate this framework, we discuss the investment objectives and constraints that may confront "typical" 25-year-old and 65-year-old investors.

2.4.1 Investment Objectives

The investor's **objectives** are his or her investment goals expressed in terms of both risk and returns. The relationship between risk and returns requires that goals not be expressed only in terms of returns. Expressing goals only in terms of returns can lead to inappropriate investment practices by the portfolio manager, such as the use of high-risk investment strategies or account "churning," which involves moving quickly in and out of investments in an attempt to buy low and sell high.

For example, a person may have a stated return goal such as "double my investment in five years." Before such a statement becomes part of the policy statement, the client must become fully informed of investment risks associated with such a goal, including the possibility of loss. *A careful analysis of the client's risk tolerance should precede any discussion of return objectives.* It makes little sense for a person who is risk averse to have his/her funds invested in high-risk assets. Investment firms survey clients to gauge their risk tolerance. Sometimes investment magazines or books contain tests that individuals can take to help them evaluate their risk tolerance (see Exhibit 2.4). Subsequently, an advisor will use the results of this evaluation to categorize a client's risk tolerance and suggest an initial asset allocation such as those contained in Exhibit 2.5.

Risk tolerance is more than a function of an individual's psychological makeup; it is affected by other factors, including a person's current insurance coverage and cash reserves. Risk tolerance is also affected by an individual's family situation (for example, marital status and the number and ages of children) and by his or her age. We know that older persons generally have shorter investment time frames within which to make up any losses; they also have years of experience, including living through various market gyrations and "corrections"

Exhibit 2.4 How Much Risk Is Right for You?

You've heard the expression "no pain, no gain"? In the investment world, the comparable phrase would be "no risk, no reward."

How you feel about risking your money will drive many of your investment decisions. The risk-comfort scale extends from very conservative (you don't want to risk losing a penny regardless of how little your money earns) to very aggressive (you're willing to risk much of your money for the possibility that it will grow tremendously). As you might guess, most investors' tolerance for risk falls somewhere in between.

If you're unsure of what your level of risk tolerance is, this quiz should help.

1. You win $300 in an office football pool. You: (a) spend it on groceries, (b) purchase lottery tickets, (c) put it in a money market account, (d) buy some stock.
2. Two weeks after buying 100 shares of a $20 stock, the price jumps to over $30. You decide to: (a) buy more stock; it's obviously a winner, (b) sell it and take your profits, (c) sell half to recoup some costs and hold the rest, (d) sit tight and wait for it to advance even more.
3. On days when the stock market jumps way up, you: (a) wish you had invested more, (b) call your financial advisor and ask for recommendations, (c) feel glad you're not in the market because it fluctuates too much, (d) pay little attention.
4. You're planning a vacation trip and can either lock in a fixed room-and-meals rate of $150 per day or book standby and pay anywhere from $100 to $300 per day. You: (a) take the fixed-rate deal, (b) talk to people who have been there about the availability of last-minute accommodations, (c) book standby and also arrange vacation insurance because you're leery of the tour operator, (d) take your chances with standby.
5. The owner of your apartment building is converting the units to condominiums. You can buy your unit for $75,000 or an option on a unit for $15,000. (Units have recently sold for close to $100,000, and prices seem to be going up.) For financing, you'll have to borrow the down payment and pay mortgage and condo fees higher than your present rent. You: (a) buy your unit, (b) buy your unit and look for another to buy, (c) sell the option and arrange to rent the unit yourself, (d) sell the option and move out because you think the conversion will attract couples with small children.
6. You have been working three years for a rapidly growing company. As an executive, you are offered the option of buying up to 2% of company stock: 2,000 shares at $10 a share. Although the company is privately owned (its stock does not trade on the open market), its majority owner has made handsome profits selling three other businesses and intends to sell this one eventually. You: (a) purchase all the shares you can and tell the owner you would invest more if allowed, (b) purchase all the shares, (c) purchase half the shares, (d) purchase a small amount of shares.
7. You go to a casino for the first time. You choose to play: (a) quarter slot machines, (b) $5 minimum-bet roulette, (c) dollar slot machines, (d) $25 minimum-bet blackjack.
8. You want to take someone out for a special dinner in a city that's new to you. How do you pick a place? (a) read restaurant reviews in the local newspaper, (b) ask coworkers if they know of a suitable place, (c) call the only other person you know in this city, who eats out a lot but only recently moved there, (d) visit the city sometime before your dinner to check out the restaurants yourself.
9. The expression that best describes your lifestyle is: (a) no guts, no glory, (b) just do it! (c) look before you leap, (d) all good things come to those who wait.
10. Your attitude toward money is best described as: (a) a dollar saved is a dollar earned, (b) you've got to spend money to make money, (c) cash and carry only, (d) whenever possible, use other people's money.

SCORING SYSTEM: Score your answers this way: (1) a-1, b-4, c-2, d-3 (2) a-4, b-1, c-3, d-2 (3) a-3, b-4, c-2, d-1 (4) a-2, b-3, c-1, d-4 (5) a-3, b-4, c-2, d-1 (6) a-4, b-3, c-2, d-1 (7) a-1, b-3, c-2, d-4 (8) a-2, b-3, c-4, d-1 (9) a-4, b-3, c-2, d-1 (10) a-2, b-3, c-1, d-4.

What your total score indicates:

- 10–17: You're not willing to take chances with your money, even though it means you can't make big gains.
- 18–25: You're semi-conservative, willing to take a small chance with enough information.
- 26–32: You're semi-aggressive, willing to take chances if you think the odds of earning more are in your favor.
- 33–40: You're aggressive, looking for every opportunity to make your money grow, even though in some cases the odds may be quite long. You view money as a tool to make more money.

Exhibit 2.5 Initial Risk and Investment Goal Categories and Asset Allocations Suggested by Investment Firms

FIDELITY INVESTMENTS SUGGESTED ASSET ALLOCATIONS:

	Cash/Short-Term	Bonds	Domestic Equities	Foreign Equities
Short-term	100%	0%	0%	0%
Conservative	30	50	20	0
Balanced	10	40	45	5
Growth	5	25	60	10
Aggressive growth	0	15	70	15
Most aggressive	0	0	80	20

VANGUARD INVESTMENTS SUGGESTED ASSET ALLOCATIONS:

Overall Objective	Risk Level	Cash/Short-Term	Bonds	Stocks
Income-oriented	Conservative	0%	100%	0%
	Moderate	0	80%	20%
	Aggressive	0	70%	30%
Balanced	Conservative	0%	60%	40%
	Moderate	0	50%	50%
	Aggressive	0	40%	60%
Growth	Conservative	0%	30%	70%
	Moderate	0	20%	80%
	Aggressive	0	0%	100%

T. ROWE PRICE MATRIX

Non-retirement-goals Matrix

Your Time Horizon

Your Risk Tolerance	3–5 years	6–10 years	11+ years
Higher	**Strategy 2** 20% cash 40% bonds 40% stocks	**Strategy 3** 10% cash 30% bonds 60% stocks	**Strategy 5** 100% stocks
Moderate	**Strategy 1** 30% cash 50% bonds 20% stocks	**Strategy 2** 20% cash 40% bonds 40% stocks	**Strategy 4** 20% bonds 80% stocks
Lower	**All Cash** 100% cash	**Strategy 1** 30% cash 50% bonds 20% stocks	**Strategy 3** 10% cash 30% bonds 60% stocks

Source: Based on data sampled from Personal Fidelity.com, Vanguard.com, and TRowePrice.com.

(a euphemism for downtrends or crashes) that younger people have not experienced or whose effect they do not fully appreciate. Risk tolerance is also influenced by one's current net worth and income expectations. All else being equal, individuals with higher incomes have a greater propensity to undertake risk because their incomes can help cover any shortfall. Likewise, individuals with larger portfolios can afford to place some assets in risky investments while the remaining assets provide a cushion against losses.

A person's return objective may be stated in terms of an absolute or a relative percentage return, but it may also be stated in terms of a general goal, such as capital preservation, current income, capital appreciation, or total return.

Capital preservation means that investors want to minimize their risk of loss, usually in real terms: They seek to maintain the purchasing power of their investment. In other words, the return needs to be no less than the rate of inflation. Generally, this is a strategy for strongly risk-averse investors or for funds needed in the short run, such as for next year's tuition payment or a down payment on a house.

Capital appreciation is an appropriate objective when the investors want the portfolio to grow in real terms over time to meet some future need. Under this strategy, growth mainly occurs through capital gains. This is an aggressive strategy for investors willing to take on risk to meet their objective. Generally, longer-term investors seeking to build a retirement or college education fund may have this goal.

When **current income** is the return objective, the investors want the portfolio to concentrate on generating income rather than capital gains. This strategy sometimes suits investors who want to supplement their earnings with income generated by their portfolio to meet their living expenses. Retirees may favor this objective for part of their portfolio to help generate spendable funds.

The objective for the **total return** strategy is similar to that of capital appreciation; namely, the investors want the portfolio to grow over time to meet a future need. Whereas the capital appreciation strategy seeks to do this primarily through capital gains, the total return strategy seeks to increase portfolio value by both capital gains and reinvesting current income. Because the total return strategy has both income and capital gains components, its risk exposure lies between that of the current income and capital appreciation strategies.

Investment Objective: 25-Year-Old What is an appropriate investment objective for our typical 25-year-old investor? Assume he holds a steady job, is a valued employee, has adequate insurance coverage, and has enough money in the bank to provide a cash reserve. Let's also assume that his current long-term, high-priority investment goal is to build a retirement fund. Depending on his risk preferences, he can select a strategy carrying moderate to high amounts of risk because the income stream from his job will probably grow over time. Further, given his young age and income growth potential, a low-risk strategy, such as capital preservation or current income, is inappropriate for his retirement fund goal; a total return or capital appreciation objective would be most appropriate. Here's a possible objective statement:

> *Invest funds in a variety of moderate- to higher-risk investments. The average risk of the equity portfolio should exceed that of a broad stock market index, such as the NYSE stock index. Foreign and domestic equity exposure should range from 80 percent to 95 percent of the total portfolio. Remaining funds should be invested in short- and intermediate-term notes and bonds.*

Investment Objective: 65-Year-Old Assume our typical 65-year-old investor likewise has adequate insurance coverage and a cash reserve. Let's also assume she is retiring this year. This individual will want less risk exposure than the 25-year-old investor because her earning power from employment will soon be ending; she will not be able to recover any investment losses by saving more out of her paycheck. Depending on her income from social security and a pension plan, she may need some current income from her retirement portfolio to meet living expenses. Given that she can be expected to live an average of another 20 years, she will need protection against inflation. A risk-averse investor will choose a combination of current income and capital preservation strategy; a more risk-tolerant investor will choose a combination of current income and total return in an attempt to have principal growth outpace inflation. Here's an example of such an objective statement:

> *Invest in stock and bond investments to meet income needs (from bond income and stock dividends) and to provide for real growth (from equities). Fixed-income securities should comprise 55–65 percent of the total portfolio; of this, 5–15 percent should be invested in short-term securities for extra liquidity and safety. The remaining 35–45 percent of the portfolio should be invested in high-quality stocks whose risk is similar to the S&P 500 index.*

More detailed analyses for our 25-year-old and our 65-year-old would make more specific assumptions about the risk tolerance of each, as well as clearly enumerate their investment goals, return objectives, the funds they have to invest at the present, the funds they expect to invest over time, and the benchmark portfolio that will be used to evaluate performance.

2.4.2 Investment Constraints

In addition to the investment objective that sets limits on risk and return, certain other constraints also affect the investment plan. Investment constraints include liquidity needs, an investment time horizon, tax factors, legal and regulatory constraints, and unique needs and preferences.

Liquidity Needs An asset is **liquid** if it can be quickly converted to cash at a price close to fair market value. Generally, assets are more liquid if many traders are interested in a fairly standardized product. Treasury bills are a highly liquid security, and real estate and venture capital are not.

Investors may have liquidity needs that the investment plan must consider. For example, although an investor may have a primary long-term goal, several near-term goals may require available funds. Wealthy individuals with sizable tax obligations need adequate liquidity to pay their taxes without upsetting their investment plan. Some retirement plans may need funds for shorter-term purposes, such as buying a car or a house or making college tuition payments.

Our typical 25-year-old investor probably has little need for liquidity as he focuses on his long-term retirement fund goal. This constraint may change, however, should he face a period of unemployment or should near-term goals, such as honeymoon expenses or a house down payment, enter the picture. Should any changes occur, the investor needs to revise his policy statement and financial plans accordingly.

Our soon-to-be-retired 65-year-old investor has a greater need for liquidity. Although she may receive regular checks from her pension plan and social security, it is not likely that they will equal her working paycheck. She will want some of her portfolio in liquid securities to meet unexpected expenses, bills, or special needs such as trips or cruises.

Time Horizon Time horizon as an investment constraint briefly entered our earlier discussion of near-term and long-term high-priority goals. A close (but not perfect) relationship exists between an investor's time horizon, liquidity needs, and ability to handle risk. Investors with long investment horizons generally require less liquidity and can tolerate greater portfolio risk: less liquidity because the funds are not usually needed for many years; greater risk tolerance because any shortfalls or losses can be overcome by earnings and returns in subsequent years.

Investors with shorter time horizons generally favor more liquid and less risky investments because losses are harder to overcome during a short time frame.

Because of life expectancies, our 25-year-old investor has a longer investment time horizon than our 65-year-old investor. But, as discussed earlier, this does not mean the 65-year-old should place all her money in short-term CDs; she needs the inflation protection that long-term investments such as common stock can provide. Still, because of the time horizon constraint, the 25-year-old can have a greater proportion of his portfolio in equities—including stocks in small firms, as well as international and emerging market firms—than the 65-year-old.

Tax Concerns Investment planning is complicated by the tax code; taxes complicate the situation even more if international investments are part of the portfolio. Taxable income from interest, dividends, or rents is taxable at the investor's marginal tax rate. The marginal tax rate is the proportion of the next one dollar in income paid as taxes. Exhibit 2.6 shows the marginal tax rates for different levels of taxable income. As of 2011, the top federal marginal tax rate was 35 percent.

Capital gains or losses arise from asset price changes. They are taxed differently than income. Income is taxed when it is received; capital gains or losses are taxed only when an asset is sold and the gain or loss, relative to its initial cost or **basis**, is realized. **Unrealized capital gains** (or *losses*) reflect the price change in currently held assets that have *not* been sold; the tax liability on unrealized capital gains can be deferred indefinitely. If appreciated assets are passed on to an heir upon the investor's death, the basis of the assets is considered to be their value on the date of the holder's death. The heirs can then sell the assets and pay lower capital gains taxes if they wish. **Realized capital gains** occur when an appreciated asset is sold; taxes are due on the realized capital gains only. As of 2011, the maximum tax rate on stock dividends and long-term capital gains is 15 percent.

Some find the difference between average and marginal income tax rates confusing. The **marginal tax rate** is the part of each additional dollar in income that is paid as tax. Thus, a married person, filing jointly, with an income of $50,000 will have a marginal tax rate of 15 percent. The 15 percent marginal tax rate should be used to determine after-tax returns on investments.

The **average tax rate** is simply a person's total tax payment divided by their total income. It represents the average tax paid on each dollar the person earned. From Exhibit 2.6, a married person, filing jointly, will pay $6,650 in tax on a $50,000 income [$1,700 + 0.15($50,000 − $17,000)]. This average tax rate is $6,650/$50,000 or 13.3 percent. Note that the average tax rate is a weighted average of the person's marginal tax rates paid on each dollar of income.

Exhibit 2.6 Individual Marginal Tax Rates, 2011

For updates, go to the IRS website, *http://www.irs.gov*.

	IF TAXABLE INCOME IS		THE TAX IS		
			THEN		
	Over	But Not Over	This Amount	Plus This %	Of the Excess Over
Single	$0	$8,500	$0	10%	$0
	$8,500	$34,500	$850	15%	$8,500
	$34,500	$83,600	$4,750	25%	$34,500
	$83,600	$174,400	$17,025	28%	$83,600
	$174,400	$379,150	$42,449	33%	$171,850
	$379,150		$110,016	35%	$379,150
Married Filing Jointly	$0	$17,000	$0	10%	$0
	$17,000	$69,000	$1,700	15%	$17,000
	$69,000	$139,350	$9,500	25%	$69,000
	$139,350	$212,300	$27,087	28%	$139,350
	$212,300	$379,150	$47,513	33%	$212,300
	$379,150		$102,574	35%	$379,150

The first $17,000 of income has a 10 percent marginal tax rate; the next $33,000 has a 15 percent marginal tax rate:

$$\frac{\$17{,}000}{\$50{,}000} \times 0.10 + \frac{\$33{,}000}{\$50{,}000} \times 0.15 = 0.133, \text{ or the average tax rate of } 13.3 \text{ percent}$$

Another tax factor is that some sources of investment income are exempt from federal and state taxes. For example, interest on federal securities, such as Treasury bills, notes, and bonds, is exempt from state taxes. Interest on municipal bonds (bonds issued by a state or other local governing body) is exempt from federal taxes. Further, if investors purchase municipal bonds issued by a local governing body of the state in which they live, the interest may be exempt from both state and federal income tax. Thus, high-income individuals have an incentive to purchase municipal bonds to reduce their tax liabilities.

The after-tax return on taxable investment income is

$$\text{After-Tax Income Return} = \text{Pre-Tax Income Return} \times (1 - \text{Marginal Tax Rate})$$

Thus, the after-tax return on a taxable bond investment should be compared to that of municipals before deciding which security a tax-paying investor should purchase.[1] Alternatively, we could compute a municipal's equivalent taxable yield, which is what a taxable bond investment would have to offer to produce the same after-tax return as the municipal. It is given by

$$\text{Equivalent Taxable Yield} = \frac{(\text{Municipal Yield})}{(1 - \text{Marginal Tax Rate})}$$

To illustrate, if an investor is in the 28 percent marginal tax bracket, a taxable investment yield of 8 percent has an after-tax yield of 8 percent × (1 − 0.28) or 5.76 percent; an equivalent-risk municipal security offering a yield greater than 5.76 percent offers the investor greater after-tax returns. On the other hand, a municipal bond yielding 6 percent has an equivalent taxable yield of: 6 percent/(1 − 0.28) = 8.33 percent; to earn more money after taxes, an equivalent-risk taxable investment has to offer a return greater than 8.33 percent.

There are other means of reducing investment tax liabilities. Contributions to an IRA (individual retirement account) may qualify as a tax deduction if certain income limits are met. Even without that deduction, taxes on any investment returns of an IRA, including any income, are deferred until the funds are withdrawn from the account. Any funds withdrawn from an IRA are taxable as current income, regardless of whether growth in the IRA occurs as a result of capital gains, income, or both. For this reason, to minimize taxes advisors recommend investing in stocks in taxable accounts and in bonds in tax-deferred accounts such as IRAs. When funds are withdrawn from a tax-deferred account such as a regular IRA, assets are taxed (at most) at a 35 percent income tax rate (Exhibit 2.6)—even if the source of the stock return is primarily capital gains. In a taxable account, capital gains are taxed at the maximum 15 percent capital gains rate. Decisions regarding IRAs (including Roth IRAs) are very important, but the details of such decisions are beyond the purpose of this book. Therefore, we recommend that investors discuss these decisions with a tax consultant or financial planner.

Legal and Regulatory Factors Both the investment process and the financial markets are highly regulated and subject to numerous laws. At times, these legal and regulatory factors constrain the investment strategies of individuals and institutions.

For example, funds removed from a regular IRA, Roth IRA, or 401(k) plan before age 59½ are taxable and subject to an additional 10 percent withdrawal penalty. You may also be

[1]Realized capital gains on municipal securities are taxed, as are all other capital gains; similarly for capital losses. Only the income from municipals is exempt from federal income tax.

familiar with the tag line in many bank CD advertisements—"substantial interest penalty upon early withdrawal." Regulations and rules such as these may make such investments unattractive for investors with substantial liquidity needs in their portfolios.

Regulations can also constrain the investment choices available to someone in a fiduciary role. A *fiduciary*, or trustee, supervises an investment portfolio of a third party, such as a trust account or discretionary account.[2] The fiduciary must make investment decisions in accordance with the owner's wishes; a properly written policy statement assists this process. In addition, trustees of a trust account must meet the prudent-man standard, which means that they must invest and manage the funds as a prudent person would manage his or her own affairs. Notably, the prudent-man standard is based on the composition of the entire portfolio, not each individual asset.[3]

All investors must respect certain laws, such as insider trading prohibitions against the purchase and sale of securities on the basis of important information that is not publicly known. Typically, the people possessing such private, or insider, information are the firm's managers, who have a fiduciary duty to their shareholders. Security transactions based on access to insider information violates the fiduciary trust the shareholders have placed with management because the managers seek personal financial gain from their privileged position as agents for the shareholders.

For our typical 25-year-old investor, legal and regulatory matters will be of little concern, with the possible exception of insider trading laws and the penalties associated with early withdrawal of funds from tax-deferred retirement accounts. Should he seek a financial advisor to assist him in constructing a financial plan, that advisor would have to obey the regulations pertinent to a client-advisor relationship. Similar concerns confront our 65-year-old investor. In addition, as a retiree, if she wants to do estate planning and set up trust accounts, she should seek legal and tax advice to ensure that her plans are properly implemented.

Unique Needs and Preferences This category covers the individual and sometimes idiosyncratic concerns of each investor. Some investors may want to exclude certain investments from their portfolio solely on the basis of personal preference or for social consciousness reasons. For example, they may request that no firms that manufacture or sell tobacco, alcohol, pornography, or environmentally harmful products be included in their portfolio. Some mutual funds screen according to this type of social responsibility criterion.

Another example of a personal constraint is the time and expertise a person has for managing his or her portfolio. Busy executives may prefer to relax during nonworking hours and let a trusted advisor manage their investments. Retirees, on the other hand, may have the time but believe they lack the expertise to choose and monitor investments, so they also may seek professional advice.

In addition, a business owner with a large portion of her wealth—and emotion—tied up in her firm's stock may be reluctant to sell even when it may be financially prudent to do so and then reinvest the proceeds for diversification purposes. Further, if the stock holdings are in a private company, it may be difficult to find a buyer unless shares are sold at a discount from their fair market value. Because each investor is unique, the implications of this final constraint differ for each person; there is no "typical" 25-year-old or 65-year-old investor. The point is, each individual will have to decide on—and then communicate—specific needs and preferences in a well-constructed policy statement.

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[2]A discretionary account is one in which the fiduciary, many times a financial planner or stockbroker, has the authority to purchase and sell assets in the owner's portfolio without first receiving the owner's approval.

[3]As we will discuss in Chapter 7, it is sometimes wise to hold assets that are individually risky in the context of a well-diversified portfolio, even if the investor is strongly risk averse.

2.5 Constructing the Policy Statement

As we have seen, the policy statement allows the investor to communicate his or her objectives (risk and return) and constraints (liquidity, time horizon, tax, legal and regulatory, and unique needs and preferences). This communication gives the advisor a better chance of implementing an investment strategy that will satisfy the investor. Even if an advisor is not used, each investor needs to take this first important step of the investment process and develop a financial plan to guide the investment strategy. To do without a plan or to plan poorly is to place the success of the financial plan in jeopardy.

2.5.1 General Guidelines

Constructing a policy statement is the investor's responsibility, but investment advisors often assist in the process. Here, for both the investor and the advisor, are guidelines for good policy statement construction.

In the process of constructing a policy statement, investors should think about the set of questions suggested previously on page 39.

When working with an investor to create a policy statement, an advisor should ensure that the policy statement satisfactorily answers the questions suggested previously on page 41.

2.5.2 Some Common Mistakes

When constructing their policy statements, participants in employer-sponsored retirement plans need to realize that in many such plans 30–40 percent of their retirement funds may be invested in their employer's stock. Having so much money invested in one asset violates diversification principles and could be costly. To put this in context, most mutual funds are limited by law to having no more than 5 percent of their assets in any one company's stock; a firm's pension plan can invest no more than 10 percent of their funds in its own stock. As noted by Schulz (1996), individuals are unfortunately doing what government regulations prevent many institutional investors from doing. In addition, some studies point out that the average stock allocation in many retirement plans is lower than it should be if the investor wants growth of principal over time—that is, investors tend to be too conservative.

Another consideration is the issue of stock trading. A number of studies by Barber and Odean (1999, 2000, 2001) and Odean (1998, 1999) have shown that individual investors typically trade stocks too often (driving up commissions), sell stocks with gains too early (prior to further price increases), and hold on to losers too long (as the price continues to fall). These costly mistakes are especially true for men and online traders.

Investors, in general, seem to neglect that important first step to achieve financial success: They do not plan for the future. Studies of retirement plans discussed by Ruffenach (2001) and Clements (1997a, b, c) show that Americans are not saving enough to finance their retirement years and they are not planning sufficiently for what will happen to their savings after they retire. Around 25 percent of workers have saved less than $50,000 for their retirement. Finally, about 60 percent of workers surveyed confessed they were "behind schedule" in planning and saving for retirement.

2.6 The Importance of Asset Allocation

A major reason why investors develop policy statements is to provide guidance for an overall investment strategy. Though a policy statement does not indicate which specific securities to purchase and when they should be sold, it should provide guidelines as to the asset classes to include and a range of percents of the investor's funds to invest in each class. How the investor divides funds into different asset classes is the process of asset allocation. Rather than provide strict percentages, asset allocation is usually expressed in ranges. This allows the investment manager some freedom, based on his or her reading of capital market trends, to invest toward the upper or lower end of the ranges. For example, suppose a policy statement requires that common stocks be 60 percent to 80 percent of the value of the portfolio and that bonds should be 20 percent to 40 percent of the portfolio's value. If a manager is particularly bullish about stocks, she will increase the allocation of stocks toward the 80 percent upper end of the equity range and decrease bonds toward the 20 percent lower end of the bond range. Should she be optimistic about bonds or bearish on stocks, that manager may shift the allocation closer to 40 percent invested in bonds with the remainder in equities.

A review of historical data and empirical studies provides strong support for the contention that the asset allocation decision is a critical component of the portfolio management process. In general, there are four decisions involved in constructing an investment strategy:

- What asset classes should be considered for investment?
- What policy weights should be assigned to each eligible asset class?
- What are the allowable allocation ranges based on policy weights?
- What specific securities or funds should be purchased for the portfolio?

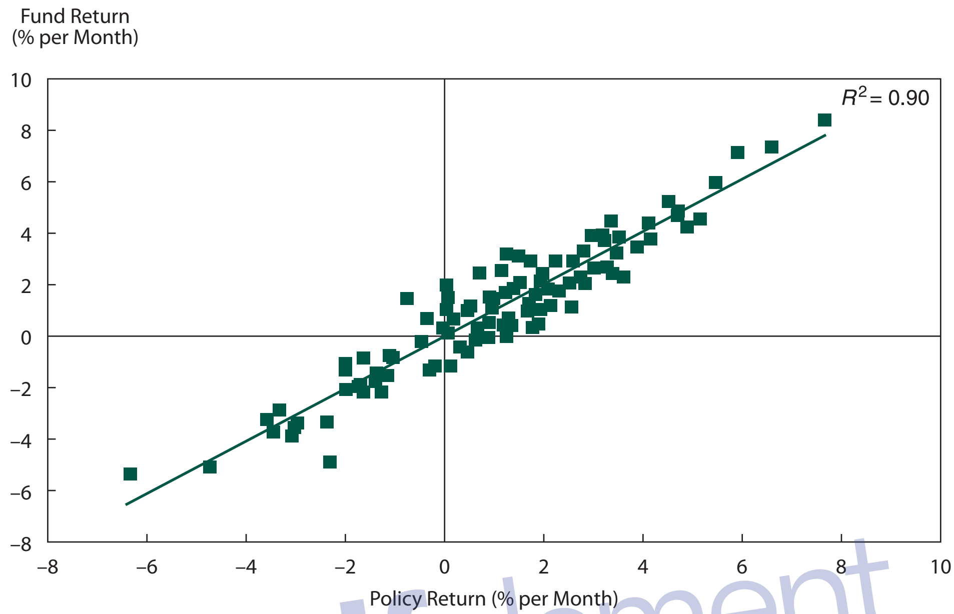
The asset allocation decision involves the first three points. How important is the asset allocation decision to an investor? In a word, *very*. Several studies by Ibbotson and Kaplan (2000); Brinson, Hood, and Beebower (1986); and Brinson, Singer, and Beebower (1991) have examined the effect of the normal policy weights on investment performance, using data from both pension funds and mutual funds, during time periods extending from the early 1970s to the late 1990s. The studies all found similar results: About 90 percent of a fund's returns over time can be explained by its target asset allocation policy. Exhibit 2.7 shows the relationship between returns on the target or policy portfolio allocation and actual returns on a sample mutual fund.

Rather than looking at just one fund and how the target asset allocation determines its returns, some studies have looked at how much the asset allocation policy affects returns on a variety of funds with different target weights. For example, Ibbotson and Kaplan (2000) found that, across a sample of funds, about 40 percent of the difference in fund returns is explained by differences in asset allocation policy. And what does asset allocation tell us about the *level* of a particular fund's returns? The studies by Brinson and colleagues (1986, 1991) and Ibbotson and Kaplan (2000) answered that question as well. They divided the policy return (what the fund return would have been had it been invested in indexes at the policy weights) by the actual fund return (which includes the effects of varying from the policy weights and security selection). Thus, a fund that was passively invested at the target weights would have a ratio value of 1.0, or 100 percent. A fund managed by someone with skill in market timing (for moving in and out of asset classes) and security selection would have a ratio less than 1.0 (or less than 100 percent); the manager's skill would result in a policy return less than the actual fund return. The studies showed the opposite: The policy-return/actual-return ratio averaged over 1.0, showing that asset allocation explains slightly more than 100 percent of the level of a fund's returns. Because of market efficiency, fund managers practicing market timing and security selection, on average, have difficulty surpassing passively invested index returns, after taking into account the expenses and fees of investing.

Thus, asset allocation is a very important decision. Across all funds, the asset allocation decision explains an average of 40 percent of the variation in fund returns. For a single fund, asset allocation explains 90 percent of the fund's variation in returns over time and slightly more than 100 percent of the average fund's level of return.

Good investment managers may add some value to portfolio performance, but the major source of investment return—and risk—over time is the asset allocation decision (Brown, 2000).

Exhibit 2.7 Time-Series Regression of Monthly Fund Return versus Fund Policy Return: One Mutual Fund, April 1988–March 1998



Note: The sample fund's policy allocations among the general asset classes were 52.4 percent U.S. large-cap stocks, 9.8 percent U.S. small-cap stocks, 3.2 percent non-U.S. stocks, 20.9 percent U.S. bonds, and 13.7 percent cash.

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2.6.1 Investment Returns after Taxes and Inflation

Exhibit 2.8 provides additional historical perspectives on returns. It indicates how an investment of $1 would have grown over the 1986–2010 period and, using fairly conservative assumptions, examines how investment returns are affected by taxes and inflation.

Focusing first on stocks, funds invested in 1986 in the Standard & Poor's 500 stocks would have averaged an 11.57 percent annual return through 2010. Unfortunately, this return is unrealistic because if the funds were invested over time, taxes would have to be paid and inflation would erode the real purchasing power of the invested funds.

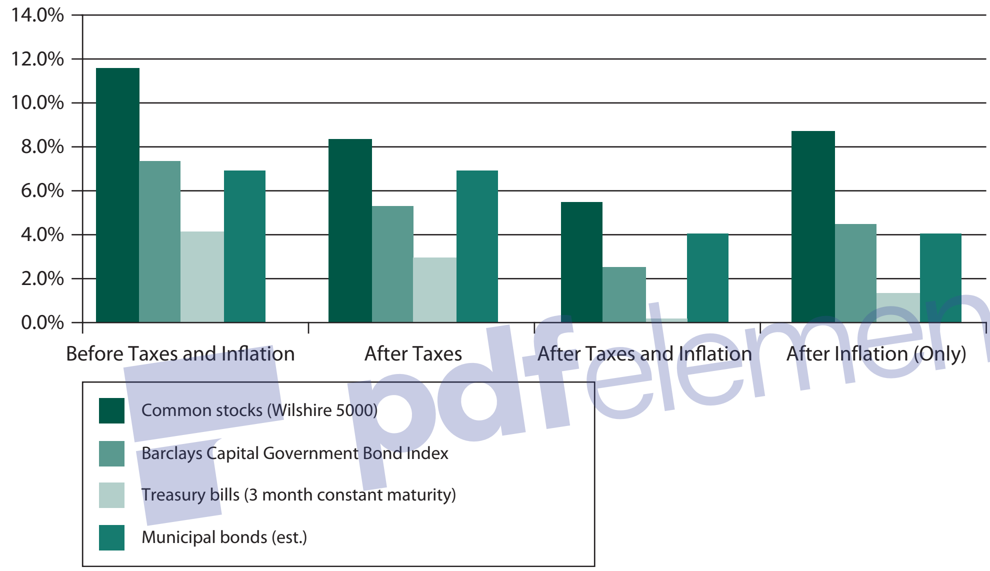
Except for tax-exempt investors and tax-deferred accounts, annual tax payments reduce investment returns. Incorporating taxes into the analysis lowers the after-tax average annual return of a stock investment to 8.33 percent.

But the major reduction in the value of our investment is caused by inflation. The real after-tax average annual return on a stock over this time frame was only 5.50 percent, which is less than half our initial unadjusted 11.57 percent return!

This example shows the long-run impact of taxes and inflation on the real value of a stock portfolio. For bonds and bills, however, the results in Exhibit 2.8 show something even more surprising. After adjusting for taxes, long-term bonds maintained their purchasing power; T-bills barely provided value in real terms. One dollar invested in long-term government bonds in 1986 gave the investor an annual average after-tax real return of 2.47 percent. An investment in Treasury bills earned an average annual rate of only 0.15 percent after taxes

Exhibit 2.8 The Effect of Taxes and Inflation on Investment Returns: 1986–2010

	Before Taxes and Inflation	After Taxes	After Taxes and Inflation	After Inflation (Only)
Common stocks (S&P 500)	11.57%	8.33%	5.50%	8.74%
Barclays Capital Government Bond Index	7.36%	5.30%	2.47%	4.53%
Treasury bills (3 month constant maturity)	4.14%	2.98%	0.15%	1.31%
Municipal bonds (est.)	6.88%	6.88%	4.05%	4.05%



Assumptions: 28 percent tax rate on income; 20 percent on price change. Compound inflation rate was 3.1 percent for full period.

Source: Computations by authors, using data indicated.

and inflation. Municipal bonds, because of the protection they offer from taxes, earned an average annual real return of almost 4.05 percent during this time.

This historical analysis demonstrates that, for taxable investments, a reasonable way to maintain purchasing power over long time periods when investing in financial assets is to invest in common stocks. Put another way, an asset allocation decision for a taxable portfolio that does not include a substantial commitment to common stocks makes it difficult for the portfolio to maintain real value over time.[4]

Notably, the fourth column, labeled "After inflation (only)," is more encouraging since it refers to results for a tax-free retirement account that is only impacted by inflation. These results should encourage investors to take advantage of tax-free opportunities.

2.6.2 Returns and Risks of Different Asset Classes

By focusing on returns, we have ignored its partner—risk. Assets with higher long-term returns have these returns to compensate for their risk. Exhibit 2.9 illustrates returns (unadjusted for

[4]Of course other equity-oriented investments, such as venture capital or real estate, may also provide inflation protection after adjusting for portfolio costs and taxes. Future studies of the performance of Treasury inflation-protected securities (TIPs) will likely show their usefulness in protecting investors from inflation as well.

Exhibit 2.9 Summary Statistics of Annual Returns, 1986–2010, U.S. Securities

	Geometric Mean (%)	Arithmetic Mean (%)	Standard Deviation (%)
Large company stocks (S&P 500)	9.94	11.57	18.23
Small company stocks (Russell 2000)	10.63	12.73	21.43
Government bonds (Barclays Capital)	7.20	7.36	5.86
Corporate bonds (Barclays Capital)	7.94	8.13	6.49
High-Yield Corporate bonds (Barclays Capital)	8.96	10.14	16.87
30-day Treasury bill (Federal Reserve)	4.12	4.14	2.24
U.S. inflation (Federal Reserve)	2.82	2.83	1.29

Source: Calculations by authors, using data noted.

inflation, transaction costs and taxes) for several asset classes over time. As expected, the higher returns available from equities (both large cap and small cap) also include higher risk. This is precisely why investors need a policy statement and why the investor and manager must understand the capital markets and have a disciplined approach to investing. Safe Treasury bills will sometimes outperform equities, and, because of their higher risk, common stocks will sometimes lose significant value. These are times when undisciplined and uneducated investors become frustrated, sell their stocks at a loss, and vow never to invest in equities again. In contrast, these are just the times when disciplined investors stick to their investment plan and position their portfolios for the next bull market.[5] By holding on to their stocks and continuing to purchasing more at depressed prices, the equity portion of the portfolio will experience a substantial increase in the future.

The asset allocation decision determines to a great extent both the returns and the volatility of the portfolio. As noted, Exhibit 2.9 indicates that stocks are riskier than bonds or T-bills. Exhibit 2.10 shows that stocks have sometimes experienced returns lower than those of T-bills for extended periods of time. Still, the long-term results in Exhibit 2.9 show that sticking with an investment policy through difficult times provides attractive rates of return over long holding periods.[6]

One popular way to measure risk is to examine the variability of returns over time by computing a standard deviation or variance of annual rates of return for an asset class. This measure, which is used in Exhibit 2.9, indicates that stocks are relatively risky and T-bills are relatively safe. Another intriguing measure of risk is the probability of *not* meeting your investment return objective. From this perspective, the results in Exhibit 2.10 show that if the investor has a long time horizon (i.e., approaching 20 years), the risk of equities is small and that of T-bills is large because of their differences in *long-term* expected returns.

2.6.3 Asset Allocation Summary

A carefully constructed policy statement determines the types of assets that should be included in a portfolio. The asset allocation decision, not the selection of specific stocks and bonds, determines most of the portfolio's returns over time. Although seemingly risky, investors seeking

[5]Newton's law of gravity seems to work two ways in financial markets. What goes up must come down; but it also appears over time that what goes down may come back up. Contrarian investors and some "value" investors use this concept of reversion to the mean to try to outperform the indexes over time.

[6]The added benefits of diversification—combining different asset classes in the portfolio—may reduce overall portfolio risk without harming potential return. The important topic of diversification is discussed in detail in Chapter 7.

Exhibit 2.10 Higher Returns Offered by Equities over Long Time Periods Time Frame: 1934–2010

Length of Holding Period (calendar years)	Percentage of Periods That Stock Returns Trailed T-Bill Returns*
1	33.80%
5	15.10
10	8.80
20	0.00
30	0.00

*Price change plus reinvested income
Source: Author calculations.

capital appreciation, income, or even capital preservation over long time periods should stipulate a sizable allocation to the equity portion in their portfolio. As noted in this section, a strategy's risk depends on the investor's goals and time horizon. As demonstrated, investing in T-bills may actually be a riskier strategy than investing in common stocks due to the risk of not meeting *long-term investment return goals* especially after considering the impact of inflation and taxes.

Asset Allocation and Cultural Differences Thus far, our analysis has focused on U.S. investors. Non-U.S. investors make their asset allocation decisions in much the same manner; but because they face different social, economic, political, and tax environments, their allocation decisions differ from those of U.S. investors. Exhibit 2.11 shows the equity allocations of pension funds in several countries. As shown, the equity allocations vary dramatically from 79 percent in Hong Kong to 37 percent in Japan and only 8 percent in Germany.

National demographic and economic differences can explain much of the divergent portfolio allocations. Of these six nations, the average age of the population is highest in Germany and Japan and lowest in the United States and the United Kingdom, which helps explain the greater use of equities in the United States and United Kingdom. Further, government privatization programs during the 1980s in the United Kingdom encouraged equity ownership among individual and institutional investors. In Germany, regulations prevent insurance firms from having more than 20 percent of their assets in equities. Both Germany and Japan have banking sectors that invest privately in firms and whose officers sit on corporate boards. Since 1980, the cost of living in the United Kingdom has increased at a rate about two times that of

Exhibit 2.11 Equity Allocations in Pension Fund Portfolios by Country

Country	Percentage in Equities
Hong Kong	79
United Kingdom	78
Ireland	68
United States	58
Japan	37
Germany	8

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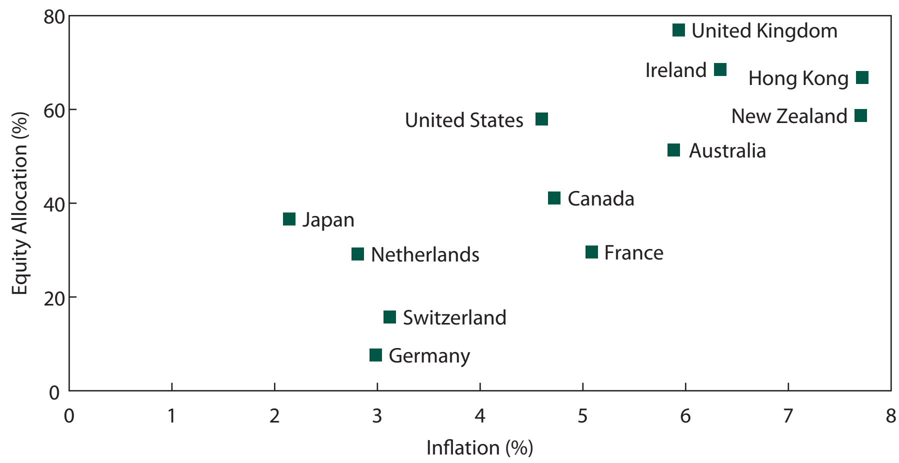
Exhibit 2.12 Asset Allocation and Inflation for Different Countries' Equity Allocation as of December 1997; Average Inflation Measured over 1980–1997



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Germany and this inflationary bias in the U.K. economy again favors higher equity allocations. Exhibit 2.12 shows the positive relationship between the level of inflation in a country and its pension fund allocation to equity. These results and many others that could be mentioned indicate that some legislation, the general economic environment, and the demographics of a country have an effect on the asset allocation by the investors in the country.

SUMMARY

- In this chapter, we saw that investors need to prudently manage risk within the context of their investment goals and preferences. Income, spending, and investing behavior will change over a person's lifetime.
- We reviewed the importance of developing an investment policy statement before implementing an investment plan. By forcing investors to examine their needs, risk tolerance, and familiarity with the capital markets, policy statements help investors correctly identify appropriate objectives and constraints. In addition, the policy statement provides a standard by which to evaluate the performance of the portfolio manager.
- We also reviewed the importance of the asset allocation decision in determining long-run portfolio investment returns and risks. Because the asset allocation decision follows setting the objectives and constraints, it is clear that the success of the investment program depends on the first step, the construction of the policy statement.

SUGGESTED READINGS

Bodie, Zvi, Dennis McLeavey, and Laurence Siegel, eds. *The Future of Life-Cycle Saving and Investing.* Charlottesville, VA: Research Foundation of CFA Institute, 2007.

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Milevsky, Mosha A., and Chris Robinson. "A Sustainable Spending Rate without Simulation," *Financial Analysts Journal* 61, no. 6 (November–December, 2005).

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QUESTIONS

1. "Young people with little wealth should not invest money in risky assets such as the stock market, because they can't afford to lose what little money they have." Do you agree or disagree with this statement? Why?

2. Your healthy 63-year-old neighbor is about to retire and comes to you for advice. From talking with her, you find out she was planning on taking all the money out of her company's retirement plan and investing it in bond mutual funds and money market funds. What advice should you give her?

3. Discuss how an individual's investment strategy may change as he or she goes through the accumulation, consolidation, spending, and gifting phases of life.

4. Why is a policy statement important?

5. Use the questionnaire "How much risk is right for you?" (Exhibit 2.4) to determine your risk tolerance. Use this information to help write a policy statement for yourself.

6. Your 45-year-old uncle is 20 years away from retirement; your 35-year-old older sister is about 30 years away from retirement. How might their investment policy statements differ?

7. What information is necessary before a financial planner can assist a person in constructing an investment policy statement?

8. Use the Internet to find the home pages for some financial-planning firms. What strategies do they emphasize? What do they say about their asset allocation strategy? What are their firms' emphases—for example, value investing, international diversification, principal preservation, retirement and estate planning?

9. Mr. Franklin is 70 years of age, is in excellent health, pursues a simple but active lifestyle, and has no children. He has interest in a private company for $90 million and has decided that a medical research foundation will receive half the proceeds now and will be the primary beneficiary of his estate upon his death. Mr. Franklin is committed to the foundation's well-being because he believes strongly that, through it, a cure will be found for the disease that killed his wife. He now realizes that an appropriate investment policy and asset allocations are required if his goals are to be met through investment of his considerable assets. Currently, the following assets are available for use in building an appropriate portfolio for him:

$45.0	million cash (from sale of the private company interest, net of a $45 million gift to the foundation)
$10.0	million stocks and bonds ($5 million each)
$ 9.0	million warehouse property (now fully leased)
$ 1.0	million value of his residence
$65.0	million total available assets

a. Formulate and justify an investment policy statement setting forth the appropriate guidelines within which future investment actions should take place. Your policy statement should encompass all relevant objective and constraint considerations.

b. Recommend and justify a long-term asset allocation that is consistent with the investment policy statement you created in Part a. Briefly explain the key assumptions you made in generating your allocation.

PROBLEMS

1. Suppose your first job pays you $28,000 annually. What percentage should your cash reserve contain? How much life insurance should you carry if you are unmarried? How much if you are married with two young children?

2. Using Exhibit 2.6, what is the marginal tax rate for a couple, filing jointly, if their taxable income is $20,000? $40,000? $60,000? What is their tax bill for each of these income levels? What is the average tax rate for each of these income levels?

3. What is the marginal tax rate for a single individual if her taxable income is $20,000? $40,000? 60,000? What is her tax bill for each of these income levels? What is her average tax rate for each of these income levels?

4. a. Someone in the 36 percent tax bracket can earn 9 percent annually on her investments in a tax-exempt IRA account. What will be the value of a one-time $10,000 investment in 5 years? 10 years? 20 years?

b. Suppose the preceding 9 percent return is taxable rather than tax-deferred and the taxes are paid annually. What will be the after-tax value of her $10,000 investment after 5, 10, and 20 years?

5. a. Someone in the 15 percent tax bracket can earn 10 percent on his investments in a tax-exempt IRA account. What will be the value of a $10,000 investment in 5 years? 10 years? 20 years?

b. Suppose the preceding 10 percent return is taxable rather than tax-deferred. What will be the after-tax value of his $10,000 investment after 5, 10, and 20 years?

6. Assume that the rate of inflation during all these periods was 3 percent a year. Compute the real value of the two tax-deferred portfolios in problems 4a and 5a.

Objectives and Constraints of Institutional Investors

Institutional investors manage large amounts of funds in the course of their business. They include mutual funds, pension funds, insurance firms, endowments, and banks. In this appendix, we review the characteristics of various institutional investors and discuss their typical investment objectives and constraints.

Mutual Funds

A mutual fund pools sums of money from investors, which are then invested in financial assets. Each mutual fund has its own investment objective, such as capital appreciation, high current income, or money market income. A mutual fund will state its investment objective, and investors choose the funds in which to invest. Two basic constraints face mutual funds: those created by law to protect mutual fund investors and those that represent choices made by the mutual fund's managers. Some of these constraints will be discussed in the mutual fund's prospectus, which must be given to all prospective investors before they purchase shares in a mutual fund. Mutual funds are discussed in more detail in Chapter 24.

Pension Funds

Pension funds are a major component of retirement planning for individuals. As of 2011, U.S. pension assets were nearly $21 trillion. Basically, a firm's pension fund receives contributions from the firm, its employees, or both. The funds are invested with the purpose of giving workers either a lump-sum payment or the promise of an income stream after their retirement. **Defined benefit pension plans** promise to pay retirees a specific income stream after retirement. The size of the benefit is usually based on factors that include the worker's salary, or time of service, or both. The company contributes a certain amount each year to the pension plan; the size of the contribution depends on assumptions concerning future salary increases and the rate of return to be earned on the plan's assets. Under a defined benefit plan, *the company carries the risk* of paying the future pension benefit to retirees; should investment performance be poor, or should the company be unable to make adequate contributions to the plan, the shortfall must be made up in future years. "Poor" investment performance means the actual return on the plan's assets fell below the assumed **actuarial rate of return**. The actuarial rate is the discount rate used to find the present value of the plan's future obligations and thus this rate determines the size of the firm's annual contribution to the pension plan.

Defined contribution pension plans do not promise set benefits but only specified contributions to the plan. As a result, employees' benefits depend on the size of the contributions made to the pension fund and the returns earned on the fund's investments. Thus, the plan's risk related to the rates of return on investments is borne by the employee. Unlike a defined benefit plan, employees' retirement income is not an obligation of the firm.

A pension plan's objectives and constraints depend on whether the plan is a *defined benefit plan* or a *defined contribution plan.* We review each separately below.

Defined Benefit The plan's risk tolerance depends on the plan's funding status and its actuarial rate. For **underfunded plans** (where the present value of the fund's liabilities to employees exceeds the value of the fund's assets), a more conservative approach toward risk is taken to ensure that the funding gap is closed over time. This may entail a strategy whereby the firm makes larger plan contributions and assumes a lower actuarial rate. **Overfunded plans** (where the present value of the pension liabilities is less than the plan's assets) allow a more aggressive investment strategy, which implies a higher actuarial rate. This allows the firm to reduce its contributions and increases the risk exposure of the plan. The return objective is to meet the plan's actuarial rate of return, which is set by actuaries who estimate future pension obligations based on assumptions about future salary increases, current salaries, retirement patterns, worker life expectancies, and the firm's benefit formula. Obviously, the actuarial rate helps determine the size of the firm's plan contributions over time.

The liquidity constraint on defined benefit funds is mainly a function of the average age of employees. A younger employee base means less liquidity is needed; an older employee base generally means more liquidity is needed to pay current pension obligations to retirees. The time horizon constraint is also affected by the average age of employees, although some experts recommend using a 5- to 10-year horizon for planning purposes. Taxes are not a major concern to the plan, because pension plans are exempt from paying tax on investment returns. The major legal constraint is that the plan must be run in accordance with the Employee Retirement and Income Security Act (ERISA), and investments must satisfy the "prudent-expert" standard when evaluated in the context of the overall pension plan's portfolio.

Defined Contribution Notably, the individual employee decides how his or her contributions to the plan are to be invested. As a result, the objectives and constraints for defined contribution plans depend on the individual. Because the employee carries the risk of inadequate retirement funding rather than the firm, defined contribution plans are generally more conservatively invested (the majority of research indicates that employees tend to be too conservative). If, however, the plan is considered part of an estate planning tool for a wealthy founder or officer of the firm, a higher risk tolerance and return objective are appropriate because most of the plan's assets will ultimately be owned by the individual's heirs.

The liquidity and time horizon needs for the plan differ depending on the average age of the individual employees and the degree of employee turnover within the firm. Similar to defined benefit plans, defined contribution plans are tax-exempt and are governed by the provisions of ERISA.

Endowment Funds

Endowment funds arise from contributions made to charitable or educational institutions. Rather than immediately spending the funds, the organization invests the money for the purpose of providing a future stream of income to the organization. The investment policy of an endowment fund is the result of a "tension" between the organization's need for current income and the desire for a growing future stream of income to protect against inflation.

To meet the institution's operating budget needs, the fund's return objective is often set by adding the spending rate (the amount taken out of the funds each year) and the expected inflation rate. Funds that have more risk-tolerant trustees may have a higher spending rate than those overseen by more risk-averse trustees. Because a total return approach usually serves to meet the return objective over time, the organization is generally withdrawing both income and capital gain returns to meet budgeted needs. The risk tolerance of an endowment fund is largely affected by the collective risk tolerance of the organization's trustees.

Due to the fund's long-term time horizon, liquidity requirements are minor except for the need to spend part of the endowment each year and maintain a cash reserve for emergencies. Many endowments are tax-exempt, although income from some private foundations can be taxed at either a 1 percent or 2 percent rate. Short-term capital gains are taxable, but long-term capital gains are not. Regulatory and legal constraints arise on the state level, where most endowments are regulated. Unique needs and preferences may affect investment strategies, especially among college or religious endowments, which may have strong preferences about social investing issues.

Insurance Companies

The investment objectives and constraints for an insurance company depend on whether it is a life insurance company or a nonlife (such as a property and casualty) insurance firm.

Life Insurance Companies Except for firms dealing only in term life insurance, life insurance firms collect premiums during a person's lifetime that must be invested until a death benefit is paid to the insurance contract's beneficiaries. At any time, the insured can turn in her policy and receive its cash surrender value. Discussing investment policy for an insurance firm is also complicated by the insurance industry's proliferation of insurance and quasi-investment products.

Basically, an insurance company wants to earn a positive "spread," which is the difference between the rate of return on investment minus the rate of return it credits its various policyholders. This concept is similar to a defined benefit pension fund that tries to earn a rate of return in excess of its actuarial rate. If the spread is positive, the insurance firm's surplus reserve account rises; if not, the surplus account declines by an amount reflecting the negative spread. A growing surplus is an important competitive tool for life insurance companies. Attractive investment returns allow the company to advertise better policy returns than those of its competitors. A growing surplus also allows the firm to offer new products and expand insurance volume.

Because life insurance companies are quasi-trust funds for savings, fiduciary principles limit the risk tolerance of the invested funds. The National Association of Insurance Commissioners (NAIC) establishes risk categories for bonds and stocks; companies with excessive investments in higher-risk categories must set aside extra funds in a mandatory securities valuation reserve (MSVR) to protect policyholders against losses.

Insurance companies' liquidity needs have increased over the years due to increases in policy surrenders and product-mix changes. A company's time horizon depends upon its specific product mix. Life insurance policies require longer-term investments, whereas guaranteed insurance contracts (GICs) and shorter-term annuities require shorter investment time horizons.

Tax rules changed considerably for insurance firms in the 1980s. For tax purposes, investment returns are divided into two components: first, the policyholder's share, which is the return portion covering the actuarially assumed rate of return needed to fund reserves; and second, the balance that is transferred to reserves. Unlike pensions and endowments, life insurance firms pay income and capital gains taxes at the corporate tax rates on the returns transferred to reserves.

Except for the NAIC, most insurance regulation is on the state level. Regulators oversee the eligible asset classes and the reserves (MSVR) necessary for each asset class and enforce the "prudent-expert" investment standard. Audits ensure that various accounting rules and investment regulations are followed.

Nonlife Insurance Companies Cash outflows are somewhat predictable for life insurance firms, based on their mortality tables. In contrast, the cash flows required by major accidents, disasters, and lawsuit settlements are not as predictable for nonlife insurance firms.

Due to their fiduciary responsibility to claimants, risk exposures are low to moderate. Depending on the specific company and competitive pressures, premiums may be affected by both the probability of a claim and the investment returns earned by the firm. Typically, casualty insurance firms invest their insurance reserves in relatively safe bonds to provide needed income to pay claims; capital and surplus funds are invested in equities for their growth potential. As with life insurers, property and casualty firms have a stronger competitive position when their surplus accounts are larger than those of their competitors. Many insurers now focus on a total return objective as a means to increase their surplus accounts over time.

Because of uncertain claim patterns, liquidity is a concern for property and casualty insurers who also want liquidity so they can switch between taxable and tax-exempt investments as their underwriting activities generate losses and profits. The time horizon for investments is typically shorter than that of life insurers, although many invest in long-term bonds to earn the higher yields available on these instruments. Investing strategy for the firm's surplus account focuses on long-term growth.

Regulation of property and casualty firms is more permissive than for life insurers. Similar to life companies, states regulate classes and quality of investments for a certain percentage of the firm's assets. Beyond this restriction, insurers can invest in many different types and qualities of instruments, although some states limit the proportion that can be invested in real estate assets.

Banks

Pension funds, endowments, and insurance firms obtain virtually free funds for investment purposes. Not so with banks. To have funds to lend, they must attract investors in a competitive interest rate environment. They compete against other banks and also against companies that offer other investment vehicles, from bonds to common stocks. A bank's success relies primarily on its ability to generate returns in excess of its funding costs.

A bank tries to maintain a positive difference between its cost of funds and its returns on assets. If banks anticipate falling interest rates, they will try to invest in longer-term assets to lock in the returns while seeking short-term deposits, whose interest cost is expected to fall over time. When banks expect rising rates, they will try to lock in longer-term deposits with fixed-interest costs, while investing funds short term to capture rising interest rates. The risk of such strategies is that losses may occur should a bank incorrectly forecast the direction of interest rates. The aggressiveness of a bank's strategy will be related to the size of its capital ratio and the oversight of regulators.

Banks need substantial liquidity to meet withdrawals and loan demand. A bank has two forms of liquidity. *Internal liquidity* is provided by a bank's investment portfolio that includes highly liquid assets. A bank has *external liquidity* if it can borrow funds in the federal funds markets (where banks lend reserves to other banks), from the Federal Reserve Bank's discount window, or if it can sell certificates of deposit at attractive rates.

Banks have a short time horizon for several reasons. First, they have a strong need for liquidity. Second, because they want to maintain an adequate interest revenue–interest expense spread, they generally focus on shorter-term investments to avoid interest rate risk and to avoid getting "locked in" to a long-term revenue source. Third, because banks typically offer short-term deposit accounts (demand deposits, NOW accounts, and such) they need to match the maturity of their assets and liabilities to avoid taking undue risks. This desire to match the maturity of assets and liabilities is shared by virtually all financial institutions.

Banks are heavily regulated by numerous state and federal agencies. The Federal Reserve Board, the Comptroller of the Currency, and the Federal Deposit Insurance Corporation all oversee various components of bank operations. The Glass-Steagall Act restricts the equity investments that banks can make. Unique situations that affect each bank's investment policy depend on their size, market, and management skills in matching asset and liability sensitivity to interest rates. For example, a bank in a small community may have many customers who deposit their money with it for the sake of convenience. A bank in a more populated area will find its deposit flows are more sensitive to interest rates and competition from nearby banks.

Institutional Investment Summary

Among the great variety of institutions, each institution has its "typical" investment objectives and constraints. This discussion has indicated the differences that exist among types of institutions and some of the major issues confronting them. Notably, just as with individual investors, "cookie-cutter" policy statements are inappropriate for institutional investors. The specific objectives, constraints, and investment strategies must be determined on a case-by-case basis.

Portfolio Theory

PART

2

The last 90 years witnessed the Great Depression, seven additional recessions of varying severity, and the deep recession that began in 2007. Yet even with these downturns, a dollar invested in a broad portfolio of stocks over this period still grew to a value about 120 times greater than a dollar invested (and reinvested) in safe assets. Why then would anyone invest in a safe asset? Because investors are risk averse, and risk is as important to them as the expected value of returns. Chapter 5, the first of five in Part Two, provides the tools needed to interpret the history of rates of return, and the lessons that history offers for how investors might go about constructing portfolios using both safe and risky assets.

Deciding the proportion an investor desires to put at risk must be augmented by a decision of how to construct an efficient portfolio of risky assets. Chapter 6 lays out modern portfolio theory (MPT), which involves the construction of the risky portfolio. It aims to accomplish efficient diversification across asset classes like bonds and stocks and across individual securities within these asset classes.

This analysis quickly leads to other questions. For example, how should one measure the risk of an individual asset held as part of a diversified portfolio? You may be surprised at the answer. Once we have an acceptable measure of risk, what precisely should be the relation between risk and return? And what is the minimally acceptable rate of return for an investment to be considered attractive? These questions also are addressed in this part of the text. Chapter 7 introduces the Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT), as well as index and multi-index models, all mainstays of applied financial economics. These models link risk with the return investors can reasonably expect on various securities.

Next, we come to one of the most controversial topics in investment management, the question of whether portfolio managers—amateur or professional—can outperform simple investment strategies such as "buy a market index fund." The evidence in Chapter 8 will at least make you pause before pursuing active strategies. You will come to appreciate how good active managers must be to outperform passive strategies. Finally, Chapter 9 on behavioral finance is concerned with lessons from psychology that have been proposed to explain irrational investor behavior that leads to observed anomalies in patterns of asset returns.

Chapters in This Part:

Risk and Return: Past and Prologue

Chapter 5

Learning Objectives:

LO5-1 Compute various measures of return on multi-year investments.

LO5-2 Use data on the past performance of stocks and bonds or scenario analysis to characterize the risk and return features of these investments.

LO5-3 Determine the expected return and risk of portfolios that are constructed by combining risky assets with risk-free investments in Treasury bills.

LO5-4 Use the Sharpe ratio to evaluate the investment performance of a portfolio and provide a guide for capital allocation.

What constitutes a satisfactory investment portfolio? Until the early 1970s, a reasonable answer would have been a federally insured bank savings account (a risk-free asset) plus a risky portfolio of U.S. stocks. Nowadays, investors have access to a vast array of assets and can easily construct portfolios that include foreign stocks and bonds, real estate, precious metals, and collectibles. Even more complex strategies may include futures, options, and other derivatives to insure portfolios against specified risks.

Clearly every individual security must be judged on its contributions to both the expected return and the risk of the entire portfolio. We begin with an examination of various conventions for measuring and reporting rates of return. Next, we turn to the historical performance of several broadly diversified investment portfolios. In doing so, we use a risk-free portfolio of Treasury bills as a benchmark to evaluate the historical performance of diversified stock and bond portfolios.

We then consider the trade-offs that arise when investors practice the simplest form of risk control, capital allocation: choosing the fraction of the portfolio invested in virtually risk-free securities versus risky securities. We show how to calculate the performance one may expect from various allocations between a risk-free asset and a risky portfolio and contemplate the mix that would best suit different investors. With this background, we can evaluate a passive strategy that will serve as a benchmark for the active strategies considered in the next chapter.

Related websites for this chapter are available at www.mhhe.com/bkm.

5.1 RATES OF RETURN

A key measure of investors' success is the rate at which their funds have grown during the investment period. The total **holding-period return (HPR)** of a share of stock depends on the increase (or decrease) in the price of the share over the investment period as well as on any dividend income the share has provided. The rate of return is defined as dollars earned over the investment period (price appreciation as well as dividends) per dollar invested:

holding-period return (HPR)

Rate of return over a given investment period.

$$\text{HPR} = \frac{\text{Ending price} - \text{Beginning price} + \text{Cash dividend}}{\text{Beginning price}} \tag{5.1}$$

This definition of the HPR assumes that the dividend is paid at the end of the holding period. When dividends are received earlier, the definition ignores reinvestment income between the receipt of the dividend and the end of the holding period. The percentage return from dividends, cash dividends/beginning price, is called the *dividend yield,* and so the dividend yield plus the capital gains yield equals the HPR.

This definition of holding return is easy to modify for other types of investments. For example, the HPR on a bond would be calculated using the same formula, except that the bond's interest or coupon payments would take the place of the stock's dividend payments.

EXAMPLE 5.1

Holding-Period Return

Consider investing some of your money, now all invested in a bank account, in a stock market index fund. The price of a share in the fund is currently $100, and your time horizon is one year. You expect the cash dividend during the year to be $4, so your expected dividend yield is 4%.

Your HPR will depend on the price one year from now. Suppose your best guess is that it will be $110 per share. Then your *capital gain* will be $10, so your capital gains yield is $10/$100 = .10, or 10%. The total holding-period rate of return is the sum of the dividend yield plus the capital gains yield, 4% + 10% = 14%.

$$\text{HPR} = \frac{\$110 - \$100 + \$4}{\$100} = .14, \text{ or } 14\%$$

Measuring Investment Returns over Multiple Periods

The holding-period return is a simple and unambiguous measure of investment return over a single period. But often you will be interested in average returns over longer periods of time. For example, you might want to measure how well a mutual fund has performed over the preceding five-year period. In this case, return measurement is more ambiguous.

Consider a fund that starts with $1 million under management. It receives additional funds from new and existing shareholders and also redeems shares of existing shareholders so that net cash inflow can be positive or negative. The fund's quarterly results are as given in Table 5.1, with negative numbers in parentheses.

The numbers indicate that when the firm does well (i.e., achieves a high HPR), it attracts new funds; otherwise it may suffer a net outflow. For example, the 10% return in the first quarter by itself increased assets under management by .10 × $1 million = $100,000; it also elicited new investments of $100,000, thus bringing assets under management to $1.2 million

TABLE 5.1 Quarterly cash flows and rates of return of a mutual fund

	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
Assets under management at start of quarter ($ million)	1.0	1.2	2.0	0.8
Holding-period return (%)	10.0	25.0	(20.0)	20.0
Total assets before net inflows	1.1	1.5	1.6	0.96
Net inflow ($ million)*	0.1	0.5	(0.8)	0.6
Assets under management at end of quarter ($ million)	1.2	2.0	0.8	1.56

*New investment less redemptions and distributions, all assumed to occur at the end of each quarter.

by the end of the quarter. An even better HPR in the second quarter elicited a larger net inflow, and the second quarter ended with $2 million under management. However, HPR in the third quarter was negative, and net inflows were negative.

How would we characterize fund performance over the year, given that the fund experienced both cash inflows and outflows? There are several candidate measures of performance, each with its own advantages and shortcomings. These are the *arithmetic average*, the *geometric average*, and the *dollar-weighted return*. These measures may vary considerably, so it is important to understand their differences.

arithmetic average

The sum of returns in each period divided by the number of periods.

Arithmetic average The **arithmetic average** of the quarterly returns is just the sum of the quarterly returns divided by the number of quarters; in the above example: $(10 + 25 - 20 + 20)/4 = 8.75\%$. Since this statistic ignores compounding, it does not represent an equivalent, single quarterly rate for the year. However, without information beyond the historical sample, the arithmetic average is the best forecast of performance for the next quarter.

geometric average

The single per-period return that gives the same cumulative performance as the sequence of actual returns.

Geometric average The **geometric average** of the quarterly returns is equal to the single per-period return that would give the same cumulative performance as the sequence of actual returns. We calculate the geometric average by compounding the actual period-by-period returns and then finding the per-period rate that will compound to the same final value. In our example, the geometric average quarterly return, r_G, is defined by:

$$(1 + .10) \times (1 + .25) \times (1 - .20) \times (1 + .20) = (1 + r_G)^4$$

The left-hand side of this equation is the compounded year-end value of a $1 investment earning the four quarterly returns. The right-hand side is the compounded value of a $1 investment earning r_G *each* quarter. We solve for r_G:

$$r_G = [(1 + .10) \times (1 + .25) \times (1 - .20) \times (1 + .20)]^{1/4} - 1 = .0719, \text{ or } 7.19\% \quad \textbf{(5.2)}$$

The geometric return is also called a *time-weighted average return* because it ignores the quarter-to-quarter variation in funds under management. In fact, an investor will obtain a larger cumulative return when high returns are earned in periods when larger sums have been invested and low returns are earned when less money is at risk. In Table 5.1, the higher returns (25% and 20%) were achieved in quarters 2 and 4, when the fund managed $1,200,000 and $800,000, respectively. The lower returns (−20% and 10%) occurred when the fund managed $2,000,000 and $1,000,000, respectively. In this case, better returns were earned when *less* money was under management—an unfavorable combination.

Published data on past returns earned by mutual funds actually are *required* to be time-weighted returns. The rationale for this practice is that since the fund manager does not have full control over the amount of assets under management, we should not weight returns in one period more heavily than those in other periods when assessing "typical" past performance.

Dollar-weighted return To account for varying amounts under management, we treat the fund cash flows as we would a capital budgeting problem in corporate finance and compute the portfolio manager's internal rate of return (IRR). The initial value of $1 million and the net cash inflows are treated as the cash flows associated with an investment "project." The year-end "liquidation value" of the portfolio is the final cash flow of the project. In our example, the investor's net cash flows are as follows:

	Quarter				
	0	1	2	3	4
Net cash flow ($ million)	−1.0	−.1	−.5	.8	−.6 + 1.56 = .96

The entry for time 0 reflects the starting contribution of $1 million; the negative entries for times 1 and 2 are additional net inflows in those quarters, while the positive value for quarter 3 signifies a withdrawal of funds. Finally, the entry for time 4 represents the sum of the final (negative) cash inflow plus the value of the portfolio at the end of the fourth quarter. The latter is the value for which the portfolio could have been liquidated at year-end.

The **dollar-weighted average return** is the internal rate of return of the project, which is 3.38%. The IRR is the interest rate that sets the present value of the cash flows realized on the portfolio (including the $1.56 million for which the portfolio can be liquidated at the end of the year) equal to the initial cost of establishing the portfolio. It therefore is the interest rate that satisfies the following equation:

dollar-weighted average return
The internal rate of return on an investment.

$$0 = -1.0 + \frac{-.1}{1 + \text{IRR}} + \frac{-.5}{(1 + \text{IRR})^2} + \frac{.8}{(1 + \text{IRR})^3} + \frac{.96}{(1 + \text{IRR})^4} \quad \textbf{(5.3)}$$

The dollar-weighted return in this example is less than the time-weighted return of 7.19% because, as we noted, the portfolio returns were higher when less money was under management. The difference between the dollar- and time-weighted average return in this case is quite large.

CONCEPT check 5.1

A fund begins with $10 million and reports the following three-month results (with negative figures in parentheses):

	Month		
	1	2	3
Net inflows (end of month, $ million)	3	5	0
HPR (%)	2	8	(4)

Compute the arithmetic, time-weighted, and dollar-weighted average returns.

Conventions for Annualizing Rates of Return

We've seen that there are several ways to compute average rates of return. There also is some variation in how the mutual fund in our example might annualize its quarterly returns.

Returns on assets with regular cash flows, such as mortgages (with monthly payments) and bonds (with semiannual coupons), usually are quoted as annual percentage rates, or APRs, which annualize per-period rates using a simple interest approach, ignoring compound interest:

$$\text{APR} = \text{Per-period rate} \times \text{Periods per year}$$

However, because it ignores compounding, the APR does not equal the rate at which your invested funds actually grow. This is called the *effective annual rate,* or EAR. When there are n compounding periods in the year, we first recover the rate per period as APR/n and then compound that rate for the number of periods in a year. (For example, $n = 12$ for monthly payment mortgages and $n = 2$ for bonds making payments semiannually.)

$$1 + \text{EAR} = (1 + \text{Rate per period})^n = \left(1 + \frac{\text{APR}}{n}\right)^n \tag{5.4}$$

Since you can earn the APR each period, after one year (when n periods have passed), your cumulative return is $(1 + \text{APR}/n)^n$. Note that one needs to know the holding period when given an APR in order to convert it to an effective rate.

Rearranging Equation 5.4, we can also find APR given EAR:

$$\text{APR} = [(1 + \text{EAR})^{1/n} - 1] \times n$$

The EAR diverges by greater amounts from the APR as n becomes larger (we compound cash flows more frequently). In the limit, we can envision continuous compounding when n becomes extremely large in Equation 5.4. With continuous compounding, the relationship between the APR and EAR becomes

$$1 + \text{EAR} = e^{\text{APR}}$$

or, equivalently,

$$\text{APR} = \ln(1 + \text{EAR})$$

More generally, the EAR of any investment can be converted to an equivalent continuously compounded rate, r_{cc}, using the relationship

$$r_{cc} = \ln(1 + \text{EAR}) \tag{5.5}$$

We will return to continuous compounding later in the chapter.

EXAMPLE 5.2

Annualizing Treasury-Bill Returns

Suppose you buy a $10,000 face value Treasury bill maturing in one month for $9,900. On the bill's maturity date, you collect the face value. Since there are no other interest payments, the holding-period return for this one-month investment is

$$\text{HPR} = \frac{\text{Cash income} + \text{Price change}}{\text{Initial price}} = \frac{\$100}{\$9{,}900} = .0101 = 1.01\%$$

The APR on this investment is therefore $1.01\% \times 12 = 12.12\%$. The effective annual rate is higher:

$$1 + \text{EAR} = (1.0101)^{12} = 1.1282$$

which implies that EAR = .1282 = 12.82%.

A warning: Terminology can be loose. Occasionally, *annual percentage yield* or *APY* and even *APR* are used interchangeably with effective annual rate, and this can lead to confusion. To avoid error, you must be alert to context.

The difficulties in interpreting rates of return over time do not end here. Two thorny issues remain: the uncertainty surrounding the investment in question and the effect of inflation.

5.2 RISK AND RISK PREMIUMS

Any investment involves some degree of uncertainty about future holding-period returns, and in many cases that uncertainty is considerable. Sources of investment risk range from macroeconomic fluctuations, to the changing fortunes of various industries, to asset-specific unexpected developments. Analysis of these multiple sources of risk is presented in Part Four, "Security Analysis."

Scenario Analysis and Probability Distributions

When we attempt to quantify risk, we begin with the question: What HPRs are possible, and how likely are they? A good way to approach this question is to devise a list of possible economic outcomes, or *scenarios,* and specify both the likelihood (probability) of each scenario and the HPR the asset will realize in that scenario. Therefore, this approach is called **scenario analysis.** The list of possible HPRs with associated probabilities is the **probability distribution** of HPRs. Consider an investment in a broad portfolio of stocks, say, an index fund, which we will refer to as the "stock market." A very simple scenario analysis for the stock market (assuming only four possible scenarios) is illustrated in Spreadsheet 5.1.

scenario analysis

Process of devising a list of possible economic scenarios and specifying the likelihood of each one, as well as the HPR that will be realized in each case.

probability distribution

List of possible outcomes with associated probabilities.

The probability distribution lets us derive measurements for both the reward and the risk of the investment. The reward from the investment is its **expected return,** which you can think of as the average HPR you would earn if you were to repeat an investment in the asset many times. The expected return also is called the *mean of the distribution* of HPRs and often is referred to as the *mean return.*

expected return

The mean value of the distribution of HPR.

To compute the expected return from the data provided, we label scenarios by s and denote the HPR in each scenario as $r(s)$, with probability $p(s)$. The expected return, denoted $E(r)$, is then the weighted average of returns in all possible scenarios, $s = 1, \ldots, S$, with weights equal to the probability of that particular scenario.

$$E(r) = \sum_{s=1}^{S} p(s)\, r(s) \tag{5.6}$$

Each entry in column D of Spreadsheet 5.1 corresponds to one of the products in the summation in Equation 5.6. The value in cell D7, which is the sum of these products, is therefore the expected return. Therefore, $E(r) = 10\%$.

Because there is risk to the investment, the actual return may be (a lot) more or less than 10%. If a "boom" materializes, the return will be better, 30%, but in a severe recession the return will be a disappointing −37%. How can we quantify this uncertainty?

The "surprise" return in any scenario is the difference between the actual return and the expected return. For example, in a boom (scenario 4) the surprise is $r(4) - E(r) = 30\% - 10\% = 20\%$. In a severe recession (scenario 1), the surprise is $r(1) - E(r) = -37\% - 10\% = -47\%$.

SPREADSHEET 5.1

Scenario analysis for the stock market

	A	B	C	D	E	F
1				Column B x	Deviation from	Column B x
2	Scenario	Probability	HPR (%)	Column C	Mean Return	Squared Deviation
3	1. Severe recession	.05	−37	−1.85	−47.00	110.45
4	2. Mild recession	.25	−11	−2.75	−21.00	110.25
5	3. Normal growth	.40	14	5.60	4.00	6.40
6	4. Boom	.30	30	9.00	20.00	120.00
7	Column sums:		Expected return =	10.00	Variance =	347.10
8			Square root of variance = Standard deviation (%) =			18.63

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variance

The expected value of the squared deviation from the mean.

Uncertainty surrounding the investment is a function of both the magnitudes and the probabilities of the possible surprises. To summarize risk with a single number, we define the **variance** as the expected value of the *squared* deviation from the mean (the expected squared "surprise" across scenarios).

$$\text{Var}(r) \equiv \sigma^2 = \sum_{s=1}^{S} p(s)[r(s) - E(r)]^2 \tag{5.7}$$

We square the deviations because negative deviations would offset positive deviations otherwise, with the result that the expected deviation from the mean return would necessarily be zero. Squared deviations are necessarily positive. Squaring (a nonlinear -transformation) exaggerates large (positive or negative) deviations and deemphasizes small deviations.

Another result of squaring deviations is that the variance has a dimension of percent squared. To give the measure of risk the same dimension as expected return (%), we use the **standard deviation**, defined as the square root of the variance:

standard deviation

The square root of the variance.

$$SD(r) \equiv \sigma = \sqrt{\text{Var}(r)} \tag{5.8}$$

EXAMPLE 5.3

Expected Return and Standard Deviation

Applying Equation 5.6 to the data in Spreadsheet 5.1, we find that the expected rate of return on the stock index fund is

$$E(r) = .05 \times (-37) + .25 \times (-11) + .40 \times 14 + .30 \times 30 = 10\%$$

We use Equation 5.7 to find the variance. First we take the difference between the holding-period return in each scenario and the mean return, then we square that difference, and finally we multiply by the probability of each scenario. The sum of the probability-weighted squared deviations is the variance.

$$\sigma^2 = .05(-37 - 10)^2 + .25(-11 - 10)^2 + .40(14 - 10)^2 + .30(30 - 10)^2 = 347.10$$

and so the standard deviation is

$$\sigma = \sqrt{347.10} = 18.63\%$$

Column F of Spreadsheet 5.1 replicates these calculations. Each entry in that column is the squared deviation from the mean multiplied by the probability of that scenario. The sum of the probability-weighted squared deviations that appears in cell F7 is the variance, and the square root of that value is the standard deviation (in cell F8).

The Normal Distribution

The normal distribution is central to the theory *and* practice of investments. Its familiar bell-shaped plot is symmetric, with identical values for all three standard measures of "typical" results: the mean (the expected value discussed earlier), the median (the value above and below which we expect 50% of the observations), and the mode (the most likely value).

Figure 5.1 illustrates a normal distribution with a mean of 10% and standard deviation (SD) of 20%. Notice that the probabilities are highest for outcomes near the mean and are significantly lower for outcomes far from the mean. But what do we mean by an outcome "far" from the mean? A return 15% below the mean would hardly be noteworthy if typical volatility were high, for example, if the standard deviation of returns were 20%, but that same outcome would be highly unusual if the standard deviation were only 5%. For this reason, it is often useful to think about deviations from the mean in terms of how many standard deviations they represent. If the standard deviation is 20%, that 15% negative surprise would be only three-fourths of a standard deviation, unfortunate perhaps but not uncommon. But if the standard deviation were only 5%, a 15% deviation would be a "three-sigma event," and quite rare.

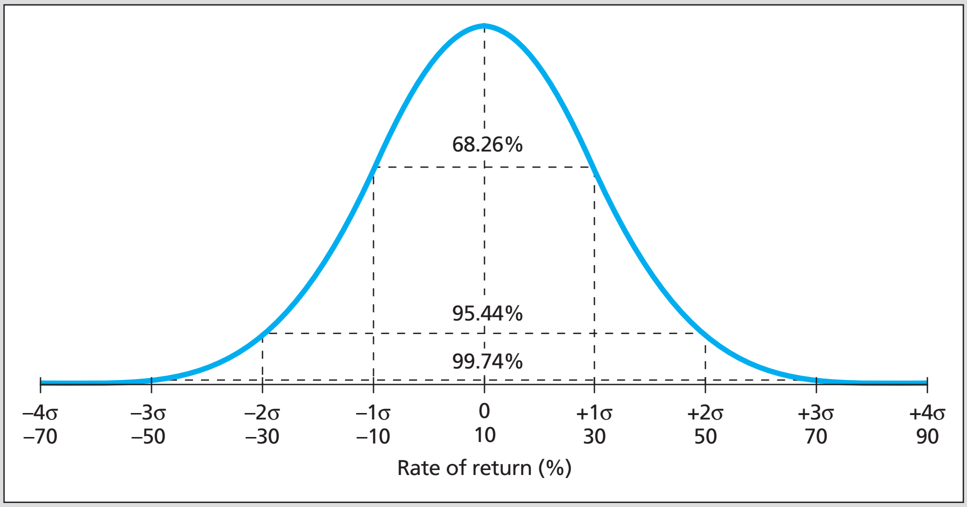

FIGURE 5.1

The normal distribution with mean return 10% and standard deviation 20%

We can transform any normally distributed return, r_i, into a "standard deviation score," by first subtracting the mean return (to obtain distance from the mean or return "surprise") and then dividing by the standard deviation (which enables us to measure distance from the mean in units of standard deviations).

$$sr_i = \frac{r_i - E(r_i)}{\sigma_i} \tag{5.9A}$$

This standardized return, which we have denoted sr_i, is normally distributed with a mean of zero and a standard deviation of 1. We therefore say that sr_i is a "standard normal" variable.

Conversely, we can start with a standard normal return, sr_i, and recover the original return by multiplying by the standard deviation and adding back the mean return:

$$r_i = E(r_i) + sr_i \times \sigma_i \tag{5.9B}$$

In fact, this is how we drew Figure 5.1. Start with a standard normal (mean = 0 and SD = 1); next, multiply the distance from the mean by the assumed standard deviation of 20%; finally, recenter the mean away from zero by adding 10%. This gives us a normal variable with mean 10% and standard deviation 20%.

Figure 5.1 shows that when returns are normally distributed, roughly two-thirds (more precisely, 68.26%) of the observations fall within one standard deviation of the mean, that is, the probability that any observation in a sample of returns would be no more than one standard deviation away from the mean is 68.26%. Deviations from the mean of more than two SDs are even rarer: 95.44% of the observations are expected to lie within this range. Finally, only 2.6 out of 1,000 observations are expected to deviate from the mean by three or more SDs.

Two special properties of the normal distribution lead to critical simplifications of investment management when returns are normally distributed:

1. The return on a portfolio comprising two or more assets whose returns are normally distributed also will be normally distributed.
2. The normal distribution is completely described by its mean and standard deviation. No other statistic is needed to learn about the behavior of normally distributed returns.

These two properties in turn imply this far-reaching conclusion:

3. The standard deviation is the appropriate measure of risk for a portfolio of assets with normally distributed returns. In this case, no other statistic can improve the risk assessment conveyed by the standard deviation of a portfolio.

value at risk (VaR)

Measure of downside risk. The worst loss that will be suffered with a given probability, often 5%.

Suppose you worry about large investment losses in worst-case scenarios for your portfolio. You might ask: "How much would I lose in a fairly extreme outcome, for example, if my return were in the fifth percentile of the distribution?" You can expect your investment experience to be worse than this value only 5% of the time and better than this value 95% of the time. In investments parlance, this cutoff is called the **value at risk** (denoted by **VaR,** to distinguish it from Var, the common notation for variance). A loss-averse investor might desire to limit portfolio VaR, that is, limit the loss corresponding to a probability of 5%.

For normally distributed returns, VaR can be derived from the mean and standard deviation of the distribution. We calculate it using Excel's standard normal function =NORMSINV(0.05). This function computes the fifth percentile of a normal distribution with a mean of zero and a variance of 1, which turns out to be −1.64485. In other words, a value that is 1.64485 standard deviations below the mean would correspond to a VaR of 5%, that is, to the fifth percentile of the distribution.

$$\text{VaR} = E(r) + (-1.64485)\sigma \qquad \textbf{(5.10)}$$

We can obtain this value directly from Excel's nonstandard normal function =NORMINV (.05, $E(r)$, σ).

When faced with a sample of actual returns that may not be normally distributed, we must estimate the VaR directly. The 5% VaR is the fifth-percentile rate of return. For a sample of 100 returns this is straightforward: If the rates are ordered from high to low, count the fifth observation from the bottom.

Calculating the 5% VaR for samples where 5% of the observations don't make an integer requires interpolation. Suppose we have 72 monthly observations so that 5% of the sample is 3.6 observations. We approximate the VaR by going .6 of the distance from the third to the fourth rate from the bottom. Suppose these rates are −42% and −37%. The interpolated value for VaR is then −42 + .6 (42 − 37) = −39%.

In practice, analysts sometimes compare the historical sample VaR to the VaR implied by a normal distribution with the same mean and SD as the sample rates. The difference between these VaR values indicates the deviation of the observed rates from normality.

CONCEPT check 5.2

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a. The current value of a stock portfolio is $23 million. A financial analyst summarizes the uncertainty about next year's holding-period return using the scenario analysis in the following spreadsheet. What are the annual holding-period returns of the portfolio in each scenario? Calculate the expected holding-period return, the standard deviation of returns, and the 5% VaR. What is the VaR of a portfolio with normally distributed returns with the same mean and standard deviation as this stock? The spreadsheet is available at the Online Learning Center (go to **www.mhhe.com/bkm,** and link to the Chapter 5 material).

	A	B	C	D	E
1	**Business Conditions**	**Scenario,** *s*	**Probability,** *p*	**End-of-Year Value ($ million)**	**Annual Dividend ($ million)**
2	High growth	1	.30	35	4.40
3	Normal growth	2	.45	27	4.00
4	No growth	3	.20	15	4.00
5	Recession	4	.05	8	2.00

b. Suppose that the worst three rates of return in a sample of 36 monthly observations are 17%, −5%, and 2%. Estimate the 5% VaR.

Normality over Time

The fact that portfolios of normally distributed assets also are normally distributed greatly simplifies analysis of risk because standard deviation, a simple-to-calculate number, is the appropriate risk measure for normally distributed portfolios.

But even if returns are normal for any particular time period, will they also be normal for other holding periods? Suppose that monthly rates are normally distributed with a mean of 1%. The expected annual rate of return is then $1.01^{12} - 1$. Can this annual rate, which is a nonlinear function of the monthly return, also be normally distributed? Unfortunately, the answer is no. Similarly, why would monthly rates be normally distributed when a monthly rate is $(1 + \text{daily rate})^{30} - 1$? Indeed, they are not. So, do we really get to enjoy the simplifications offered by the normal distribution?

Despite these potential complications, when returns over very short time periods (e.g., an hour or even a day) are normally distributed, then HPRs up to holding periods as long as a month will be *nearly* normal, and we can treat them as if they are normal. Longer-term, for example, annual, HPRs will indeed deviate more substantially from normality, but even here, if we expressed those HPRs as continuously compounded rates, they will remain normally distributed. The practical implication is this: Use continuously compounded rates in all work where normality plays a crucial role, as in estimating VaR from actual returns.

To see why relatively short-term rates are still nearly normal, consider these calculations: Suppose that rates are normally distributed over an infinitesimally short period. Beyond that, compounding, strictly speaking, takes them adrift from normality. But those deviations will be very small. Suppose that on an annual basis the continuously compounded rate of return has a mean of .12 (i.e., 12%; we must work with decimals when using continuously compounded rates). Equivalently, the effective annual rate has an expected value of $E(r) = e^{.12} - 1 = 0.1275$. So the difference between the effective annual rate and continuously compounded rate is meaningful, .75%, or 75 basis points. On a monthly basis, however, the equivalent continuously compounded expected holding-period return is 1%, implying an expected monthly effective rate of $e^{.01} - 1 = .01005$. The difference between effective annual and continuously compounded rates here is trivial, only one-half of a basis point. For shorter periods the difference will be smaller still. So, when continuously compounded rates are exactly normal, rates over periods up to a month are so close to those continuously compounded values that we can treat them as if they are effectively normal.

Another important aspect of (normal) continuously compounded rates over time is this: Just as the total continuously compounded rate and the risk premium grow in direct proportion to the length of the investment period, so does the variance (*not* the standard deviation) of the total continuously compounded return and the risk premium. Hence, for an asset with annual continuously compounded SD of .20 (20%), the variance is .04, and the quarterly variance will be .01, implying a quarterly standard deviation of .10, or 10%. (Verify that the monthly standard deviation is 5.77%.) Because variance grows in direct proportion to time, the standard deviation grows in proportion to the square root of time.

Deviation from Normality and Value at Risk

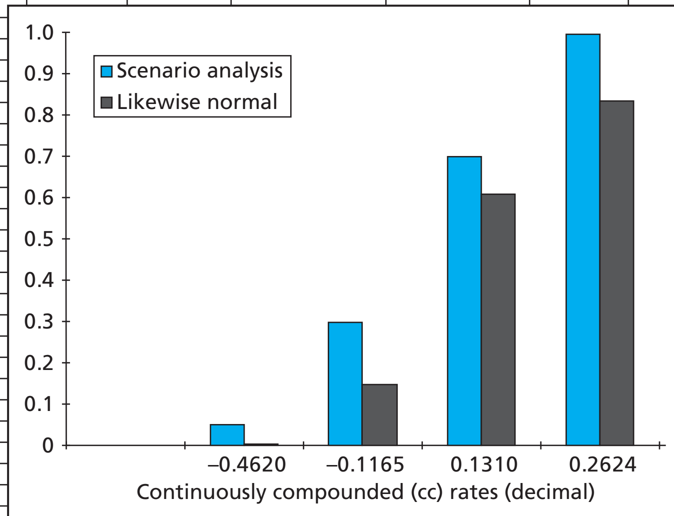
The scenario analysis laid out in Spreadsheet 5.1 offers insight about the issue of normality in practice. While a four-scenario analysis is quite simplistic, even this simple example can nevertheless shed light on how practical analysis might take shape.[1]

How can the returns specified in the scenario analysis in Spreadsheet 5.1 be judged against the normal distribution? (As prescribed above, we first convert the effective rates specified

[1]You may wonder: Is the fact that the probability of the worst-case scenario is .05 in Spreadsheet 5.1 just a lucky happenstance given our interest in the 5% VaR? The answer is no. Given investor concern about VaR, it is fair (in fact, necessary) to demand of analysts that their scenario analysis explicitly take a stand on the rate of return corresponding to the probability of the VaR of interest, here .05.

FIGURE 5.2

Comparing scenario analysis (from Spreadsheet 5.1) to a normal distribution with the same mean and standard deviation



	A	B	C	D	E	F	G
27							Cumulative
28				Scenario Analysis		Squared	Corresponding
29		Probability	Cumulative	Effective Rate	cc Rate (decimal)	Deviations	Normal
30		0.05	0.05	–0.37	−0.4620	0.2926	0.0020
31		0.25	0.3	–0.11	−0.1165	0.0382	0.1494
32		0.40	0.7	0.14	0.1310	0.0027	0.6092
33		0.30	1	0.30	0.2624	0.0337	0.8354
34			Mean	0.10	0.0789		
35					SD	0.1881	
36							
37		VaR Scenario Analysis		–0.37	−0.4620		
38		VaR corresponding Normal		–0.2058	−0.2304		
39							

in each scenario to their equivalent continuously compounded rates using Equation 5.5.) Obviously, it is naive to believe that this simple analysis includes all possible rates. But while we cannot explicitly pin down probabilities of rates other than those given in the table, we can get a good sense of the entire spectrum of potential outcomes by examining the distribution of the assumed scenario rates, as well as their mean and standard deviation.

Figure 5.2 shows the known points from the cumulative distribution of the scenario analysis next to the corresponding points from a "likewise normal distribution" (a normal distribution with the same mean and standard deviation, SD). Below the graph, we see a table of the actual distributions. The mean in cell D34 is computed from the formula =SUMPRODUCT($B$30:$B$33, D30:D33), where the probability cells B30:B33 are fixed to allow copying to the right.[2] Similarly, the SD in cell F35 is computed from =SUMPRODUCT(B30:B33, F30:F33)^0.5. The 5% VaR of the normal distribution in cell E38 is computed from =NORMINV(0.05, E34, F35).

VaR values appear in cells D37 and D38. The VaR from the scenario analysis, −37%, is far worse than the VaR derived from the corresponding normal distribution, −20.58%. This immediately suggests that the scenario analysis entails a higher probability of extreme losses than would be consistent with a normal distribution. On the other hand, the normal distribution allows for the possibility of extremely large returns, beyond the maximum return of 30% envisioned in the scenario analysis. We conclude that the scenario analysis has a distribution that is skewed to the left compared to the normal. It has a longer left tail (larger losses) and a

[2]The Excel function SUMPRODUCT multiplies each term in the first column specified (in this case, the probabilities in column B) with the corresponding terms in the second column specified (in this case, the returns in column D), and then adds up those products. This gives us the expected rate of return across scenarios.

shorter right tail (smaller gains). It makes up for this negative attribute with a larger probability of positive, but not extremely large, gains (14% and 30%).

This example shows when and why the VaR is an important statistic. When returns are normal, knowing just the mean and standard deviation allows us to fully describe the entire distribution. In that case, we do not need to estimate VaR explicitly—we can calculate it exactly from the properties of the normal distribution. But when returns are not normal, the VaR conveys important additional information beyond mean and standard deviation. It gives us additional insight into the shape of the distribution, for example, skewness or risk of extreme negative outcomes.[3]

Because risk is largely driven by the likelihood of extreme negative returns, two additional statistics are used to indicate whether a portfolio's probability distribution differs significantly from normality with respect to potential extreme values. The first is **kurtosis,** which compares the frequency of extreme values to that of the normal distribution. The kurtosis of the normal distribution is zero, so positive values indicate higher frequency of extreme values than this benchmark. A negative value suggests that extreme values are less frequent than with the normal distribution. Kurtosis sometimes is called "fat tail risk," as plots of probability distributions with higher likelihood of extreme events will be higher than the normal distribution at both ends or "tails" of the distribution; in other words, the distributions exhibit "fat tails." Similarly, exposure to extreme events is often called *tail risk,* because these are outcomes in the far reaches or "tail" of the probability distribution.

kurtosis

Measure of the fatness of the tails of a probability distribution relative to that of a normal distribution. Indicates likelihood of extreme outcomes.

The second statistic is the **skew,** which measures the asymmetry of the distribution. Skew takes on a value of zero if, like the normal, the distribution is symmetric. Negative skew suggests that extreme negative values are more frequent than extreme positive ones. Nonzero values for kurtosis and skew indicate that special attention should be paid to the VaR, in addition to the use of standard deviation as measure of portfolio risk.

skew

Measure of the asymmetry of a probability distribution.

Using Time Series of Return

Scenario analysis postulates a probability distribution of future returns. But where do the probabilities and rates of return come from? In large part, they come from observing a sample history of returns. Suppose we observe a 10-year time series of monthly returns on a diversified portfolio of stocks. We can interpret each of the 120 observations as one potential "scenario" offered to us by history. Adding judgment to this history, we can develop a scenario analysis of future returns.

As a first step, we estimate the expected return, standard deviation, and VaR for the sample history. We assume that each of the 120 returns represents one independent draw from the historical probability distribution. Hence, each return is assigned an equal probability of $1/120 = .0083$. When you use a fixed probability in Equation 5.6, you obtain the simple average of the observations, often used to estimate the mean return.

As mentioned earlier, the same principle applies to the VaR. We sort the returns from high to low. The bottom six observations comprise the lower 5% of the distribution. The sixth observation from the bottom is just at the fifth percentile, and so would be the 5% VaR for the historical sample.

Estimating variance from Equation 5.7 requires a minor correction. Remember that variance is the expected value of squared deviations from the mean return. But the true mean is not observable; we *estimate* it using the sample average. If we compute variance as the average of squared deviations from the sample average, we will slightly underestimate it because this procedure ignores the fact that the average necessarily includes some estimation error. The necessary correction turns out to be simple: With a sample of n observations, we divide the sum of the squared deviations from the sample average by $n - 1$ instead of n. Thus, the estimates of variance and standard deviation from a time series of returns, r_t, are

$$\mathrm{Var}(r_t) = \frac{1}{n-1}\Sigma(r_t - \bar{r}_t)^2 \quad \mathrm{SD}(r_t) = \sqrt{\mathrm{Var}(r_t)} \quad \bar{r}_t = \frac{1}{n}\Sigma r_t \tag{5.11}$$

[3]The financial crisis of 2008–2009 demonstrated that bank portfolio returns are far from normally distributed, with exposure to unlikely but catastrophic returns in extreme market meltdowns. The international Basel accord on bank regulation requires banks to monitor portfolio VaR to better control risk.

EXAMPLE 5.4

Historical Means and Standard Deviations

To illustrate how to calculate average returns and standard deviations from historical data, let's compute these statistics for the returns on the S&P 500 portfolio using five years of data from the following table. The average return over this period is 16.7%, computed by dividing the sum of column (1), below, by the number of observations. In column (2), we take the deviation of each year's return from the 16.7% average return. In column (3), we calculate the squared deviation. The variance is, from Equation 5.11, the sum of the five squared deviations divided by (5 − 1). The standard deviation is the square root of the variance. If you input the column of rates into a spreadsheet, the AVERAGE and STDEV functions will give you the statistics directly.

Year	(1) Rate of Return	(2) Deviation from Average Return	(3) Squared Deviation
1	16.9%	0.2%	0.0
2	31.3	14.6	213.2
3	−3.2	−19.9	396.0
4	30.7	14.0	196.0
5	7.7	−9.0	81.0
Total	83.4%		886.2

$$\text{Average rate of return} = 83.4/5 = 16.7$$

$$\text{Variance} = \frac{1}{5-1} \times 886.2 = 221.6$$

$$\text{Standard deviation} = \sqrt{221.6} = 14.9\%$$

Risk Premiums and Risk Aversion

How much, if anything, would you invest in the index stock fund described in Spreadsheet 5.1? First, you must ask how much of an expected reward is offered to compensate for the risk involved in stocks.

We measure the "reward" as the difference between the expected HPR on the index fund and the **risk-free rate,** the rate you can earn on Treasury bills. We call this difference the **risk premium.** If the risk-free rate in the example is 4% per year, and the expected index fund return is 10%, then the risk premium on stocks is 6% per year.

risk-free rate

The rate of return that can be earned with certainty.

risk premium

An expected return in excess of that on risk-free securities.

The rate of return on Treasury bills also varies over time. However, we know the rate of return on T-bills *at the beginning* of the holding period, while we can't know the return we will earn on risky assets until the end of the holding period. Therefore, to study the risk premium on risky assets we compile a series of **excess returns,** that is, returns in excess of the T-bill rate in each period. A reasonable forecast of an asset's risk premium is the average of its historical excess returns.

excess return

Rate of return in excess of the risk-free rate.

The degree to which investors are willing to commit funds to stocks depends on **risk aversion.** It seems obvious that investors are risk averse in the sense that, without a positive risk premium, they would not be willing to invest in stocks. In theory then, there must always be a positive risk premium on all risky assets in order to induce risk-averse investors to hold the existing supply of these assets.

risk aversion

Reluctance to accept risk.

A positive risk premium distinguishes speculation from gambling. Investors taking on risk to earn a risk premium are speculating. Speculation is undertaken *despite* the risk because of a favorable risk-return trade-off. In contrast, gambling is the assumption of risk for no purpose beyond the enjoyment of the risk itself. Gamblers take on risk even without a risk premium.[4]

[4]Sometimes a gamble might *seem* like speculation to the participants. If two investors differ in their forecasts of the future, they might take opposite positions in a security, and both may have an expectation of earning a positive risk premium. In such cases, only one party can, in fact, be correct.

To determine an investor's optimal portfolio strategy, we need to quantify his degree of risk aversion. To do so, we look at how he is willing to trade off risk against expected return. An obvious benchmark is the risk-free asset, which has neither volatility nor risk premium: It pays a certain rate of return, r_f. Risk-averse investors will not hold risky assets without the prospect of earning some premium above the risk-free rate. An individual's degree of risk aversion can be inferred by contrasting the risk premium on the investor's entire wealth (the complete portfolio, C), $E(r_C) - r_f$, against the variance of the portfolio return, σ_C^2. Notice that the risk premium and the level of risk that can be attributed to *individual* assets in the complete wealth portfolio are of no concern to the investor here. All that counts is the bottom line: *complete portfolio* risk premium versus *complete portfolio* risk.

A natural way to proceed is to measure risk aversion by the risk premium necessary to compensate an investor for investing his entire wealth in a portfolio, say Q, with a variance, σ_Q^2. This approach relies on the principle of *revealed preference:* We infer preferences from the choices individuals are willing to make. We will measure risk aversion by the risk premium offered by the complete portfolio per unit of variance. This ratio measures the compensation that an investor has apparently required (per unit of variance) to be induced to hold this portfolio. For example, if we were to observe that the entire wealth of an investor is held in a portfolio with annual risk premium of .10 (10%) and variance of .0256 (SD = 16%), we would infer this investor's degree of risk aversion as:

$$A = \frac{E(r_Q) - r_f}{\sigma_Q^2} = \frac{0.10}{0.0256} = 3.91 \tag{5.12}$$

We call the ratio of a portfolio's risk premium to its variance the **price of risk.**[5] Later in the section, we turn the question around and ask how an investor with a given degree of risk aversion, say, $A = 3.91$, should allocate wealth between the risky and risk-free assets.

price of risk

The ratio of portfolio risk premium to variance.

To get an idea of the level of the risk aversion exhibited by investors in U.S. capital markets, we can look at a representative portfolio held by these investors. Assume that all short-term borrowing offsets lending; that is, average borrowing/lending is zero. In that case, the average investor holds a complete portfolio represented by a stock-market index;[6] call it M. A common proxy for the market index is the S&P 500 Index. Using a long-term series of historical returns on the S&P 500 to estimate investors' expectations about mean return and variance, we can recast Equation 5.12 with these stock market data to obtain an estimate of average risk aversion:

$$\bar{A} = \frac{\text{Average}(r_M) - r_f}{\text{Sample } \sigma_M^2} \approx \frac{0.08}{0.04} = 2 \tag{5.13}$$

The price of risk of the market index portfolio, which reflects the risk aversion of the average investor, is sometimes called the *market price of risk.* Conventional wisdom holds that plausible estimates for the value of A lie in the range of 1.5−4. (Take a look at average excess returns and SD of the stock portfolios in Table 5.2, and compute the risk aversion implied by their histories to investors that invested in them their entire wealth.)

The Sharpe (Reward-to-Volatility) Ratio

Risk aversion implies that investors will accept a lower reward (as measured by their portfolio risk premium) in exchange for a sufficient reduction in the standard deviation. A statistic

[5]Notice that when we use variance rather than the SD, the price of risk of a portfolio does not depend on the holding period. The reason is that variance is proportional to the holding period. Since portfolio return and risk premium also are proportional to the holding period, the portfolio pays the same price of risk for any holding period.

[6]In practice, a broad market index such as the S&P 500 often is taken as representative of the entire market.

TABLE 5.2 Annual rate-of-return statistics for diversified portfolios for 1926–2010 and three subperiods (%)

	World Portfolio		U.S. Market		
	Equity Return in U.S. Dollars	**Bond Return in U.S. Dollars**	**Small Stocks**	**Large Stocks**	**Long-Term T-Bonds**
Total Return—Geometric Average					
1926–2010	9.21	5.42	11.80	9.62	5.12
1926–1955	8.31	2.54	11.32	9.66	3.46
1956–1985	10.28	5.94	13.81	9.52	4.64
1986–2010	9.00	8.34	9.99	9.71	7.74
Total Real Return—Geometric Average					
1926–2010	6.03	2.35	8.54	6.43	2.06
1926–1955	6.86	1.16	9.82	8.18	2.07
1956–1985	5.23	1.09	8.60	4.51	−0.15
1986–2010	5.99	5.36	6.96	6.68	4.77
Excess Return Statistics					
Arithmetic average					
1926–2010	7.22	2.09	13.91	8.00	1.76
1926–1955	9.30	1.75	20.02	11.67	2.43
1956–1985	5.55	0.38	12.18	5.01	−0.87
1986–2010	6.74	4.54	8.66	7.19	4.11
Standard deviation					
1926–2010	18.98	8.50	37.56	20.70	7.93
1926–1955	21.50	8.10	49.25	25.40	4.12
1956–1985	16.33	8.42	32.31	17.58	8.29
1986–2010	19.27	8.81	25.82	17.83	10.07
Minimum (lowest excess return)					
1926–2010	−41.97	−18.50	−55.34	−46.65	−13.43
1926–1955	−41.03	−13.86	−55.34	−46.65	−6.40
1956–1985	−32.49	−18.50	−45.26	−34.41	−13.09
1986–2010	−41.97	−11.15	−41.47	−38.44	−13.43
Maximum (highest excess return)					
1926–2010	70.51	28.96	152.88	54.26	26.07
1926–1955	70.51	28.96	152.88	54.26	10.94
1956–1985	35.25	26.40	99.94	42.25	24.96
1986–2010	36.64	24.40	73.73	32.11	26.07
Deviation from the Normal Distribution*					
Kurtosis					
1926–2010	1.49	1.01	0.65	1.05	0.24
1926–1955	1.88	3.05	0.03	0.97	−0.24
1956–1985	0.25	1.52	−0.08	0.04	0.99
1986–2010	1.85	−0.31	0.53	1.93	−0.45
Skew					
1926–2010	−0.83	0.44	−0.40	−0.86	0.16
1926–1955	−0.67	0.64	−0.49	−1.01	−0.20
1956–1985	−0.61	0.44	−0.31	−0.52	0.79
1986–2010	−1.36	0.26	−0.45	−1.30	−0.26
Performance Statistics					
Sharpe ratio					
1926–2010	0.38	0.25	0.37	0.39	0.22
1926–1955	0.43	0.22	0.41	0.46	0.59

(continued)

TABLE 5.2 (concluded)

	World Portfolio		U.S. Market		
	Equity Return in U.S. Dollars	**Bond Return in U.S. Dollars**	**Small Stocks**	**Large Stocks**	**Long-Term T-Bonds**
1956–1985	0.34	0.05	0.38	0.28	−0.11
1986–2010	0.35	0.51	0.34	0.40	0.41
VaR*					
1926–2010	−27.41	−10.81	−65.13	−36.86	−11.69
1926–1955	−40.04	−14.55	−78.60	−53.43	−5.48
1956–1985	−29.08	−13.53	−49.53	−30.51	−12.46
1986–2010	−46.35	−10.25	−49.16	−42.28	−13.85
Difference of actual VaR from VaR of a Normal distribution with same mean and SD					
1926–2010	−2.62	0.34	−18.22	−9.40	−0.99
1926–1955	−13.58	−3.32	−16.51	−20.34	−1.22
1956–1985	−8.19	−1.15	−10.38	−6.89	1.16
1986–2010	−18.66	−1.03	−15.33	−18.26	−1.83

*Applied to continuously compounded (cc) excess returns (= cc total return − cc T-bill rates).

Source: Inflation data: BLS; T-bills and U.S. small stocks: Fama and French, **http://mba.tuck.dart mouth.edu/pages/faculty/ken.french/data_library.html;** Large U.S. stocks: S&P500; Long-term U.S. government bonds: 1926–2003 return on 20-Year U.S. Treasury bonds, and 2004–2008 Lehman Brothers long-term Treasury index; World portfolio of large stocks: Datastream; World portfolio of Treasury bonds: 1926–2003 Dimson, Elroy, and Marsh, and 2004–2008 Datastream.

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commonly used to rank portfolios in terms of this risk-return trade-off is the **Sharpe (or reward-to-volatility) ratio,** defined as

Sharpe (or reward-to-volatility) ratio
Ratio of portfolio risk premium to standard deviation.

$$S = \frac{\text{Portfolio risk premium}}{\text{Standard deviation of portfolio excess return}} = \frac{E(r_P) - r_f}{\sigma_P} \tag{5.14}$$

A risk-free asset would have a risk premium of zero and a standard deviation of zero. Therefore, the reward-to-volatility ratio of a risky portfolio quantifies the incremental reward (the increase in risk premium) for each increase of 1% in the portfolio standard deviation. For example, the Sharpe ratio of a portfolio with an annual risk premium of 8% and standard deviation of 20% is 8/20 = 0.4. A higher Sharpe ratio indicates a better reward per unit of volatility, in other words, a more efficient portfolio. Portfolio analysis in terms of mean and standard deviation (or variance) of excess returns is called **mean-variance analysis.**

mean-variance analysis
Ranking portfolios by their Sharpe ratios.

A warning: We will see in the next chapter that while standard deviation and VaR of returns are useful risk measures for diversified portfolios, these are not useful ways to think about the risk of individual securities. Therefore, the Sharpe ratio is a valid statistic only for ranking portfolios; it is *not* valid for individual assets. For now, therefore, let's examine the historical reward-to-volatility ratios of broadly diversified portfolios that reflect the performance of some important asset classes.

CONCEPT check 5.3

a. A respected analyst forecasts that the return of the S&P 500 Index portfolio over the coming year will be 10%. The one-year T-bill rate is 5%. Examination of recent returns of the S&P 500 Index suggests that the standard deviation of returns will be 18%. What does this information suggest about the degree of risk aversion of the average investor, assuming that the average portfolio resembles the S&P 500?

b. What is the Sharpe ratio of the portfolio in (*a*)?

5.3 THE HISTORICAL RECORD

World and U.S. Risky Stock and Bond Portfolios

We begin our examination of risk with an analysis of a long sample of return history (85 years) for five risky asset classes. These include three well-diversified stock portfolios—world large stocks, U.S. large stocks, and U.S. small stocks—as well as two long-term bond portfolios—world and U.S. Treasury bonds. The 85 annual observations for each of the five time series of returns span the period 1926–2010.

Until 1969, the "World Portfolio" of stocks was constructed from a diversified sample of large capitalization stocks of 16 developed countries weighted in proportion to the relative size of gross domestic product. Since 1970 this portfolio has been diversified across 24 developed countries (almost 6,000 stocks) with weights determined by the relative capitalization of each market. "Large Stocks" is the Standard & Poor's market value–weighted portfolio of 500 U.S. common stocks selected from the largest market capitalization stocks. "Small U.S. Stocks" are the smallest 20% of all stocks trading on the NYSE, NASDAQ, and Amex (currently almost 1,000 stocks).

The World Portfolio of bonds was constructed from the same set of countries as the World Portfolio of stocks, using long-term bonds from each country. Until 1996, "Long-Term T-Bonds" were represented by U.S. government bonds with at least a 20-year maturity and approximately current-level coupon rate.[7] Since 1996, this bond series has been measured by the Barclay's (formerly the Lehman Brothers) Long-Term Treasury Bond Index.

Look first at Figure 5.3, which shows histograms of total (continuously compounded) returns of the five risky portfolios and of Treasury bills. Notice the hierarchy of risk: Small stocks are the most risky, followed by large stocks and then long-term bonds. At the same time, the higher average return offered by riskier assets is evident, consistent with investor risk aversion. T-bill returns are by far the least volatile. In fact, despite the variability in their returns, bills are actually riskless, since you know the return you will earn at the beginning of the holding period. The small dispersion in these returns reflects the variation in interest rates over time.

Figure 5.4 provides another view of the hierarchy of risk. Here we plot the year-by-year returns on U.S. large stocks, long-term Treasury bonds, and T-bills. Risk is reflected by wider swings of returns from year to year.

Table 5.2 presents statistics of the return history of the five portfolios over the full 85-year period, 1926–2010, as well as for three subperiods.[8] The first 30-year subperiod, 1926–1955, includes the Great Depression (1929–1939), World War II, the postwar boom, and a subsequent recession. The second subperiod (1956–1985) includes four recessions (1957–1958, 1960–1961, 1973–1975, and 1980–1982) and a period of "stagflation" (poor growth combined with high inflation (1974–1980). Finally, the most recent 25-year subperiod (1986–2010) included two recessions (1990–1991, 2001–2003) bracketing the so-called high-tech bubble of the 1990s, and a severe recession that started in December 2007 and is estimated to have ended in the second half of 2009. Let us compare capital asset returns in these three subperiods.

We start with the geometric averages of total returns in the top panel of the table. This is the equivalent, constant annual rate of return that an investor would have earned over the period. To appreciate these rates, you must consider the power of compounding. Think about an investor who might have chosen to invest in either large U.S. stocks or U.S. long-term T-bonds at the end of 1985. The geometric averages for 1986–2010 tell us that over the most recent 25-year period, the stock portfolio would have turned \$1 into $\$1 \times 1.0971^{25} = \$10.13$, while the same investment in the T-bond portfolio would have brought in $\$1 \times 1.0774^{25} = \$6.45$. We will see later that T-bills would have provided only \$2.74.

[7] The importance of the coupon rate when comparing returns on bonds is discussed in Part Three.

[8] Year-by-year returns are available on the Online Learning Center. Go to **www.mhhe.com/bkm,** and link to material for Chapter 5.

FIGURE 5.3

Frequency distribution of annual, continuously compounded rates of return, 1926–2010

Source: Prepared from data used in Table 5.2.

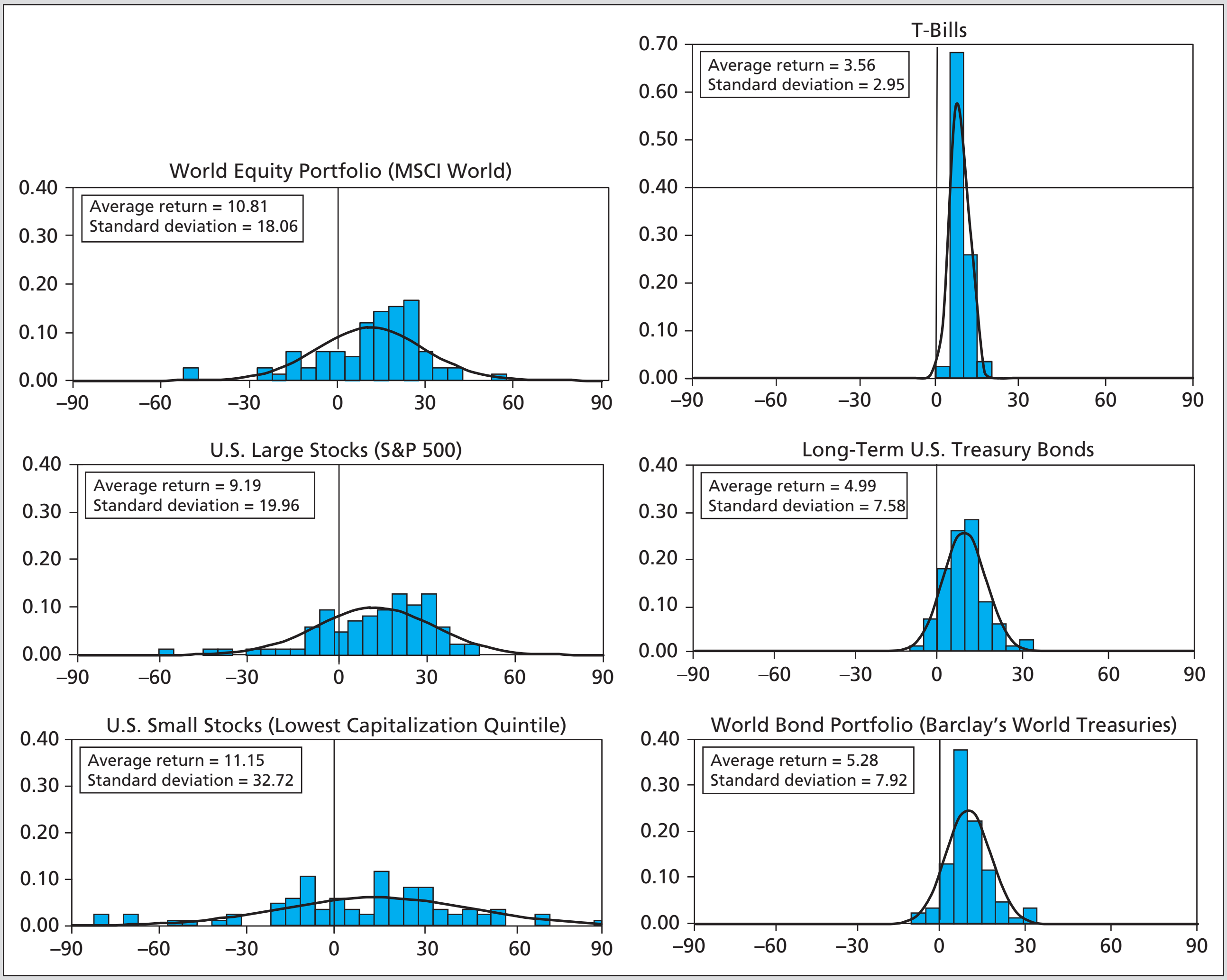
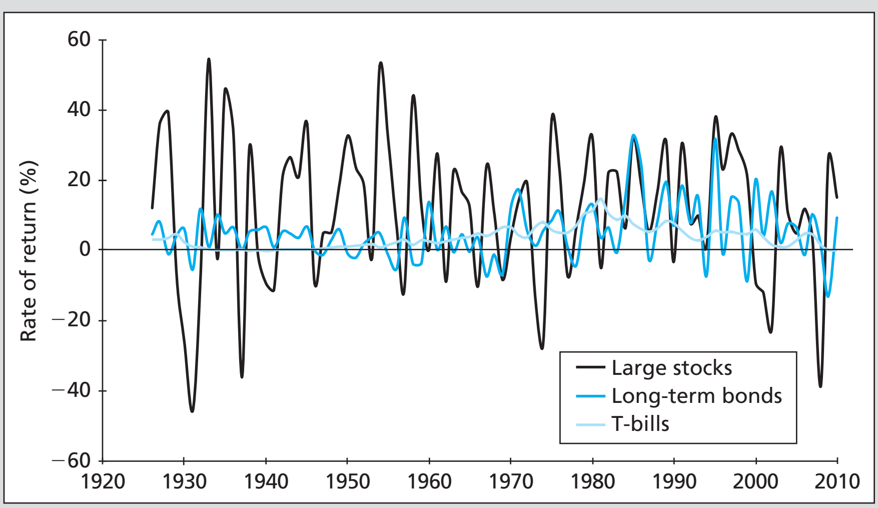


FIGURE 5.4

Rates of return on stocks, bonds, and bills, 1926–2010

Source: Prepared from data used in Table 5.2.



Thus, while the differences in average returns in Table 5.2 may seem modest at first glance, they imply great differences in long-term results. Naturally, the reason all investors don't invest everything in stocks is the higher risk that strategy would entail.

The geometric average is always less than the arithmetic average. For a normal distribution, the difference is exactly half the variance of the return (with returns measured as decimals, not percentages). Here are the arithmetic averages (from Figure 5.3) and geometric averages (from Table 5.2) for the three stock portfolios over the period (1926–2010), the differences between the two averages, as well as half the variance computed from the respective standard deviations.

	Average Portfolio Return (%)		
	World Stocks	**U.S. Small Stocks**	**U.S. Large Stocks**
Arithmetic average	10.89	17.57	11.67
Geometric average	9.21	11.80	9.62
Difference	1.68	5.78	2.04
Half historical variance	1.75	6.84	2.09

You can see that the differences between the geometric and arithmetic averages are consequential and generally close to one-half the variance of returns, suggesting that these distributions may be approximately normal, but there is a greater discrepancy for small stocks; therefore, VaR will still add important information about risk beyond standard deviation, at least for this asset class.

We have suggested that the geometric average is the correct measure for historical perspective. But investors are concerned about their *real* (inflation-adjusted) rates of return, not the paper profits indicated by the nominal (dollar) return. The real geometric averages suggest that the real cost of equity capital for large corporations has been about 6%. Notice from Table 5.2 that the average real rate on small stocks has been consistently declining, steadily approaching that of large stocks. One reason is that the average size of small, publicly traded firms has grown tremendously. Although they are still far smaller than the larger firms, their size apparently has reached the level where there is little remaining small-firm premium. The higher-than-historical-average returns recently provided by long-term bonds are due largely to capital gains earned as interest rates plunged in the recessions of the decade ending in 2010.

In the previous section we discussed the importance of risk and risk premiums. Let us now turn to the excess-return panel of Table 5.2. Notice first that excess returns do not need to be adjusted for inflation because they are returns over and above the nominal risk-free rate. Second, bond portfolios, albeit an important asset class, are not really candidates for an investor's sole-investment vehicle, because they are not sufficiently diversified. Third, the large differences in average returns across historical periods reflect the tremendous volatility of annual returns. One might wonder whether the differences across subperiods are statistically significant. Recalling that the standard deviation of the average return is the annual standard deviation divided by the square root of the number of observations, none of the differences between these subperiod averages and the 1926–2010 average exceeds one standard deviation for stocks and 1.8 standard deviations for bonds. Thus, differences in these subperiod results might well reflect no more than statistical noise.

The minimum and maximum historical returns also reflect the large variability in annual returns. Notice the large worst-case annual losses (around 50%) and even larger best-case gains (50%–150%) on the stock portfolios, as well as the more moderate extreme returns on the bond portfolios. Interestingly, the small and large U.S. stock portfolios each experienced both their maximum and minimum returns during the Great Depression; indeed, that period is also associated with the largest standard deviations of stock portfolio returns.

EXAMPLE 5.5

The Risk Premium and Growth of Wealth

The potential import of the risk premium can be illustrated with a simple example. Consider two investors with \$1 million as of December 31, 2000. One invests in the small-stock portfolio, and the other in T-bills. Suppose both investors reinvest all income from their portfolios and liquidate their investments 10 years later, on December 31, 2010. We can find the annual rates of return for this period from the spreadsheet of returns at the Online Learning Center. (Go to **www.mhhe.com/bkm.** Look for the link to Chapter 5 material.) We compute a "wealth index" for each investment by compounding wealth at the end of each year by the return earned in the following year. For example, we calculate the value of the wealth index for small stocks as of 2003 by multiplying the value as of 2002 (1.1404) by 1 plus the rate of return earned in 2003 (measured in decimals), that is, by 1 + .7475, to obtain 1.9928.

	Small Stocks		T-Bills	
Year	**Return (%)**	**Wealth Index**	**Return (%)**	**Wealth Index**
2000		1		1
2001	29.25	1.2925	3.86	1.0386
2002	−11.77	1.1404	1.63	1.0555
2003	74.75	1.9928	1.02	1.0663
2004	14.36	2.2790	1.19	1.0790
2005	3.26	2.3533	2.98	1.1111
2006	17.69	2.7696	4.81	1.1646
2007	−8.26	2.5408	4.67	1.2190
2008	−39.83	1.5288	1.64	1.2390
2009	36.33	2.0842	0.05	1.2396
2010	29.71	2.7034	0.08	1.2406

The final value of each portfolio as of December 31, 2010, equals its initial value (\$1 million) multiplied by the wealth index at the end of the period:

Date	Small Stocks	T-Bills
December 31, 2000	\$1,000,000	\$1,000,000
December 31, 2010	\$2,703,420	\$1,240,572

The difference in total return is dramatic. Even with its devasting 2008 return, the value of the small-stock portfolio after 10 years is 118% more than that of the T-bill portfolio.

We can also calculate the geometric average return of each portfolio over this period. For T-bills, the geometric average over the 10-year period is computed from:

$$(1 + r_G)^{10} = 1.2406$$
$$1 + r_G = 1.2406^{1/10} = 1.0218$$
$$r_G = 2.18\%$$

Similarly, the geometric average for small stocks is 10.46%. The difference in geometric average reflects the difference in cumulative wealth provided by the small-stock portfolio over this period.

Are these portfolios normally distributed? The next section of Table 5.2 shows the kurtosis and skew of the distributions. As discussed earlier, testing for normality requires us to use continuously compounded rates. Accordingly, we use Equation 5.5 to compute continuously compounded rates of return. We calculate ln(1 + annual rate) for each asset and compute excess returns by subtracting the continuously compounded rate of return on T-bills. Because these measures derive from higher exponents of deviations from the mean (the cubed deviation for skew and the fourth power of the deviation for kurtosis), these measures are highly sensitive to rare but extreme outliers; therefore, we can rely on these measures only in very large samples that allow for sufficient observations to be taken as exhibiting a "representative" number of such events. You can see that these measures also vary considerably across subperiods. The picture is quite unambiguous with respect to stock portfolios. There is excess positive kurtosis and negative skew. These indicate extreme gains and, even more so, extreme losses that are significantly more likely than would be predicted by the normal distribution. We must conclude that VaR (and similar risk measures) to augment standard deviation is in order.

The last section in Table 5.2 presents performance statistics, Sharpe ratios, and value at risk. Sharpe ratios of stock portfolios are in the range of 0.37–0.39 for the overall history and range between 0.34–0.46 across all subperiods. We can estimate that the return-risk tradeoff in stocks on an annual basis is about a .4% risk premium for each increment of 1% to standard deviation. In fact, just as with the average excess return, the differences between subperiods are not significant. The same can be said about the three stock portfolios: None showed significant superior performance. Bonds can outperform stocks in periods of falling interest rates, as we see from the Sharpe ratios in the most recent subperiod. But, as noted earlier, bond portfolios are not sufficiently diversified to allow for the use of the Sharpe ratio as a performance measure. (As we will discuss in later chapters, standard deviation as a risk measure makes sense for an investor's overall portfolio but not for one relatively narrow component of it.)

The VaR panel in Table 5.2 shows unambiguously for stocks, and almost so for bonds, that potential losses are larger than suggested by likewise normal distributions. To highlight this observation, the last panel of the table shows the difference of actual 5% VaR from likewise normal distributions; the evidence is quite clear and consistent with the kurtosis and skew statistics.

Finally, investing internationally is no longer considered exotic, and Table 5.2 also provides some information on the historical results from international investments. It appears that for passive investors who focus on investments in index funds, international diversification doesn't deliver impressive improvement over investments in the U.S. alone. However, international investments do hold large potential for active investors. We elaborate on these observations in Chapter 19, which is devoted to international investing.

CONCEPT check 5.4

Compute the average excess return on large-company stocks (over the T-bill rate) and the standard deviation for the years 1926–1934. You will need to obtain data from the spreadsheet available at the Online Learning Center at **www.mhhe.com/bkm.** Look for Chapter 5 material.

5.4 INFLATION AND REAL RATES OF RETURN

A 10% annual rate of return means that your investment was worth 10% more at the end of the year than it was at the beginning of the year. This does not necessarily mean, however, that you could have bought 10% more goods and services with that money, for it is possible that in the course of the year prices of goods also increased. If prices have changed, the increase in your purchasing power will not match the increase in your dollar wealth.

At any time, the prices of some goods may rise while the prices of other goods may fall; the *general* trend in prices is measured by examining changes in the consumer price index, or CPI. The CPI measures the cost of purchasing a representative bundle of goods, the "consumption basket" of a typical urban family of four. The **inflation rate** is measured by the rate of increase of the CPI.

inflation rate

The rate at which prices are rising, measured as the rate of increase of the CPI.

Suppose the rate of inflation (the percentage change in the CPI, denoted by i) for the last year amounted to $i = 6\%$. The purchasing power of money was thus reduced by 6%. Therefore, part of your investment earnings were offset by the reduction in the purchasing power of the dollars you received at the end of the year. With a 10% interest rate, for example, after you netted out the 6% reduction in the purchasing power of money, you were left with a net increase in purchasing power of about 4%. Thus, we need to distinguish between a **nominal interest rate**—the growth rate of money—and a **real interest rate**—the growth rate of purchasing power. If we call R the nominal rate, r the real rate, and i the inflation rate, then we conclude

nominal interest rate

The interest rate in terms of nominal (not adjusted for purchasing power) dollars.

$$r \approx R - i \tag{5.15}$$

real interest rate

The excess of the interest rate over the inflation rate. The growth rate of purchasing power derived from an investment.

In words, the real rate of interest is the nominal rate reduced by the loss of purchasing power resulting from inflation.

In fact, the exact relationship between the real and nominal interest rates is given by

$$1 + r = \frac{1 + R}{1 + i} \tag{5.16}$$

In words, the growth factor of your purchasing power, $1 + r$, equals the growth factor of your money, $1 + R$, divided by the new price level that is $1 + i$ times its value in the previous period. The exact relationship can be rearranged to

$$r = \frac{R - i}{1 + i} \tag{5.17}$$

which shows that the approximate rule overstates the real rate by the factor $1 + i$.[9]

EXAMPLE 5.6

Real versus Nominal Rates

If the interest rate on a one-year CD is 8%, and you expect inflation to be 5% over the coming year, then using the approximation given in Equation 5.15, you expect the real rate to be $r = 8\% - 5\% = 3\%$. Using the exact formula given in Equation 5.17, the real rate is $r = \frac{.08 - .05}{1 + .05} = .0286$, or 2.86%. Therefore, the approximation rule overstates the expected real rate by only .14 percentage points. The approximation rule of Equation 5.16 is more accurate for small inflation rates and is perfectly exact for continuously compounded rates.

The Equilibrium Nominal Rate of Interest

We've seen that the real rate of return is approximately the nominal rate minus the inflation rate. Because investors should be concerned with real returns—the increase in their purchasing power—they will demand higher nominal rates of return on their investments. This higher rate is necessary to maintain the expected real return as inflation increases.

Irving Fisher (1930) argued that the nominal rate ought to increase one-for-one with increases in the expected inflation rate. Using $E(i)$ to denote the current expected inflation over the coming period, then the so-called Fisher equation is

$$R = r + E(i) \tag{5.18}$$

Suppose the real rate of interest is 2%, and the inflation rate is 4%, so that the nominal interest rate is about 6%. If the expected inflation rate rises to 5%, the nominal interest rate should climb to roughly 7%. The increase in the nominal rate offsets the increase in expected inflation, giving investors an unchanged growth of purchasing power at a 2% rate.

[9]Notice that for continuously compounded rates, Equation 5.16 is perfectly accurate. Because $\ln(x/y) = \ln(x) - \ln(y)$, the continuously compounded real rate of return, r_{cc}, can be derived from the annual rates as

$$r_{cc} = \ln(1 + r) = \ln\left(\frac{1 + R}{1 + i}\right) = \ln(1 + R) - \ln(1 + i) = R_{cc} - i_{cc}$$

CONCEPT check 5.5

a. Suppose the real interest rate is 3% per year, and the expected inflation rate is 8%. What is the nominal interest rate?

b. Suppose the expected inflation rate rises to 10%, but the real rate is unchanged. What happens to the nominal interest rate?

U.S. History of Interest Rates, Inflation, and Real Interest Rates

Figure 5.5 plots nominal interest rates, inflation rates, and real rates in the U.S. between 1926 and 2010. Since the mid-1950s, nominal rates have increased roughly in tandem with inflation, broadly consistent with the Fisher equation. The 1930s and 1940s, however, show us that very volatile levels of unexpected inflation can play havoc with realized *real* rates of return.

Table 5.3 quantifies what we see in Figure 5.5. One interesting pattern that emerges is the steady increase in the average real interest rate across the three subperiods reported in the table. Perhaps this reflects the shrinking national savings rate (and therefore reduced availability of funds to borrowers) over this period. Another striking observation from Table 5.3 is the dramatic reduction in the variability of the inflation rate and the real interest rate. This is reflected in the decline in standard deviations as well as in the steady attenuation of minimum and maximum values. This reduction in variability also is related to the patterns in correlation that we observe. According to the Fisher equation, an increase in expected inflation translates directly into an increase in nominal interest rates; therefore, the correlation between nominal rates and inflation rates should be positive and high. In contrast, the correlation between real rates and inflation should be zero, because expected inflation is fully factored into the nominal interest rate and does not affect the expected real rate of return. The table indicates that during the early period, 1926–1955, market rates did not accord to this logic, possibly due to the extraordinarily high and almost certainly unforeseen variability in inflation rates. Since 1955, however, the nominal T-bill rate and inflation rate have tracked each other far more closely (as is clear from Figure 5.5), and the correlations show greater consistency with Fisher's logic.

Inflation-indexed bonds called Treasury Inflation-Protected Securities (TIPS) were introduced in the U.S. in 1997. These are bonds of 5- to 30-year original maturities with coupons and principal that increase at the rate of inflation. (We discuss these bonds in more detail in Chapter 10.) The difference between nominal rates on conventional T-bonds and the rates on equal-maturity TIPS provides a measure of expected inflation (often called *break-even inflation*) over that maturity.

FIGURE 5.5

Interest rates, inflation, and real interest rates, 1926–2010

Source: T-bills: Prof. Kenneth French, **http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html;** Inflation: Bureau of Labor Statistics, **www.bls.gov;** Real rate: authors' calculations.

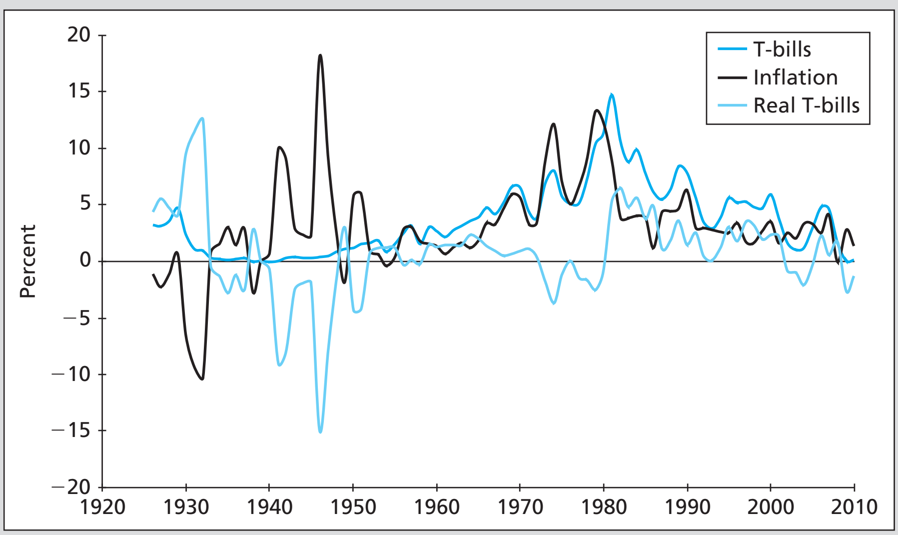


TABLE 5.3 Annual rates of return statistics for U.S. T-bills, inflation, and real interest rates, 1926–2010 and three subperiods (%)

	U.S. Market		
	T-Bills	**Inflation**	**Real T-Bills**
Arithmetic average			
1926–2010	3.66	3.08	0.68
1926–1955	1.10	1.51	−0.11
1956–1985	5.84	4.85	0.98
1986–2010	4.14	2.84	1.26
Standard deviation			
1926–2010	3.09	4.17	3.89
1926–1955	1.22	5.55	5.84
1956–1985	3.19	3.50	2.39
1986–2010	2.25	1.31	1.87
Correlations	**T-bills+inflation**	**Real bills+inflation**	
1926–2010	0.41	−0.46	
1926–1955	−0.30	−0.59	
1956–1985	0.72	−0.53	
1986–2010	0.53	0.35	
Minimum (lowest rate)			
1926–2010	−0.04	−10.27	−15.04
1926–1955	−0.04	−10.27	−15.04
1956–1985	1.53	0.67	−3.65
1986–2010	0.05	−0.04	−2.64
Maximum (highest rate)			
1926–2010	14.72	18.13	12.50
1926–1955	4.74	18.13	12.50
1956–1985	14.72	13.26	6.45
1986–2010	8.38	6.26	4.91

* Two slightly negative interest rates occurred in the 1930s, before T-bills were introduced. In those days, the Treasury instead guaranteed short-term bonds. In highly uncertain times, great demand for these bonds could result in a negative rate.

Source: T-bills: Fama and French risk-free rate; Inflation data: Bureau of Labor Statistics (inflation-cpiu-dec2dec).

5.5 ASSET ALLOCATION ACROSS RISKY AND RISK-FREE PORTFOLIOS

History shows us that long-term bonds have been riskier investments than investments in Treasury bills and that stock investments have been riskier still. On the other hand, the riskier investments have offered higher average returns. Investors, of course, do not make all-or-nothing choices from these investment classes. They can and do construct their portfolios using securities from all asset classes.

A simple strategy to control portfolio risk is to specify the fraction of the portfolio invested in broad asset classes such as stocks, bonds, and safe assets such as Treasury bills. This aspect of portfolio management is called **asset allocation** and plays an important role in the determination of portfolio performance. Consider this statement by John Bogle, made when he was the chairman of the Vanguard Group of Investment Companies:

asset allocation

Portfolio choice among broad investment classes.

> The most fundamental decision of investing is the allocation of your assets: How much should you own in stock? How much should you own in bonds? How much should you own in cash reserves? . . . That decision [has been shown to account] for an astonishing 94% of the differences

> in total returns achieved by institutionally managed pension funds. . . . There is no reason to believe that the same relationship does not also hold true for individual investors.[10]

capital allocation

The choice between risky and risk-free assets.

complete portfolio

The entire portfolio including risky and risk-free assets.

The most basic form of asset allocation envisions the portfolio as dichotomized into risky versus risk-free assets. The fraction of the portfolio placed in risky assets is called the **capital allocation** to risky assets and speaks directly to investor risk aversion.

To focus on the capital allocation decision, we think about an investor who allocates funds between T-bills and a portfolio of risky assets. We can envision the risky portfolio, *P*, as a mutual fund or ETF (exchange-traded fund) that includes a bundle of risky assets in desired, fixed proportions. Thus, when we shift wealth into and out of *P*, we do not change the relative proportion of the various securities within the risky portfolio. We put off until the next chapter the question of how to best construct the risky portfolio. We call the overall portfolio composed of the risk-free asset and the risky portfolio, *P*, the **complete portfolio** that includes the entire investor's wealth.

The Risk-Free Asset

The power to tax and to control the money supply lets the government, and only the government, issue default-free (Treasury) bonds. The default-free guarantee by itself is not sufficient to make the bonds risk-free in real terms, since inflation affects the purchasing power of the proceeds from the bonds. The only risk-free asset in real terms would be a price-indexed government bond such as TIPS. Even then, a default-free, perfectly indexed bond offers a guaranteed real rate to an investor only if the maturity of the bond is identical to the investor's desired holding period. These qualifications notwithstanding, it is common to view Treasury bills as *the* risk-free asset. Any inflation uncertainty over the course of a few weeks, or even months, is negligible compared to the uncertainty of stock market returns.[11]

In practice, most investors treat a broader range of money market instruments as effectively risk-free assets. All the money market instruments are virtually immune to interest rate risk (unexpected fluctuations in the price of a bond due to changes in market interest rates) because of their short maturities, and all are fairly safe in terms of default or credit risk.

Money market mutual funds hold, for the most part, three types of securities: Treasury bills, bank certificates of deposit (CDs), and commercial paper. The instruments differ slightly in their default risk. The yields to maturity on CDs and commercial paper, for identical maturities, are always slightly higher than those of T-bills. A history of this yield spread for 90-day CDs is shown in Figure 2.2 in Chapter 2.

Money market funds have changed their relative holdings of these securities over time, but by and large, the risk of such blue-chip, short-term investments as CDs and commercial paper is minuscule compared to that of most other assets, such as long-term corporate bonds, common stocks, or real estate. Hence, we treat money market funds, as well as T-bills, as representing the most easily accessible risk-free asset for most investors.

Portfolio Expected Return and Risk

We can examine the risk-return combinations that result from various capital allocations in the complete portfolio to risky versus risk-free assets. Finding the available combinations of

[10]John C. Bogle, *Bogle on Mutual Funds* (Burr Ridge, IL: Irwin Professional Publishing, 1994), p. 235.

[11]In the wake of the euro crisis as well as the credit downgrade of the United States in the summer of 2011, one clearly needs to consider whether (or when) sovereign debt can be treated as risk-free. Governments that issue debt in their home currency can in principle always repay that debt, if need be by printing more money in that currency. This strategy, however, can lead to runaway inflation, so the real return on that debt would hardly be risk-free. Moreover, the cost of possible hyperinflation can be so great that they might justifiably conclude that default is the lesser of the two evils. Governments that issue debt in currencies they do not control (e.g., euro-denominated Greek debt) cannot fall back on the printing press, even under extreme duress, so default in that situation is certainly possible. Since the euro crisis, analysts have focused considerable attention on measures of sovereign fiscal health such as the ratio of indebtedness to GDP. As is also true of corporate debt, long- and medium-term debt issues are typically riskier, as they allow more time for credit conditions to deteriorate before the loan is paid off.

risk and return is the "technical" part of capital allocation; it deals only with the opportunities available to investors. In the next section, we address the "personal preference" part of the problem, the individual's choice of the preferred risk-return combination, given his degree of risk aversion.

Since we assume that the composition of the risky portfolio, *P*, already has been determined, the only concern here is with the proportion of the investment budget (y) to be allocated to it. The remaining proportion $(1 - y)$ is to be invested in the risk-free asset, which has a rate of return denoted r_f.

We denote the *actual* risky rate of return by r_P, the *expected* rate of return on *P* by $E(r_P)$, and its standard deviation by σ_P. In the numerical example, $E(r_P) = 15\%$, $\sigma_P = 22\%$, and $r_f = 7\%$. Thus, the risk premium on the risky asset is $E(r_P) - r_f = 8\%$.

Let's start with two extreme cases. If you invest all of your funds in the risky asset, that is, if you choose $y = 1$, the expected return on your complete portfolio will be 15% and the standard deviation will be 22%. This combination of risk and return is plotted as point *P* in Figure 5.6. At the other extreme, you might put all of your funds into the risk-free asset, that is, you choose $y = 0$. In this case, you would earn a riskless return of 7%. (This choice is plotted as point *F* in Figure 5.6.)

Now consider more moderate choices. For example, if you allocate equal amounts of your *complete portfolio, C,* to the risky and risk-free assets, that is, you choose $y = .5$, the expected return on the complete portfolio will be the average of $E(r_P)$ and r_f. Therefore, $E(r_C) = .5 \times 7\% + .5 \times 15\% = 11\%$. The risk premium of the complete portfolio is therefore $11\% - 7\% = 4\%$, which is half of the risk premium of *P*. The standard deviation of the portfolio also is one-half of *P*'s, that is, 11%. When you reduce the fraction of the complete portfolio allocated to the risky asset by half, you reduce both the risk and risk premium by half.

To generalize, the risk premium of the complete portfolio, *C,* will equal the risk premium of the risky asset times the fraction of the portfolio invested in the risky asset.

$$E(r_C) - r_f = y[E(r_P) - r_f] \tag{5.19}$$

The standard deviation of the complete portfolio will equal the standard deviation of the risky asset times the fraction of the portfolio invested in the risky asset.

$$\sigma_C = y\,\sigma_P \tag{5.20}$$

In sum, both the risk premium and the standard deviation of the complete portfolio increase in proportion to the investment in the risky portfolio. Therefore, the points that describe the risk and return of the complete portfolio for various capital allocations of y all plot on the

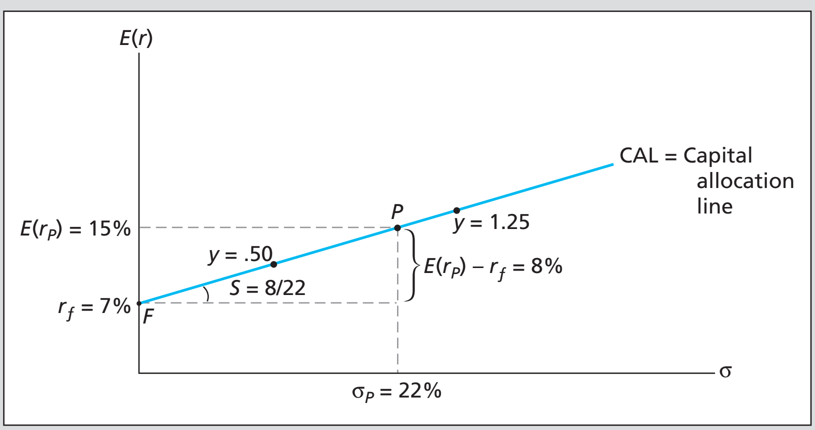


FIGURE 5.6

The investment opportunity set with a risky asset and a risk-free asset straight line connecting F and P, as shown in Figure 5.6, with an intercept of r_f and slope (rise/run) equal to the familiar Sharpe ratio of P:

$$S = \frac{E(r_P) - r_f}{\sigma_P} = \frac{15 - 7}{22} = .36 \qquad \textbf{(5.21)}$$

CONCEPT check 5.6

What are the expected return, risk premium, standard deviation, and ratio of risk premium to standard deviation for a complete portfolio with $y = .75$?

The Capital Allocation Line

capital allocation line (CAL)

Plot of risk-return combinations available by varying portfolio allocation between a risk-free asset and a risky portfolio.

The line plotted in Figure 5.6 depicts the risk-return combinations available by varying capital allocation, that is, by choosing different values of y. For this reason it is called the **capital allocation line,** or **CAL.** The slope, S, of the CAL equals the increase in expected return that an investor can obtain per unit of additional standard deviation or extra return per extra risk. It is obvious why it is also called the *reward-to-volatility ratio,* or Sharpe ratio, after William Sharpe who first suggested its use.

Notice that the Sharpe ratio is the same for risky portfolio P and the complete portfolio that mixes P and the risk-free asset in equal proportions.

	Expected Return	Risk Premium	Standard Deviation	Reward-to-Volatility Ratio
Portfolio P:	15%	8%	22%	$\frac{8}{22} = 0.36$
Portfolio C:	11%	4%	11%	$\frac{4}{11} = 0.36$

In fact, the reward-to-volatility ratio is the same for all complete portfolios that plot on the capital allocation line. While the risk-return combinations differ according to the investor's choice of y, the *ratio* of reward to risk is constant.

What about points on the CAL to the right of portfolio P in the investment opportunity set? You can construct complete portfolios to the right of point P by borrowing, that is, by choosing $y > 1$. This means that you borrow a proportion of $y - 1$ and invest both the borrowed funds and your own wealth in the risky portfolio P. If you can borrow at the risk-free rate, $r_f = 7\%$, then your rate of return will be $r_C = -(y - 1)r_f + y\, r_P = r_f + y(r_P - r_f)$. This complete portfolio has risk premium of $y[E(r_P) - r_f]$ and SD $= y\,\sigma_P$. Verify that your Sharpe ratio equals that of any other portfolio on the same CAL.

EXAMPLE 5.7

Levered Complete Portfolios

Suppose the investment budget is \$300,000, and an investor borrows an additional \$120,000, investing the \$420,000 in the risky asset. This is a levered position in the risky asset, which is financed in part by borrowing. In that case

$$y = \frac{420{,}000}{300{,}000} = 1.4$$

and $1 - y = 1 - 1.4 = -.4$, reflecting a short position in the risk-free asset, or a borrowing position. Rather than lending at a 7% interest rate, the investor borrows at 7%. The portfolio rate of return is

$$E(r_C) = 7 + (1.4 \times 8) = 18.2$$

Another way to find this portfolio rate of return is as follows: You expect to earn \$63,000 (15% of \$420,000) and pay \$8,400 (7% of \$120,000) in interest on the loan. Simple subtraction yields an

(continued)

EXAMPLE 5.7

Levered Complete Portfolios (concluded)

expected profit of \$54,600, which is 18.2% of your investment budget of \$300,000. Therefore, $E(r_C) = 18.2\%$.

Your portfolio still exhibits the same reward-to-volatility ratio:

$$\sigma_C = 1.4 \times 22 = 30.8$$

$$S = \frac{E(r_C) - r_f}{\sigma_C} = \frac{11.2}{30.8} = .36$$

As you might have expected, the levered portfolio has both a higher expected return and a higher standard deviation than an unlevered position in the risky asset.

Risk Aversion and Capital Allocation

We have developed the CAL, the graph of all feasible risk-return combinations available from allocating the complete portfolio between a risky portfolio and a risk-free asset. The investor confronting the CAL now must choose one optimal combination from the set of feasible choices. This choice entails a trade-off between risk and return. Individual investors with different levels of risk aversion, given an identical capital allocation line, will choose different positions in the risky asset. Specifically, the more risk-averse investors will choose to hold *less* of the risky asset and *more* of the risk-free asset.

How can we find the best allocation between the risky portfolio and risk-free asset? Recall that a particular investor's degree of risk aversion (A) measures the price of risk she demands from the complete portfolio in which her entire wealth is invested. The compensation for risk demanded by the investor must be compared to the price of risk offered by the risky portfolio, P. We can find the investor's preferred capital allocation, y, by dividing the risky portfolio's price of risk by the investor's risk aversion, her *required* price of risk:

$$y = \frac{\text{Available risk premium to variance ratio}}{\text{Required risk premium to variance ratio}} = \frac{[E(r_P) - r_f]/\sigma_P^2}{A} = \frac{[E(r_P) - r_f]}{A\sigma_P^2} \quad \textbf{(5.22)}$$

Notice that when the price of risk of the available risky portfolio exactly matches the investor's degree of risk aversion, her entire wealth will be invested in it ($y = 1$).

What would the investor of Equation 5.12 (with $A = 3.91$) do when faced with the market index portfolio of Equation 5.13 (with price of risk = 2)? Equation 5.22 tells us that this investor would invest $y = 2/3.91 = 0.51$ (51%) in the market index portfolio and a proportion $1 - y = 0.49$ in the risk-free asset.

Graphically, more risk-averse investors will choose portfolios near point F on the capital allocation line plotted in Figure 5.6. More risk-tolerant investors will choose points closer to P, with higher expected return and higher risk. The most risk-tolerant investors will choose portfolios to the right of point P. These levered portfolios provide even higher expected returns, but even greater risk.

The investor's asset allocation choice also will depend on the trade-off between risk and return. When the reward-to-volatility ratio increases, investors might well decide to take on riskier positions. Suppose an investor reevaluates the probability distribution of the risky portfolio and now perceives a greater expected return without an accompanying increase in the standard deviation. This amounts to an increase in the reward-to-volatility ratio or, equivalently, an increase in the slope of the CAL. As a result, this investor will choose a higher y, that is, a greater position in the risky portfolio.

One role of a professional financial adviser is to present investment opportunity alternatives to clients, obtain an assessment of the client's risk tolerance, and help determine the appropriate complete portfolio.[12]

[12]"Risk tolerance" is simply the flip side of "risk aversion." Either term is a reasonable way to describe attitudes toward risk. We generally find it easier to talk about risk *aversion,* but practitioners often use the term risk *tolerance.*

5.6 PASSIVE STRATEGIES AND THE CAPITAL MARKET LINE

passive strategy

Investment policy that avoids security analysis.

A **passive strategy** is based on the premise that securities are fairly priced, and it avoids the costs involved in undertaking security analysis. Such a strategy might at first blush appear to be naive. However, we will see in Chapter 8 that intense competition among professional money managers might indeed force security prices to levels at which further security analysis is unlikely to turn up significant profit opportunities. Passive investment strategies may make sense for many investors.

To avoid the costs of acquiring information on any individual stock or group of stocks, we may follow a "neutral" diversification approach. Select a diversified portfolio of common stocks that mirrors the corporate sector of the broad economy. This results in a value-weighted portfolio, which, for example, invests a proportion in GE stock that equals the ratio of GE's market value to the market value of all listed stocks.

Such strategies are called *indexing*. The investor chooses a portfolio of all the stocks in a broad market index such as the S&P 500. The rate of return on the portfolio then replicates the return on the index. Indexing has become a popular strategy for passive investors. We call the capital allocation line provided by one-month T-bills and a broad index of common stocks the **capital market line** (CML). That is, a passive strategy using the broad stock market index as the risky portfolio generates an investment opportunity set that is represented by the CML.

capital market line

The capital allocation line using the market index portfolio as the risky asset.

Historical Evidence on the Capital Market Line

Table 5.4 is a small cut-and-paste from Table 5.3, which concentrates on S&P 500 data, a popular choice for a broad stock-market index. As we discussed earlier, the large standard deviation of its rate of return implies that we cannot reject the hypothesis that the entire 85-year period is characterized by the same Sharpe ratio. Using this history as a guide, investors might reasonably forecast a risk premium of around 8% coupled with a standard deviation of approximately 20%, resulting in a Sharpe ratio of .4.

We also have seen that to hold a complete portfolio with these risk-return characteristics, the "average" investor (with $y = 1$) would need to have a coefficient of risk aversion of $.08/.20^2 = 2$. But that average investor would need some courage. As the VaR figures in Table 5.4 indicate, the market index has exhibited a probability of 5% of a 36.86% or worse loss in a year; surely this is no picnic. This substantial risk, together with differences in risk aversion across individuals, might explain the large differences we observe in portfolio positions across investors.

Finally, notice the instability of the excess returns on the S&P 500 across the 30-year subperiods in Table 5.4. The great variability in excess returns raises the question of whether the 8% historical average really is a reasonable estimate of the risk premium looking into the future. It also suggests that different investors may come to different conclusions about future excess returns, another reason for capital allocations to vary.

In fact, there has been considerable recent debate among financial economists about the "true" equity risk premium, with an emerging consensus that the historical average may be an unrealistically high estimate of the future risk premium. This argument is based on several

TABLE 5.4 Excess return statistics for the S&P 500

	Excess Return (%)			
	Average	Std Dev	Sharpe Ratio	5% VaR
1926–2010	8.00	20.70	.39	−36.86
1926–1955	11.67	25.40	.46	−53.43
1956–1985	5.01	17.58	.28	−30.51
1986–2010	7.19	17.83	.40	−42.28

On the MARKET FRONT

TRIUMPH OF THE OPTIMISTS

As a whole, the last eight decades have been very kind to U.S. equity investors. Even accounting for miserable 2008 returns, stock investments have outperformed investments in safe Treasury bills by more than 7% per year. The real rate of return averaged more than 6%, implying an expected doubling of the real value of the investment portfolio about every 12 years!

Is this experience representative? A book by three professors at the London Business School, Elroy Dimson, Paul Marsh, and Mike Staunton, extends the U.S. evidence to other countries and to longer time periods. Their conclusion is given in the book's title, *Triumph of the Optimists**: In every country in their study (which included markets in North America, Europe, Asia, and Africa), the investment optimists—those who bet on the economy by investing in stocks rather than bonds or bills—were vindicated. Over the long haul, stocks beat bonds everywhere.

On the other hand, the equity risk premium is probably not as large as the post-1926 evidence from Table 5.3 would seem to indicate. First, results from the first 25 years of the last century (which included the first World War) were less favorable to stocks. Second, U.S. returns have been better than those of most other countries, and so a more representative value for the historical risk premium may be lower than the U.S. experience. Finally, the sample that is amenable to historical analysis suffers from a self-selection problem. Only those markets that have survived to be studied can be included in the analysis. This leaves out countries such as Russia or China, whose markets were shut down during communist rule, and whose results if included would surely bring down the average historical performance of equity investments. Nevertheless, there is powerful evidence of a risk premium that shows its force everywhere the authors looked.

*Elroy Dimson, Paul Marsh, Mike Staunton, *Triumph of the Optimists: 101 Years of Global Investment Returns* (Princeton, NJ: Princeton University Press, 2002).

factors: the use of longer time periods in which equity returns are examined; a broad range of countries rather than just the U.S. in which excess returns are computed (Dimson, Marsh, and Staunton, 2001); direct surveys of financial executives about their expectations for stock market returns (Graham and Harvey, 2001); and inferences from stock market data about investor expectations (Jagannathan, McGrattan, and Scherbina, 2000; Fama and French, 2002). The nearby box discusses some of this evidence.

Costs and Benefits of Passive Investing

The fact that an individual's capital allocation decision is hard does not imply that its implementation needs to be complex. A passive strategy is simple and inexpensive to implement: Choose a broad index fund or ETF and divide your savings between it and a money market fund. To justify spending your own time and effort or paying a professional to pursue an active strategy requires some evidence that those activities are likely to be profitable. As we shall see later in the text, this is much harder to come by than you might expect!

To choose an active strategy, an investor must be convinced that the benefits outweigh the cost, and the cost can be quite large. As a benchmark, annual expense ratios for index funds are around 20 and 50 basis points for U.S. and international stocks, respectively. The cost of utilizing a money market fund is smaller still, and T-bills can be purchased at no cost.

Here is a very cursory idea of the cost of active strategies: The annual expense ratio of an active stock mutual fund averages around 1% of invested assets, and mutual funds that invest in more exotic assets such as real estate or precious metals can be more expensive still. A hedge fund will cost you 1% to 2% of invested assets plus 10% or more of any returns above the risk-free rate. If you are wealthy and seek more dedicated portfolio management, costs will be even higher.

Because of the power of compounding, an extra 1% of annual costs can have large consequences for the future value of your portfolio. With a risk-free rate of 2% and a risk premium of 8%, you might expect your wealth to grow by a factor of $1.10^{30} = 17.45$ over a 30-year investment horizon. If fees are 1%, then your net return is reduced to 9%, and your wealth grows by a factor of only $1.09^{30} = 13.26$ over that same horizon. That seemingly small management fee reduces your final wealth by about one-quarter.

The potential benefits of active strategies are discussed in detail in Chapter 8. The news is generally not that good for active investors. However, the factors that keep the active management industry going are (1) the large potential of enrichment from successful investments—the same power of compounding works in your favor if you can add even a few basis points to total return, (2) the difficulty in assessing performance (discussed in Chapter 18), and (3) uninformed investors who are willing to pay for professional money management. There is no question that some money managers can outperform passive strategies. The problem is (1) how do you identify them and (2) do their fees outstrip their potential. Whatever the choice one makes, one thing is clear: The CML using the passive market index is not an obviously inferior choice.

SUMMARY

- Investors face a trade-off between risk and expected return. Historical data confirm our intuition that assets with low degrees of risk should provide lower returns on average than do those of higher risk.
- Shifting funds from the risky portfolio to the risk-free asset is the simplest way to reduce risk. Another method involves diversification of the risky portfolio. We take up diversification in later chapters.
- U.S. T-bills provide a perfectly risk-free asset in nominal terms only. Nevertheless, the standard deviation of real rates on short-term T-bills is small compared to that of assets such as long-term bonds and common stocks, so for the purpose of our analysis, we consider T-bills the risk-free asset. Besides T-bills, money market funds hold short-term, safe obligations such as commercial paper and CDs. These entail some default risk but relatively little compared to most other risky assets. For convenience, we often refer to money market funds as risk-free assets.
- A risky investment portfolio (referred to here as the risky asset) can be characterized by its reward-to-volatility ratio. This ratio is the slope of the capital allocation line (CAL), the line connecting the risk-free asset to the risky asset. All combinations of the risky and risk-free asset lie on this line. Investors would prefer a steeper-sloping CAL, because that means higher expected returns for any level of risk.
- An investor's preferred choice among the portfolios on the capital allocation line will depend on risk aversion. Risk-averse investors will weight their complete portfolios more heavily toward Treasury bills. Risk-tolerant investors will hold higher proportions of their complete portfolios in the risky asset.
- The capital market line is the capital allocation line that results from using a passive investment strategy that treats a market index portfolio, such as the Standard & Poor's 500, as the risky asset. Passive strategies are low-cost ways of obtaining well-diversified portfolios with performance that will reflect that of the broad stock market.

KEY TERMS

arithmetic average, 112
asset allocation, 133
capital allocation, 134
capital allocation line (CAL), 136
capital market line, 138
complete portfolio, 134
dollar-weighted average return, 113
excess return, 122
expected return, 115
geometric average, 112
holding-period return (HPR), 111
inflation rate, 130
kurtosis, 121
mean-variance analysis, 125
nominal interest rate, 131
passive strategy, 138
price of risk, 123
probability distribution, 115
real interest rate, 131
risk aversion, 122
risk-free rate, 122
risk premium, 122
scenario analysis, 115
Sharpe (or reward-to-volatility) ratio, 125
skew, 121
standard deviation, 116
value at risk (VaR), 118
variance, 116

KEY FORMULAS

Arithmetic average of n returns: $(r_1 + r_2 + \cdots + r_n)/n$

Geometric average of n returns: $[(1 + r_1)(1 + r_2) \cdots (1 + r_n)]^{1/n} - 1$

Continuously compounded rate of return, r_{cc}: $\ln(1 + \text{Effective annual rate})$

Expected return: Σ [prob(Scenario) $\times$ Return in scenario]

Variance: Σ [prob(Scenario) $\times$ (Deviation from mean in scenario)^2]

Standard deviation: $\sqrt{\text{Variance}}$

Sharpe ratio: $\dfrac{\text{Portfolio risk premium}}{\text{Standard deviation of excess return}} = \dfrac{E(r_P) - r_f}{\sigma_P}$

Real rate of return: $\dfrac{1 + \text{Nominal return}}{1 + \text{Inflation rate}} - 1$

Real rate of return (continuous compounding): $r_{\text{nominal}} - \text{Inflation rate}$

Optimal capital allocation to the risky asset, y: $\dfrac{E(r_P) - r_f}{A\sigma_P^2}$

Mc Graw Hill connect® Select problems are available in McGraw-Hill's *Connect Finance.* Please see the Supplements section of the book's frontmatter for more information.

PROBLEM SETS

Basic

1. Suppose you've estimated that the fifth-percentile value at risk of a portfolio is −30%. Now you wish to estimate the portfolio's first-percentile VaR (the value below which lie 1% of the returns). Will the 1% VaR be greater or less than −30%? *(LO 5-2)*
2. To estimate the Sharpe ratio of a portfolio from a history of asset returns, we use the difference between the simple (arithmetic) average rate of return and the T-bill rate. Why not use the geometric average? *(LO 5-4)*
3. When estimating a Sharpe ratio, would it make sense to use the average excess real return that accounts for inflation? *(LO 5-4)*
4. You've just decided upon your capital allocation for the next year, when you realize that you've underestimated both the expected return and the standard deviation of your risky portfolio by 4%. Will you increase, decrease, or leave unchanged your allocation to risk-free T-bills? *(LO 5-4)*

Intermediate

5. Suppose your expectations regarding the stock market are as follows:

State of the Economy	Probability	HPR
Boom	0.3	44%
Normal growth	0.4	14
Recession	0.3	−16

Use Equations 5.6–5.8 to compute the mean and standard deviation of the HPR on stocks. *(LO 5-4)*

6. The stock of Business Adventures sells for $40 a share. Its likely dividend payout and end-of-year price depend on the state of the economy by the end of the year as follows: *(LO 5-2)*

	Dividend	Stock Price
Boom	$2.00	$50
Normal economy	1.00	43
Recession	.50	34

a. Calculate the expected holding-period return and standard deviation of the holding-period return. All three scenarios are equally likely.

b. Calculate the expected return and standard deviation of a portfolio invested half in Business Adventures and half in Treasury bills. The return on bills is 4%.

7. XYZ stock price and dividend history are as follows:

Year	Beginning-of-Year Price	Dividend Paid at Year-End
2010	$100	$4
2011	$110	$4
2012	$ 90	$4
2013	$ 95	$4

An investor buys three shares of XYZ at the beginning of 2010, buys another two shares at the beginning of 2011, sells one share at the beginning of 2012, and sells all four remaining shares at the beginning of 2013. ***(LO 5-1)***

a. What are the arithmetic and geometric average time-weighted rates of return for the investor?

b. What is the dollar-weighted rate of return? (*Hint:* Carefully prepare a chart of cash flows for the *four* dates corresponding to the turns of the year for January 1, 2010, to January 1, 2013. If your calculator cannot calculate internal rate of return, you will have to use a spreadsheet or trial and error.)

8. *a.* Suppose you forecast that the standard deviation of the market return will be 20% in the coming year. If the measure of risk aversion in Equation 5.13 is $A = 4$, what would be a reasonable guess for the expected market risk premium?

b. What value of A is consistent with a risk premium of 9%?

c. What will happen to the risk premium if investors become more risk tolerant? ***(LO 5-4)***

9. Using the historical risk premiums as your guide, what is your estimate of the expected annual HPR on the S&P 500 stock portfolio if the current risk-free interest rate is 5%? ***(LO 5-3)***

10. What has been the historical average *real* rate of return on stocks, Treasury bonds, and Treasury bills? ***(LO 5-2)***

11. Consider a risky portfolio. The end-of-year cash flow derived from the portfolio will be either $50,000 or $150,000, with equal probabilities of .5. The alternative riskless investment in T-bills pays 5%. ***(LO 5-3)***

a. If you require a risk premium of 10%, how much will you be willing to pay for the portfolio?

b. Suppose the portfolio can be purchased for the amount you found in (*a*). What will the expected rate of return on the portfolio be?

c. Now suppose you require a risk premium of 15%. What is the price you will be willing to pay now?

d. Comparing your answers to (*a*) and (*c*), what do you conclude about the relationship between the required risk premium on a portfolio and the price at which the portfolio will sell?

For Problems 12–16, assume that you manage a risky portfolio with an expected rate of return of 17% and a standard deviation of 27%. The T-bill rate is 7%.

12. Your client chooses to invest 70% of a portfolio in your fund and 30% in a T-bill money market fund. ***(LO 5-3)***

a. What is the expected return and standard deviation of your client's portfolio?

b. Suppose your risky portfolio includes the following investments in the given proportions:

Stock A	27%
Stock B	33%
Stock C	40%

What are the investment proportions of your client's overall portfolio, including the position in T-bills?

c. What is the reward-to-volatility ratio (S) of your risky portfolio and your client's overall portfolio?

d. Draw the CAL of your portfolio on an expected return/standard deviation diagram. What is the slope of the CAL? Show the position of your client on your fund's CAL.

13. Suppose the same client in the previous problem decides to invest in your risky portfolio a proportion (y) of his total investment budget so that his overall portfolio will have an expected rate of return of 15%. *(LO 5-3)*
    a. What is the proportion y?
    b. What are your client's investment proportions in your three stocks and the T-bill fund?
    c. What is the standard deviation of the rate of return on your client's portfolio?
14. Suppose the same client as in the previous problem prefers to invest in your portfolio a proportion (y) that maximizes the expected return on the overall portfolio subject to the constraint that the overall portfolio's standard deviation will not exceed 20%. *(LO 5-3)*
    a. What is the investment proportion, y?
    b. What is the expected rate of return on the overall portfolio?
15. You estimate that a passive portfolio invested to mimic the S&P 500 stock index yields an expected rate of return of 13% with a standard deviation of 25%. Draw the CML and your fund's CAL on an expected return/standard deviation diagram. *(LO 5-4)*
    a. What is the slope of the CML?
    b. Characterize in one short paragraph the advantage of your fund over the passive fund.
16. Your client (see previous problem) wonders whether to switch the 70% that is invested in your fund to the passive portfolio. *(LO 5-4)*
    a. Explain to your client the disadvantage of the switch.
    b. Show your client the maximum fee you could charge (as a percent of the investment in your fund deducted at the end of the year) that would still leave him at least as well off investing in your fund as in the passive one. (*Hint:* The fee will lower the slope of your client's CAL by reducing the expected return net of the fee.)
17. What do you think would happen to the expected return on stocks if investors perceived an increase in the volatility of stocks? *(LO 5-4)*
18. You manage an equity fund with an expected risk premium of 10% and a standard deviation of 14%. The rate on Treasury bills is 6%. Your client chooses to invest $60,000 of her portfolio in your equity fund and $40,000 in a T-bill money market fund. What is the expected return and standard deviation of return on your client's portfolio? *(LO 5-3)*
19. What is the reward-to-volatility ratio for the *equity fund* in the previous problem? *(LO 5-4)*

For Problems 20–22, download the spreadsheet containing the data for Table 5.2, "Rates of return, 1926–2010," from www.mhhe.com/bkm.

20. Calculate the same subperiod means and standard deviations for small stocks as Table 5.4 of the text provides for large stocks. *(LO 5-2)*
    a. Have small stocks provided better reward-to-volatility ratios than large stocks?
    b. Do small stocks show a similar higher standard deviation in the earliest subperiod as Table 5.4 documents for large stocks?
21. Convert the nominal returns on both large and small stocks to real rates. Reproduce Table 5.4 using real rates instead of excess returns. Compare the results to those of Table 5.4. *(LO 5-1)*
22. Repeat the previous problem for small stocks and compare with the results for nominal rates. *(LO 5-1)*

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Challenge

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23. Download the annual returns on the combined NYSE/NASDAQ/AMEX markets as well as the S&P 500 from the Online Learning Center at **www.mhhe.com/bkm.** For both indexes, calculate: *(LO 5-2)*
   a. Average return.
   b. Standard deviation of return.
   c. Skew of return.
   d. Kurtosis of return.
   e. The 5% value at risk.
   f. Based on your answers to parts (*b*)–(*e*), compare the risk of the two indexes.

CFA® PROBLEMS

CFA Problems

1. A portfolio of nondividend-paying stocks earned a geometric mean return of 5% between January 1, 2005, and December 31, 2011. The arithmetic mean return for the same period was 6%. If the market value of the portfolio at the beginning of 2005 was $100,000, what was the market value of the portfolio at the end of 2011? *(LO 5-1)*
2. Which of the following statements about the standard deviation is/are *true?* A standard deviation: *(LO 5-2)*
   a. Is the square root of the variance.
   b. Is denominated in the same units as the original data.
   c. Can be a positive or a negative number.
3. Which of the following statements reflects the importance of the asset allocation decision to the investment process? The asset allocation decision: *(LO 5-3)*
   a. Helps the investor decide on realistic investment goals.
   b. Identifies the specific securities to include in a portfolio.
   c. Determines most of the portfolio's returns and volatility over time.
   d. Creates a standard by which to establish an appropriate investment time horizon.

Use the following data in answering CFA Questions 4–6.

Investment	Expected Return, $E(r)$	Standard Deviation, σ
1	.12	.30
2	.15	.50
3	.21	.16
4	.24	.21

Investor "satisfaction" with portfolio increases with expected return and decreases with variance according to the "utility" formula: $U = E(r) - \frac{1}{2} A\sigma^2$ where $A = 4$.

4. Based on the formula for investor satisfaction or "utility," which investment would you select if you were risk averse with $A = 4$? *(LO 5-4)*
5. Based on the formula above, which investment would you select if you were risk neutral? *(LO 5-4)*
6. The variable (A) in the utility formula represents the: *(LO 5-4)*
   a. Investor's return requirement.
   b. Investor's aversion to risk.
   c. Certainty equivalent rate of the portfolio.
   d. Preference for one unit of return per four units of risk.

Use the following scenario analysis for stocks *X* and *Y* to answer CFA Questions 7 through 9.

	Bear Market	Normal Market	Bull Market
Probability	.2	.5	.3
Stock *X*	−20%	18%	50%
Stock *Y*	−15%	20%	10%

7. What are the expected returns for stocks *X* and *Y*? *(LO 5-2)*
8. What are the standard deviations of returns on stocks *X* and *Y*? *(LO 5-2)*
9. Assume that of your $10,000 portfolio, you invest $9,000 in stock *X* and $1,000 in stock *Y*. What is the expected return on your portfolio? *(LO 5-3)*
10. Probabilities for three states of the economy and probabilities for the returns on a particular stock in each state are shown in the table below.

State of Economy	Probability of Economic State	Stock Performance	Probability of Stock Performance in Given Economic State
Good	.3	Good	.6
		Neutral	.3
		Poor	.1
Neutral	.5	Good	.4
		Neutral	.3
		Poor	.3
Poor	.2	Good	.2
		Neutral	.3
		Poor	.5

What is the probability that the economy will be neutral *and* the stock will experience poor performance? *(LO 5-2)*

11. An analyst estimates that a stock has the following probabilities of return depending on the state of the economy. What is the expected return of the stock? *(LO 5-2)*

State of Economy	Probability	Return
Good	.1	15%
Normal	.6	13
Poor	.3	7

WEB *master*

1. Use data from **finance.yahoo.com** to answer the following questions.
   a. Select the Company tab and enter the ticker symbol "ADBE." Click on the *Profile* tab to see an overview of the company.
   b. What is the latest price reported in the *Summary* section? What is the 12-month target price? Calculate the expected holding-period return based on these prices.
   c. Use the *Historical Prices* section to answer the question "How much would I have today if I invested $10,000 in ADBE five years ago?" Using this information, calculate the five-year holding-period return on Adobe's stock.
2. From the *Historical Prices* tab, download Adobe's dividend-adjusted stock price for the last 24 months into an Excel spreadsheet. Calculate the monthly rate of return for each month, the average return, and the standard deviation of returns over that period.
3. Calculating the real rate of return is an important part of evaluating an investment's performance. To do this, you need to know the nominal return on your investment and

the rate of inflation during the corresponding period. To estimate the expected real rate of return before you make an investment, you can use the promised yield and the expected inflation rate.

a. Go to **www.bankrate.com** and click on the *CDs and Investments* tab. Using *Compare CDs & Investment Rates* box, find the average one-year CD rate from banks across the nation (these will be nominal rates).

b. Use the St. Louis Federal Reserve's website at **research.stlouisfed.org/fred2** as a source for data about expected inflation. Search for "MICH inflation," which will provide you with the University of Michigan Inflation Expectation data series (MICH). Click on the *View Data* link and find the latest available data point. What is the expected inflation rate for the next year?

c. On the basis of your answers to parts *(a)* and *(b),* calculate the expected real rate of return on a one-year CD investment.

d. What does the result tell you about real interest rates? Are they positive or negative, and what does this mean?

SOLUTIONS TO CONCEPT checks

5.1 *a.* The arithmetic average is $(2 + 8 - 4)/3 = 2\%$ per month.

b. The time-weighted (geometric) average is
$[(1 + .02) \times (1 + .08) \times (1 - .04)]^{1/3} - 1 = .0188 = 1.88\%$ per month.

c. We compute the dollar-weighted average (IRR) from the cash flow sequence (in $ millions):

	Month		
	1	2	3
Assets under management at beginning of month	10.0	13.2	19.256
Investment profits during month (HPR × Assets)	0.2	1.056	(0.77)
Net inflows during month	3.0	5.0	0.0
Assets under management at end of month	13.2	19.256	18.486

	Time			
	0	1	2	3
Net cash flow*	−10	−3.0	−5.0	+18.486

*Time 0 is today. Time 1 is the end of the first month. Time 3 is the end of the third month, when net cash flow equals the ending value (potential liquidation value) of the portfolio.

The IRR of the sequence of net cash flows is 1.17% per month.

The dollar-weighted average is less than the time-weighted average because the negative return was realized when the fund had the most money under management.

5.2 *a.* Computing the HPR for each scenario, we convert the price and dividend data to rate-of-return data:

Scenario	Prob	Ending Value ($ million)	Dividend ($ million)	HPR	HPR × Prob	Deviation: HPR-mean	Prob × Dev'n Squared
1	.30	$35	$4.40	.713	.214	.406	.049
2	.45	27	4.00	.348	.157	.040	.001
3	.20	15	4.00	−.174	−.035	−.481	.046
4	.05	8	2.00	−.565	−.028	−.873	.038
Sum:					.307		.135

Expected HPR = .307 = 30.7%.

Variance = .135.

Standard deviation $= \sqrt{.135} = .367 = 36.7\%$.

5% VaR = −56.5%.

For the corresponding normal distribution, VaR would be 30.7% − 1.64485 × 36.7% = −29.67%.

b. With 36 returns, 5% of the sample would be .05 × 36 = 1.8 observations. The worst return is −17%, and the second-worst is −5%. Using interpolation, we estimate the fifth-percentile return as:

$$-17\% + .8[-5\% - (-17\%)] = -7.4\%$$

5.3 *a.* If the average investor chooses the S&P 500 portfolio, then the implied degree of risk aversion is given by Equation 5.13:

$$A = \frac{.10 - .05}{.18^2} = 1.54$$

b. $S = \dfrac{10 - 5}{18} = .28$

5.4 The mean excess return for the period 1926–1934 is 3.56% (below the historical average), and the standard deviation (using $n - 1$ degrees of freedom) is 32.55% (above the historical average). These results reflect the severe downturn of the great crash and the unusually high volatility of stock returns in this period.

5.5 *a.* Solving:

$$1 + R = (1 + r)(1 + i) = (1.03)(1.08) = 1.1124$$
$$R = 11.24\%$$

b. Solving:

$$1 + R = (1.03)(1.10) = 1.133$$
$$R = 13.3\%$$

5.6 $E(r) = 7 + .75 \times 8\% = 13\%$

$\sigma = .75 \times 22\% = 16.5\%$

Risk premium = 13 − 7 = 6%

$$\frac{\text{Risk premium}}{\text{Standard deviation}} = \frac{13 - 7}{16.5} = .36$$

Chapter 6

Efficient Diversification

Learning Objectives:

- **LO6-1** Show how covariance and correlation affect the power of diversification to reduce portfolio risk.
- **LO6-2** Calculate mean, variance, and covariance using either historical data or scenario analysis.
- **LO6-3** Construct efficient portfolios and use the Sharpe ratio to evaluate portfolio efficiency.
- **LO6-4** Calculate the composition of the optimal risky portfolio.
- **LO6-5** Use index models to analyze the risk and return characteristics of securities and portfolios.

In this chapter we describe how investors can construct the best possible risky portfolio. The key concept is efficient diversification.

The notion of diversification is age-old. The adage "Don't put all your eggs in one basket" obviously predates formal economic theory. However, a rigorous model showing how to make the most of the power of diversification was not devised until 1952, a feat for which Harry Markowitz eventually won the Nobel Prize in Economics. This chapter is largely developed from his work, as well as from later insights that built on his work.

We start with a bird's-eye view of how diversification reduces the variability of portfolio returns. We then turn to the construction of optimal risky portfolios. We follow a top-down approach, starting with asset allocation across a small set of broad asset classes, such as stocks, bonds, and money market securities. Then we show how the principles of optimal asset allocation can easily be generalized to solve the problem of security selection among many risky assets. We discuss the efficient set of risky portfolios and show how it leads us to the best attainable capital allocation. Finally, we show how index models of security returns can simplify the search for efficient portfolios and the interpretation of the risk characteristics of individual securities.

The last section examines the common fallacy that long-term investment horizons mitigate the impact of asset risk. We argue that the common belief in "time diversification" is in fact an illusion and is not real diversification.

Related websites for this chapter are available at www.mhhe.com/bkm.

6.1 DIVERSIFICATION AND PORTFOLIO RISK

Suppose you have in your risky portfolio only one stock, say, Dell Computers. What are the sources of risk affecting this "portfolio"?

We can identify two broad sources of uncertainty. The first is the risk from general economic conditions, such as business cycles, inflation, interest rates, exchange rates, and so forth. None of these macroeconomic factors can be predicted with certainty, and all affect Dell stock. Then you must add firm-specific influences, such as Dell's success in R&D, its management style and philosophy, and so on. Firm-specific factors are those that affect Dell without noticeably affecting other firms.

Now consider adding another security to the risky portfolio. If you invest half of your risky portfolio in ExxonMobil, leaving the other half in Dell, what happens to portfolio risk? Because the firm-specific influences on the two stocks differ (statistically speaking, the influences are independent), this strategy should reduce portfolio risk. For example, when oil prices fall, hurting ExxonMobil, computer prices might rise, helping Dell. The two effects are offsetting, which stabilizes portfolio return.

But why stop at only two stocks? Diversifying into many more securities continues to reduce exposure to firm-specific factors, so portfolio volatility should continue to fall. Ultimately, however, there is no way to avoid all risk. To the extent that virtually all securities are affected by common (risky) macroeconomic factors, we cannot eliminate exposure to general economic risk, no matter how many stocks we hold.

Figure 6.1 illustrates these concepts. When all risk is firm-specific, as in Figure 6.1A, diversification can reduce risk to low levels. With all risk sources independent, and with investment spread across many securities, exposure to any particular source of risk is negligible. This is an application of the law of large numbers. The reduction of risk to very low levels because of independent risk sources is called the *insurance principle.*

When a common source of risk affects all firms, however, even extensive diversification cannot eliminate all risk. In Figure 6.1B, portfolio standard deviation falls as the number of securities increases, but it is not reduced to zero. The risk that remains even after diversification is called **market risk,** risk that is attributable to marketwide risk sources. Other terms are **systematic risk** or **nondiversifiable risk.** The risk that *can* be eliminated by diversification is called **unique risk, firm-specific risk, nonsystematic risk,** or **diversifiable risk.**

This analysis is borne out by empirical studies. Figure 6.2 shows the effect of portfolio diversification, using data on NYSE stocks. The figure shows the average standard deviations of equally weighted portfolios constructed by selecting stocks at random as a function of the number of stocks in the portfolio. On average, portfolio risk does fall with diversification, but

market risk, systematic risk, nondiversifiable risk

Risk factors common to the whole economy.

unique risk, firm-specific risk, nonsystematic risk, diversifiable risk

Risk that can be eliminated by diversification.

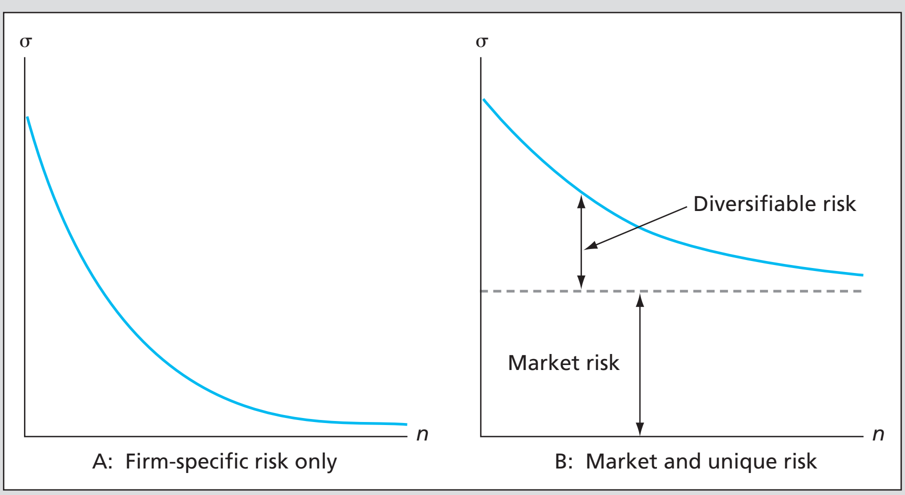


FIGURE 6.1

Portfolio risk as a function of the number of stocks in the portfolio

On the MARKET FRONT

DANGER: HIGH LEVELS OF COMPANY STOCK

Q: I'm 48 years old and have about 90% of my 401(k) invested in my company's stock and the rest in an international equity fund. I want to diversify further, but don't know where to turn. Any suggestions?

A: Diversify further? That's an understatement. You, my friend, need a total 401(k) portfolio makeover.

The glaring trouble spot, of course, is your huge concentration of company stock. Generally, I recommend that, to the extent you own your employer's stock at all in your 401(k), you limit it to 10% or so of your account's value.

The problem is that once you get beyond a small holding of company stock—or the shares of any one company for that matter—you dramatically increase the riskiness of your portfolio in two ways.

First, you expose yourself to the possibility that your company may simply implode, decimating the stock's value (and your 401(k)'s balance along with it) virtually overnight. But even if that doesn't happen, there's another risk: heightened volatility. A single stock is typically two to three times more volatile than a diversified portfolio. And when you load up your 401(k) with the stock of one company, it subjects your account value to the possibility of much wider swings.

In short, the payoff you'll likely get from investing in company stock doesn't adequately compensate you for the risk you're taking.

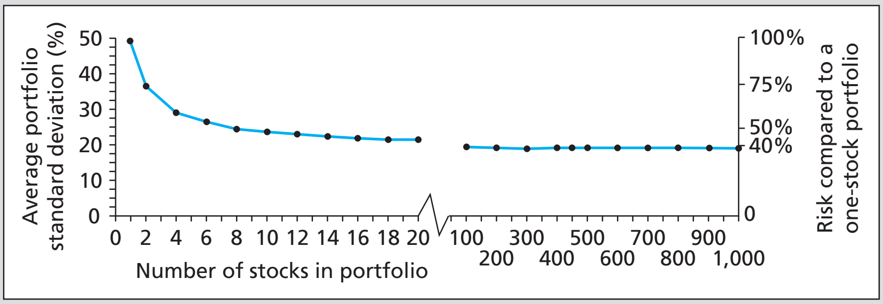
So, about that further diversification. Basically, you need to rebuild your portfolio from the ground up so that you not only own a broad range of stocks, but bonds as well. As for which investments you should choose from your 401(k)'s lineup, I'd recommend sticking as much as possible to low-cost funds and particularly index funds to the extent they're available.

SOURCE: Walter Updegrave, "Danger: High Levels of Company Stock," **http://money.cnn.com,** January 19, 2009.

FIGURE 6.2

Portfolio risk decreases as diversification increases

Source: Meir Statman, "How Many Stocks Make a Diversified Portfolio?" *Journal of Financial and Quantitative Analysis* 22, September 1987.



the power of diversification to reduce risk is limited by common sources of risk. The nearby box, "Danger: High Levels of Company Stock," highlights the dangers of neglecting diversification.

In light of this discussion, it is worth pointing out that general macroeconomic conditions in the U.S. do not move in lockstep with those in other countries. International diversification may further reduce portfolio risk, but here too, global economic and political factors affecting all countries to various degrees will limit the extent of risk reduction.

6.2 ASSET ALLOCATION WITH TWO RISKY ASSETS

In the last chapter we examined the capital allocation decision, how much of the portfolio to place in risk-free securities versus in a risky portfolio. Of course, investors need to choose the precise composition of the risky portfolio. In a top-down process, the first step would be an asset allocation decision. As the other nearby box, "First Take Care of Asset Allocation Needs," emphasizes, most investment professionals recognize that the asset allocation decision must take precedence over the choice of particular stocks.

We turn first to asset allocation between only two risky assets, still assumed to be a bond fund and a stock fund. Once we understand the portfolios of two risky assets, we will

FIRST TAKE CARE OF ASSET ALLOCATION NEEDS

If you want to build a top-performing mutual-fund portfolio, you should start by hunting for top-performing funds, right?

Wrong.

Too many investors gamely set out to find top-notch funds without first settling on an overall portfolio strategy. Result? These investors wind up with a mishmash of funds that don't add up to a decent portfolio.

So what should you do? With thousands of stock, bond, and money-market funds to choose from, you couldn't possibly analyze all the funds available. Instead, to make sense of the bewildering array of funds available, you should start by deciding what basic mix of stock, bond, and money-market funds you want to hold. This is what experts call your "asset allocation."

This asset allocation has a major influence on your portfolio's performance. The more you have in stocks, the higher your likely long-run return.

But with the higher potential return from stocks come sharper short-term swings in a portfolio's value. As a result, you may want to include a healthy dose of bond and money-market funds, especially if you are a conservative investor or you will need to tap your portfolio for cash in the near future.

Once you have settled on your asset allocation mix, decide what sort of stock, bond, and money-market funds you want to own. This is particularly critical for the stock portion of your portfolio. One way to damp the price swings in your stock portfolio is to spread your money among large, small, and foreign stocks.

You could diversify even further by making sure that, when investing in U.S. large- and small-company stocks, you own both growth stocks with rapidly increasing sales or earnings and also beaten-down value stocks that are inexpensive compared with corporate assets or earnings.

Similarly, among foreign stocks, you could get additional diversification by investing in both developed foreign markets such as France, Germany, and Japan, and also emerging markets like Argentina, Brazil, and Malaysia.

SOURCE: Abridged from Jonathan Clements, "It Pays for You to Take Care of Asset-Allocation Needs Before Latching onto Fads," *The Wall Street Journal,* April 6, 1998. Reprinted by permission of *The Wall Street Journal,*

reintroduce the choice of the risk-free asset. This will complete the asset allocation problem across the three key asset classes: stocks, bonds, and T-bills. Constructing efficient portfolios of many risky securities is a straightforward extension of this asset allocation exercise.

Covariance and Correlation

To optimally construct a portfolio from risky assets, we need to understand how the uncertainties of asset returns interact. A key determinant of portfolio risk is the extent to which the returns on the two assets vary either in tandem or in opposition. Portfolio risk depends on the *covariance* between the returns of the assets in the portfolio. We can see why using a simple scenario analysis.

The scenario analysis in Spreadsheet 6.1 posits four possible scenarios for the economy: a severe recession, a mild recession, normal growth, and a boom. The performance of stocks follows the broad economy, returning, respectively, −37%, −11%, 14%, and 30% in the four scenarios. In contrast, bonds perform best in a mild recession, returning 15% (since falling interest rates create capital gains), and in the normal growth scenario, where their return is 8%. They suffer from defaults in severe recession, resulting in a negative return, −9%, and from

SPREADSHEET 6.1

Capital market expectations for the stock and bond funds

	A	B	C	D	E	F
1			Stock Fund		Bond Fund	
2	Scenario	Probability	Rate of Return	Col B x Col C	Rate of Return	Col B x Col E
3	Severe recession	.05	−37	−1.9	−9	−0.45
4	Mild recession	.25	−11	−2.8	15	3.8
5	Normal growth	.40	14	5.6	8	3.2
6	Boom	.30	30	9.0	−5	−1.5
7	Expected or Mean Return:		SUM:	10.0	SUM:	5.0

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inflation in the boom scenario, where their return is −5%. Notice that bonds outperform stocks in both the mild and severe recession scenarios. In both normal growth and boom scenarios, stocks outperform bonds.

The expected return on each fund equals the probability-weighted average of the outcomes in the four scenarios. The last row of Spreadsheet 6.1 shows that the expected return of the stock fund is 10% and that of the bond fund is 5%. The variance is the probability-weighted average of the squared deviation of actual return from the expected return; the standard deviation is the square root of the variance. These values are computed in Spreadsheet 6.2.

What about the risk and return characteristics of a portfolio made up from the stock and bond funds? The portfolio return is the weighted average of the returns on each fund with weights equal to the proportion of the portfolio invested in each fund. Suppose we form a portfolio with 40% invested in the stock fund and 60% in the bond fund. Then the portfolio return in each scenario is the weighted average of the returns on the two funds. For example

$$\text{Portfolio return in mild recession} = .40 \times (-11\%) + .60 \times 15\% = 4.6\%$$

which appears in cell C6 of Spreadsheet 6.3.

Spreadsheet 6.3 shows the rate of return of the portfolio in each scenario. Notice that both funds suffer in a severe downturn and, therefore, the portfolio also experiences a substantial loss of 20.2%. This is a manifestation of systematic risk affecting a broad spectrum of securities. Declines of more than 25% in the S&P 500 Index have occurred five times in the past 86 years (1930, 1931, 1937, 1974, and 2008), roughly once every 17 years. Avoiding losses in these extreme outcomes would require one to devote a large allocation of the portfolio to risk-free (low return) investments or (expensive) portfolio insurance (which we will discuss in Chapter 16).

SPREADSHEET 6.2

Variance of returns

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	A	B	C	D	E	F	G	H	I	J
1				Stock Fund			Bond Fund			
2				Deviation				Deviation		
3			Rate	from		Column B	Rate	from		Column B
4			of	Expected	Squared	×	of	Expected	Squared	×
5	Scenario	Prob.	Return	Return	Deviation	Column E	Return	Return	Deviation	Column I
6	Severe recession	.05	−37	−47	2209	110.45	−9	−14	196	9.80
7	Mild recession	.25	−11	−21	441	110.25	15	10	100	25.00
8	Normal growth	.40	14	4	16	6.40	8	3	9	3.60
9	Boom	.30	30	20	400	120.00	−5	−10	100	30.00
10				Variance = SUM		347.10			Variance:	68.40
11			Standard deviation = SQRT(Variance)			18.63			Std. Dev.:	8.27

SPREADSHEET 6.3

Performance of a portfolio invested in the stock and bound funds

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	A	B	C	D	E	F	G
1			Portfolio invested 40% in stock fund and 60% in bond fund				
2			Rate	Column B	Deviation from		Column B
3			of	×	Expected	Squared	×
4	Scenario	Probability	Return	Column C	Return	Deviation	Column F
5	Severe recession	.05	−20.2	−1.01	−27.2	739.84	36.99
6	Mild recession	.25	4.6	1.15	−2.4	5.76	1.44
7	Normal growth	.40	10.4	4.16	3.4	11.56	4.62
8	Boom	.30	9.0	2.70	2.0	4.00	1.20
9		Expected return:		7.00		Variance:	44.26
10						Standard deviation:	6.65

Extreme events such as a severe recession make for the large standard deviation of stocks, 18.63%, and even of bonds, 8.27%. Still, the overall standard deviation of the diversified portfolio, 6.65%, is considerably smaller than that of stocks and even smaller than that of bonds.

The low risk of the portfolio is due to the inverse relationship between the performances of the stock and bond funds. In a mild recession, stocks fare poorly, but this is offset by the large positive return of the bond fund. Conversely, in the boom scenario, bond prices fall, but stocks do very well. Notice that while the portfolio's expected return is just the weighted average of the expected return of the two assets, *the portfolio standard deviation is actually lower than that of either component fund.*

Portfolio risk is reduced most when the returns of the two assets most reliably offset each other. The natural question investors should ask, therefore, is how one can measure the tendency of the returns on two assets to vary either in tandem or in opposition to each other. The statistics that provide this measure are the covariance and the correlation coefficient.

The covariance is calculated in a manner similar to the variance. Instead of multiplying the difference of an asset return from its expected value by itself (i.e., squaring it), we multiply it by the deviation of the *other* asset return from *its* expectation. The sign and magnitude of this product are determined by whether deviations from the mean move together (i.e., are both positive or negative in the same scenarios) and whether they are small or large at the same time.

We start in Spreadsheet 6.4 with the deviation of the return on each fund from its expected value. For each scenario, we multiply the deviation of the stock fund return from its mean by the deviation of the bond fund. The product will be positive if both asset returns exceed their respective means or if both fall short of their respective means. The product will be negative if one asset exceeds its mean return when the other falls short. Spreadsheet 6.4 shows that the stock fund return in a mild recession falls short of its expected value by 21%, while the bond fund return exceeds its mean by 10%. Therefore, the product of the two deviations is $-21 \times 10 = -210$, as reported in column E. The product of deviations is negative if one asset performs well when the other is performing poorly. It is positive if both assets perform well or poorly in the same scenarios.

The probability-weighted average of the products is called *covariance* and measures the *average* tendency of the asset returns to vary in tandem, that is, to co-vary. The formula for the covariance of the returns on the stock and bond funds is given in Equation 6.1. Each particular scenario in this equation is labeled or "indexed" by i. In general, i ranges from scenario 1 to S (the total number of scenarios; here, $S = 4$). The probability of each scenario is denoted $p(i)$.

$$\text{Cov}(r_S, r_B) = \sum_{i=1}^{S} p(i)[r_S(i) - E(r_S)][r_B(i) - E(r_B)] \tag{6.1}$$

The covariance of the stock and bond funds is computed in the next-to-last line of Spreadsheet 6.4 using Equation 6.1. The negative value for the covariance indicates that the two assets, on average, vary inversely; when one performs well, the other tends to perform poorly.

SPREADSHEET 6.4

Covariance between the returns of the stock and bond funds

	A	B	C	D	E	F
1			Deviation from Mean Return		Covariance	
2	Scenario	Probability	Stock Fund	Bond Fund	Product of Dev	Col B × Col E
3	Severe recession	.05	−47	−14	658	32.9
4	Mild recession	.25	−21	10	−210	−52.5
5	Normal growth	.40	4	3	12	4.8
6	Boom	.30	20	−10	−200	−60.0
7				Covariance =	SUM:	−74.8
8	Correlation coefficient = Covariance/(StdDev(stocks)*StdDev(bonds)) =					−0.49

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Like variance, the unit of covariance is percent square, which is why it is difficult to interpret its magnitude. For instance, does the covariance of −74.8 in cell F7 indicate that the inverse relationship between the returns on stock and bond funds is strong? It's hard to say. An easier statistic to interpret is the *correlation coefficient,* which is the covariance divided by the product of the standard deviations of the returns on each fund. We denote the correlation coefficient by the Greek letter rho, ρ.

$$\text{Correlation coefficient} = \rho_{SB} = \frac{\text{Cov}(r_S, r_B)}{\sigma_S \sigma_B} = \frac{-74.8}{18.63 \times 8.27} = -.49 \qquad \textbf{(6.2)}$$

Correlation is a pure number and can range from values of −1 to +1. A correlation of −1 indicates that one asset's return varies perfectly inversely with the other's. If you were to do a linear regression of one asset's return on the other, the slope coefficient would be negative and the R-square of the regression would be 100%, indicating a perfect fit. The R-square is the square of the correlation coefficient and tells you the fraction of the variance of one return explained by the other return. With a correlation of −1, you could predict 100% of the variability of one asset's return if you knew the return on the other asset. Conversely, a correlation of +1 would indicate perfect positive correlation and also would imply an R-square of 100%. A correlation of zero indicates that the returns on the two assets are unrelated. The correlation coefficient of $\rho_{SB} = -.49$ in Equation 6.2 confirms the tendency of the returns on the stock and bond funds to vary inversely. In fact, a fraction of $(-.49)^2 = .24$ of the variance of stocks can be explained by the returns on bonds.

Equation 6.2 shows that whenever the covariance is called for in a calculation we can replace it with the following expression using the correlation coefficient:

$$\text{Cov}(r_S, r_B) = \rho_{SB}\sigma_S\sigma_B \qquad \textbf{(6.3)}$$

We are now in a position to derive the risk and return features of portfolios of risky assets.

CONCEPT check 6.1

Suppose the rates of return of the bond portfolio in the four scenarios of Spreadsheet 6.1 are −10% in a severe recession, 10% in a mild recession, 7% in a normal period, and 2% in a boom. The stock returns in the four scenarios are −37%, −11%, 14%, and 30%. What are the covariance and correlation coefficient between the rates of return on the two portfolios?

Using Historical Data

We've seen that portfolio risk and return depend on the means and variances of the component securities, as well as on the covariance between their returns. One way to obtain these inputs is a scenario analysis as in Spreadsheets 6.1–6.4. A common alternative approach to produce these inputs is to make use of historical data. The idea is that variability and covariability change slowly over time. Thus, if we estimate these statistics from recent data, our estimates will provide useful predictions for the near future—perhaps next month or next quarter.

In this approach, we use realized returns to estimate variances and covariances. Means cannot be as precisely estimated from past returns. We discuss mean returns in great detail later. The estimate of variance is the average value of the squared deviations around the sample average; the estimate of the covariance is the average value of the cross-product of deviations. Notice that, as in scenario analysis, the focus for risk is on deviations of returns from their average value. Instead of using mean returns based on the scenario analysis, we use average returns during the sample period. We can illustrate this approach with a simple example.

EXAMPLE 6.1

Using Historical Data to Estimate Means, Standard Deviations, Covariance, and Correlation

Consider the 10 years of returns for the two mutual funds presented in the following spreadsheet. While these are far less data than most analysts would use, for the sake of illustration we will pretend that they are adequate to estimate mean returns and relevant risk statistics. In practice, analysts would use higher-frequency data (e.g., monthly or even daily data) to estimate risk coefficients and would, as well, supplement historical data with fundamental analysis to forecast future returns.

The spreadsheet starts with the raw return data in columns B and C. We use standard Excel functions to obtain average returns, standard deviation, covariance, and correlation (see rows 18–21). We also confirm (in cell F14) that covariance is the average value of the product of each asset's deviation from its mean return.

The average returns and standard deviations in this spreadsheet are similar to those of our previous scenario analysis. However, the correlation between stock and bond returns in this example is low but positive, which is more consistent with historical experience than the strongly negative correlation of −.49 implied by our scenario analysis.

	A	B	C	D	E	F
1		**Rates of Return**		**Deviations from Average Returns**		Products of
2	**Year**	Stock Fund	Bond Fund	Stock Fund	Bond Fund	Deviations
3	2006	30.17	5.08	20.17	0.08	1.53
4	2007	32.97	7.52	22.97	2.52	57.78
5	2008	21.04	−8.82	11.04	−13.82	−152.56
6	2009	−8.10	5.27	−18.10	0.27	−4.82
7	2010	−12.89	12.20	−22.89	7.20	−164.75
8	2011	−28.53	−7.79	−38.53	−12.79	493.00
9	2012	22.49	6.38	12.49	1.38	17.18
10	2013	12.58	12.40	2.58	7.40	19.05
11	2014	14.81	17.29	4.81	12.29	59.05
12	2015	15.50	0.51	5.50	−4.49	−24.70
13						
14	Average	10.00	5.00	Covariance = average product of deviations:		30.08
15	SD	19.00	8.00	Correlation = Covariance/(SD stocks*SD bonds):		0.20
16						
17	**Excel formulas**					
18	Average	=average(B3:B12)				
19	Std deviation	=stdevp(B3:B12)				
20	Covariance	=covar(B3:B12,C3:C12)				
21	Correlation	=correl(B3:B12,C3:C12)				
22						
23						

Two comments on Example 6.1 are in order. First, you may recall from a statistics class and from Chapter 5 that when variance is estimated from a sample of n observed returns, it is common to divide the squared deviations by $n - 1$ rather than by n. This is because we take deviations from an estimated average return rather than the true (but unknown) expected return; this procedure is said to adjust for a "lost degree of freedom." In Excel, the function STDEVP computes standard deviation dividing by n, while the function STDEV uses $n - 1$. Excel's covariance and correlation functions both use n. In Example 6.1, we ignored this fine point, and divided by n throughout. In any event, the correction for the lost degree of freedom is negligible when there are plentiful observations. For example with 60 returns (e.g., five years of monthly data), the difference between dividing by 60 or 59 will affect variance or covariance by a factor of only 1.017.

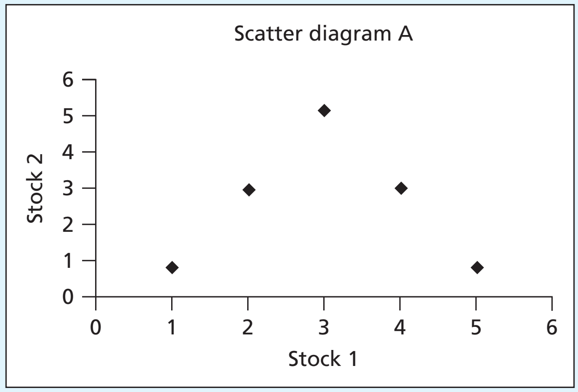
Second, we repeat the warning about the statistical reliability of historical estimates. Estimates of variance and covariance from past data are generally reliable forecasts (at least for the short term). However, averages of past returns typically provide highly noisy (i.e., imprecise) forecasts of future expected returns. In this example, we use past averages from small samples because our objective is to demonstrate the methodology. In practice, professional investors spend most of their resources on macroeconomic and security analysis to improve their estimates of mean returns.

CONCEPT check 6.2

The following tables present returns on various pairs of stocks in several periods. In part A, we show you a scatter diagram of the returns on the first pair of stocks. Draw (or prepare in Excel) similar scatter diagrams for cases B through E. Match up your diagrams (A–E) to the following list of correlation coefficients by choosing the correlation that best describes the relationship between the returns on the two stocks: $\rho = -1, 0, .2, .5, 1.0$.

A.

% Return	
Stock 1	**Stock 2**
5	1
1	1
4	3
2	3
3	5



B.

% Return	
Stock 1	**Stock 2**
1	1
2	2
3	3
4	4
5	5

C.

% Return	
Stock 1	**Stock 2**
1	5
2	4
3	3
4	2
5	1

D.

% Return	
Stock 1	**Stock 2**
5	5
1	3
4	3
2	0
3	5

E.

% Return	
Stock 1	**Stock 2**
5	4
1	3
4	1
2	0
3	5

The Three Rules of Two-Risky-Assets Portfolios

Suppose a proportion denoted by w_B is invested in the bond fund and the remainder $1 - w_B$, denoted by w_S, is invested in the stock fund. The properties of the portfolio are determined by the following three rules governing combinations of random variables:

Rule 1: The rate of return on a portfolio is the weighted average of returns on the component securities, with the investment proportions as weights.

$$r_P = w_B r_B + w_S r_S \tag{6.4}$$

Rule 2: The expected rate of return on a portfolio is the weighted average of the expected *returns on the component securities, with the portfolio proportions as weights.*

$$E(r_P) = w_B E(r_B) + w_S E(r_S) \tag{6.5}$$

Rules 1 and 2 say that a portfolio's actual return and its mean return are linear functions of the component security returns and portfolio weights. This is not so for portfolio variance, as the third rule shows.

Rule 3: The variance of the rate of return on a two-risky-asset portfolio is

$$\sigma_P^2 = (w_B\sigma_B)^2 + (w_S\sigma_S)^2 + 2(w_B\sigma_B)(w_S\sigma_S)\rho_{BS} \tag{6.6}$$

where ρ_{BS} is the correlation coefficient between the returns on the stock and bond funds. Notice that using Equation 6.3, we may replace the last term in Equation 6.6 with $2w_B w_S Cov(r_B, r_S)$.

The variance of a portfolio is the *sum* of the contributions of the component security variances *plus* a term that involves the correlation coefficient (and hence, covariance) between the returns on the component securities. We know from the last section why this last term arises. When the correlation between the component securities is small or negative, there will be a greater tendency for returns on the two assets to offset each other. This will reduce portfolio risk. Notice in Equation 6.6 that portfolio variance is lower when the correlation coefficient is lower.

The formula describing portfolio variance is more complicated than that describing portfolio return. This complication has a virtue, however: a tremendous potential for gains from diversification.

The Risk-Return Trade-Off with Two-Risky-Assets Portfolios

We can assess the benefit from diversification by using Rules 2 and 3 to compare the risk and expected return of a better-diversified portfolio to a less diversified benchmark. Suppose an investor estimates the following input data:

$$E(r_B) = 5\% \quad \sigma_B = 8\% \quad E(r_S) = 10\% \quad \sigma_S = 19\% \quad \rho_{BS} = .2$$

Currently, all funds are invested in the bond fund, but the invester ponders a portfolio invested 40% in stock and 60% in bonds. Using Rule 2, the expected return of this portfolio is

$$E(r_P) = .4 \times 10\% + .6 \times 5\% = 7\%$$

which represents a gain of 2% compared to a bond-only investment. Using Rule 3, the portfolio standard deviation is

$$\sigma = \sqrt{(.4 \times 19)^2 + (.6 \times 8)^2 + 2(.4 \times 19) \times (.6 \times 8) \times .2} = 9.76\%$$

which is less than the weighted average of the component standard deviations: $.4 \times 19 + .6 \times 8 = 12.40\%$. The difference of 2.64% reflects the benefits of diversification. This benefit is cost-free in the sense that diversification allows us to experience the full contribution of the stock's higher expected return, while keeping the portfolio standard deviation below the average of the component standard deviations.

EXAMPLE 6.2

Benefits from Diversification

Suppose we invest 85% in bonds and only 15% in stocks. We can construct a portfolio with an expected return higher than bonds $(.85 \times 5) + (.15 \times 10) = 5.75\%$ and, at the same time, a standard deviation less than bonds. Using Equation 6.6 again, we find that the portfolio variance is

$$(.85 \times 8)^2 + (.15 \times 19)^2 + 2(.85 \times 8)(.15 \times 19) \times .2 = 62.1$$

and, accordingly, the portfolio standard deviation is $\sqrt{62.1} = 7.88\%$, which is less than the standard deviation of either bonds or stocks alone. Taking on a more volatile asset (stocks) actually reduces portfolio risk! Such is the power of diversification.

We can find investment proportions that will reduce portfolio risk even further. The risk-minimizing proportions are 90.7% in bonds and 9.3% in stocks.[1] With these proportions, the portfolio standard deviation will be 7.80%, and the portfolio's expected return will be 5.47%.

Is this portfolio preferable to the one considered in Example 6.2, with 15% in the stock fund? That depends on investor preferences, because the portfolio with the lower variance also has a lower expected return.

What the analyst can and must do is show investors the entire **investment opportunity set.** This is the set of all attainable combinations of risk and return offered by portfolios formed using the available assets in differing proportions. We find the investment opportunity

investment opportunity set

Set of available portfolio risk-return combinations.

[1]The minimum-variance portfolio minimizes the variance (and hence standard deviation) of returns, regardless of the expected return. The formula for the weight in bonds is $w_B = \dfrac{\sigma_S^2 - \sigma_B\sigma_S\rho_{BS}}{\sigma_S^2 + \sigma_B^2 - 2\sigma_B\sigma_S\rho_{BS}}$, and the weight in stocks is $w_S = 1 - w_B$. Notice that when correlation is zero, the variance-minimizing weight simplifies to the ratio of stock variance to the sum of the variances of stocks and bonds: $w_B = \dfrac{\sigma_S^2}{\sigma_S^2 + \sigma_B^2}$.

SPREADSHEET 6.5

The investment opportunity set with the stock and bond funds

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	A	B	C	D	E
1			**Input Data**		
2	$E(r_S)$	$E(r_B)$	σ_S	σ_B	ρ_{BS}
3	10	5	19	8	0.2
4	Portfolio Weights		Expected Return, $E(r_p)$		Std Dev
5	$w_S = 1 - w_B$	w_B	Col A*A3 + Col B*B3		(Equation 6.6)
6	−0.2	1.2	4.0		9.59
7	−0.1	1.1	4.5		8.62
8	0.0	1.0	5.0		8.00
9	0.0932	0.9068	5.5		7.804
10	0.1	0.9	5.5		7.81
11	0.2	0.8	6.0		8.07
12	0.3	0.7	6.5		8.75
13	0.4	0.6	7.0		9.77
14	0.5	0.5	7.5		11.02
15	0.6	0.4	8.0		12.44
16	0.7	0.3	8.5		13.98
17	0.8	0.2	9.0		15.60
18	0.9	0.1	9.5		17.28
19	1.0	0.0	10.0		19.00
20	1.1	−0.1	10.5		20.75
21	1.2	−0.2	11.0		22.53
22	Notes:				
23	1. Negative weights indicate short positions.				
24	2. The weights of the minimum-variance portfolio are computed using the formula in Footnote 1.				

set using Spreadsheet 6.5. Columns A and B set out several different proportions for investments in the stock and bond funds. The next columns present the portfolio expected return and standard deviation corresponding to each allocation. These risk-return combinations are plotted in Figure 6.3.

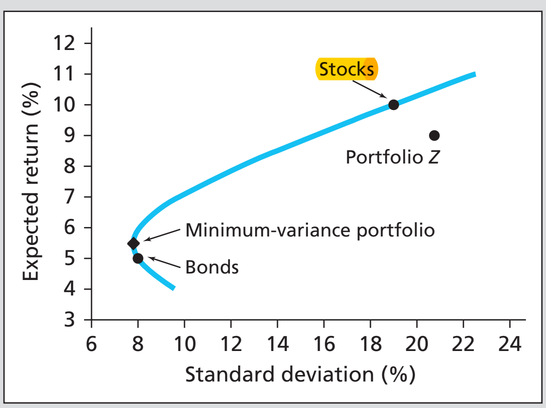
The Mean-Variance Criterion

Investors desire portfolios that lie to the "northwest" in Figure 6.3. These are portfolios with high expected returns (toward the "north" of the figure) and low volatility (to the "west"). These preferences mean that we can compare portfolios using a *mean-variance criterion* in the following way: Portfolio A is said to dominate portfolio B if all investors prefer A over B. This will be the case if it has higher mean return and lower variance or standard deviation:

$$E(r_A) \geq E(r_B) \quad \text{and} \quad \sigma_A \leq \sigma_B$$

FIGURE 6.3

The investment opportunity set with the stock and bond funds



Graphically, when we plot the expected return and standard deviation of each portfolio in Figure 6.3, portfolio A will lie to the northwest of B. Given a choice between portfolios A and B, *all* investors would choose A. For example, the stock fund in Figure 6.3 dominates portfolio Z; the stock fund has higher expected return and lower volatility.

Portfolios that lie below the minimum-variance portfolio in the figure can therefore be rejected out of hand as inefficient. Any portfolio on the downward-sloping portion of the curve (including the bond fund) is "dominated" by the portfolio that lies directly above it on the upward-sloping portion of the curve since that portfolio has higher expected return and equal standard deviation. The best choice among the portfolios on the upward-sloping portion of the curve is not as obvious, because in this region higher expected return is accompanied by greater risk. We will discuss the best choice when we introduce the risk-free asset to the portfolio decision.

So far we have assumed a correlation of .2 between stock and bond returns. We know that low correlations aid diversification and that a higher correlation coefficient results in a reduced effect of diversification. What are the implications of perfect positive correlation between bonds and stocks?

A correlation coefficient of 1 simplifies Equation 6.6 for portfolio variance. Looking at it again, you will see that substitution of $\rho_{BS} = 1$ allows us to "complete the square" of the quantities $w_B\sigma_B$ and $w_S\sigma_S$ to obtain

$$\sigma_P^2 = w_B^2\sigma_B^2 + w_S^2\sigma_S^2 + 2w_B\sigma_B w_S\sigma_S = (w_B\sigma_B + w_S\sigma_S)^2$$
$$\sigma_P = w_B\sigma_B + w_S\sigma_S$$

The portfolio standard deviation is a weighted average of the component security standard deviations only in the special case of perfect positive correlation. In this circumstance, there are no gains to be had from diversification. Both the portfolio mean and the standard deviation are simple weighted averages. Figure 6.4 shows the opportunity set with perfect positive correlation—a straight line through the component securities. No portfolio can be discarded as inefficient in this case, and the choice among portfolios depends only on risk aversion. Diversification in the case of perfect positive correlation is not effective.

Perfect positive correlation is the *only* case in which there is no benefit from diversification. Whenever $\rho < 1$, the portfolio standard deviation is less than the weighted average of the standard deviations of the component securities. Therefore, *there are benefits to diversification whenever asset returns are less than perfectly positively correlated*.

Our analysis has ranged from very attractive diversification benefits ($\rho_{BS} < 0$) to no benefits at all ($\rho_{BS} = 1$). For ρ_{BS} within this range, the benefits will be somewhere in between.

A realistic correlation coefficient between stocks and bonds based on historical experience is actually around .20. The expected returns and standard deviations that we have so far assumed also reflect historical experience, which is why we include a graph for $\rho_{BS} = .2$ in Figure 6.4. Spreadsheet 6.6 enumerates some of the points on the various opportunity sets in Figure 6.4. As the figure illustrates, $\rho_{BS} = .2$ is a lot better for diversification than perfect positive correlation and a bit worse than zero correlation.

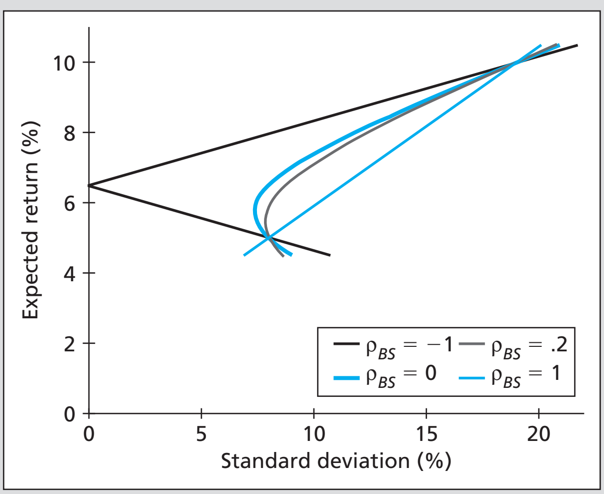


FIGURE 6.4

Investment opportunity sets for bonds and stocks with various correlation coefficients

SPREADSHEET 6.6

Investment opportunity set for stocks and bonds with various correlation coefficients

	A	B	C	D	E	F	G
1		Input Data					
2	$E(r_S)$	$E(r_B)$	σ_S	σ_B			
3	10	5	19	8			
4							
5	Weights in Stocks	Portfolio Expected Return	Portfolio Standard Deviation[1] for Given Correlation, ρ				
6	w_S	$E(r_P)$ = Col A*A3 + (1 − Col A)*B3	−1	0	0.2	0.5	1
7	−0.1	4.5	10.70	9.00	8.62	8.02	6.90
8	0.0	5.0	8.00	8.00	8.00	8.00	8.00
9	0.1	5.5	5.30	7.45	7.81	8.31	9.10
10	0.2	6.0	2.60	7.44	8.07	8.93	10.20
11	0.3	6.5	0.10	7.99	8.75	9.79	11.30
12	0.4	7.0	2.80	8.99	9.77	10.83	12.40
13	0.6	8.0	8.20	11.84	12.44	13.29	14.60
14	0.8	9.0	13.60	15.28	15.60	16.06	16.80
15	1.0	10.0	19.00	19.00	19.00	19.00	19.00
16	1.1	10.5	21.70	20.92	20.75	20.51	20.10
17			Minimum-Variance Portfolio[2,3,4,5]				
18	w_S(min) = (σ_B^2 − $\sigma_B\sigma_S\rho$)/(σ_S^2 + σ_B^2 − 2*$\sigma_B\sigma_S\rho$) =		0.2963	0.1506	0.0923	−0.0440	−0.7273
19	$E(r_P)$ = w_S (min)*A3 + (1 − w_S (min))*B3 =		6.48	5.75	5.46	4.78	1.36
20	σ_P =		0.00	7.37	7.80	7.97	0.00

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Notes:

1. σ_P = SQRT[(Col A*C3)^2 + ((1 − Col A)*D3)^2 + 2*Col A*C3*(1 − Col A)*D3*ρ]
2. The standard deviation is calculated from Equation 6.6 using the weights of the minimum-variance portfolio:

$$\sigma_P = \text{SQRT}[(w_S(\text{min})*\text{C3})\text{^}2 + ((1 - w_S(\text{min}))*\text{D3})\text{^}2 + 2*w_S(\text{min})*\text{C3}*(1 - w_S(\text{min}))*\text{D3}*\rho]$$

3. As the correlation coefficient grows, the minimum-variance portfolio requires a smaller position in stocks (even a negative position for higher correlations), and the performance of this portfolio becomes less attractive.
4. Notice that with correlation of .5 or higher, minimum variance is achieved with a short position in stocks. The standard deviation is lower than that of bonds, but the mean is lower as well.
5. With perfect positive correlation (column G), you can drive the standard deviation to zero by taking a large, short position in stocks. The mean return is then as low as 1.36%.

Negative correlation between a pair of assets is also possible. When correlation is negative, there will be even greater diversification benefits. Again, let us start with the extreme. With perfect negative correlation, we substitute $\rho_{BS} = -1$ in Equation 6.6 and simplify it by completing the square:

$$\sigma_P^2 = (w_B\sigma_B - w_S\sigma_S)^2$$

and, therefore,

$$\sigma_P = \text{ABS}[w_B\sigma_B - w_S\sigma_S] \qquad \textbf{(6.7)}$$

The right-hand side of Equation 6.7 denotes the absolute value of $w_B\sigma_B - w_S\sigma_S$. The solution involves the absolute value because standard deviation cannot be negative.

With perfect negative correlation, the benefits from diversification stretch to the limit. Equation 6.7 yields the proportions that will reduce the portfolio standard deviation all the way to zero.[2] With our data, this will happen when $w_B = 70.37\%$. While exposing us to zero risk, investing 29.63% in stocks (rather than placing all funds in bonds) will still increase the portfolio expected return from 5% to 6.48%. Of course, we can hardly expect results this attractive in reality.

[2]The proportion in bonds that will drive the standard deviation to zero when $\rho = -1$ is

$$w_B = \frac{\sigma_S}{\sigma_B + \sigma_S}$$

Compare this formula to the formula in footnote 1 for the variance-minimizing proportions when $\rho = 0$.

CONCEPT check 6.3

Suppose that for some reason you are *required* to invest 50% of your portfolio in bonds and 50% in stocks. Use the data on mean returns and standard deviations in Spreadsheet 6.5 to answer the following questions.

a. If the standard deviation of your portfolio is 10%, what must be the correlation coefficient between stock and bond returns?

b. What is the expected rate of return on your portfolio?

c. Now suppose that the correlation between stock and bond returns is .22 instead of the value you found in part (*a*) but that you are free to choose whatever portfolio proportions you desire. Are you likely to be better or worse off than you were in part (*a*)?

6.3 THE OPTIMAL RISKY PORTFOLIO WITH A RISK-FREE ASSET

Now we can expand the asset allocation problem to include a risk-free asset. Let us continue to use the input data from Spreadsheet 6.5. Suppose then that we are still confined to the risky bond and stock funds but now can also invest in T-bills yielding 3%. When we add the risk-free asset to a stock-plus-bond risky portfolio, the resulting opportunity set is the straight line that we called the CAL (capital allocation line) in Chapter 5. We now consider various CALs constructed from risk-free bills and a variety of possible risky portfolios, each formed by combining the stock and bond funds in alternative proportions.

We start in Figure 6.5 with the opportunity set of risky assets constructed only from the bond and stock funds. The lowest-variance risky portfolio is labeled MIN (denoting the *minimum-variance portfolio*). CAL_{MIN} is drawn through it and shows the risk-return trade-off with various positions in T-bills and portfolio MIN. It is immediately evident from the figure that we could do better (i.e., obtain a higher Sharpe ratio) by using portfolio A instead of MIN as the risky portfolio. CAL_A dominates CAL_{MIN}, offering a higher expected return for any level of volatility. Spreadsheet 6.6 (see bottom panel of column E) shows that portfolio MIN's expected return is 5.46% and its standard deviation (SD) is 7.80%. Portfolio A (row 10 in Spreadsheet 6.6) offers an expected return of 6% with an SD of 8.07%.

The slope of the CAL is the Sharpe ratio of the risky portfolio, that is, the ratio of excess return to standard deviation:

$$S_P = \frac{E(r_P) - r_f}{\sigma_P} \tag{6.8}$$

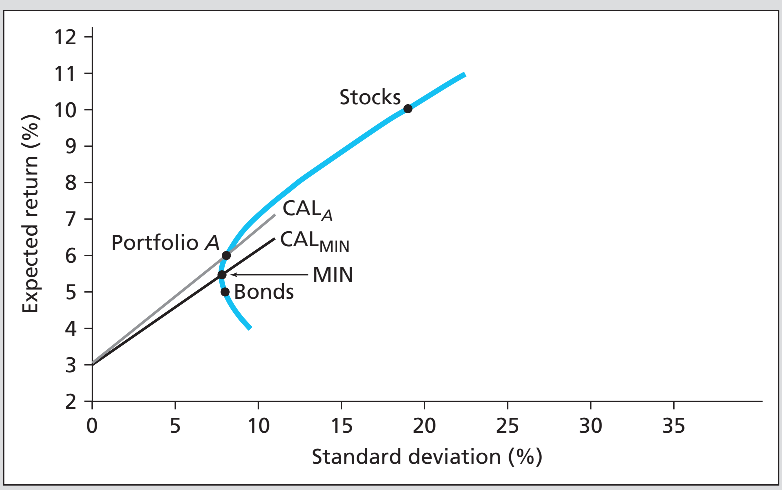
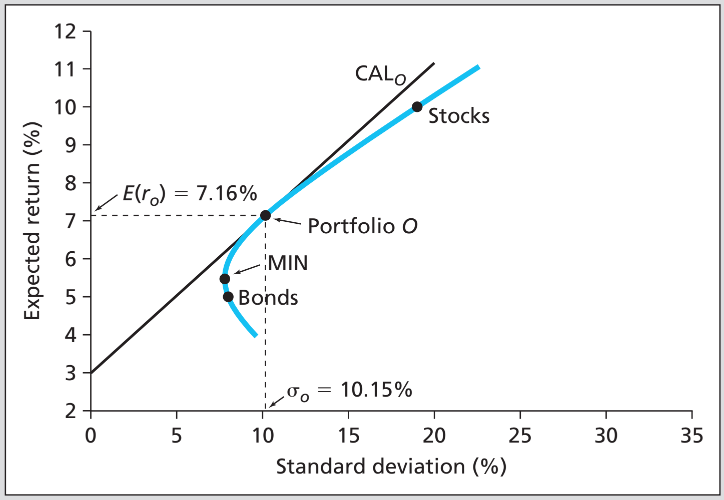


FIGURE 6.5

The opportunity set of stocks, bonds, and a risk-free asset with two capital allocation lines

FIGURE 6.6

The optimal capital allocation line with bonds, stocks, and T-bills



This is the rate at which the investor can increase expected return by accepting higher portfolio standard deviation. With a T-bill rate of 3% we obtain the Sharpe ratio of the two portfolios:

$$S_{\text{MIN}} = \frac{5.46 - 3}{7.80} = .32 \quad S_A = \frac{6 - 3}{8.07} = .37 \tag{6.9}$$

The higher ratio for portfolio A compared to MIN measures the improvement it offers in the risk-return trade-off.

But why stop at portfolio A? We can continue to ratchet the CAL upward until it reaches the ultimate point of tangency with the investment opportunity set. This must yield the CAL with the highest feasible reward-to-volatility (Sharpe) ratio. Therefore, the tangency portfolio (O) in Figure 6.6 is the **optimal risky portfolio** to mix with T-bills, which may be defined as the risky portfolio resulting in the highest possible CAL.

optimal risky portfolio

The best combination of risky assets to be mixed with safe assets to form the complete portfolio.

Figure 6.6 clearly shows the improvement in the risk-return trade-off obtained with CAL$_O$. For any portfolio standard deviation, CAL$_O$ offers a higher expected return than is attainable from the opportunity set constructed only from the risky bond and stock funds.

To find the composition of the optimal risky portfolio, O, we search for weights in the stock and bond funds that maximize the portfolio's Sharpe ratio. With only two risky assets, we can solve for the optimal portfolio weights using the following formula:

$$w_B = \frac{[E(r_B) - r_f]\sigma_S^2 - [E(r_S) - r_f]\sigma_B\sigma_S\rho_{BS}}{[E(r_B) - r_f]\sigma_S^2 + [E(r_S) - r_f]\sigma_B^2 - [E(r_B) - r_f + E(r_S) - r_f]\sigma_B\sigma_S\rho_{BS}}$$

$$w_S = 1 - w_B \tag{6.10}$$

Using the risk premiums (expected excess return over the risk-free rate) of the stock and bond funds, their standard deviations, and the correlation between their returns in Equation 6.10, we find that the weights of the optimal portfolio are $w_B(O) = .568$ and $w_S(O) = .432$. Using these weights, Equations 6.5, 6.6, and 6.8 imply that $E(r_O) = 7.16\%$, $\sigma_O = 10.15\%$, and therefore the Sharpe ratio of the optimal portfolio (the slope of its CAL) is

$$S_O = \frac{E(r_O) - r_f}{\sigma_O} = \frac{7.16 - 3}{10.15} = .41$$

This Sharpe ratio is significantly higher than those provided by either the bond or stock portfolios alone.

In the last chapter we saw that the preferred *complete* portfolio formed from a risky portfolio and a risk-free asset depends on the investor's risk aversion. More risk-averse investors prefer low-risk portfolios despite the lower expected return, while more risk-tolerant investors choose higher-risk, higher-return portfolios. Both investors, however, will choose portfolio *O* as their risky portfolio since it results in the highest return per unit of risk, that is, the steepest capital allocation line. Investors will differ only in their allocation of investment funds between portfolio *O* and the risk-free asset.

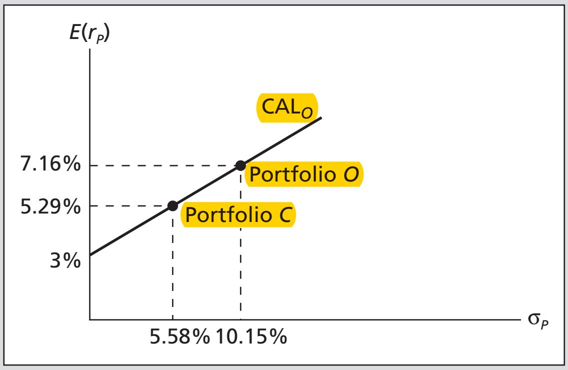


FIGURE 6.7

The complete portfolio

Figure 6.7 shows one possible choice for the preferred complete portfolio, *C*. The investor places 55% of wealth in portfolio *O* and 45% in Treasury bills. The rate of return and volatility of the portfolio are

$$E(r_C) = 3 + .55 \times (7.16 - 3) = 5.29\%$$

$$\sigma_C = .55 \times 10.15 = 5.58\%$$

We found above that the optimal risky portfolio *O* is formed by mixing the bond fund and stock fund with weights of 56.8% and 43.2%. Therefore, the overall asset allocation of the *complete* portfolio is as follows:

Weight in risk-free asset		45.00%
Weight in bond fund	.568 × 55% =	31.24
Weight in stock fund	.432 × 55% =	23.76
Total		100.00%

Figure 6.8 depicts the overall asset allocation. The allocation reflects considerations of both efficient diversification (the construction of the optimal risky portfolio, *O*) and risk aversion (the allocation of funds between the risk-free asset and the risky portfolio *O* to form the complete portfolio, *C*).

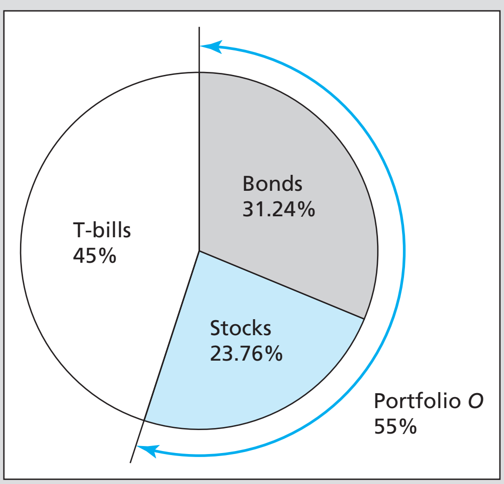


FIGURE 6.8

The composition of the complete portfolio: The solution to the asset allocation problem

CONCEPT check 6.4

A universe of securities includes a risky stock (*X*), a stock-index fund (*M*), and T-bills. The data for the universe are:

	Expected Return	Standard Deviation
X	15%	50%
M	10	20
T-bills	5	0

The correlation coefficient between *X* and *M* is −.2.

a. Draw the opportunity set of securities *X* and *M.*
b. Find the optimal risky portfolio (*O*), its expected return, standard deviation, and Sharpe ratio. Compare with the Sharpe ratio of *X* and *M.*
c. Find the slope of the CAL generated by T-bills and portfolio *O.*
d. Suppose an investor places 2/9 (i.e., 22.22%) of the complete portfolio in the risky portfolio *O* and the remainder in T-bills. Calculate the composition of the complete portfolio, its expected return, SD, and Sharpe ratio.

6.4 EFFICIENT DIVERSIFICATION WITH MANY RISKY ASSETS

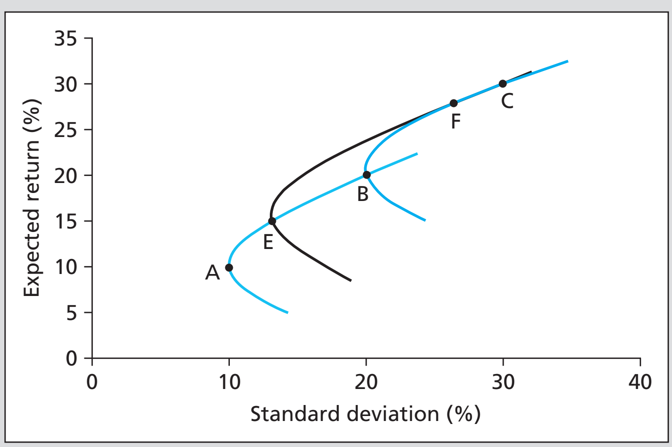
We extend the two-risky-assets portfolio methodology to the case of many risky assets and a risk-free asset in three steps. First, we extend the two-risky-assets opportunity set to many assets. Next we determine the optimal risky portfolio that supports the steepest CAL, that is, maximizes its Sharpe ratio. Finally, we choose a complete portfolio on CAL_O based on the investor's risk aversion by mixing the risk-free asset with the optimal risky portfolio.

The Efficient Frontier of Risky Assets

To get a sense of how additional risky assets can improve investment opportunities, look at Figure 6.9. Points *A, B,* and *C* represent the expected returns and standard deviations of three stocks. The curve passing through *A* and *B* shows the risk-return combinations of portfolios formed from those two stocks. Similarly, the curve passing through *B* and *C* shows portfolios formed from those two stocks. Now observe point *E* on the *AB* curve and point *F* on the *BC* curve. These points represent two portfolios chosen from the set of *AB* and *BC* combinations. The curve that passes through *E* and *F* in turn represents portfolios constructed from portfolios *E* and *F.* Since *E* and *F* are themselves constructed from *A, B,* and *C,*

FIGURE 6.9

Portfolios constructed with three stocks (A, B, and C)



this curve shows some of the portfolios constructed from these *three* stocks. Notice that curve *EF* extends the investment opportunity set to the northwest, which is the desired direction.

Now we can continue to take other points (each representing portfolios) from these three curves and further combine them into new portfolios, thus shifting the opportunity set even farther to the northwest. You can see that this process would work even better with more stocks. Moreover, the boundary or "envelope" of all the curves thus developed, will lie quite away from the individual stocks in the northwesterly direction, as shown in Figure 6.10.

The analytical technique to derive the efficient set of risky assets was developed by Harry Markowitz in 1951 and ultimately earned him the Nobel Prize in Economics. We sketch his approach here.

First, we determine the risk-return opportunity set. The aim is to construct the northwesternmost portfolios in terms of expected return and standard deviation from the universe of securities. The inputs are the expected returns and standard deviations of each asset in the universe, along with the correlation coefficients between each pair of assets. These data come from security analysis, to be discussed in Part Four. The graph that connects all the northwesternmost portfolios is called the **efficient frontier** of risky assets. It represents the set of portfolios that offers the highest possible expected rate of return for each level of portfolio standard deviation. These portfolios may be viewed as efficiently diversified. One such frontier is shown in Figure 6.10.

efficient frontier

Graph representing a set of portfolios that maximizes expected return at each level of portfolio risk.

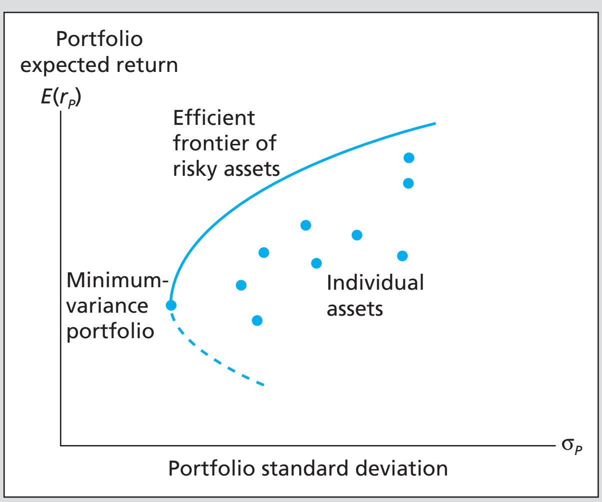
There are three ways to produce the efficient frontier. We will sketch each in a way that allows you to participate and gain insight into the logic and mechanics of the efficient frontier: Please take a pencil and paper and draw the graph as you follow along with our discussion. For each method, first draw the horizontal axis for portfolio standard deviation and the vertical axis for risk premium. We focus on the risk premium (expected excess returns), R, rather than total returns, r, so that the risk-free asset will lie at the origin (with zero SD and zero risk premium). We begin with the minimum-variance portfolio—mark it as point G (for *global* minimum variance). Imagine that G's coordinates are .10 (SD = 10%) and .03 (risk premium = 3%); this is your first point on the efficient frontier. Later, we add detail about how to find these coordinates.

The three ways to draw the efficient frontier are (1) maximize the risk premium for any level of SD; (2) minimize the SD for any level of risk premium; and (3) maximize the Sharpe ratio for any level of SD (or risk premium).

For the first method, maximizing the risk premium for any level of SD, draw a few vertical lines to the right of G (there can be no portfolio with SD less than G's). Choose the vertical line drawn at SD = 12%; we therefore search for the portfolio with the highest possible expected return consistent with an SD of 12%. So we give the computer an assignment to maximize the risk premium subject to two constraints: (i) The portfolio weights sum to 1

FIGURE 6.10

The efficient frontier of risky assets and individual assets



(this is called the *feasibility* constraint, since any legitimate portfolio must have weights that sum to 1), and (ii) the portfolio SD must match the constraint value, $\sigma = .12$. The optimization software searches over all portfolios with $\sigma = .12$ and finds the highest feasible portfolio on the vertical line drawn at $\sigma = .12$; this is the portfolio with the highest risk premium. Assume that for this portfolio $R = .04$. You now have your second point on the efficient frontier. Do the same for other vertical lines to the right of .12, and when you "connect the dots," you will have drawn a frontier like that in Figure 6.10.

The second method is to minimize the SD for any level of risk premium. Here, you need to draw a few horizontal lines above *G* (portfolios lying below *G* are inefficient because they offer a *lower* risk premium and *higher* variance than *G*). Draw the first horizontal line at $R = .04$. Now the computer's assignment is to minimize the SD subject to the usual feasibility constraint. But in this method, we replace the constraint on SD by one on the portfolio's risk premium ($R = .04$). Now the computer seeks the portfolio that is farthest to the left along the horizontal line—this is the portfolio with the lowest SD consistent with a risk premium of 4%. You already know that this portfolio must be at $\sigma = .12$, since the first point on the efficient frontier that you found using method 1 was $(\sigma, R) = (.12, .04)$. Repeat this approach using other risk premiums, and you will find other points along the efficient frontier. Again, connect the dots and you will have the frontier of Figure 6.10.

The third approach to forming the efficient frontier, maximizing the Sharpe ratio for any SD or risk premium, is easiest to visualize by revisiting Figure 6.5. Observe that each portfolio on the efficient frontier provides the highest Sharpe ratio, the slope of a ray from the risk-free rate, for any choice of SD or expected return. Let's start by specifying the SD constraint, achieved by using the vertical lines to the right of *G.* To each line, we draw rays from the origin at ever-increasing slopes, and we assign the computer to find the *feasible* portfolio with the highest slope. This is similar to sliding up the vertical line to find the highest risk premium. We must find the same frontier as that found with either of the first two methods. Similarly, we could instead specify a risk-premium constraint and construct rays from the origin to horizontal lines. We assign the computer to find the *feasible* portfolio with the highest slope to the given horizontal line. This is similar to sliding to the left on horizontal lines in method 2.

We started the efficient frontier from the minimum-variance portfolio, *G. G* is found with a program that minimizes SD subject *only* to the feasibility constraint. This portfolio has the lowest SD for *any* risk premium, which is why it is called the "global" minimum-variance portfolio. By the same principle, the optimal portfolio, *O,* will maximize the Sharpe ratio globally, subject only to the feasibility constraint. Any *individual* asset ends up inside the efficient frontier, because single-asset portfolios are inefficient—they are not efficiently diversified.

Various constraints may preclude a particular investor from choosing portfolios on the efficient frontier, however. If an institution is prohibited by law from taking short positions in any asset, for example, the portfolio manager must add constraints to the computer-optimization program that rule out negative (short) positions.

Short-sale restrictions are only one possible constraint. Some clients may want to ensure a minimum level of expected dividend yield. In this case, input data must include a set of expected dividend yields. The optimization program is made to include a constraint to ensure that the expected *portfolio* dividend yield will equal or exceed the desired level. Another common constraint forbids investments in companies engaged in "undesirable social activity." This constraint implies that portfolio weights in these companies must equal zero.

In principle, portfolio managers can tailor an efficient frontier to meet any particular objective. Of course, satisfying constraints carries a price tag. An efficient frontier subject to additional constraints will offer a lower reward-to-volatility (Sharpe) ratio. Clients should be aware of this cost and may want to think twice about constraints that are not mandated by law.

Deriving the efficient frontier and graphing it with any number of assets and any set of constraints is quite straightforward. For a not-too-large number of assets, the efficient frontier can be computed and graphed even with a spreadsheet program.

The spreadsheet program, available at **www.mhhe.com/bkm,** can easily incorporate restrictions against short sales imposed on some portfolio managers. To impose this restriction, the program simply requires that each weight in the optimal portfolio be greater than or equal to zero. One way to see whether the short-sale constraint actually matters is to find the efficient portfolio without it. If one or more of the weights in the optimal portfolio turn out negative, we know the short-sale restrictions will result in a different efficient frontier with a less attractive risk-return trade-off.

Choosing the Optimal Risky Portfolio

The second step of the optimization plan involves the risk-free asset. Using the current risk-free rate, we search for the capital allocation line with the highest Sharpe ratio (the steepest slope), as shown in Figures 6.5 and 6.6.

The CAL formed from the optimal risky portfolio (*O*) will be tangent to the efficient frontier of risky assets discussed above. This CAL dominates all feasible CALs. Portfolio *O*, therefore, is the optimal risky portfolio. Because we know that an investor will choose a point on the CAL that mixes the *optimal* risky portfolio with T-bills, there is actually no need to either provide access to or derive the entire efficient frontier. Therefore, as a practical matter, rather than solving for the entire efficient frontier, we can proceed directly to determining the optimal portfolio. This requires maximizing the Sharpe ratio subject only to the feasibility constraint. The "global" maximum-Sharpe-ratio portfolio is the optimal portfolio *O*. The ray from the origin to *O* and beyond is the optimal CAL.

The Preferred Complete Portfolio and the Separation Property

Finally, in the third step, the investor chooses the appropriate mix between the optimal risky portfolio (*O*) and T-bills, exactly as in Figure 6.7.

A portfolio manager will offer the same risky portfolio (*O*) to all clients, no matter what their degrees of risk aversion. Risk aversion comes into play only when clients select their desired point on the CAL. More risk-averse clients will invest more in the risk-free asset and less in the optimal risky portfolio *O* than less risk-averse clients, but both will use portfolio *O* as the optimal risky investment vehicle.

This result is called a **separation property,** introduced by James Tobin (1958), the 1983 Nobel Laureate for Economics: It implies that portfolio choice can be separated into two independent tasks. The first task, to determine the optimal risky portfolio (*O*), is purely technical. Given a particular set of input data, the best risky portfolio is the same for all clients regardless of risk aversion. The second task, construction of the complete portfolio from bills and portfolio *O*, is personal and depends on risk aversion. Here the client is the decision maker.

separation property

The property that implies portfolio choice can be separated into two independent tasks: (1) determination of the optimal risky portfolio, which is a purely technical problem, and (2) the personal choice of the best mix of the risky portfolio and the risk-free asset.

Optimal risky portfolios for different clients may vary because of constraints on short sales, dividend yield, tax considerations, or other client preferences. Our analysis, though, suggests that a few portfolios may be sufficient to serve the demands of a wide range of investors. We see here the theoretical basis of the mutual fund industry. If the optimal portfolio is the same for all clients, professional management is more efficient and less costly. One management firm can serve a number of clients with relatively small incremental administrative costs.

The (computerized) optimization technique is the easiest part of portfolio construction. When different managers use different input data, they will develop different efficient frontiers and offer different "optimal" portfolios. Therefore, the real arena of the competition among portfolio managers is in the sophisticated security analysis that produces the input estimates. The rule of GIGO (garbage in–garbage out) applies fully to portfolio selection. If the quality of the security analysis is poor, a passive portfolio such as a market-index fund will yield better results than an active portfolio tilted toward *seemingly* favorable securities.

Constructing the Optimal Risky Portfolio: An Illustration

To illustrate how the optimal risky portfolio might be constructed, suppose an analyst wished to construct an efficiently diversified global portfolio using the stock market indices of six countries. The top panel of Table 6.1 shows the input list. The values for standard deviations and the correlation matrix are estimated from recent historical data, while forecasts of risk premiums are generated from fundamental analysis. Examination of the table shows the U.S. index portfolio has the highest Sharpe ratio. China and Japan have the lowest, and the correlation of

TABLE 6.1 Efficient frontiers for international diversification with and without short sales and CAL with short sales

A. Input list

	Excess Returns			
	Mean	**SD**	**Sharpe Ratio**	**INPUT LIST**
U.S.	0.0600	0.1495	0.4013	**Expected excess returns from fundamental analysis.**
U.K.	0.0530	0.1493	0.3551	**Standard deviations and correlation matrix from**
FRANCE	0.0680	0.2008	0.3386	**econometric estimates.**
GERMANY	0.0800	0.2270	0.3525	
JAPAN	0.0450	0.1878	0.2397	
CHINA	0.0730	0.3004	0.2430	

Correlation Matrix

	U.S.	**U.K.**	**France**	**Germany**	**Japan**	**China**
U.S.	1					
U.K.	0.83	1				
FRANCE	0.83	0.92	1			
GERMANY	0.85	0.88	0.96	1		
JAPAN	0.43	0.44	0.47	0.43	1	
CHINA	0.16	0.28	0.26	0.29	0.14	1

B. Efficient frontier—short sales allowed

Portfolio:	**(1)**	**(2)**	**G**	**(4)**	**(5)**	**(6)**	**(7)**	**O**	**(9)**	**(10)**	**(11)**	**(12)**	**(13)**
Risk premium	0.0325	0.0375	0.0410	0.0425	0.0450	0.0500	0.0550	0.058474	0.0600	0.0650	0.0700	0.0800	0.0850
SD	0.1147	0.1103	0.1094	0.1095	0.1106	0.1154	0.1234	0.130601	0.1341	0.1469	0.1612	0.1933	0.2104
Slope (Sharpe)	0.2832	0.3400	0.3749	0.3880	0.4070	0.4334	0.4457	0.447733	0.4474	0.4425	0.4341	0.4140	0.4040
Portfolio weights													
U.S.	0.5948	0.6268	0.6476	0.6569	0.6724	0.7033	0.7342	0.755643	0.7651	0.7960	0.8269	0.8887	0.9196
U.K.	1.0667	0.8878	0.7681	0.7155	0.6279	0.4527	0.2775	0.155808	0.1023	−0.0728	−0.2480	−0.5984	−0.7736
FRANCE	−0.1014	−0.1308	−0.1618	−0.1727	−0.1908	−0.2272	−0.2635	−0.2888	−0.2999	−0.3362	−0.3725	−0.4452	−0.4816
GERMANY	−0.8424	−0.6702	−0.5431	−0.4901	−0.4019	−0.2253	−0.0487	0.0740	0.1278	0.3044	0.4810	0.8341	1.0107
JAPAN	0.2158	0.1985	0.1866	0.1815	0.1729	0.1558	0.1386	0.126709	0.1215	0.1043	0.0872	0.0529	0.0357
CHINA	0.0664	0.0879	0.1025	0.1089	0.1195	0.1407	0.1619	0.176649	0.1831	0.2043	0.2256	0.2680	0.2892

C. Capital allocation line (CAL) with short sales

Risk premium	0.0000	0.0494	0.0490	0.0490	0.0495	0.0517	0.0553	0.0585	0.0600	0.0658	0.0722	0.0865	0.1343
SD	0.0000	0.1103	0.1094	0.1095	0.1106	0.1154	0.1234	0.1306	0.1341	0.1469	0.1612	0.1933	0.3000

D. Efficient frontier—no short sales

Portfolio	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**Min Var**	**(7)**	**(8)**	**Optimum**	**(10)**	**(11)**	**(12)**	**(13)**
Risk premium	0.0450	0.0475	0.0490	0.0510	0.0535	0.0560	0.0573	0.0590	0.0607	0.0650	0.07	0.0750	0.0800
SD	0.1878	0.1555	0.1435	0.1372	0.1330	0.131648	0.1321	0.1337	0.1367	0.1493	0.1675332	0.1893	0.2270
Slope (Sharpe)	0.2397	0.3055	0.3414	0.3718	0.4022	0.425089	0.4339	0.4411	0.4439	0.4353	0.4178277	0.3963	0.3525
Portfolio weights													
U.S.	0.0000	0.0000	0.0000	0.0671	0.2375	0.4052	0.4964	0.6122	0.7067	0.6367	0.4223	0.1680	0.0000
U.K.	0.0000	0.3125	0.5000	0.5465	0.3967	0.2491	0.1689	0.0670	0.0000	0.0000	0.0000	0.0000	0.0000
FRANCE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
GERMANY	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1324	0.3558	0.5976	1.0000
JAPAN	1.0000	0.6875	0.5000	0.3642	0.3029	0.2424	0.2096	0.1679	0.1114	0.0232	0.0000	0.0000	0.0000
CHINA	0.0000	0.0000	0.0000	0.0222	0.0630	0.1032	0.1251	0.1529	0.1819	0.2077	0.2219	0.2343	0.0000

France and Germany with the U.S. is high. Given these data, one might be tempted to conclude that, *perhaps,* U.S. investors may not benefit much from international diversification during this period. But even in this sample period, we will see that diversification is beneficial.

Panel B shows the efficient frontier developed as follows: First we generate the global minimum-variance portfolio *G* by minimizing the SD with just the feasibility constraint, and then we find portfolio *O* by maximizing the Sharpe ratio subject only to the same constraint. To fill out the curve, we choose more risk premiums; for each, we maximize the Sharpe ratio subject to the feasibility constraint as well as the appropriate risk-premium constraint. In all, we have 13 points to draw the graph in Figure 6.11, one of which is the global maximum-Sharpe-ratio portfolio, *O*.

The results are quite striking. Observe that the SD of the global minimum-variance portfolio of 10.94% is far lower than that of the lowest-variance country (the U.K.), which has an SD of 14.93%. *G* is formed by taking short positions in Germany and France, as well as a large position in the relatively low-risk U.K. Moreover, the Sharpe ratio of this portfolio is higher than that of all countries but the U.S! Still, even this portfolio will be rejected in favor of the highest Sharpe-ratio portfolio.

Portfolio *O* attains a Sharpe ratio of .4477, compared to the U.S. ratio of .4013, a significant improvement that can be verified from the CAL shown in Panel C. The points shown on the CAL have the same SD as those on the efficient frontier portfolios, so the risk premium for each equals the SD times the Sharpe ratio of portfolio *O*.[3] Notice that portfolio (9) on the CAL has the same risk premium as the U.S., 6%, but an SD of 13.41%, fully 1.5% less than the 14.95% SD of the U.S. All this is achieved while still investing 76% of the portfolio in the U.S., although it does require a large short position in France (−29.99%).

Many institutional investors are prohibited from taking short positions, and individuals may be averse to large short positions because the unlimited upward potential of stock prices implies unlimited potential losses on short sales. Panel D shows the efficient frontier when an additional constraint is applied to each portfolio, namely, that all weights must be nonnegative.

Take a look at the two frontiers in Figure 6.11. The no-short-sale frontier is clearly inferior on both ends. This is because both very low-return and very high-return frontier portfolios will typically entail short positions. At the low-return/low-volatility end of the frontier,

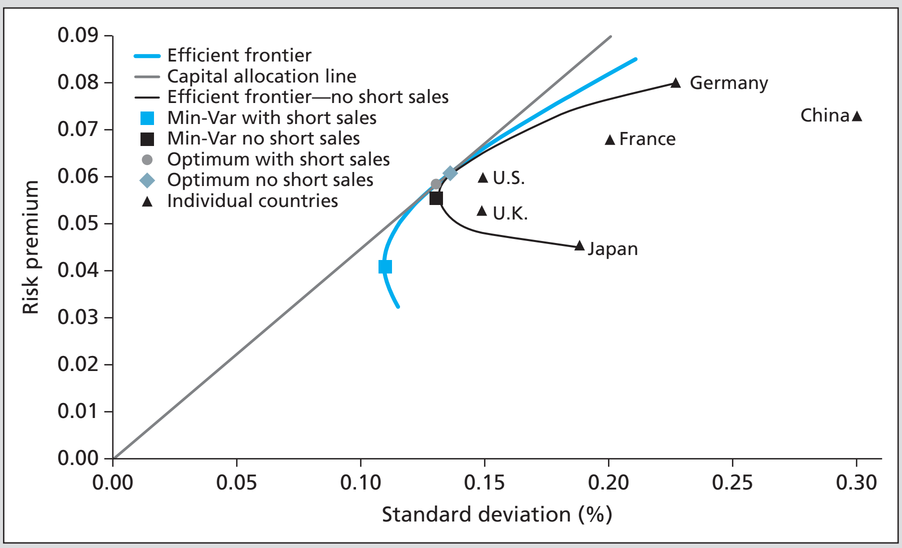


FIGURE 6.11

Efficient frontiers and CAL from Table 6.1

[3]Since the Sharpe ratio is S = risk premium/SD, we can rearrange to show that SD = risk premium/S_O. The CAL has the same slope everywhere, equal to the Sharpe ratio of portfolio *O* that supports it.

portfolios have short positions in stocks with a high correlation and low risk premium that reduce variance at low cost to expected return. At the other (high expected return) end of the frontier, we find short positions in low-risk-premium stocks in favor of larger positions in high-risk-premium stocks. At the same time, the no-short-sale frontier is restricted to begin with the lowest-risk-premium country (Japan) and end with the highest (Germany). Without short sales, we cannot achieve lower or higher risk premiums than are offered by these portfolios. Intermediate-return portfolios on each frontier, including the optimal portfolio, *O,* are not far apart. Thus, even under the no-short-sale constraint, the Sharpe ratio (.4439) is still higher than that of the U.S. portfolio. The no-short-sale CAL can match the U.S. risk premium of 6% with an SD of only 13.52%, still 1.4% less than the SD of the U.S.

CONCEPT check 6.5

Two portfolio managers work for competing investment management houses. Each employs security analysts to prepare input data for the construction of the optimal portfolio. When all is completed, the efficient frontier obtained by manager A dominates that of manager B in that A's optimal risky portfolio lies northwest of B's. Is the more attractive efficient frontier asserted by manager A evidence that she really employs better security analysts?

6.5 A SINGLE-INDEX STOCK MARKET

index model

Model that relates stock returns to returns on both a broad market index and firm-specific factors.

excess return

Rate of return in excess of the risk-free rate.

beta

The sensitivity of a security's returns to the market factor.

We started this chapter with the distinction between systematic and firm-specific risk. Systematic risk is macroeconomic, affecting all securities, while firm-specific risk factors affect only one particular firm or, at most, a cluster of firms. **Index models** are statistical models designed to estimate these two components of risk for a particular security or portfolio. The first to use an index model to explain the benefits of diversification was another Nobel Prize winner, William F. Sharpe (1963). We will introduce his major work (the capital asset pricing model) in the next chapter.

The popularity of index models is due to their practicality. To construct the efficient frontier from a universe of 100 securities, we would need to estimate 100 expected returns, 100 variances, and $100 \times 99/2 = 4{,}950$ covariances. And a universe of 100 securities is actually quite small. A universe of 1,000 securities would require estimates of $1{,}000 \times 999/2 = 499{,}500$ covariances, as well as 1,000 expected returns and variances. Assuming that one common factor is responsible for all the covariability of stock returns, with all other variability due to firm-specific factors, dramatically simplifies the analysis.

Let us use R_i to denote the **excess return** on a security, that is, the rate of return in excess of the risk-free rate: $R_i = r_i - r_f$. Then we can express the distinction between macroeconomic and firm-specific factors by decomposing this excess return in some holding period into three components:[4]

$$R_i = \beta_i R_M + e_i + \alpha_i \tag{6.11}$$

The first two terms on the right-hand side of Equation 6.11 reflect the impact of two sources of uncertainty. R_M is the excess return on a broad market index (the S&P 500 is commonly used for this purpose), so variation in this term reflects the influence of economywide or macroeconomic events that generally affect all stocks to greater or lesser degrees. The security's **beta,** β_i, is the typical response of that particular stock's excess return to changes in the market index's excess return. As such, beta measures a stock's comparative sensitivity to macroeconomic news. A value greater than 1 would indicate a stock with greater sensitivity to the

[4]Equation 6.11 is surprisingly simple and would appear to require very strong assumptions about security market equilibrium. But, in fact, if rates of return are normally distributed, then returns will be linear in one or more indexes. Statistics theory tells us that, when rates of return on a set of securities are *joint-normally* distributed, then the rate of return on each asset is linear in one identical index as in Equation 6.11. When rates of return exhibit a multivariate normal distribution, we can use a multi-index generalization of Equation 6.11. Practitioners employ index models such as 6.11 extensively because of the ease of use as we just noted, but they would not do so unless empirical evidence supported them.

economy than the average stock. These are known as *cyclical stocks.* Betas less than 1 indicate below-average sensitivity and therefore are known as *defensive stocks.* Recall that the risk attributable to the stock's exposure to uncertain market returns is called market or *systematic* risk, because it relates to the uncertainty that pervades the whole economic system.

The term e_i in Equation 6.11 represents the impact of **firm-specific** or **residual risk.** The expected value of e_i is zero, as the impact of unexpected events must average out to zero. Both residual risk and systematic risk contribute to the total volatility of returns.

firm-specific or **residual risk**

Component of return variance that is independent of the market factor.

The term α_i in Equation 6.11 is not a risk measure. Instead, α_i represents the expected return on the stock *beyond* any return induced by movements in the market index. This term is called the security **alpha.** A positive alpha is attractive to investors and suggests an underpriced security: Among securities with identical sensitivity (beta) to the market index, securities with higher alpha values will offer higher expected returns. Conversely, stocks with negative alphas are apparently overpriced; for any value of beta, they offer lower expected returns.

alpha

A stock's expected return beyond that induced by the market index; its expected excess return when the market's excess return is zero.

In sum, the index model separates the realized rate of return on a security into macro (systematic) and micro (firm-specific) components. The excess rate of return on each security is the sum of three components:

	Symbol
1. The component of return due to movements in the overall market (as represented by the index R_M); β_i is the security's responsiveness to the market.	$\beta_i R_M$
2. The component attributable to unexpected events that are relevant only to this security (firm-specific).	e_i
3. The stock's expected excess return if the market factor is neutral, that is, if the market-index excess return is zero.	α_i

Because the firm-specific component of the stock return is uncorrelated with the market return, we can write the variance of the excess return of the stock as[5]

$$\begin{aligned}\text{Variance}(R_i) &= \text{Variance}(\alpha_i + \beta_i R_M + e_i)\\ &= \text{Variance}(\beta_i R_M) + \text{Variance}(e_i)\\ &= \beta_i^2\sigma_M^2 + \sigma^2(e_i)\\ &= \text{Systematic risk} + \text{Firm-specific risk}\end{aligned} \tag{6.12}$$

Therefore, the total variance of the rate of return of each security is a sum of two components:

1. The variance attributable to the uncertainty of the entire market. This variance depends on both the variance of R_M, σ_M^2, *and* the beta of the stock on R_M.
2. The variance of the firm-specific return, e_i, which is independent of market performance.

This single-index model is convenient. It relates security returns to a market index that investors follow. Moreover, as we soon shall see, its usefulness goes beyond mere convenience.

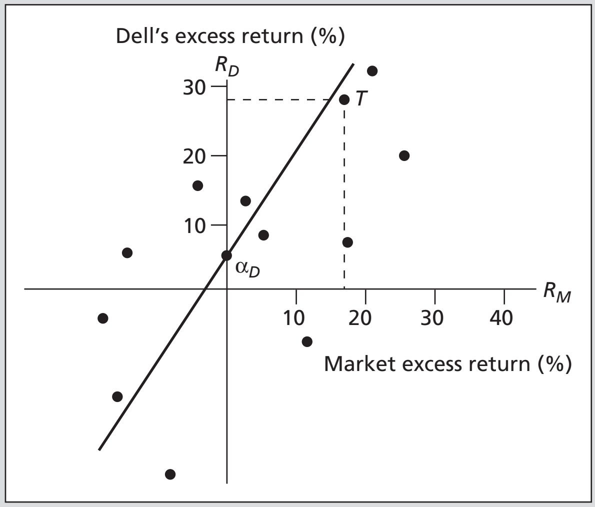
Statistical and Graphical Representation of the Single-Index Model

Equation 6.11, $R_i = \alpha_i + \beta_i R_M + e_i$, may be interpreted as a single-variable *regression equation* of R_i on the market excess return R_M. The excess return on the security (R_i) is the dependent variable that is to be explained by the regression. On the right-hand side of the equation are the intercept α_i; the regression (slope) coefficient beta, β_i, multiplying the independent (explanatory) variable R_M; and the residual (unexplained) return, e_i. We plot this regression in Figure 6.12, which shows a scatter diagram for Dell's excess return against the excess return of the market index.

[5]Notice that because α_i is a constant, it has no bearing on the variance of R_i.

FIGURE 6.12

Scatter diagram for Dell



The horizontal axis of the scatter diagram measures the explanatory variable, here the market excess return, R_M. The vertical axis measures the dependent variable, here Dell's excess return, R_D. Each point on the scatter diagram represents a sample pair of returns (R_M, R_D) observed over a particular holding period. Point T, for instance, describes a holding period when the excess return was 17% for the market index and 27% for Dell.

Regression analysis uses a sample of historical returns to estimate the coefficients (alpha and beta) of the index model. The analysis finds the regression line, shown in Figure 6.12, that minimizes the sum of the squared deviations around it. Hence, we say the regression line "best fits" the data in the scatter diagram. The line is called the **security characteristic line,** or SCL.

security characteristic line

Plot of a security's *predicted* excess return from the excess return of the market.

The regression intercept (α_D) is measured from the origin to the intersection of the regression line with the vertical axis. Any point on the vertical axis represents zero market excess return, so the intercept gives us the *expected excess* return on Dell when market return was "neutral," that is, equal to the T-bill return. The intercept in Figure 6.12 is 4.5%.

The slope of the regression line, the ratio of the rise to the run, is called the *regression coefficient* or simply the beta. In Figure 6.12, Dell's beta is 1.4. A stock beta measures systematic risk since it predicts the response of the security to each extra 1% return on the market index.

The regression line does not represent *actual* returns; points on the scatter diagram almost never lie exactly on the regression line. Rather, the line represents average tendencies; it shows the *expectation* of R_D given the market excess return, R_M. The algebraic representation of the regression line is

$$E(R_D|R_M) = \alpha_D + \beta_D R_M \tag{6.13}$$

which reads: The expectation of R_D *given* a value of R_M equals the intercept plus the slope coefficient times the value of R_M.

Because the regression line represents expectations and these expectations may not be realized (as the scatter diagram shows), the *actual* returns also include a residual, e_i. This surprise (at point T, for example) is measured by the vertical distance between the point of the scatter diagram and the regression line. The expected return on Dell, given a market return of 17%, would have been 4.5% + 1.4 × 17% = 28.3%. The actual return was only 27%, so point T falls below the regression line by 1.3%.

Equation 6.12 shows that the greater the beta of a security, that is, the greater the slope of the regression, the greater the systematic risk and total variance. Because the market is composed of all securities, the typical response to a market movement must be one for one. An "aggressive" investment will have a beta higher than 1; that is, the security has above-average market risk.[6] Conversely, securities with betas lower than 1 are called defensive.

A security may have a negative beta. Its regression line will then slope downward, meaning that, for more favorable macro events (higher R_M), we would expect a *lower* return, and vice versa. The latter means that when the macro economy goes bad (negative R_M) and securities with positive beta are expected to have negative excess returns, the negative-beta security will shine. The result is that a negative-beta security provides a hedge against systematic risk.

The dispersion of the scatter of actual returns about the regression line is determined by the residual variance $\sigma^2(e_D)$. The magnitude of firm-specific risk varies across securities. One way to measure the relative importance of systematic risk is to measure the ratio of systematic variance to total variance.

$$\begin{aligned}\rho^2 &= \frac{\text{Systematic (or explained) variance}}{\text{Total variance}} \\ &= \frac{\beta_D^2\sigma_M^2}{\sigma_D^2} = \frac{\beta_D^2\sigma_M^2}{\beta_D^2\sigma_M^2 + \sigma^2(e_D)}\end{aligned} \tag{6.14}$$

where ρ is the correlation coefficient between R_D and R_M. Its square measures the ratio of explained variance to total variance, that is, the proportion of total variance that can be attributed to market fluctuations. But if beta is negative, so is the correlation coefficient, an indication that the explanatory and dependent variables are expected to move in opposite directions.

At the extreme, when the correlation coefficient is either 1 or -1, the security return is fully explained by the market return and there are no firm-specific effects. All the points of the scatter diagram will lie exactly on the line. This is called *perfect correlation* (either positive or negative); the return on the security is perfectly predictable from the market return. A large correlation coefficient (in absolute value terms) means systematic variance dominates the total variance; that is, firm-specific variance is relatively unimportant. When the correlation coefficient is small (in absolute value terms), the market factor plays a relatively unimportant part in explaining the variance of the asset, and firm-specific factors dominate.

CONCEPT check 6.6

Interpret the eight scatter diagrams of Figure 6.13 in terms of systematic risk, diversifiable risk, and the intercept.

Example 6.3 on the following page illustrates how you can use a spreadsheet to estimate the single-index model from historical data.

Diversification in a Single-Index Security Market

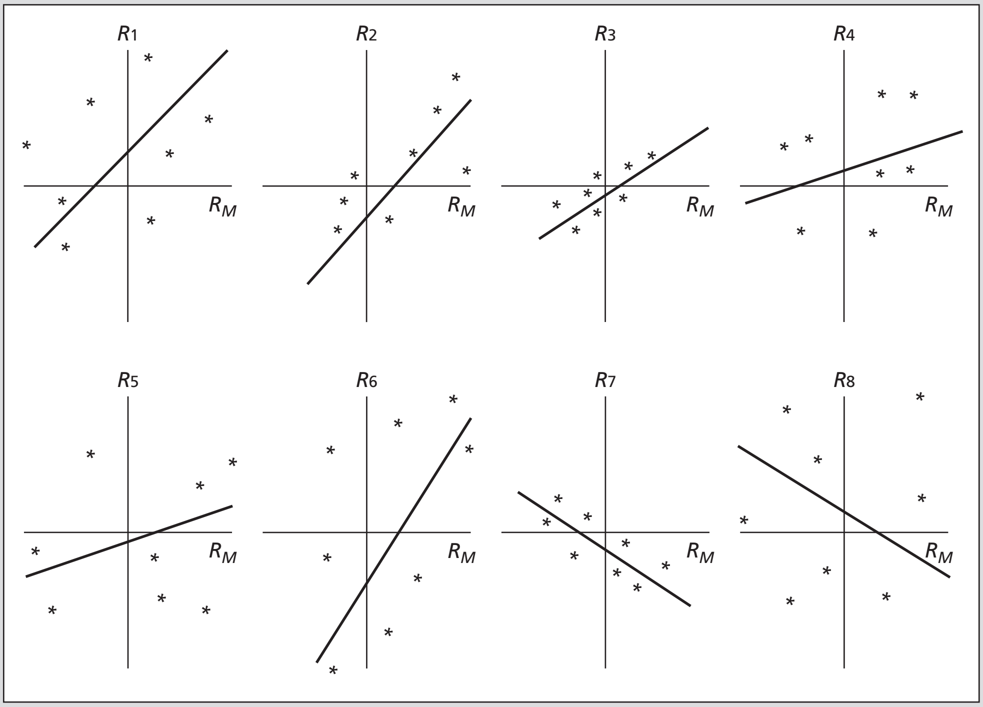
Imagine a portfolio that is divided equally among securities whose returns follow the single-index model of Equation 6.11. What are the systematic and nonsystematic variances of this portfolio?

The beta of the portfolio is a simple average of the individual security betas; hence, the systematic variance equals $\beta_P^2\sigma_M^2$. This is the level of market risk in Figure 6.1B. The market variance (σ_M^2) and the beta of the portfolio determine its market risk.

[6] Note that only the *weighted* average of betas (using market values as weights) will be 1, since the stock market index is value-weighted. We know from Chapter 5 that the distribution of securities by market value is not symmetric: There are relatively few large corporations and many more smaller ones. As a result, the simple average of the betas of individual securities, when computed against a value-weighted index such as the S&P 500, will be greater than 1, pushed up by the tendency for stocks of low-capitalization companies to have betas greater than 1.

FIGURE 6.13

Various scatter diagrams



The systematic component of each security return, $\beta_i R_M$, is driven by the market factor and therefore is perfectly correlated with the systematic part of any other security's return. Hence, there are no diversification effects on systematic risk no matter how many securities are involved. As far as *market risk* goes, a single security has the same systematic risk as a diversified portfolio with the same beta. The number of securities makes no difference.

EXAMPLE 6.3

Estimating the Index Model Using Historical Data

The direct way to calculate the slope and intercept of the characteristic lines for ABC and XYZ is from the variances and covariances. Here, we use the Data Analysis menu of Excel to obtain the covariance matrix in the following spreadsheet.

The slope coefficient for ABC is given by the formula

$$\beta_{ABC} = \frac{\text{Cov}(R_{ABC}, R_{Market})}{\text{Var}(R_{Market})} = \frac{773.31}{669.01} = 1.156$$

The intercept for ABC is

$$\alpha_{ABC} = \text{Average}(R_{ABC}) - \beta_{ABC} \times \text{Average}(R_{Market})$$
$$= 15.20 - 1.156 \times 9.40 = 4.33$$

Therefore, the security characteristic line of ABC is given by

$$R_{ABC} = 4.33 + 1.156 R_{Market}$$

This result also can be obtained by using the "Regression" command from Excel's Data Analysis menu, as we show at the bottom of the spreadsheet. The minor differences between the direct regression output and our calculations above are due to rounding error.

(continued)

EXAMPLE 6.3

Estimating the Index Model Using Historical Data

(concluded)

	A	B	C	D	E	F	G	H	I
2			Annualized Rates of Return					Excess Returns	
3	Week	ABC	XYZ	Mkt. Index	Risk free		ABC	XYZ	Market
4	1	65.13	-22.55	64.40	5.23		59.90	-27.78	59.17
5	2	51.84	31.44	24.00	4.76		47.08	26.68	19.24
6	3	-30.82	-6.45	9.15	6.22		-37.04	-12.67	2.93
7	4	-15.13	-51.14	-35.57	3.78		-18.91	-54.92	-39.35
8	5	70.63	33.78	11.59	4.43		66.20	29.35	7.16
9	6	107.82	32.95	23.13	3.78		104.04	29.17	19.35
10	7	-25.16	70.19	8.54	3.87		-29.03	66.32	4.67
11	8	50.48	27.63	25.87	4.15		46.33	23.48	21.72
12	9	-36.41	-48.79	-13.15	3.99		-40.40	-52.78	-17.14
13	10	-42.20	52.63	20.21	4.01		-46.21	48.62	16.20
14	**Average:**						**15.20**	**7.55**	**9.40**
15									
16	**COVARIANCE MATRIX**								
17		ABC	XYZ	Market					
18	ABC	3020.933							
19	XYZ	442.114	1766.923						
20	Market	773.306	396.789	669.010					
21									
22	**SUMMARY OUTPUT OF EXCEL REGRESSION**								
23									
24	Regression Statistics								
25	Multiple R	0.544							
26	R-Square	0.296							
27	Adj. R-Square	0.208							
28	Standard Error	48.918							
29	Observations	10.000							
30									
31									
32		Coefficients	Std. Error	t-Stat	p-value				
33	Intercept	4.336	16.564	0.262	0.800				
34	Market return	1.156	0.630	1.834	0.104				
35									

Note: This is the output provided by the Data Analysis tool in Excel. As a technical aside, we should point out that the covariance matrix produced by Excel does not adjust for degrees of freedom. In other words, it divides total squared deviations from the sample average (for variance) or total cross product of deviations from sample averages (for covariance) by total observations, despite the fact that sample averages are estimated parameters. This procedure does not affect regression coefficients, however, because in the formula for beta, both the numerator (i.e., the covariance) and denominator (i.e., the variance) are affected equally.

It is quite different with firm-specific risk. Consider a portfolio of n securities with weights, $w_i \left(\text{where} \sum_{i=1}^{n} w_i = 1\right)$, in securities with nonsystematic risk, $\sigma^2_{e_i}$. The nonsystematic portion of the portfolio return is

$$e_P = \sum_{i=1}^{n} w_i e_i$$

Because the firm-specific terms, e_i, are uncorrelated, the portfolio nonsystematic variance is the weighted sum of the individual firm-specific variances:[7]

$$\sigma^2_{e_P} = \sum_{i=1}^{n} w_i^2 \sigma^2_{e_i} \tag{6.15}$$

Each individual nonsystematic variance is multiplied by the *square* of the portfolio weight. With diversified portfolios, the squared weights are very small. For example, if $w_i = .01$ (think of a portfolio with 100 securities), then $w_i^2 = .0001$. The sum in Equation 6.15 is far less than

[7]We use the result from statistics that when we multiply a random variable (in this case, e_i) by a constant (in this case, w_i), the variance is multiplied by the *square* of the constant. The variance of the sum in Equation 6.15 equals the sum of the variances because in this case all covariances are zero.

the average firm-specific variance of the stocks in the portfolio. We conclude that the impact of nonsystematic risk becomes negligible as the number of securities grows and the portfolio becomes ever-more diversified. This is why the number of securities counts more than the size of their nonsystematic variance.

In sum, when we control the systematic risk of the portfolio by manipulating the average beta of the component securities, the number of securities is of no consequence. But for *nonsystematic* risk the number of securities is more important than the firm-specific variance of the securities. Sufficient diversification can virtually eliminate firm-specific risk. Understanding this distinction is essential to understanding the role of diversification.

We have just seen that when forming highly diversified portfolios, firm-specific risk becomes *irrelevant.* Only systematic risk remains. This means that for diversified investors, the relevant risk measure for a security will be the security's beta, β, since firms with higher β have greater sensitivity to market risk. As Equation 6.12 makes clear, systematic risk will be determined by both market volatility, σ_M^2, and the firm's β.

CONCEPT check 6.7

a. What is the characteristic line of XYZ in Example 6.3?
b. Does ABC or XYZ have greater systematic risk?
c. What proportion of the variance of XYZ is firm-specific risk?

Using Security Analysis with the Index Model

Imagine that you are a portfolio manager in charge of the endowment of a small charity. Without the resources to engage in security analysis, you would choose a passive portfolio comprising one or more index funds and T-bills. Denote this portfolio as M. You estimate its standard deviation as σ_M and acquire a forecast of its risk premium as R_M. Now you find that you have sufficient resources to perform fundamental analysis on one stock, say Google. You forecast Google's risk premium as R_G and estimate its beta (β_G) and residual SD, $\sigma(e_G)$, against the benchmark portfolio M. How should you proceed?

Without access to other securities, all you can do is construct the optimal portfolio (with the highest Sharpe ratio) from M and Google using Equation 6.10. It turns out that the index model allows us to further simplify Equation 6.10.

Notice that your forecast of R_G implies that Google's alpha is $\alpha_G = R_G - \beta_G R_M$. We use two key statistics $\alpha_G/\sigma^2(e_G)$ and R_M/σ_M^2, to find the position of Google in the optimal risky portfolio in two steps. In step 1, we compute

$$w_G^0 = \frac{\alpha_G/\sigma^2(e_G)}{R_M/\sigma_M^2} \tag{6.16}$$

In step 2, we adjust the value from Equation 6.16 for the beta of Google:

$$w_G^* = \frac{w_G^0}{1 + w_G^0(1 - \beta_G)} \qquad w_M^* = 1 - w_G^* \tag{6.17}$$

The Sharpe ratio of this portfolio exceeds that of the passive portfolio M, S_M, according to

$$S_O^2 - S_M^2 = \left(\frac{\alpha_G}{\sigma(e_G)}\right)^2 \tag{6.18}$$

We see that the improvement over the passive benchmark is determined by the ratio $\alpha_G/\sigma(e_G)$, which is called Google's **information ratio.** This application of the index model is called the Treynor-Black model, after Fischer Black and Jack Treynor who proposed it in 1973.

information ratio

Ratio of alpha to the standard deviation of the residual.

The value of the Treynor-Black model becomes dramatic when you analyze more than one stock. To compute the optimal portfolio comprising the benchmark portfolio and more than two stocks, you would need to use the involved Markowitz methodology of Section 6.4. But with the Treynor-Black model, the task is straightforward. You can view Google in the previous discussion as your **active portfolio.** If instead of Google alone you analyze several stocks, a portfolio of these stocks would make up your active portfolio, which then would be mixed with the passive index. You would use the alpha, beta, and residual SD of the active portfolio in Equations 6.16–6.18 to obtain the weights of the optimal portfolio, *O,* and its Sharpe ratio. Thus, the only task left is to determine the exact composition of the active portfolio, as well as its alpha, beta, and residual standard deviation.

active portfolio

The portfolio formed by optimally combining analyzed stocks.

Suppose that in addition to analyzing Google, you analyze Dell's stock (D) and estimate its alpha, beta, and residual variance. You estimate the ratio for Google, $\alpha_G/\sigma^2(e_G)$, the corresponding ratio for Dell, and the sum of these ratios for all stocks in the active portfolio. Using Google and Dell,

$$\sum_i \alpha_i/\sigma^2(e_i) = \alpha_G/\sigma^2(e_G) + \alpha_D/\sigma^2(e_D) \qquad \textbf{(6.19)}$$

Treynor and Black showed that the optimal weight of each security in the active portfolio should be

$$w_G(\text{active}) = \frac{\alpha_G/\sigma^2(e_G)}{\sum_i \alpha_i/\sigma^2(e_i)} \quad w_D(\text{active}) = \frac{\alpha_D/\sigma^2(e_D)}{\sum_i \alpha_i/\sigma^2(e_i)} \qquad \textbf{(6.20)}$$

Notice that the active portfolio entails two offsetting considerations. On the one hand, a stock with a higher alpha value calls for a high weight to take advantage of its attractive expected return. On the other hand, a high residual variance leads us to temper our position in the stock to avoid bearing firm-specific risk.

The alpha and beta of the active portfolio are weighted averages of each component stock's alpha and beta, and the residual variance is the weighted sum of each stock's residual variance, using the squared portfolio weights:

$$\begin{aligned} \alpha_A &= w_{GA}\alpha_{GA} + w_{DA}\alpha_{DA} \quad \beta_A = w_{GA}\beta_{GA} + w_{DA}\beta_{DA} \\ \sigma^2(e_A) &= w_{GA}^2\sigma^2(e_G) + w_{DA}^2\sigma^2(e_D) \end{aligned} \qquad \textbf{(6.21)}$$

Given these parameters, we can now use Equations 6.16–6.18 to determine the weight of the active portfolio in the optimal portfolio and the Sharpe ratio it achieves.

EXAMPLE 6.5

The Treynor-Black Model

Suppose your benchmark portfolio is the S&P 500 Index. The input list in Panel A of Table 6.2 includes the data for the passive index as well as the two stocks, Google and Dell. Both stocks have positive alpha values, so you would expect the optimal portfolio to be tilted toward these stocks. However, the tilt will be limited to avoid excessive exposure to otherwise-diversifiable firm-specific risk. The optimal trade-off maximizes the Sharpe ratio. We use the Treynor-Black model to accomplish this task.

We begin in Panel B assuming that the *active portfolio* comprises solely Google, which has an information ratio of .115. This "portfolio" is then combined with the passive index to form the optimal risky portfolio as in Equations 6.16–6.18. The calculations in Table 6.2 show that the optimal portfolio achieves a Sharpe ratio of .20, compared with .16 for the passive benchmark. This optimal portfolio is invested 43.64% in Google and 56.36% in the benchmark.

In Panel C, we add Dell to the list of actively analyzed stocks. The optimal weights of each stock in the active portfolio are 55.53% in Google and 44.47% in Dell. This gives the active portfolio an information ratio of .14, which improves the Sharpe ratio of the optimal portfolio to .24. The optimal portfolio invests 91.73% in the active portfolio and 8.27% in the index. This large tilt is acceptable because the residual standard deviation of the active portfolio (6.28%) is far less than that of either stock. Finally, the optimal portfolio weight in Google is 50.94% and in Dell, 40.79%. Notice that the weight in Google is now *larger* than its weight without Dell! This, too, is a result of diversification within the active position that allows a larger tilt toward Google's large alpha.

TABLE 6.2 Construction of optimal portfolios using the index model

Input List

		Active Portfolio		
	Benchmark Portfolio (S&P 500)	**Google**	**Dell**	
A. Input data				
Risk premium	0.7	2.20	1.74	
Standard deviation	4.31	11.39	10.49	
Sharpe ratio	0.16	not applicable		
Alpha		1.04	0.75	
Beta		1.65	1.41	
Residual standard deviation		9.01	8.55	
Information ratio = alpha/residual SD		0.1154	0.0877	
Alpha/residual variance		0.0128	0.0103	
Portfolio Construction				
B. Optimal portfolio with Google only in active portfolio				
Performance data				
Sharpe ratio = SQRT (index Sharpe^2 + Google information ratio^2)	0.20			
Composition of optimal portfolio				
w^0 = (alpha/residual SD)/(index risk premium/index variance)		0.3400		
$w^* = w^0/(1 + w^0(1 - \text{beta}))$	0.5636	0.4364		
C. Optimal portfolio with Google and Dell in the active portfolio				**Active Portfolio (sum)**
Composition of the active portfolio				
w^0 of stock (Equation 6.15)		0.3400	0.2723	0.6122
w^0/Sum w^0 of analyzed stocks		0.5553	0.4447	1.0000
Performance of the active portfolio				
alpha = weight in active portfolio × stock alpha		0.58	0.33	0.91
beta = weight in active portfolio × stock beta		0.92	0.63	1.54
Residual variance = square weight × stock residual variance		25.03	14.46	39.49
Residual SD = SQRT (active portfolio residual variance)				6.28
Information ratio = active portfolio alpha/residual SD				0.14
Performance of the optimal portfolio				
Sharpe ratio	0.24			
	Index	**Active**		
Composition of optimal portfolio				
w^0		0.6122		
w^*	0.0827	0.9173		
		Google	**Dell**	
Weight of active portfolio × weight of stock in active portfolio		0.5094	0.4079	

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6.6 RISK OF LONG-TERM INVESTMENTS

So far we have envisioned portfolio investment for one period. We have not made any explicit assumptions about the duration of that period, so one might take it to be of any length, and thus our analysis would seem to apply as well to long-term investments. Yet investors are frequently advised that stock investments for the long run are not as risky as it might appear from the statistics presented in this chapter and the previous one. To understand this widespread misconception, we must first understand what the alternative long-term investment strategies are.

Risk and Return with Alternative Long-Term Investments

We have not yet had much to say about the investor's time horizon. From the standpoint of risk and return,[8] does it matter whether an investor's horizon is long or short? A common misconception is that long-term investors should allocate a greater proportion of wealth into stocks simply because in some sense stocks are less risky over long-term horizons. This belief that stocks become less risky over longer horizons is based on a notion of "time diversification," that spreading your risky investments over many time periods offers a similar benefit in risk reduction as spreading an investment budget over many assets in a given period (the subject we have worked on throughout this chapter). That belief, however, is incorrect.

We can gain insight into risk in the long run by comparing one-year ("short-term") and two-year ("longer-term") *risky* investments. Imagine an investment opportunity set that is identical in each of the two years. It includes a risky portfolio with a normally distributed, continuously compounded, annual risk premium of R and variance of σ^2. The one-year Sharpe ratio is therefore $S_1 = R/\sigma$, and the one-year price of risk is $P_1 = R/\sigma^2$. Investors can allocate their portfolios between that risky portfolio and a risk-free asset with zero risk premium and variance.[9] As we learned from Table 5.2, we can safely assume that the stock portfolio returns are serially uncorrelated.

Of course, you cannot properly compare a one-year to a two-year investment without specifying what the one-year investor will do in the second year. To make the comparisons meaningful, we compare the strategies of three investment companies that advertise three alternative two-year investment strategies: Company 1 calls its strategy "Two-In": Invest everything in the risky portfolio for two years. Company 2 touts its "One-In" strategy: In one year invest fully in the risk-free asset, and in the other year invest fully in the risky portfolio. Finally, Company 3 advocates a "Half-in-Two" strategy: In both years, invest half the investment budget in the risk-free asset and the other half in the risky portfolio. We must decide which strategy is best.

Recall that both the mean and variance of continuously compounded, serially uncorrelated returns (or excess returns) grow in proportion to the length of the holding period. We show the rate-of-return statistics for the three strategies in Table 6.3. The risk premium is zero for bills and R for the risky portfolio, so the first row of the table shows the accumulation of the investor's risk premium over two years using each strategy. Similarly, the second row shows the accumulation of the variance of the investor's wealth.

We see immediately that risk is *not* lower for longer-term investors. A two-year investment in stocks (the Two-In strategy) has twice the variance as the One-In strategy. This observation already should settle the debate of whether total risk in the long run is smaller—it clearly is not. Rather, it grows proportionally over time: The two-year investment in the risky portfolio has double the variance of the one-year investment.

[8]An investor may choose a specific investment horizon for a number of reasons, such as the target retirement age. Other factors that we have not considered, for example, the magnitude of one's human capital (the value of future earning power) versus current financial wealth, also may affect investment horizon and portfolio allocations.

[9]The hierarchy of portfolio choice we developed for any given holding period is this: Construct a risky portfolio with the highest Sharpe ratio. Allocate the entire investment budget between this portfolio and the risk-free asset. The optimal weight in the risky portfolio is $y = P/A$, where P is the price of risk and A is the investor's risk aversion.

TABLE 6.3 Two-year risk premium, variance, Sharpe ratio, and price of risk for three strategies

Strategy:	Two-In[1]	One-In[2]	Half-in-Two[3]
Risk premium	$R + R = 2R$	$0 + R = R$	$2^* \tfrac{1}{2} R = R$
Variance	$\sigma^2 + \sigma^2 = 2\sigma^2$	$0 + \sigma^2 = \sigma^2$	$2^* \tfrac{1}{4} \sigma^2 = \sigma^2/2$
Sharpe ratio[4]	$\dfrac{2R}{\sigma\sqrt{2}} = S_1\sqrt{2}$	$R/\sigma = S_1$	$\dfrac{R}{\sigma/\sqrt{2}} = S_1\sqrt{2}$
Price of risk[5]	$R/\sigma^2 = P_1$	$R/\sigma^2 = P_1$	$\dfrac{R}{\sigma^2/2} = 2P_1$

[1]Two-In: Invest entirely in the risky portfolio for two years.

[2]One-In: Invest entirely in the risky portfolio for one year and in the risk-free asset in the other.

[3]Half-in-Two: Invest half the budget in the risky portfolio for two years.

[4]Sharpe Ratio $= \dfrac{\text{Risk premium}}{\text{Standard deviation}}$

[5]Price of risk $= \dfrac{\text{Risk premium}}{\text{Variance}}$

While the One-In investment is less risky than the Two-In strategy, an even safer investment strategy that still offers the same risk premium as One-In is the Half-in-Two strategy, which invests half the investor's wealth in stocks in *each* of the two years. This has only one-half the variance of One-In and only one-fourth the variance of Two-In. When less risk than Two-In is desired, spreading the risk evenly over time, rather than lumping all the risk into a concentrated period (as the One-In strategy does), is the best strategy. This is evident from the Sharpe ratios of each strategy (line 3 of the table): The Sharpe ratio of Half-in-Two exceeds that of One-In by a multiple of $\sqrt{2}$. Does this mean that Half-in-Two actually does offer a meaningful benefit of time diversification? Put differently, does Half-in-Two allow investors to prudently allocate greater portfolio shares to the risky portfolio? Surprisingly, the answer is no. Even this more limited notion of time diversification is faulty.

We will compare investors' optimal capital allocations under each of these strategies. However, we can dismiss the One-In strategy out of hand, as it clearly is dominated by the Half-in-Two strategy, which has equal risk premium with only half the variance. Therefore, we need only compare Two-In with Half-in-Two. We established earlier that an investor with a degree of risk aversion A will allocate a fraction of overall wealth to the risky portfolio equal to $y = \dfrac{\text{Price of risk}}{A}$. For Two-In, that fraction is $\dfrac{R/\sigma^2}{A}$, while for Half-in-Two, it is $\dfrac{2R/\sigma^2}{A}$. So the Half-in-Two strategy gets double the allocation as All-In; but remember that Half-in-Two is only half as heavily invested in the risky portfolio. These effects precisely cancel out: The higher risk and return of Two-In are precisely offset by the investor's reduced allocation to it. So the Half-in-Two strategy that *seems* to offer the benefit of time diversification does not in fact elicit a greater overall allocation to the risky portfolio.

Would it matter if we extended the horizon from two years to some greater value? With a horizon of T years, a "time diversification strategy" (we must now change our strategy from "Half in Two" to "$1/T$ in T") puts at risk $1/T$ of the budget each year. The price of risk of the time-diversified strategy is TR/σ^2, compared to only R/σ^2 for the All-In strategy. Will this elicit greater investments in the risky portfolio as T increases? Once again, the telling point is that the time-diversified portfolio has no better Sharpe ratio than the fully invested All-In portfolio: The Sharpe ratio for both strategies is now $S_1\sqrt{T}$. Any investor will invest with the time-diversified strategy T times the fraction he would with the All-In T-year strategy; the net effect is that these alternatives are for all practical purposes equivalent. The investor choosing between these two alternatives is simply sliding up or down the CAL. Despite its higher price of risk, the overall allocation to the risky portfolio is not higher for the time-diversified portfolio.

Why the Unending Confusion?

It is no secret that the vast majority of financial advisers believe that "stocks are less risky if held for the long run," and so advise their clients. Their reasoning is this: The risk premium grows at the rate of the horizon, T. The standard deviation grows at the slower rate of only $\sqrt{T}$. The fact that risk grows more slowly than the risk premium is evident from the Sharpe ratio, $S_1\sqrt{T}$, which grows with the investment horizon.

This story sounds compelling, and in fact it contains no mathematical error. However, it is only half the story. Time diversification seems to offer a better risk-return trade-off (a higher Sharpe ratio) if you compare the All-In to the "One-In" strategy that invests all in one year and nothing later. But the relevant alternative to All-In is $1/T$ in T. The long-term $1/T$ in T investment strategy falls on the same "time CAL" as the All-In strategy, since it uses the same risky portfolio for the entire horizon. When this strategy is on the menu, complete portfolio allocations will *not* shift toward risky investments even as the investor's horizon extends.[10]

[10]You can verify that, more generally, a $1/n$ in T strategy (investing a portfolio weight of $1/n$ in the risky portfolio over T years) would give us the same results. You will find that the Sharpe ratio is $S_1\sqrt{T}$, while the price of risk is nP_1. Hence, you would invest n times more in the low-risk strategy and end with the same complete portfolio as the All-In strategy.

SUMMARY

- The expected rate of return of a portfolio is the weighted average of the component asset expected returns with the investment proportions as weights.
- The variance of a portfolio is a sum of the contributions of the component-security variances *plus* terms involving the covariance among assets.
- Even if correlations are positive, the portfolio standard deviation will be less than the weighted average of the component standard deviations, as long as the assets are not *perfectly* positively correlated. Thus, portfolio diversification is of value as long as assets are less than perfectly correlated.
- The contribution of an asset to portfolio variance depends on its correlation with the other assets in the portfolio, as well as on its own variance. An asset that is perfectly negatively correlated with a portfolio can be used to reduce the portfolio variance to zero. Thus, it can serve as a perfect hedge.
- The efficient frontier of risky assets is the graphical representation of the set of portfolios that maximizes portfolio expected return for a given level of portfolio standard deviation. Rational investors will choose a portfolio on the efficient frontier.
- A portfolio manager identifies the efficient frontier by first establishing estimates for the expected returns and standard deviations and determining the correlations among them. The input data are then fed into an optimization program that produces the investment proportions, expected returns, and standard deviations of the portfolios on the efficient frontier.
- In general, portfolio managers will identify different efficient portfolios because of differences in the methods and quality of security analysis. Managers compete on the quality of their security analysis relative to their management fees.
- If a risk-free asset is available and input data are identical, all investors will choose the same portfolio on the efficient frontier, the one that is tangent to the CAL. All investors with identical input data will hold the identical risky portfolio, differing only in how much each allocates to this optimal portfolio and to the risk-free asset. This result is characterized as the separation principle of portfolio selection.
- The single-index model expresses the excess return on a security as a function of the market excess return: $R_i = \alpha_i + \beta_i R_M + e_i$. This equation also can be interpreted as a

regression of the security excess return on the market-index excess return. The regression line has intercept α_i and slope β_i and is called the security characteristic line.

- In a single-index model, the variance of the rate of return on a security or portfolio can be decomposed into systematic and firm-specific risk. The systematic component of variance equals β^2 times the variance of the market excess return. The firm-specific component is the variance of the residual term in the index-model equation.
- The beta of a portfolio is the weighted average of the betas of the component securities. A security with negative beta reduces the portfolio beta, thereby reducing exposure to market volatility. The unique risk of a portfolio approaches zero as the portfolio becomes more highly diversified.

KEY TERMS

active portfolio, 177
alpha, 171
beta, 170
diversifiable risk, 149
efficient frontier, 165
excess return, 170
firm-specific risk, 149
index model, 170
information ratio, 176
investment opportunity set, 157
market risk, 149
nondiversifiable risk, 149
nonsystematic risk, 149
optimal risky portfolio, 162
residual risk, 171
security characteristic line, 172
separation property, 167
systematic risk, 149
unique risk, 149

KEY FORMULAS

The expected rate of return on a portfolio: $E(r_P) = w_B E(r_B) + w_S E(r_S)$

The variance of the return on a portfolio: $\sigma_P^2 = (w_B\sigma_B)^2 + (w_S\sigma_S)^2 + 2(w_B\sigma_B)(w_S\sigma_S)\rho_{BS}$

The Sharpe ratio of a portfolio: $S_P = \dfrac{E(r_P) - r_f}{\sigma_P}$

Sharpe ratio maximizing portfolio weights with two risky assets (B and S) and a risk-free asset:

$$w_B = \frac{[E(r_B) - r_f]\sigma_S^2 - [E(r_S) - r_f]\sigma_B\sigma_S\rho_{BS}}{[E(r_B) - r_f]\sigma_S^2 + [E(r_S) - r_f]\sigma_B^2 - [E(r_B) - r_f + E(r_S) - r_f]\sigma_B\sigma_S\rho_{BS}}$$

$$w_S = 1 - w_B$$

The index-model equation: $R_i = \beta_i R_M + \alpha_i + e_i$

Decomposition of variance based on the index-model equation:

$$\text{Variance}(R_i) = \beta_i^2\sigma_M^2 + \sigma^2(e_i)$$

Percent of security variance explained by the index return = the square of the correlation coefficient of the regression of the security on the market:

$$\rho^2 = \frac{\text{Systematic (or explained) variance}}{\text{Total variance}}$$

$$= \frac{\beta_D^2\sigma_M^2}{\sigma_D^2} = \frac{\beta_D^2\sigma_M^2}{\beta_D^2\sigma_M^2 + \sigma^2(e_D)}$$

Optimal position in the active portfolio, A:

$$w_A^* = \frac{w_A^0}{1 + w_A^0(1 - \beta_A)} \qquad w_M^* = 1 - w_A^*$$

$$w_A^0 = \frac{\alpha_A/\sigma^2(e_A)}{R_M/\sigma_M^2}$$

Optimal weight of a security, G, in the active portfolio: $w_G(\text{active}) = \dfrac{\alpha_G/\sigma^2(e_G)}{\sum_i \alpha_i/\sigma^2(e_i)}$

Select problems are available in McGraw-Hill's *Connect Finance.* Please see the Supplements section of the book's frontmatter for more information.

PROBLEM SETS

Basic

1. In forming a portfolio of two risky assets, what must be true of the correlation coefficient between their returns if there are to be gains from diversification? Explain. *(LO 6-1)*
2. When adding a risky asset to a portfolio of many risky assets, which property of the asset is more important, its standard deviation or its covariance with the other assets? Explain. *(LO 6-1)*
3. A portfolio's expected return is 12%, its standard deviation is 20%, and the risk-free rate is 4%. Which of the following would make for the greatest increase in the portfolio's Sharpe ratio? *(LO 6-3)*
   a. An increase of 1% in expected return.
   b. A decrease of 1% in the risk-free rate.
   c. A decrease of 1% in its standard deviation.
4. An investor ponders various allocations to the optimal risky portfolio and risk-free T-bills to construct his complete portfolio. How would the Sharpe ratio of the complete portfolio be affected by this choice? *(LO 6-3)*

Intermediate

5. The standard deviation of the market-index portfolio is 20%. Stock A has a beta of 1.5 and a residual standard deviation of 30%. *(LO 6-5)*
   a. What would make for a larger increase in the stock's variance: an increase of .15 in its beta or an increase of 3% (from 30% to 33%) in its residual standard deviation?
   b. An investor who currently holds the market-index portfolio decides to reduce the portfolio allocation to the market index to 90% and to invest 10% in stock A. Which of the changes in (*a*) will have a greater impact on the portfolio's standard deviation?
6. Suppose that the returns on the stock fund presented in Spreadsheet 6.1 were −40%, −14%, 17%, and 33% in the four scenarios. *(LO 6-2)*
   a. Would you expect the mean return and variance of the stock fund to be more than, less than, or equal to the values computed in Spreadsheet 6.2? Why?
   b. Calculate the new values of mean return and variance for the stock fund using a format similar to Spreadsheet 6.2. Confirm your intuition from part (*a*).
   c. Calculate the new value of the covariance between the stock and bond funds using a format similar to Spreadsheet 6.4. Explain intuitively the change in the covariance.
7. Use the rate-of-return data for the stock and bond funds presented in Spreadsheet 6.1, but now assume that the probability of each scenario is as follows: severe recession: .10; mild recession: .20; normal growth: .35; boom: .35. *(LO 6-2)*
   a. Would you expect the mean return and variance of the stock fund to be more than, less than, or equal to the values computed in Spreadsheet 6.2? Why?
   b. Calculate the new values of mean return and variance for the stock fund using a format similar to Spreadsheet 6.2. Confirm your intuition from part (*a*).
   c. Calculate the new value of the covariance between the stock and bond funds using a format similar to Spreadsheet 6.4. Explain intuitively why the absolute value of the covariance has changed.

The following data apply to Problems 8–12.

A pension fund manager is considering three mutual funds. The first is a stock fund, the second is a long-term government and corporate bond fund, and the third is a T-bill money market fund that yields a sure rate of 5.5%. The probability distributions of the risky funds are:

	Expected Return	Standard Deviation
Stock fund (S)	15%	32%
Bond fund (B)	9	23

The correlation between the fund returns is .15.

8. Tabulate and draw the investment opportunity set of the two risky funds. Use investment proportions for the stock fund of 0% to 100% in increments of 20%. What expected return and standard deviation does your graph show for the minimum-variance portfolio? *(LO 6-2)*
9. Draw a tangent from the risk-free rate to the opportunity set. What does your graph show for the expected return and standard deviation of the optimal risky portfolio? *(LO 6-3)*
10. What is the reward-to-volatility ratio of the best feasible CAL? *(LO 6-3)*
11. Suppose now that your portfolio must yield an expected return of 12% and be efficient, that is, on the best feasible CAL. *(LO 6-4)*
    a. What is the standard deviation of your portfolio?
    b. What is the proportion invested in the T-bill fund and each of the two risky funds?
12. If you were to use only the two risky funds and still require an expected return of 12%, what would be the investment proportions of your portfolio? Compare its standard deviation to that of the optimal portfolio in the previous problem. What do you conclude? *(LO 6-4)*
13. Stocks offer an expected rate of return of 10% with a standard deviation of 20%, and gold offers an expected return of 5% with a standard deviation of 25%. *(LO 6-3)*
    a. In light of the apparent inferiority of gold to stocks with respect to both mean return and volatility, would anyone hold gold? If so, demonstrate graphically why one would do so.
    b. How would you answer (*a*) if the correlation coefficient between gold and stocks were 1? Draw a graph illustrating why one would or would not hold gold. Could these expected returns, standard deviations, and correlation represent an equilibrium for the security market?
14. Suppose that many stocks are traded in the market and that it is possible to borrow at the risk-free rate, r_f. The characteristics of two of the stocks are as follows:

Stock	Expected Return	Standard Deviation
A	8%	40%
B	13	60
Correlation = −1		

Could the equilibrium r_f be greater than 10%? (*Hint:* Can a particular stock portfolio be substituted for the risk-free asset?) *(LO 6-3)*

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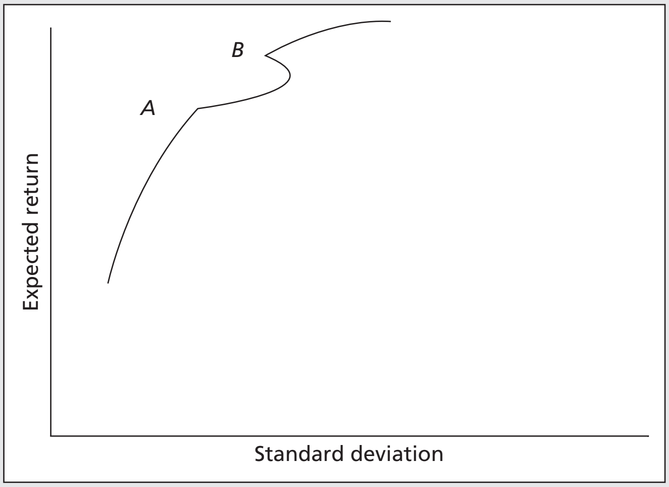
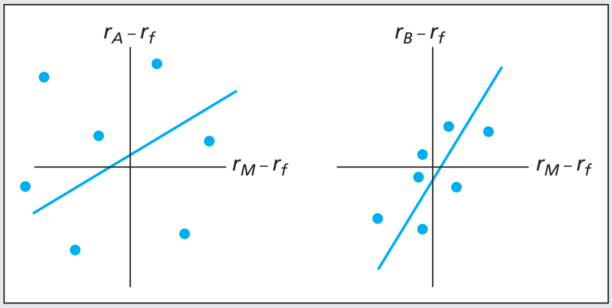
15. You can find a spreadsheet containing the historic returns presented in Table 5.2 on the text's website at **www.mhhe.com/bkm.** (Look for the link to Chapter 5 material.) Copy the data for the last 20 years into a new spreadsheet. Analyze the risk-return trade-off that would have characterized portfolios constructed from large stocks and long-term Treasury bonds over the last 20 years. What was the average rate of return and standard deviation of each asset? What was the correlation coefficient of their annual returns? What would have been the average return and standard deviation of portfolios with differing weights in the two assets? For example, consider weights in stocks starting at zero and incrementing by .10 up to a weight of 1. What was the

average return and standard deviation of the minimum-variance combination of stocks and bonds? *(LO 6-2)*

16. Assume expected returns and standard deviations for all securities, as well as the risk-free rate for lending and borrowing, are known. Will investors arrive at the same optimal risky portfolio? Explain. *(LO 6-4)*
17. Your assistant gives you the following diagram as the efficient frontier of the group of stocks you asked him to analyze. The diagram looks a bit odd, but your assistant insists he double-checked his analysis. Would you trust him? Is it possible to get such a diagram? *(LO 6-4)*



18. What is the relationship of the portfolio standard deviation to the weighted average of the standard deviations of the component assets? *(LO 6-1)*
19. A project has a .7 chance of doubling your investment in a year and a .3 chance of halving your investment in a year. What is the standard deviation of the rate of return on this investment? *(LO 6-2)*
20. Investors expect the market rate of return this year to be 10%. The expected rate of return on a stock with a beta of 1.2 is currently 12%. If the market return this year turns out to be 8%, how would you revise your expectation of the rate of return on the stock? *(LO 6-5)*
21. The following figure shows plots of monthly rates of return and the stock market for two stocks. *(LO 6-5)*
    a. Which stock is riskier to an investor currently holding her portfolio in a diversified portfolio of common stock?
    b. Which stock is riskier to an undiversified investor who puts all of his funds in only one of these stocks?



22. Go to **www.mhhe.com/bkm** and link to the material for Chapter 6, where you will find a spreadsheet containing monthly rates of return for GM, the S&P 500, and T-bills over a recent five-year period. Set up a spreadsheet just like that of Example 6.3 and find the beta of GM. *(LO 6-5)*

eXcel
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23. Here are rates of return for six months for Generic Risk, Inc. What is Generic's beta? (*Hint:* Find the answer by plotting the scatter diagram.) *(LO 6-5)*

Month	Market Return	Generic Return
1	0%	+2%
2	0	0
3	−1	0
4	−1	−2
5	+1	+4
6	+1	+2

Challenge

24. Go to the Online Learning Center at **www.mhhe.com/bkm,** where you will find rate-of-return data over 60 months for Google, the T-bill rate, and the S&P 500, which we will use as the market-index portfolio. *(LO 6-4)*
   a. Use these data and Excel's regression function to compute Google's excess return each period as well as its alpha, beta, and residual standard deviation, $\sigma(e)$.
   b. What was the Sharpe ratio of the S&P 500 over this period?
   c. What was Google's information ratio over this period?
   d. If someone whose risky portfolio is currently invested in an index portfolio such as the S&P 500 wishes to take a position in Google based on the estimates from parts (*a*)–(*c*), what would be the optimal fraction of the risky portfolio to invest in Google? Use Equations 6.16 and 6.17.
   e. Based on Equation 6.18 and your answer to part (*d*), by how much would the Sharpe ratio of the optimal risky portfolio increase given the incremental position in Google?

CFA® PROBLEMS

CFA Problems

1. A three-asset portfolio has the following characteristics:

Asset	Expected Return	Standard Deviation	Weight
X	15%	22%	0.50
Y	10	8	0.40
Z	6	3	0.10

What is the expected return on this three-asset portfolio? *(LO 6-1)*

2. George Stephenson's current portfolio of $2 million is invested as follows:

Summary of Stephenson's Current Portfolio				
	Value	Percent of Total	Expected Annual Return	Annual Standard Deviation
Short-term bonds	$ 200,000	10%	4.6%	1.6%
Domestic large-cap equities	600,000	30	12.4	19.5
Domestic small-cap equities	1,200,000	60	16.0	29.9
Total portfolio	$2,000,000	100%	13.8%	23.1%

Stephenson soon expects to receive an additional $2 million and plans to invest the entire amount in an index fund that best complements the current portfolio. Stephanie Coppa, CFA, is evaluating the four index funds shown in the following table for their ability to produce a portfolio that will meet two criteria relative to the current portfolio: (1) maintain or enhance expected return and (2) maintain or reduce volatility.

Each fund is invested in an asset class that is not substantially represented in the current portfolio.

Index Fund Characteristics			
Index Fund	Expected Annual Return	Expected Annual Standard Deviation	Correlation of Returns with Current Portfolio
Fund A	15%	25%	+0.80
Fund B	11	22	+0.60
Fund C	16	25	+0.90
Fund D	14	22	+0.65

State which fund Coppa should recommend to Stephenson. Justify your choice by describing how your chosen fund *best* meets both of Stephenson's criteria. No calculations are required. *(LO 6-4)*

3. Abigail Grace has a $900,000 fully diversified portfolio. She subsequently inherits ABC Company common stock worth $100,000. Her financial adviser provided her with the following estimates: *(LO 6-5)*

Risk and Return Characteristics		
	Expected Monthly Returns	Standard Deviation of Monthly Returns
Original Portfolio	0.67%	2.37%
ABC Company	1.25	2.95

The correlation coefficient of ABC stock returns with the original portfolio returns is .40.

a. The inheritance changes Grace's overall portfolio and she is deciding whether to keep the ABC stock. Assuming Grace keeps the ABC stock, calculate the:
   i. Expected return of her new portfolio which includes the ABC stock.
   ii. Covariance of ABC stock returns with the original portfolio returns.
   iii. Standard deviation of her new portfolio which includes the ABC stock.

b. If Grace sells the ABC stock, she will invest the proceeds in risk-free government securities yielding .42% monthly. Assuming Grace sells the ABC stock and replaces it with the government securities, calculate the:
   i. Expected return of her new portfolio which includes the government securities.
   ii. Covariance of the government security returns with the original portfolio returns.
   iii. Standard deviation of her new portfolio which includes the government securities.

c. Determine whether the beta of her new portfolio, which includes the government securities, will be higher or lower than the beta of her original portfolio.

d. Based on conversations with her husband, Grace is considering selling the $100,000 of ABC stock and acquiring $100,000 of XYZ Company common stock instead. XYZ stock has the same expected return and standard deviation as ABC stock. Her husband comments, "It doesn't matter whether you keep all of the ABC stock or replace it with $100,000 of XYZ stock." State whether her husband's comment is correct or incorrect. Justify your response.

e. In a recent discussion with her financial adviser, Grace commented, "If I just don't lose money in my portfolio, I will be satisfied." She went on to say, "I am more afraid of losing money than I am concerned about achieving high returns." Describe *one* weakness of using standard deviation of returns as a risk measure for Grace.

The following data apply to CFA Problems 4–6:

Hennessy & Associates manages a $30 million equity portfolio for the multimanager Wilstead Pension Fund. Jason Jones, financial vice president of Wilstead, noted that Hennessy had rather consistently achieved the best record among the Wilstead's six equity managers. Performance of the Hennessy portfolio had been clearly superior to that of the S&P 500 in four of the past five years. In the one less favorable year, the shortfall was trivial.

Hennessy is a "bottom-up" manager. The firm largely avoids any attempt to "time the market." It also focuses on selection of individual stocks, rather than the weighting of favored industries.

There is no apparent conformity of style among the six equity managers. The five managers, other than Hennessy, manage portfolios aggregating $250 million, made up of more than 150 individual issues.

Jones is convinced that Hennessy is able to apply superior skill to stock selection, but the favorable results are limited by the high degree of diversification in the portfolio. Over the years, the portfolio generally held 40–50 stocks, with about 2% to 3% of total funds committed to each issue. The reason Hennessy seemed to do well most years was that the firm was able to identify each year 10 or 12 issues that registered particularly large gains.

Based on this overview, Jones outlined the following plan to the Wilstead pension committee:

> Let's tell Hennessy to limit the portfolio to no more than 20 stocks. Hennessy will double the commitments to the stocks that it really favors and eliminate the remainder. Except for this one new restriction, Hennessy should be free to manage the portfolio exactly as before.

All the members of the pension committee generally supported Jones's proposal, because all agreed that Hennessy had seemed to demonstrate superior skill in selecting stocks. Yet the proposal was a considerable departure from previous practice, and several committee members raised questions.

4. Answer the following: ***(LO 6-1)***
   a. Will the limitation of 20 stocks likely increase or decrease the risk of the portfolio? Explain.
   b. Is there any way Hennessy could reduce the number of issues from 40 to 20 without significantly affecting risk? Explain.
5. One committee member was particularly enthusiastic concerning Jones's proposal. He suggested that Hennessy's performance might benefit further from reduction in the number of issues to 10. If the reduction to 20 could be expected to be advantageous, explain why reduction to 10 might be less likely to be advantageous. (Assume that Wilstead will evaluate the Hennessy portfolio independently of the other portfolios in the fund.) ***(LO 6-1)***
6. Another committee member suggested that, rather than evaluate each managed portfolio independently of other portfolios, it might be better to consider the effects of a change in the Hennessy portfolio on the total fund. Explain how this broader point of view could affect the committee decision to limit the holdings in the Hennessy portfolio to either 10 or 20 issues. ***(LO 6-1)***
7. Dudley Trudy, CFA, recently met with one of his clients. Trudy typically invests in a master list of 30 equities drawn from several industries. As the meeting concluded, the client made the following statement: "I trust your stock-picking ability and believe that you should invest my funds in your five best ideas. Why invest in 30 companies when you obviously have stronger opinions on a few of them?" Trudy plans to respond to his client within the context of Modern Portfolio Theory. ***(LO 6-1)***
   a. Contrast the concepts of systematic risk and firm-specific risk, and give an example of each type of risk.
   b. Critique the client's suggestion. Discuss how both systematic and firm-specific risk change as the number of securities in a portfolio is increased.

WEB *master*

1. Go to **finance.yahoo.com** and download five years of monthly closing prices for Eli Lilly (ticker = LLY), Alcoa (AA), and the S&P 500 Index (GSPC). Download the data into an Excel file and use the Adjusted-Close prices, which adjust for dividend payments, to calculate the monthly rate of return for each price series. Use an XY Scatter Plot chart with no line joining the points to plot Alcoa's returns against the S&P 500. Now select

one of the data points, and right-click to obtain a shortcut menu allowing you to enter a trend line. This is Alcoa's characteristic line, and the slope is Alcoa's beta. Repeat this process for Lilly. What conclusions can you draw from each company's characteristic line?

2. Following the procedures in the previous question, find five years of monthly returns for Staples. Using the first two years of data, what is Staples' beta? What is the beta using the latest two years of data? How stable is the beta estimate? If you use all five years of data, how close is your estimate of beta to the estimate reported in Yahoo's *Key Statistics* section?
3. Following the procedures in the previous questions, find five years of monthly returns for the following firms: Genzyme Corporation, Sony, Cardinal Health, Inc., Black & Decker Corporation, and Kellogg Company. Copy the returns from these five firms into a single Excel workbook, with the returns for each company properly aligned. Use the full range of available data. Then do the following:
   a. Using the Excel functions for average (AVERAGE) and sample standard deviation (STDEV), calculate the average and the standard deviation of the returns for each of the firms.
   b. Using Excel's correlation function (CORREL), construct the correlation matrix for the five stocks based on their monthly returns for the entire period. What are the lowest and the highest individual pairs of correlation coefficients? (*Alternative:* You may use Excel's Data Analysis Tool to generate the correlation matrix.)
4. There are some free online tools that will calculate the optimal asset weights and draw the efficient frontier for the assets that you specify. One of the sites is **www.investorcraft.com/PortfolioTools/EfficientFrontier.aspx.**

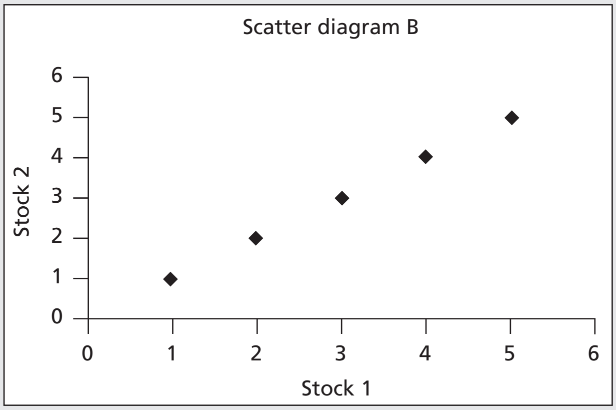
   Go to this site and enter at least eight assets in the selection box. You can search for the companies by name or by symbol. Click on the *Next Step* button and select one of the time spans offered. Specify an appropriate risk-free rate, a minimum allowable asset weight of 0, and a maximum allowable asset weight of 100. Click on *Calculate* to get your results.
   a. What are the expected return and the standard deviation of the portfolio based on adjusted weights?
   b. How do they compare to those for the optimal portfolio and the minimum variance portfolio?
   c. Of the three portfolios shown, with which one would you feel most comfortable as an investor?

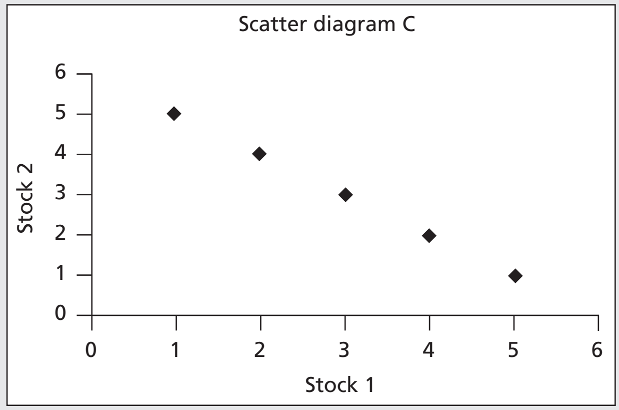
SOLUTIONS TO CONCEPT *checks*

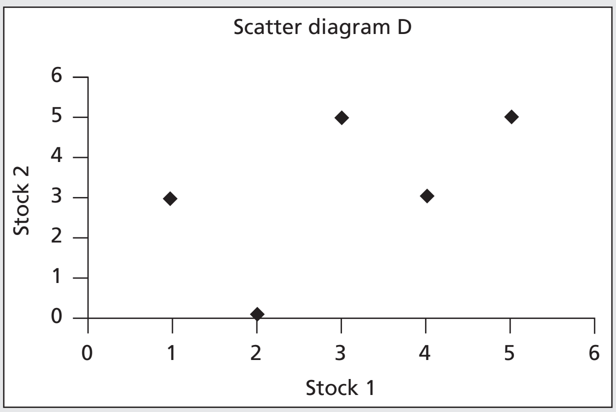
6.1 Recalculation of Spreadsheets 6.1 and 6.4 shows that the covariance is now −5.80 and the correlation coefficient is −.07.

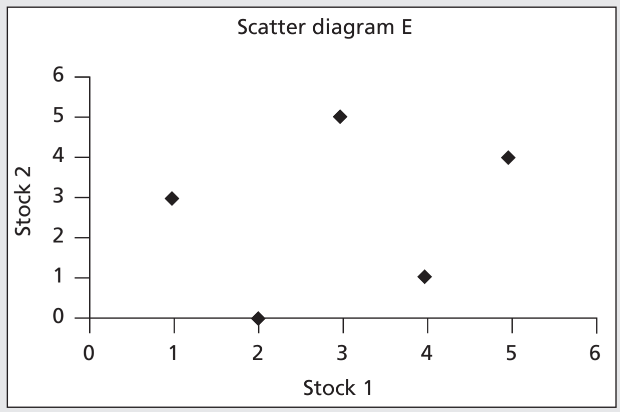
	A	B	C	D	E	F
1			Stock Fund		Bond Fund	
2	Scenario	Probability	Rate of Return	Col B × Col C	Rate of Return	Col B × Col E
3	Severe recession	.05	−37.0	−1.9	−10	−0.5
4	Mild recession	.25	−11.0	−2.8	10	2.5
5	Normal growth	.40	14.0	5.6	7	2.8
6	Boom	.30	30.0	9.0	2	0.6
7	Expected or Mean Return:		SUM:	10.0	SUM:	5.4
8						
9			Deviation from Mean Return		Covariance	
10	Scenario	Probability	Stock Fund	Bond Fund	Product of Dev	Col B × Col E
11	Severe recession	.05	−47.0	−15.4	723.8	36.19
12	Mild recession	.25	−21.0	4.6	−96.6	−24.15
13	Normal growth	.40	4.0	1.6	6.4	2.56
14	Boom	.30	20.0	−3.4	−68.0	−20.40
15		SD =	18.63	4.65	Covariance =	−5.80
16	Correlation coefficient = Covariance/(StdDev(stocks)*StdDev(bonds)) =					−0.07

6.2 The scatter diagrams for pairs B–E are shown below. Scatter diagram A (presented with the Concept Check) shows an exact mirror image between the pattern of points 1,2,3 versus 3,4,5. Therefore, the correlation coefficient is zero. Scatter diagram B shows perfect positive correlation (1). Similarly, C shows perfect negative correlation (−1). Now compare the scatters of D and E. Both show a general positive correlation, but scatter D is tighter. Therefore, D is associated with a correlation of about .5 (use a spreadsheet to show that the exact correlation is .54), and E is associated with a correlation of about .2 (show that the exact correlation coefficient is .23).









6.3 *a.* Using Equation 6.6 with the data $\sigma_B = 8$; $\sigma_S = 19$; $w_B = .5$; and $w_S = 1 - w_B = .5$, we obtain the equation

$$\sigma_P^2 = 10^2 = (w_B\sigma_B)^2 + (w_S\sigma_S)^2 + 2(w_B\sigma_B)(w_S\sigma_S)\rho_{BS}$$
$$= (.5 \times 8)^2 + (.5 \times 19)^2 + 2(.5 \times 8)(.5 \times 19)\rho_{BS}$$

which yields $\rho = .1728$.

b. Using Equation 6.5 and the additional data $E(r_B) = 5\%$; $E(r_S) = 10\%$, we obtain

$$E(r_P) = w_B E(r_B) + w_S E(r_S) = (.5 \times 5) + (.5 \times 10) = 7.5\%$$

c. On the one hand, you should be happier with a correlation of .17 than with .22 since the lower correlation implies greater benefits from diversification and means that, for any level of expected return, there will be lower risk. On the other hand, the constraint that you must hold 50% of the portfolio in bonds represents a cost to you since it prevents you from choosing the risk-return trade-off most suited to your tastes. Unless you would choose to hold about 50% of the portfolio in bonds anyway, you are better off with the slightly higher correlation but with the ability to choose your own portfolio weights.

6.4 *a.* Implementing Equations 6.5 and 6.6, we generate data for the graph. See Spreadsheet 6.7 and Figure 6.14 on the following pages.

b. Implementing the formulas indicated in Spreadsheet 6.6, we generate the optimal risky portfolio (*O*) and the minimum-variance portfolio.

c. The slope of the CAL is equal to the risk premium of the optimal risky portfolio divided by its standard deviation, $(11.28 - 5)/17.59 = .357$.

d. The mean of the complete portfolio is $.2222 \times 11.28 + .7778 \times 5 = 6.395\%$, and its standard deviation is $.2222 \times 17.58 = 3.91\%$. Sharpe ratio $= (6.395 - 5)/3.91 = .357$.

SPREADSHEET 6.7

For Concept Check 4. Mean and standard deviation for various portfolio applications

	A	B	C	D	E	F	G
5		Data	X	M	T-Bills		
6		Mean (%)	15	10	5		
7		Std. Dev. (%)	50	20	0		
8		Corr. Coeff. X and S		-0.20			
9		Portfolio Opportunity set					
10		Weight in X	Weight in S	Pf Mean (%)	Pf Std Dev (%)		
11		-1.00	2.00	5.00	70.00		
12		-0.90	1.90	5.50	64.44		
13		-0.80	1.80	6.00	58.92		
14		-0.70	1.70	6.50	53.45		
15		-0.60	1.60	7.00	48.04		
16		-0.50	1.50	7.50	42.72		
17		-0.40	1.40	8.00	37.52		
18		-0.30	1.30	8.50	32.51		
19		-0.20	1.20	9.00	27.78		
20		-0.10	1.10	9.50	23.52		
21		0.00	1.00	10.00	20.00		
22		0.10	0.90	10.50	17.69		
23		0.20	0.80	11.00	17.09		
24		0.30	0.70	11.50	18.36		
25		0.40	0.60	12.00	21.17		
26		0.50	0.50	12.50	25.00		
27		0.60	0.40	13.00	29.46		
28		0.70	0.30	13.50	34.31		
29		0.80	0.20	14.00	39.40		
30		0.90	0.10	14.50	44.64		
31		1.00	0.00	15.00	50.00		
32		1.10	-0.10	15.50	55.43		
33		1.20	-0.20	16.00	60.93		
34		1.30	-0.30	16.50	66.46		
35		1.40	-0.40	17.00	72.03		
36		1.50	-0.50	17.50	77.62		
37		1.60	-0.60	18.00	83.23		
38		1.70	-0.70	18.50	88.87		
39		1.80	-0.80	19.00	94.51		
40		1.90	-0.90	19.50	100.16		
41		2.00	-1.00	20.00	105.83		
42	**Min. Var Pf**	0.18	0.82	10.91	17.06		
43	**Optimal Pf**	0.26	0.74	11.28	17.59		
44							
45							
46							
47							
48							
49							

=B13*$C$6+C13*$D$6

=(B15^2*$C$7^2
+C15^2*$D$7^2
+2*B15*C15
*$C$7*$D$7*$D$8)^0.5

=((C6-E6)*D7^2-(D6-E6)*C7*D7*D8)/
((C6-E6)*D7^2+(D6-E6)*C7^2-(C6-E6+D6-E6)*C7*D7*D8)

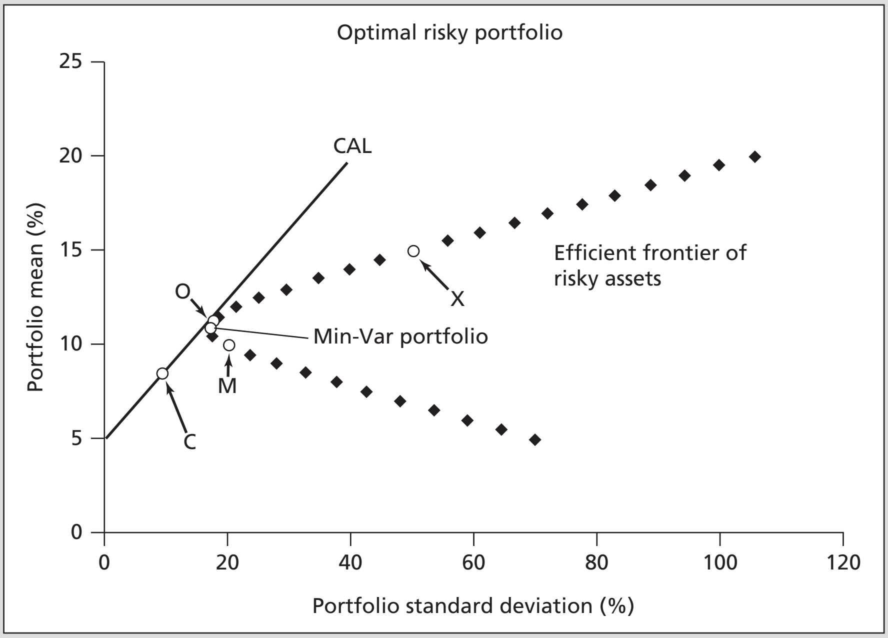
The composition of the complete portfolio is
$.2222 \times .26 = .06$ (i.e., 6%) in X
$.2222 \times .74 = .16$ (i.e., 16%) in M
and 78% in T-bills.

6.5 Efficient frontiers derived by portfolio managers depend on forecasts of the rates of return on various securities and estimates of risk, that is, standard deviations and correlation coefficients. The forecasts themselves do not control outcomes. Thus, to prefer a manager with a rosier forecast (northwesterly frontier) is tantamount to rewarding the bearers of good news and punishing the bearers of bad news. What the investor wants is to reward bearers of *accurate* news. Investors should monitor forecasts of portfolio managers on a regular basis to develop a track record of their forecasting accuracy. Portfolio choices of the more accurate forecasters will, in the long run, outperform the field.

6.6 *a.* Beta, the slope coefficient of the security on the factor: Securities $R_1 - R_6$ have a positive beta. These securities move, on average, in the same direction as the market (R_M). R_1, R_2, R_6 have large betas, so they are "aggressive" in that they carry more systematic risk than R_3, R_4, R_5, which are "defensive." R_7 and R_8 have a negative beta. These are hedge assets that carry negative systematic risk.

FIGURE 6.14

For Concept Check 6.4. Plot of mean return versus standard deviation using data from spreadsheet.



b. Intercept, the expected return when the market is neutral: The estimates show that R_1, R_4, R_8 have a positive intercept, while R_2, R_3, R_5, R_6, R_7 have negative intercepts. To the extent that one believes these intercepts will persist, a positive value is preferred.

c. Residual variance, the nonsystematic risk: R_2, R_3, R_7 have a relatively low residual variance. With sufficient diversification, residual risk eventually will be eliminated, and, hence, the difference in the residual variance is of little economic significance.

d. Total variance, the sum of systematic and nonsystematic risk: R_3 has a low beta and low residual variance, so its total variance will be low. R_1, R_6 have high betas and high residual variance, so their total variance will be high. But R_4 has a low beta and high residual variance, while R_2 has a high beta with a low residual variance. In sum, total variance often will misrepresent systematic risk, which is the part that matters.

6.7. *a.* To obtain the characteristic line of XYZ, we continue the spreadsheet of Example 6.3 and run a regression of the excess return of XYZ on the excess return of the market-index fund.

Summary Output	
Regression Statistics	
Multiple R	0.363
R-square	0.132
Adjusted R-square	0.023
Standard error	41.839
Observations	10

	Coefficients	Standard Error	*t*-Stat	*p*-Value	Lower 95%	Upper 95%
Intercept	3.930	14.98	0.262	0.800	−30.62	38.48
Market	0.582	0.528	1.103	0.302	−0.635	1.798

The regression output shows that the slope coefficient of XYZ is .582 and the intercept is 3.93%; hence the characteristic line is $R_{XYZ} = 3.93 + .582R_{\text{Market}}$.

b. The beta coefficient of ABC is 1.156, greater than XYZ's .582, implying that ABC has greater systematic risk.

c. The regression of XYZ on the market index shows an R-square of .132. Hence the proportion of unexplained variance (nonsystematic risk) is .868, or 86.8%.

Capital Asset Pricing and Arbitrage Pricing Theory

Chapter 7

Learning Objectives:

LO7-1 Use the implications of capital market theory to estimate security risk premiums.

LO7-2 Construct and use the security market line.

LO7-3 Specify and use a multifactor security market line.

LO7-4 Take advantage of an arbitrage opportunity with a portfolio that includes mispriced securities.

LO7-5 Use arbitrage pricing theory with more than one factor to identify mispriced securities.

The capital asset pricing model, almost always referred to as the CAPM, is a centerpiece of modern financial economics. It was first proposed by William F. Sharpe, who was awarded the 1990 Nobel Prize in Economics.

The CAPM provides a precise prediction of the relationship we should observe between the risk of an asset and its expected return. This relationship serves two vital functions.

First, it provides a benchmark rate of return for evaluating possible investments. For example, a security analyst might want to know whether the expected return she forecasts for a stock is more or less than its "fair" return given its risk. Second, the model helps us make an educated guess as to the expected return on assets that have not yet been traded in the marketplace. For example, how do we price an initial public offering of stock? How will a major new investment project affect the return investors require on a company's stock? Although the CAPM does not fully withstand empirical tests, it is widely used because of the insight it offers and because its accuracy suffices for many important applications.

Once you understand the intuition behind the CAPM, it becomes clear that the model may be improved by generalizing it to allow for multiple sources of risk. Therefore, we turn next to multifactor models of risk and return, and we show how these result in richer descriptions of the risk-return relationship.

Finally, we consider an alternative derivation of the risk-return relationship known as arbitrage pricing theory, or APT. Arbitrage is the exploitation of security mispricing to earn risk-free economic profits. The most basic principle of capital market theory is that prices ought to be aligned to eliminate risk-free profit opportunities. If actual prices allowed for such arbitrage, the resulting opportunities for profitable trading would lead to strong pressure on security prices that would persist until equilibrium was restored and the opportunities were eliminated. We will see that this no-arbitrage principle leads to a risk-return relationship like that of the CAPM. Like the generalized version of the CAPM, the simple APT is easily extended to accommodate multiple sources of systematic risk.

Related websites for this chapter are available at www.mhhe.com/bkm.

7.1 THE CAPITAL ASSET PRICING MODEL

Historically, the CAPM was developed prior to the index model introduced in the previous chapter (Equation 6.11). The index model was widely adopted as a natural description of the stock market immediately on the heels of the CAPM because the CAPM implications so neatly match the intuition underlying the model. So it makes sense to use the index model to help understand the lessons of the CAPM.

The index model describes an empirical relationship between the excess return on an individual stock, R_i, and that of a broad market-index portfolio, R_M: $R_i = \beta_i R_M + \alpha_i + e_i$, where alpha is the expected firm-specific return and e_i is zero-mean "noise," or firm-specific risk. Therefore, the expected excess return on a stock, given (conditional on) the market excess return, R_M, is $E(R_i \mid R_M) = \beta_i R_M + \alpha_i$.

What does this mean to portfolio managers? Hunt for positive-alpha stocks, don't invest in negative-alpha stocks, and, better yet, sell short negative-alpha stocks if short sales are not prohibited. Investor demand for a positive-alpha stock will increase its price. As the price of a stock rises, other things being equal, the expected return falls, reducing and ultimately eliminating the very alpha that first created the excess demand. Conversely, the drop in demand for a negative-alpha stock will reduce its price, pushing its alpha back toward zero. In the end, such buying or selling pressure will leave most securities with zero alpha values most of the time. Put another way, unless and until your own analysis of a stock tells you otherwise, you should assume alpha is zero.

When alpha is zero, there is no reward from bearing firm-specific risk; the only way to earn a higher expected return than the T-bill rate is by bearing systematic risk. Recall the Treynor-Black model, in which the position in any active portfolio is zero if the alpha is zero. In that case, the best portfolio is the one that completely eliminates nonsystematic risk, and that portfolio is an indexed portfolio that mimics the broad market. This is the conclusion of the CAPM. But science demands more than a story like this. It requires a carefully set up model with explicit assumptions in which an outcome such as the one we describe will be the only possible result. Here goes.

The Model: Assumptions and Implications

capital asset pricing model (CAPM)

A model that relates the required rate of return on a security to its systematic risk as measured by beta.

The **capital asset pricing model,** or **CAPM,** was developed by Treynor, Sharpe, Lintner, and Mossin in the early 1960s, and further refined later. The model predicts the relationship between the risk and equilibrium expected returns on risky assets. It begins by laying down the necessary, albeit unrealistic, assumptions that are necessary for the validity of the model. Thinking about an admittedly unrealistic world allows a relatively easy leap to the solution. With this accomplished, we can add realism to the environment, one step at a time, and see how the theory must be amended. This process allows us to develop a reasonably realistic model.

The conditions that lead to the CAPM ensure competitive security markets and investors who choose from identical efficient portfolios using the mean-variance criterion:

1. Markets for securities are perfectly competitive and equally profitable to all investors.
   1.A. No investor is sufficiently wealthy that his or her actions alone can affect market prices.
   1.B. All information relevant to security analysis is publicly available at no cost.
   1.C. All securities are publicly owned and traded, and investors may trade all of them. Thus, all risky assets are in the investment universe.
   1.D. There are no taxes on investment returns. Thus, all investors realize identical returns from securities.
   1.E. Investors confront no transaction costs that inhibit their trading.
   1.F. Lending and borrowing at a common risk-free rate are unlimited.
2. Investors are alike in every way except for initial wealth and risk aversion; hence, they all choose investment portfolios in the same manner.
   2.A. Investors plan for the same (single-period) horizon.
   2.B. Investors are rational, mean-variance optimizers.
   2.C. Investors are efficient users of analytical methods, and by assumption 1.B they have access to all relevant information. Hence, they use the same inputs and consider identical portfolio opportunity sets. This assumption is often called *homogeneous expectations.*

Obviously, these assumptions ignore many real-world complexities. However, they lead to powerful insights into the nature of equilibrium in security markets.

Given these assumptions, we summarize the equilibrium that will prevail in this hypothetical world of securities and investors. We elaborate on these implications in the following sections.

1. All investors will choose to hold the **market portfolio (M),** which includes all assets of the security universe. For simplicity, we shall refer to all assets as stocks. The proportion of each stock in the market portfolio equals the market value of the stock (price per share times the number of shares outstanding) divided by the total market value of all stocks.
2. The market portfolio will be on the efficient frontier. Moreover, it will be the optimal risky portfolio, the tangency point of the capital allocation line (CAL) to the efficient frontier. As a result, the capital market line (CML), the line from the risk-free rate through the market portfolio, M, is also the best attainable capital allocation line. All investors hold M as their optimal risky portfolio, differing only in the amount invested in it as compared to investment in the risk-free asset.
3. The risk premium on the market portfolio will be proportional to the variance of the market portfolio and investors' typical degree of risk aversion. Mathematically,

$$E(r_M) - r_f = \overline{A}\sigma_M^2 \tag{7.1}$$

   where σ_M is the standard deviation of the return on the market portfolio and $\overline{A}$ represents the degree of risk aversion of the average investor.
4. The risk premium on individual assets will be proportional to the risk premium on the market portfolio (M) and to the *beta coefficient* of the security on the market portfolio. Beta measures the extent to which returns respond to the market portfolio. Formally, beta is the regression (slope) coefficient of the security return on the market return, representing sensitivity to fluctuations in the overall security market.

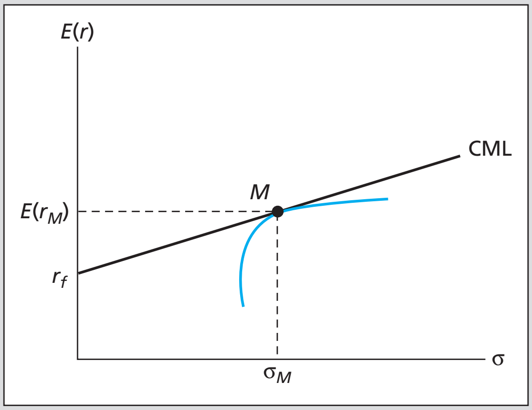
market portfolio (M)
The portfolio for which each security is held in proportion to its total market value.

Why All Investors Would Hold the Market Portfolio

Given all our assumptions, it is easy to see why all investors hold identical risky portfolios. If all investors use mean-variance analysis (assumptions 2.A and 2.B), apply it to the same universe of securities (assumptions 1.C and 1.F) with an identical time horizon (assumption 2.A),

FIGURE 7.1

The efficient frontier and the capital market line



use the same security analysis (assumption 2.C), and experience identical net returns from the same securities (assumptions 1.A, 1.D, and 1.E), they all must arrive at the same determination of the optimal risky portfolio.

With everyone choosing to hold the same risky portfolio, stocks will be represented in the aggregate risky portfolio in the same proportion as they are in each investor's (common) risky portfolio. If Google represents 1% in each common risky portfolio, Google will be 1% of the aggregate risky portfolio. This in fact is the market portfolio since the market is no more than the aggregate of all individual portfolios. Because each investor uses the market portfolio for the optimal risky portfolio, the CAL in this case is called the *capital market line,* or CML, as in Figure 7.1.

Suppose the optimal portfolio of our investors does not include the stock of some company, say, Southwest Airlines. When no investor is willing to hold Southwest stock, the demand is zero, and the stock price will take a free fall. As Southwest stock gets progressively cheaper, it begins to look more attractive, while all other stocks look (relatively) less attractive. Ultimately, Southwest will reach a price at which it is desirable to include it in the optimal stock portfolio, and investors will buy.

This price adjustment process guarantees that all stocks will be included in the optimal portfolio. The only issue is the price. At a given price level, investors will be willing to buy a stock; at another price, they will not. The bottom line is this: If all investors hold an *identical* risky portfolio, this portfolio must be the *market* portfolio.

The Passive Strategy Is Efficient

The CAPM implies that a passive strategy, using the CML as the optimal CAL, is a powerful alternative to an active strategy. The market portfolio proportions are a result of profit-oriented "buy" and "sell" orders that cease only when there is no more profit to be made. And in the simple world of the CAPM, all investors use precious resources in security analysis. A passive investor who takes a free ride by simply investing in the market portfolio benefits from the efficiency of that portfolio. In fact, an active investor who chooses any other portfolio will end on a CAL that is inferior to the CML used by passive investors.

mutual fund theorem

States that all investors desire the same portfolio of risky assets and can be satisfied by a single mutual fund composed of that portfolio.

We sometimes call this result a **mutual fund theorem** because it implies that only one mutual fund of risky assets—the market index fund—is sufficient to satisfy the investment demands of all investors. The mutual fund theorem is another incarnation of the separation property discussed in Chapter 6. Assuming all investors choose to hold a market-index mutual fund, we can separate portfolio selection into two components: (1) a technical side, in which an efficient mutual fund is created by professional management; and (2) a personal side, in which an investor's risk aversion determines the allocation of the complete portfolio between the mutual fund and the risk-free asset. Here, all investors agree that the mutual fund they would like to hold is invested in the market portfolio.

While investment managers in the real world generally construct risky portfolios that differ from the market index, we attribute this to the differences in their estimates of risk and expected return (in violation of assumption 2.C). Nevertheless, a passive investor may view the market index as a reasonable first approximation to an efficient risky portfolio.

The logical inconsistency of the CAPM is this: If a passive strategy is costless *and* efficient, why would anyone follow an active strategy? But if no one does any security analysis, what brings about the efficiency of the market portfolio?

We have acknowledged from the outset that the CAPM simplifies the real world in its search for a tractable solution. Its applicability to the real world depends on whether its predictions are accurate enough. The model's use is some indication that its predictions are reasonable. We discuss this issue in Section 7.3 and in greater depth in Chapter 8.

CONCEPT check 7.1

If only some investors perform security analysis while all others hold the market portfolio (*M*), would the CML still be the efficient CAL for investors who do not engage in security analysis? Explain.

The Risk Premium of the Market Portfolio

In Chapter 5 we showed how individual investors decide how much to invest in the risky portfolio when they can include a risk-free asset in the investment budget. Returning now to the decision of how much to invest in the market portfolio *M* and how much in the risk-free asset, what can we deduce about the equilibrium risk premium of portfolio *M*?

We asserted earlier that the equilibrium risk premium of the market portfolio, $E(r_M) - r_f$, will be proportional to the degree of risk aversion of the average investor and to the risk of the market portfolio, σ_M^2. Now we can explain this result.

When investors purchase stocks, their demand drives up prices, thereby lowering expected rates of return and risk premiums. But when risk premiums fall, investors will move some of their funds from the risky market portfolio into the risk-free asset. In equilibrium, the risk premium on the market portfolio must be just high enough to induce investors to hold the available supply of stocks. If the risk premium is too high, there will be excess demand for securities, and prices will rise; if it is too low, investors will not hold enough stock to absorb the supply, and prices will fall. The *equilibrium* risk premium of the market portfolio is therefore proportional both to the risk of the market, as measured by the variance of its returns, and to the degree of risk aversion of the average investor, denoted by $\overline{A}$ in Equation 7.1.

EXAMPLE 7.1

Market Risk, the Risk Premium, and Risk Aversion

Suppose the risk-free rate is 5%, the average investor has a risk-aversion coefficient of $\overline{A} = 2$, and the standard deviation of the market portfolio is 20%. Then, from Equation 7.1, we estimate the equilibrium value of the market risk premium[1] as $2 \times .20^2 = .08$. So the expected rate of return on the market must be

$$\begin{aligned} E(r_M) &= r_f + \text{Equilibrium risk premium} \\ &= .05 + .08 = .13 = 13\% \end{aligned}$$

If investors were more risk averse, it would take a higher risk premium to induce them to hold shares. For example, if the average degree of risk aversion were 3, the market risk premium would be $3 \times .20^2 = .12$, or 12%, and the expected return would be 17%.

CONCEPT check 7.2

Historical data for the S&P 500 Index show an average excess return over Treasury bills of about 7.5% with standard deviation of about 20%. To the extent that these averages approximate investor expectations for the sample period, what must have been the coefficient of risk aversion of the average investor? If the coefficient of risk aversion were 3.5, what risk premium would have been consistent with the market's historical standard deviation?

Expected Returns on Individual Securities

The CAPM is built on the insight that the appropriate risk premium on an asset will be determined by its contribution to the risk of investors' overall portfolios. Portfolio risk is what matters to investors, and portfolio risk is what governs the risk premiums they demand.

[1]To use Equation 7.1, we must express returns in decimal form rather than as percentages.

We know that nonsystematic risk can be reduced to an arbitrarily low level through diversification (Chapter 6); therefore, investors do not require a risk premium as compensation for bearing nonsystematic risk. They need to be compensated only for bearing systematic risk, which cannot be diversified. We know also that the contribution of a single security to the risk of a large diversified portfolio depends only on the systematic risk of the security as measured by its beta, as we saw in Section 6.5. Therefore, it should not be surprising that the risk premium of an asset is proportional to its beta; a security with double the systematic risk of another must pay twice the risk premium. Thus, the ratio of risk premium to beta should be the same for any two securities or portfolios.

If we equate the ratio of risk premium to systematic risk for the market portfolio, which has a beta of 1, to the corresponding ratio for a particular stock, for example, Dell, we find that

$$\frac{E(r_M) - r_f}{1} = \frac{E(r_D) - r_f}{\beta_D}$$

expected return (mean return)–beta relationship

Implication of the CAPM that security risk premiums (expected excess returns) will be proportional to beta.

Rearranging results in the CAPM's **expected return–beta relationship:**

$$E(r_D) = r_f + \beta_D[E(r_M) - r_f] \tag{7.2}$$

In words, an asset's risk premium equals the asset's systematic risk measure (its beta) times the risk premium of the (benchmark) market portfolio. This expected return (or mean return)–beta relationship is the most familiar expression of the CAPM.

The mean–beta relationship of the CAPM makes a powerful economic statement. It implies, for example, that a security with a high variance but a relatively low beta of .5 will carry one-third the risk premium of a low-variance security with a beta of 1.5. Equation 7.2 quantifies the conclusion we reached in Chapter 6: Only systematic risk matters to investors who can diversify, and systematic risk is measured by beta.

EXAMPLE 7.2

Expected Returns and Risk Premiums

Suppose the risk premium of the market portfolio is 9%, and we estimate the beta of Dell as $\beta_D = 1.3$. The risk premium predicted for the stock is therefore 1.3 times the market risk premium, or $1.3 \times 9\% = 11.7\%$. The expected rate of return on Dell is the risk-free rate plus the risk premium. For example, if the T-bill rate were 5%, the expected rate of return would be 5% + 11.7% = 16.7% or, using Equation 7.2 directly,

$$\begin{aligned} E(r_D) &= r_f + \beta_D[\text{Market risk premium}] \\ &= 5\% + 1.3 \times 9\% = 16.7\% \end{aligned}$$

If the estimate of the beta of Dell were only 1.2, the required risk premium for Dell would fall to 10.8%. Similarly, if the market risk premium were only 8% and $\beta_D = 1.3$, Dell's risk premium would be only 10.4%.

The fact that many investors hold active portfolios that differ from the market portfolio does not necessarily invalidate the CAPM. Recall that reasonably well-diversified portfolios shed almost all firm-specific risk and are subject to only systematic risk. Even if one does not hold the precise market portfolio, a well-diversified portfolio will be so highly correlated with the market that a stock's beta relative to the market still will be a useful risk measure.

In fact, several researchers have shown that modified versions of the CAPM will hold despite differences among individuals that may cause them to hold different portfolios. A study by Brennan (1970) examines the impact of differences in investors' personal tax rates on market equilibrium. Another study by Mayers (1972) looks at the impact of nontraded assets such as human capital (earning power). Both find that while the market portfolio is no longer each investor's optimal risky portfolio, a modified version of the mean–beta relationship still holds.

If the mean–beta relationship holds for any individual asset, it must hold for any combination of assets. The beta of a portfolio is simply the weighted average of the betas of the stocks in the portfolio, using as weights the portfolio proportions. Thus, beta also predicts a portfolio's risk premium in accordance with Equation 7.2.

EXAMPLE 7.3

Portfolio Beta and Risk Premium

Consider the following portfolio:

Asset	Beta	Risk Premium	Portfolio Weight
Microsoft	1.2	9.0%	0.5
American Electric Power	0.8	6.0	0.3
Gold	0.0	0.0	0.2
Portfolio	0.84	?	1.0

If the market risk premium is 7.5%, the CAPM predicts that the risk premium on each stock is its beta times 7.5%, and the risk premium on the portfolio is $.84 \times 7.5\% = 6.3\%$. This is the same result that is obtained by taking the weighted average of the risk premiums of the individual stocks. (Verify this for yourself.)

A word of caution: We often hear that a well-managed firm will provide a high rate of return. This is true when referring to the *firm's* accounting return on investments in plant and equipment. The CAPM, however, predicts returns on investments in the *securities* of the firm that trade in capital markets.

Say everyone knows a firm is well run. Its stock price will be bid up, and returns to stockholders at those high prices will not be extreme. Security *prices* reflect public information about a firm's prospects, but only the risk of the company (as measured by beta) should affect *expected returns.* In a rational market, investors receive high expected returns only if they bear systematic risk.

CONCEPT check 7.3

Suppose the risk premium on the market portfolio is estimated at 8% with a standard deviation of 22%. What is the risk premium on a portfolio invested 25% in GE with a beta of 1.15 and 75% in Dell with a beta of 1.25?

The Security Market Line

The expected return–beta relationship is a reward-risk equation. The beta of a security is the appropriate measure of its risk because beta is proportional to the variance the security contributes to the optimal risky portfolio.[2]

With approximately normal returns, we measure the risk of a portfolio by its standard deviation. Because the beta of a stock measures the stock's contribution to the standard deviation of the market portfolio, we expect the required risk premium to be a function of beta. The CAPM confirms this intuition, stating further that the security's risk premium is directly proportional to both the beta and the risk premium of the market portfolio; that is, the risk premium equals $\beta[E(r_M) - r_f]$.

The mean–beta relationship is called the **security market line (SML)** in Figure 7.2. Its slope is the risk premium of the market portfolio. At the point where $\beta = 1$ (the beta of the market portfolio), we can read off the vertical axis the expected return on the market portfolio.

security market line (SML)

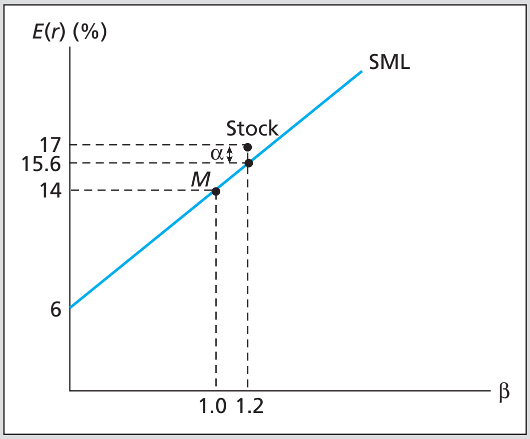
Graphical representation of the expected return–beta relationship of the CAPM.

It is useful to compare the SML to the capital market line. The CML graphs the risk premiums of efficient complete portfolios (made up of the market portfolio and the risk-free asset) as a function of portfolio standard deviation. This is appropriate because standard deviation is a valid measure of risk for portfolios that are candidates for an investor's complete portfolio.

[2]The contribution of a security to portfolio variance equals the variance of the portfolio when the security is included minus the variance when the security is excluded, with the weights of all other securities increased proportionally to bring total weights to 1.

FIGURE 7.2

The security market line and a positive-alpha stock



The SML, in contrast, graphs *individual-asset* risk premiums as a function of asset risk. The relevant measure of risk for an individual asset (which is held as part of a well-diversified portfolio) is not the asset standard deviation but rather the asset beta. The SML is valid both for individual assets and portfolios.

The security market line provides a benchmark for evaluation of investment performance. The SML provides the required rate of return that will compensate investors for the beta risk of that investment, as well as for the time value of money.

Because the SML is the graphical representation of the mean–beta relationship, "fairly priced" assets plot exactly on the SML. The expected returns of such assets are commensurate with their risk. Whenever the CAPM holds, all securities must lie on the SML in equilibrium. Underpriced stocks plot above the SML: Given beta, their expected returns are greater than is indicated by the CAPM. Overpriced stocks plot below the SML. The difference between fair and actual expected rates of return on a stock is the **alpha,** denoted α. The expected return on a mispriced security is given by $E(r_s) = \alpha_s + r_f + \beta_s[E(r_M) - r_f]$.

alpha

The abnormal rate of return on a security in excess of what would be predicted by an equilibrium model such as the CAPM.

EXAMPLE 7.4

The Alpha of a Security

Suppose the return on the market is expected to be 14%, a stock has a beta of 1.2, and the T-bill rate is 6%. The SML would predict an expected return on the stock of

$$E(r) = r_f + \beta[E(r_M) - r_f]$$
$$= 6 + 1.2(14 - 6) = 15.6\%$$

If one believes the stock will provide instead a return of 17%, its implied alpha would be 1.4%, as shown in Figure 7.2. If instead the expected return were only 15%, the stock alpha would be negative, −.6%.

Applications of the CAPM

One place the CAPM may be used is in the investment management industry. Suppose the SML is taken as a benchmark to assess the *fair* expected return on a risky asset. Then an analyst calculates the return she actually expects. Notice that we depart here from the simple CAPM world in that active investors apply their own analysis to derive a private "input list." If a stock is perceived to be a good buy, or underpriced, it will provide a positive alpha, that is, an expected return in excess of the fair return stipulated by the SML.

The CAPM is also useful in capital budgeting decisions. When a firm is considering a new project, the SML provides the required return demanded of the project. This is the cutoff internal rate of return (IRR) or "hurdle rate" for the project.

EXAMPLE 7.5

The CAPM and Capital Budgeting

Suppose Silverado Springs Inc. is considering a new spring-water bottling plant. The business plan forecasts an internal rate of return of 14% on the investment. Research shows the beta of similar products is 1.3. Thus, if the risk-free rate is 4%, and the market risk premium is estimated at 8%, the hurdle rate for the project should be 4 + 1.3 × 8 = 14.4%. Because the IRR is less than the risk-adjusted discount or hurdle rate, the project has a negative net present value and ought to be rejected.

Yet another use of the CAPM is in utility rate-making cases. Here the issue is the rate of return a regulated utility should be allowed to earn on its investment in plant and equipment.

EXAMPLE 7.6

The CAPM and Regulation

Suppose shareholder equity invested in a utility is \$100 million, and the equity beta is .6. If the T-bill rate is 6%, and the market risk premium is 8%, then a fair annual profit will be $6 + (.6 \times 8) = 10.8\%$ of \$100 million, or \$10.8 million. Since regulators accept the CAPM, they will allow the utility to set prices at a level expected to generate these profits.

CONCEPT check 7.4

a. Stock XYZ has an expected return of 12% and $\beta = 1$. Stock ABC is expected to return 13% with a beta of 1.5. The market's expected return is 11% and $r_f = 5\%$. According to the CAPM, which stock is a better buy? What is the alpha of each stock? Plot the SML and the two stocks. Show the alphas of each on the graph.

b. The risk-free rate is 8% and the expected return on the market portfolio is 16%. A firm considers a project with an estimated beta of 1.3. What is the required rate of return on the project? If the IRR of the project is 19%, what is the project alpha?

7.2 THE CAPM AND INDEX MODELS

The CAPM has two limitations: It relies on the theoretical market portfolio, which includes *all* assets (such as real estate, foreign stocks, etc.), and it applies to *expected* as opposed to actual returns. To implement the CAPM, we cast it in the form of an *index model* and use realized, not expected, returns.

An index model replaces the theoretical all-inclusive portfolio with a market index such as the S&P 500. An important advantage of index models is that the composition and rate of return of the index is unambiguous and widely published, hence providing a clear benchmark for performance evaluation.

In contrast to an index model, the CAPM revolves around the elusive "market portfolio." However, because many assets are not traded, investors would not have full access to the market portfolio even if they could exactly identify it. Thus, the theory behind the CAPM rests on a shaky real-world foundation. But, as in all science, a theory is legitimate if it predicts real-world outcomes with sufficient accuracy. In particular, the reliance on the market portfolio shouldn't faze us if the predictions are sufficiently accurate when the index portfolio is substituted for the CAPM market portfolio.

We can start with the central prediction of the CAPM: The market portfolio is mean-variance efficient. An index model can be used to test this hypothesis by verifying that an index chosen to be representative of the full market is mean-variance efficient.

To test mean-variance efficiency of an index portfolio, we must show that the Sharpe ratio of the index is not surpassed by any other portfolio. We will examine this question in the next chapter.

The CAPM predicts relationships among *expected* returns. However, all we can observe are realized (historical) holding-period returns, which in a particular holding period seldom, if ever, match initial expectations. For example, the S&P 500 returned −39% in 2008. Could this possibly have been expected when investors could have invested in risk-free Treasury bills? In fact, this logic implies that any stock-index return less than T-bills must entail a negative departure from expectations. Since expectations must be realized on average, this means that more often than not, positive excess returns exceeded expectations.

The Index Model, Realized Returns, and the Mean–Beta Equation

To move from a model cast in expectations to a realized-return framework, we start with the single-index regression equation in realized excess returns, Equation 6.11:

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{Mt} - r_{ft}) + e_{it} \tag{7.3}$$

On the MARKET FRONT

ALPHA BETTING

IT HAS never been easier to pay less to invest. No fewer than 136 exchange-traded funds (ETFs) were launched in the first half of 2006, more than in the whole of 2005.

For those who believe in efficient markets, this represents a triumph. ETFs are quoted securities that track a particular index, for a fee that is normally just a fraction of a percentage point. They enable investors to assemble a low-cost portfolio covering a wide range of assets from international equities, through government and corporate bonds, to commodities.

But as fast as the assets of ETFs and index-tracking mutual funds are growing, another section of the industry seems to be flourishing even faster. Watson Wyatt, a firm of actuaries, estimates that "alternative asset investment" (ranging from hedge funds through private equity to property) grew by around 20% in 2005, to \$1.26 trillion. Investors who take this route pay much higher fees in the hope of better performance. One of the fastest-growing assets, funds of hedge funds, charge some of the highest fees of all.

Why are people paying up? In part, because investors have learned to distinguish between the market return, dubbed beta, and managers' outperformance, known as alpha. "Why wouldn't you buy beta and alpha separately?" asks Arno Kitts of Henderson Global Investors, a fund-management firm. "Beta is a commodity and alpha is about skill."

Clients have become convinced that no one firm can produce good performance in every asset class. That has led to a "core and satellite" model, in which part of the portfolio is invested in index trackers with the rest in the hands of specialists. But this creates its own problems. Relations with a single balanced manager are simple. It is much harder to research and monitor the performance of specialists. That has encouraged the middlemen—managers of managers (in the traditional institutional business) and funds-of-funds (in the hedge-fund world), which are usually even more expensive.

That their fees endure might suggest investors can identify outperforming fund managers in advance. However, studies suggest this is extremely hard. And even where you can spot talent, much of the extra performance may be siphoned off into higher fees. "A disproportionate amount of the benefits of alpha go to the manager, not the client," says Alan Brown at Schroders, an asset manager.

In any event, investors will probably keep pursuing alpha, even though the cheaper alternatives of ETFs and tracking funds are available. Craig Baker of Watson Wyatt says that, although above-market returns may not be available to all, clients who can identify them have a "first mover" advantage. As long as that belief exists, managers can charge high fees.

SOURCE: *The Economist,* September 14, 2006.

where r_{it} is the holding-period return (HPR) on asset i in period t and α_i and β_i are the intercept and slope of the security characteristic line that relates asset i's realized excess return to the realized excess return of the index. We denote the index return by r_M to emphasize that the index portfolio is proxying for the market. The e_{it} measures firm-specific effects during holding period t; it is the deviation in that period of security i's realized HPR from the regression line, the forecast of return based on the index's actual HPR. We set the relationship in terms of *excess* returns (over r_{ft}), consistent with the CAPM's logic of risk premiums.

To compare the index model with the CAPM predictions about expected asset returns, we take expectations in Equation 7.3. Recall that the expectation of e_{it} is zero, so in terms of expectations, Equation 7.3 becomes

$$E(r_{it}) - r_{ft} = \alpha_i + \beta_i[E(r_{Mt}) - r_{ft}] \quad \textbf{(7.4)}$$

Comparing Equation 7.4 to Equation 7.2 reveals that the CAPM predicts $\alpha_i = 0$. Thus, we have converted the CAPM prediction about unobserved expectations of security returns relative to an unobserved market portfolio into a prediction about the intercept in a regression of observed variables: realized excess returns of a security relative to those of an observed index.

Operationalizing the CAPM in the form of an index model has a drawback, however. If intercepts of regressions of returns on an index differ substantially from zero, you will not be able to tell whether it is because you chose a bad index to proxy for the market or because the theory is not useful.

In actuality, some instances of persistent, positive significant alpha values have been identified; these will be discussed in Chapter 8. Among these are (1) small versus large stocks; (2) stocks of companies that have recently announced unexpectedly good earnings; (3) stocks with high ratios of book value to market value; and (4) stocks with "momentum" that have experienced recent advances in price. In general, however, future alphas are practically impossible to predict from past values. The result is that index models are widely used to operationalize capital asset pricing theory (see the nearby box).

Estimating the Index Model

To illustrate how to estimate the index model, we will use actual data and apply the model to the stock of Google (G), in a manner similar to that followed by practitioners. Let us rewrite Equation 7.3 for Google, denoting Google's excess return as $R_G = r_G - r_f$ and denoting months using the subscript t.

$$R_{Gt} = \alpha_G + \beta_G R_{Mt} + e_{Gt}$$

The dependent variable in this regression equation is Google's excess return in each month, explained by the excess return on the market index in that month, R_{Mt}. The regression coefficients are intercept α_G and slope β_G.

The alpha of Google is the average of the firm-specific factors during the sample period; the zero-average surprise in each month is captured by the last term in the equation, e_{Gt}. This residual is the difference between Google's actual excess return and the excess return that would be predicted from the regression line:

$$\begin{aligned} \text{Residual} &= \text{Actual return} - \text{Predicted return for Google based on market return} \\ e_{Gt} &= R_{Gt} \qquad\qquad - (\alpha_G + \beta_G R_{Mt}) \end{aligned}$$

We are interested in estimating the intercept α_G and Google's beta as measured by the slope coefficient, β_G. We estimate Google's firm-specific risk by *residual standard deviation,* which is just the standard deviation of e_{Gt}.

We conduct the analysis in three steps: Collect and process relevant data; feed the data into a statistical program (here we use Excel) to estimate the regression Equation 7.3; and use the results to answer these questions about Google's stock: (*a*) What have we learned about the behavior of Google's returns, (*b*) what required rate of return is appropriate for investments with the same risk as Google's equity, and (*c*) how might we assess the performance of a portfolio manager who invested heavily in Google stock during this period?

Collecting and processing data We start with the monthly series of Google stock prices and the S&P 500 Index, adjusted for stock splits and dividends over the period January 2006–December 2010.[3] From these series we computed monthly holding-period returns on Google and the market index.

For the same period we compiled monthly rates of return on one-month T-bills.[4] With these three series of returns we generate monthly excess return on Google's stock and the market index. Some statistics for these returns are shown in Table 7.1. Notice that the monthly variation in the T-bill return reported in Table 7.1 does not reflect risk, as investors knew the return on bills at the beginning of each month.

The period of January 2006–December 2010 includes the late stage of recovery from the mild 2001 recession, the severe recession that officially began in December 2007, as well as the first stage of the recovery that began in June 2009. Table 7.1 shows that the effect of the financial crisis was so severe that the monthly geometric average return of the market index, .107%, was less than that of T-bills, .180%. We noted in Chapter 5 that arithmetic averages exceed geometric averages, with the difference between them increasing with return volatility.[5] In

[3]Returns are available from **finance.yahoo.com.** We need to use the price series adjusted for dividends and splits in order to obtain holding-period returns (HPRs). The unadjusted price series would tell us about capital gains alone rather than total returns.

[4]We downloaded these rates from Professor Kenneth French's website: **mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.**

[5]When returns are normally distributed, the relation between the arithmetic average return, r_A, and the geometric average return, r_G (expressed as decimals, not percentages), is arithmetic average = geometric average plus one-half variance of returns. This relation holds approximately when returns are not precisely normally distributed.

TABLE 7.1 Monthly return statistics: T-bills, S&P 500, and Google, January 2006–December 2010

Statistic (%)	T-bills	S&P 500	Google
Average rate of return	0.184	0.239	1.125
Average excess return	–	0.054	0.941
Standard deviation*	0.177	5.11	10.40
Geometric average	0.180	0.107	0.600
Cumulative total 5-year return	11.65	6.60	43.17
Gain Jan 2006–Oct 2007	9.04	27.45	70.42
Gain Nov 2007–May 2009	2.29	−38.87	−40.99
Gain June 2009–Dec 2010	0.10	36.83	42.36

*The rate on T-bills is known in advance, hence SD does not reflect risk.

this period, the monthly SD of the market index, 5.11%, was large enough that despite the market return's lower geometric average, its monthly arithmetic average, .239%, was greater than that of T-bills, .185%, resulting in a positive average excess return of .054% per month.

Google had a cumulative five-year return of 43.17%, a lot better than T-bills (11.65%) or the S&P 500 (6.60%). Its monthly standard deviation of 10.40%, about double that of the market, raises the question of how much of that volatility is systematic.

Google's returns over subperiods within these five years illustrate a common illusion. Observe in Table 7.1 that Google's prerecession increase between January 2006 and October 2007 was 70.42%. The subsequent financial crisis decline (November 2007–May 2009) and recovery (June 2009–December 2010) were of similar magnitudes of −40.99% and 42.36%, respectively, and you might think they should have just about canceled out. Yet the total five-year return was "only" 43.17%, around 27% less than the prerecession gain of 70.42%. Where did that 27% go? It went in the crisis: The decline and subsequent increase had a total impact on cumulative return of $(1 - .4099) \times (1 + .4236) = .8401$, resulting in a loss of about 16%. Apply that loss to the prerecession value of stock, and you obtain $.8401 \times (1 + .7042) = 1.43$, just equal to the five-year cumulative return.

Why didn't the 40.99% loss and 42.36% gain (roughly) cancel out? In general, a large gain following a large loss has a muted impact on cumulative return because it acts on a diminished investment base, while a large loss following a large gain has an amplified impact because it acts on a greater investment base. The greater the fluctuations, the greater the impact on final investment value, which is why the spread between the geometric average (which reflects cumulative return) and the arithmetic average grows with stock volatility.

Figure 7.3 Panel A shows the monthly return on the securities during the sample period. The significantly higher volatility of Google is evident, and the graph suggests that its beta is greater than 1: When the market moves, Google tends to move in the same direction, but by greater amounts.

Figure 7.3 Panel B shows the evolution of cumulative returns. It illustrates the positive index returns in the early years of the sample, the steep decline during the recession, and the significant partial recovery of losses at the end of the sample period. Whereas Google outperforms T-bills, T-bills outperform the market index over the period, highlighting the worse-than-expected realizations in the capital market.

security characteristic line (SCL)

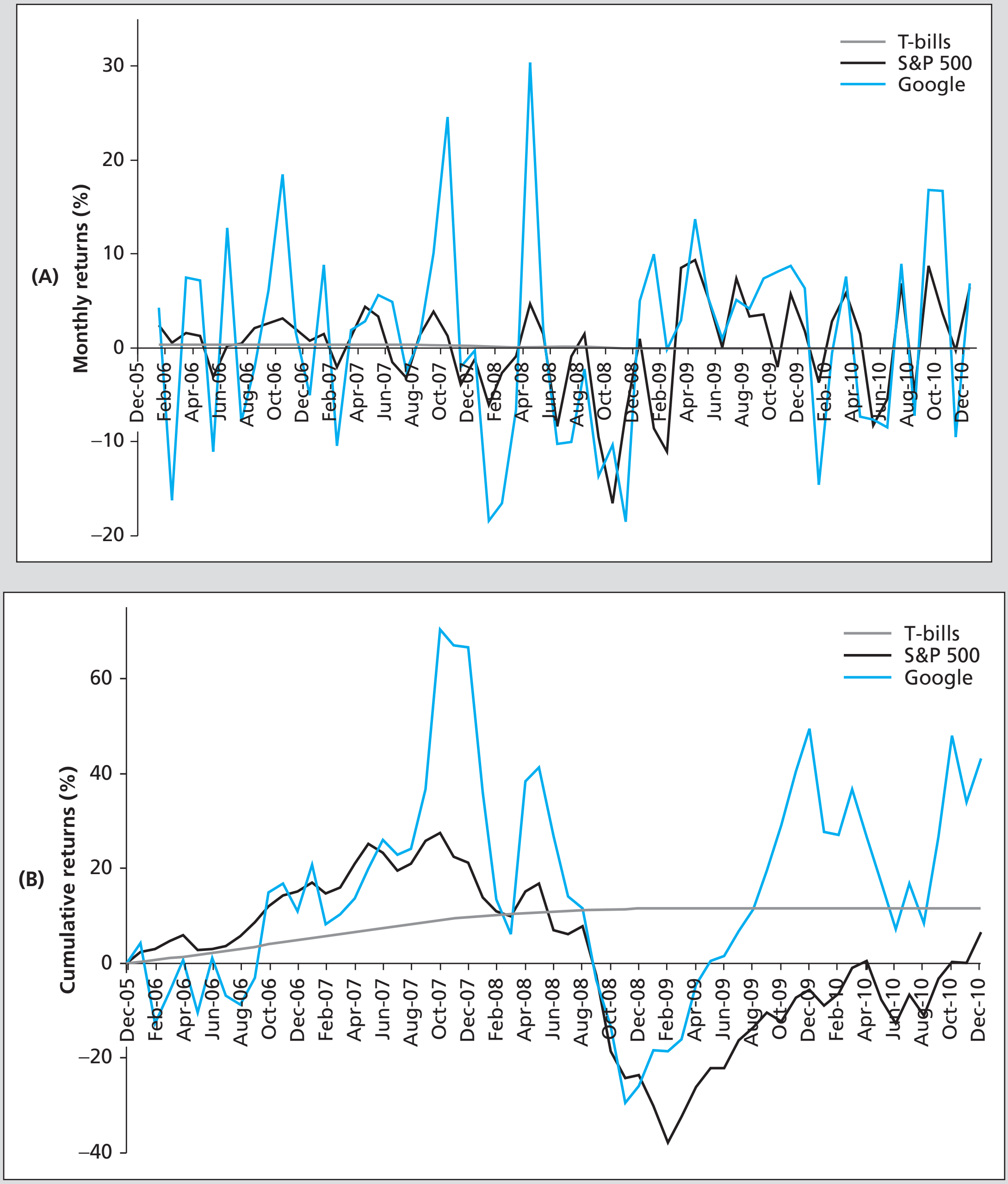
A plot of a security's expected excess return over the risk-free rate as a function of the excess return on the market.

Estimation results We regressed Google's excess returns against those of the index using the Regression command from the Data Analysis menu of Excel.[6] The scatter diagram in Figure 7.4 shows the data points for each month as well as the regression line that best fits the data. As noted in the previous chapter, this is called the **security characteristic line (SCL),** because it describes the relevant characteristics of the stock. Figure 7.4 allows us to view the residuals, the deviation of Google's return each month from the prediction of

[6]Mac users can download a free data analysis tool kit called StatPlus from **www.AnalystSoft.com.**

FIGURE 7.3

Returns for T-bills, S&P 500 Index, and Google stock. **Panel A:** monthly returns; **Panel B:** cumulative returns.



the regression equation. By construction, these residuals *average* to zero, but in any particular month, the residual may be positive or negative.

The residuals for April 2008 (23.81%) and November 2008 (−10.97%) are labeled explicitly. The April 2008 point lies above the regression line, indicating that in this month, Google's return was better than predicted from the market return. The distance between the point and the regression line is Google's firm-specific return, which is the residual for April.

The standard deviation of the residuals indicates the accuracy of predictions from the regression line. If there is a lot of firm-specific risk, there will be a wide scatter of points around the line (a high residual standard deviation), indicating that the market return will not enable a precise forecast of Google's return.

FIGURE 7.4

Scatter diagram and security characteristic line for Google against the S&P 500, Jan 2006–Dec 2010

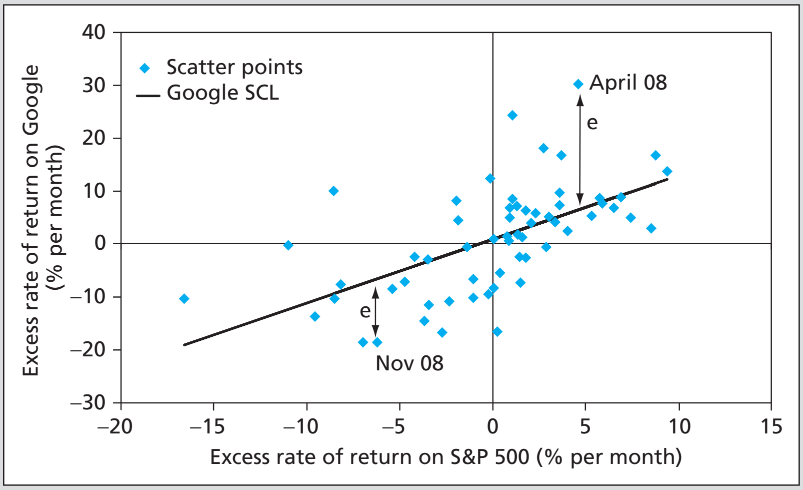


Table 7.2 is the regression output from Excel. The first line shows that the correlation coefficient between the excess returns of Google and the index was .59. The more relevant statistic, however, is the *adjusted R-square* (.3497). It is the square of the correlation coefficient, adjusted downward for the number of coefficients or "degrees of freedom" used to estimate the regression line.[7] The adjusted R-square tells us that 34.97% of the variance of Google's excess returns is explained by the variation in the excess returns of the index, and hence the remainder, or 65.03%, of the variance is firm specific, or unexplained by market movements. The dominant contribution of firm-specific factors to variation in Google's returns is typical of individual stocks, reminding us why diversification can greatly reduce risk.

The standard deviation of the residuals is referred to in the output (below the adjusted R-square) as the "standard error" of the regression (8.46%). In roughly two-thirds of the months, the firm-specific component of Google's excess return was between ±8.46%. This wide spread is more evidence of Google's considerable firm-specific volatility.

The middle panel of Table 7.2, labeled "ANOVA" (for "analysis of variance"), analyzes the sources of variability in Google returns, those two sources being variation in market returns and variation due to firm-specific factors. For the most part, these statistics are not essential for our analysis. You can, however, use the total sum of squares, labeled *SS,* to find Google's variance over this period. Divide the total *SS,* or 6,381.15, by the degrees of freedom, *df,* or 59, to find that variance of excess returns was 108.16, implying a monthly standard deviation of 10.40%, as reported in Table 7.1.

Finally, the bottom panel of the table shows the estimates of the regression intercept and slope (alpha = .88% and beta = 1.20). The positive alpha means that, measured by *realized* returns, Google stock was above the security market line (SML) for this period. But the next column shows considerable imprecision in this estimate as measured by its standard error, 1.09, considerably larger than the estimate itself. The *t*-statistic (the ratio of the estimate of alpha to its standard error) is only .801, indicating low statistical significance. This is reflected in the large *p*-value in the next column, .426, which indicates the probability that an estimate of alpha this large could have resulted from pure chance even if the true alpha were zero. The last two columns give the upper and lower bounds of the 95% confidence interval around the coefficient estimate. This confidence interval tells us that, with a probability of .95, the true alpha lies in the wide interval from −1.74 to 3.49. Thus, we cannot conclude from this particular

[7]The relationship between the adjusted R-square (R_A^2) and the unadjusted (R^2) with n observations and k independent variables (plus intercept) is $1 - R_A^2 = (1 - R^2)\dfrac{n-1}{n-k-1}$, and thus a greater k will result in a larger downward adjustment to R_A^2. While R^2 cannot fall when you add an additional independent variable to a regression, R_A^2 can actually fall, indicating that the explanatory power of the added variable is not enough to compensate for the extra degree of freedom it uses. The more "parsimonious" model (without the added variable) would be considered statistically superior.

TABLE 7.2 Security characteristic line for Google (S&P 500 used as market index), January 2006–December 2010

Linear Regression

Regression Statistics	(This table produced by StatPlus patch for Mac Excel, which lacks the Data Analysis tool of Windows Excel)
R (correlation)	0.5914
R-square	0.3497
Adjusted *R*-square	0.3385
SE of regression	8.4585
Total number of observations	60

Regression equation: Google (excess return) = 0.8751 + 1.2031 * S&P 500 (excess return)

ANOVA

	df	*SS*	*MS*	*F*	*p*-level
Regression	1	2231.50	2231.50	31.19	0.0000
Residual	58	4149.65	71.55		
Total	59	6381.15			

	Coefficients	Standard Error	*t*-Statistic	*p*-value	LCL	UCL
Intercept	0.8751	1.0920	0.8013	0.4262	−1.7375	3.4877
S&P 500	1.2031	0.2154	5.5848	0.0000	0.6877	1.7185
t-Statistic (2%)	2.3924					

LCL—Lower confidence interval (95%)
UCL—Upper confidence interval (95%)

sample, with any degree of confidence, that Google's true alpha was not zero, which would be the prediction of the CAPM.

The second line in the panel gives the estimate of Google's beta, which is 1.20. The standard error of this estimate is .215, resulting in a *t*-statistic of 5.58, and a practically zero *p*-value for the hypothesis that the true beta is in fact zero. In other words, the probability of observing an estimate this large if the true beta were zero is negligible. Another important question is whether Google's beta is significantly different from the average stock beta of 1. This hypothesis can be tested by computing the *t*-statistic:

$$t = \frac{\text{Estimated value} - \text{Hypothesis value}}{\text{Standard error of estimate}} = \frac{1.2031 - 1}{.2154} = .94$$

This value is considerably below the conventional threshold for statistical significance; we cannot say with confidence that Google's beta differs from 1. The 95% confidence interval for beta ranges from .69 to 1.72.

What we learn from this regression The regression analysis reveals much about Google, but we must temper our conclusions by acknowledging that the tremendous volatility in stock market returns makes it difficult to derive strong statistical conclusions about the parameters of the index model, at least for individual stocks. With such noisy variables we must expect imprecise estimates; such is the reality of capital markets.

Despite these qualifications, we can safely say that Google is a cyclical stock, that is, its returns vary equally with or more than the overall market, as its beta is higher than the average value of 1, albeit not significantly so. Thus, we would expect Google's excess return to respond, on average, more than one-for-one with the market index. Without additional information, if we had to forecast the volatility of a portfolio that includes Google, we would use the beta estimate of 1.20 to compute the contribution of Google to portfolio variance.

Moreover, if we had to advise Google's management of the appropriate discount rate for a project that is similar in risk to its equity,[8] we would use this beta estimate in conjunction with the prevailing risk-free rate and our forecast of the expected excess return on the market index. Suppose the current T-bill rate is 2.75%, and our forecast for the market excess return is 5.5%. Then the required rate of return for an investment with the same risk as Google's equity would be

$$\begin{aligned}\text{Required rate} &= \text{Risk-free rate} + \beta \times \text{Expected excess return of index} \\ &= r_f + \beta(r_M - r_f) = 2.75 + 1.20 \times 5.5 = 9.35\%\end{aligned}$$

In light of the imprecision of both the market risk premium and Google's beta estimate, we would try to bring more information to bear on these estimates. For example, we would compute the betas of other firms in the industry, which ought to be similar to Google's, to sharpen our estimate of Google's systematic risk.

Finally, suppose we were asked to determine whether, given Google's positive alpha, a portfolio manager was correct in loading up a managed portfolio with Google stock over the period 2006–2010.

To answer this question, let's find the optimal position in Google that would have been prescribed by the Treynor-Black model of the previous chapter. Let us assume that the manager had an accurate estimate of Google's alpha and beta, as well as its residual standard deviation and correlation with the index (from Tables 7.1 and 7.2). We still need information about the manager's forecast for the index, since we know it was *not* the actual return. Suppose the manager assumed a market-index risk premium of .6%/month (near the historical average) and correctly estimated the index standard deviation of 5.11%/month. Thus, the manager's input list would have included:

Security	Risk Premium (%)	Standard Deviation (%)	Correlation
Index	0.7	5.11	
Google	0.875 + 1.203 × 0.6 = 1.60	10.40	0.59

Using Equation 6.10 we calculate for the optimized portfolio (P):

$$w_M = .3911 \quad w_G = .6089 \quad E(R_P) = 1.21\% \quad \sigma_P = 7.69\%$$

Thus, it appears the manager would have been quite right to tilt the portfolio heavily toward Google during this period. This reflects its large positive alpha over the sample period.

We can also measure the improvement in portfolio performance. Using Equation 6.8, the Sharpe ratio of the index and the optimized portfolios based on expected returns are

$$S_M = .12 \quad S_P = .16$$

So the position in Google substantially increased the Sharpe ratio.

This exercise would not be complete without the next step, where we observe the performance of the proposed "optimal" portfolio. After all, analysts commonly use available data to construct portfolios for a future period. We put *optimal* in quotes because everyone in the profession knows that past alpha values do not predict future values. Hence, a portfolio formed solely, or even primarily, by extrapolating past alpha would never qualify as optimal. However, if we treat this alpha as though it came from security analysis, we can paint a picture of what might go on in the trenches of portfolio management.

[8]A word of caution: Remember that as a general rule, equity beta is greater than asset beta, because leverage increases the exposure of equity to business risk. The required rate of return on Google's *stock* would be appropriate for an investment with the same risk as Google's *equity.* In this instance, Google has virtually no debt so this issue may be moot, but this is not generally the case.

At this writing, we have 10 additional months of returns (January 2011–October 2011) for the S&P 500, Google, and T-bills. We test the proposed portfolio for three future periods following the data collection and analysis period: the next quarter, next semiannual period, and next 10 months. For each of these periods we compare the performance of the proposed portfolio to the passive index portfolio and to T-bills. The results are as follows:

Cumulative Returns (%) of Three Alternative Strategy Portfolios

Portfolio	Proposed Google-S&P 500	Passive: S&P 500 Index	T-bills
2011 Q1	1.36	5.42	0.01
2011 First half	−7.35	5.01	0.01
January–October 2011	0.41	−0.35	0.01

We see that 2011 began as a good year for the market, with a half-year cumulative excess return of 5.01%. Up to this point, the proposed portfolio stumbled badly. Yet the next four months bring a complete reversal of fortune: The market stumbled badly, dragging its cumulative return into negative territory, while Google shined and returned the proposed "optimal" portfolio into positive territory. It is clear that evaluating performance is fraught with enormous potential estimation error. Even a nonsense portfolio can have its day when volatility is so high. This basic fact of investment life makes portfolio performance evaluation hazardous, as we discuss in Chapter 18.

Predicting Betas

A single-index model may not be fully consistent with the CAPM, which may not be a sufficiently accurate predictor of risk premiums. Still the concept of systematic versus diversifiable risk is useful. Systematic risk is approximated well by the regression equation beta and nonsystematic risk by the residual variance of the regression.

As an empirical rule, it appears that betas exhibit a statistical property called *mean reversion.* This suggests that high-β (that is, $\beta > 1$) securities tend to exhibit a lower β in the future, while low-β (that is, $\beta < 1$) securities exhibit a higher β in future periods. Researchers who desire predictions of future betas often adjust beta estimates from historical data to account for regression toward 1. For this reason, it is necessary to verify whether the estimates are already "adjusted betas."

A simple way to account for mean reversion is to forecast beta as a weighted average of the sample estimate with the value 1.

EXAMPLE 7.7

Forecast of Beta

Suppose that past data yield a beta estimate of .65. A common weighting scheme is $^2/_3$ on the sample estimate and $^1/_3$ on the value 1. Thus, the adjusted forecast of beta will be

$$\text{Adjusted beta} = {}^2/_3 \times .65 + {}^1/_3 \times 1 = .77$$

The final forecast of beta is in fact closer to 1 than the sample estimate.

A more sophisticated technique would base the weight of the sample beta on its statistical quality. A more precise estimate of beta will get a higher weight.

However, obtaining a precise statistical estimate of beta from past data on individual stocks is a formidable task, because the volatility of rates of return is so large. In particular, there is a lot of "noise" in the data due to firm-specific events. The problem is less severe with diversified portfolios because diversification reduces firm-specific variance.

One might hope that more precise estimates of beta could be obtained by using a long time series of returns. Unfortunately, this is not a solution because betas change over time[9] and old data can provide a misleading guide to current betas.

Two methods can help improve forecasts of beta. The first is an application of a technique that goes by the name of ARCH models. ARCH models better predict variance and covariance using high-frequency (daily) historical data to identify persistent changes in variance and covariance. The second method involves an additional step where beta estimates from time series regressions are augmented by other information about the firm, for example, P/E ratios.

7.3 THE CAPM AND THE REAL WORLD

In limited ways, portfolio theory and the CAPM have become accepted tools in the practitioner community. Many investment professionals think about the distinction between firm-specific and systematic risk and are comfortable with the use of beta to measure systematic risk. Still, the nuances of the CAPM are not nearly as well established in the community. For instance, compensation of portfolio managers is not based on appropriate risk-adjusted performance (see Chapter 18). What can we make of this?

New ways of thinking about the world (that is, new models or theories) displace old ones when the old models become either intolerably inconsistent with data or when the new model is demonstrably more consistent with available data. When Copernicus overthrew the age-old belief that stars orbit about the sun in circular motions, it took many years before navigators replaced old astronomical tables with superior ones based on his theory. The old tools fit the data with sufficient precision. To some extent, the slowness with which modern portfolio theory has permeated daily practice in the money management industry also has to do with its precision in fitting data and explaining variation in rates of return across assets. Let's review some of the evidence on this score.

The CAPM was first published by Sharpe in the *Journal of Finance* (the journal of the American Finance Association) in 1964 and took the world of finance by storm. Early tests by Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) were only partially supportive of the CAPM: Average returns were higher for higher-beta portfolios, but the reward for beta risk was less than predicted by the simple version of the CAPM.

While this sort of evidence against the CAPM remained largely within the ivory towers of academia, Roll's (1977) paper "A Critique of Capital Asset Pricing Tests" shook the practitioner world as well. Roll argued that since the true market portfolio can never be observed, the CAPM is *necessarily* untestable.

The publicity given the now classic "Roll's critique" resulted in popular articles such as "Is Beta Dead?" that effectively slowed the permeation of portfolio theory through the world of finance.[10] Although Roll is absolutely correct on theoretical grounds, some tests suggest that the error introduced by using a broad market index as proxy for the true, unobserved market portfolio is perhaps not the greatest problem involved in testing the CAPM.

Fama and French (1992) published a study that dealt the CAPM an even harsher blow. They claimed that in contradiction to the CAPM, certain characteristics of the firm, namely, size and the ratio of market to book value, were far more useful in predicting future returns than beta.

Fama and French and several others have published many follow-up studies of this topic. We will review some of this literature later in the chapter, and the nearby box discusses controversies about the risk-return relationship that have been reinforced in the wake of the financial crisis of 2008. It seems clear from these studies that beta does not tell the whole story of risk. There seem to be risk factors that affect security returns beyond beta's one-dimensional measurement of market sensitivity. In the next section, we introduce a theory of risk premiums that explicitly allows for multiple risk factors.

[9] *ARCH* stands for "autoregressive conditional heteroskedasticity." (The model was developed by Robert F. Engle, who received the 2003 Nobel Prize in Economics.) This is a fancy way of saying that the volatility (and covariance) of stocks changes over time in ways that can be at least partially predicted from past data.

[10] A. Wallace, "Is Beta Dead?" *Institutional Investor* 14 (July 1980), pp. 22–30.

On the MARKET FRONT

TAKING STOCK

Since the stock market bubble of the late 1990s burst, investors have had ample time to ponder where to put the remains of their money. Economists and analysts too have been revisiting old ideas. None has been dearer to them than the capital asset pricing model (CAPM), a formula linking movements in a single share price to those of the market as a whole. The key statistic here is "beta."

Many investors and managers have given up on beta, however. Although it is useful for working out overall correlation with the market, it tells you little about share-price performance in absolute terms. In fact, the CAPM's obituary was already being written more than a decade ago when a paper by Eugene Fama and Kenneth French showed that the shares of small companies and "value stocks" (shares with low price–earnings ratios or high ratios of book value to market value) do much better over time than their betas would predict.

Another paper, by John Campbell and Tuomo Vuolteenaho of Harvard University, tries to resuscitate beta by splitting it into two.* The authors start from first principles. In essence, the value of a company depends on two things: its expected profits and the interest rate used to discount these profits. Changes in share prices therefore stem from changes in one of these factors.

From this observation, these authors propose two types of beta: one to gauge shares' responses to changes in profits; the other to pick up the effects of changes in the interest rate. Allowing for separate cash flow versus interest rate betas helps better explain the performance of small and value companies. Shares of such companies are more sensitive than the average to news about profits, in part because they are bets on future growth. Shares with high price–earnings ratios vary more with the discount rate. In all cases, above-average returns compensate investors for above-average risks.

EQUITY'S ALLURE

Beta is a tool for comparing shares with each other. Recently, however, investors have been worried about equity as an asset class. The crash left investors asking what became of the fabled equity premium, the amount by which they can expect returns on shares to exceed those from government bonds.

History says that shareholders have a lot to be optimistic about. Over the past 100 years, investors in American shares have enjoyed a premium, relative to Treasury bonds, of around seven percentage points. Similar effects have been seen in other countries. Some studies have reached less optimistic conclusions, suggesting a premium of four or five points. But even this premium seems generous.

Many answers have been put forward to explain the premium. One is that workers cannot hedge against many risks, such as losing their jobs, which tend to hit at the same time as stock market crashes; this means that buying shares would increase the volatility of their income, so that investors require a premium to be persuaded to hold them. Another is that shares, especially in small companies, are much less liquid than government debt. It is also sometimes argued that in extreme times—in depression or war, or after bubbles—equities fare much worse than bonds, so that equity investors demand higher returns to compensate them for the risk of catastrophe.

Yes, over long periods equities have done better than bonds. But the equity "premium" is unpredictable. Searching for a consistent, God-given premium is a fool's errand.

*John Campbell and Tuomo Vuolteenaho, "Bad Beta, Good Beta," *American Economic Review* 94 (December 2004), pp. 1249–1275.

Liquidity, a different kind of risk factor, was ignored for a long time. Although first analyzed by Amihud and Mendelson as early as 1986, it is yet to be accurately measured and incorporated in portfolio management. Measuring liquidity and the premium commensurate with illiquidity is part of a larger field in financial economics, namely, market structure. We now know that trading mechanisms on stock exchanges can affect the liquidity of assets traded on these exchanges and thus significantly affect their market value.

Despite all these issues, beta is not dead. Research shows that when we use a more inclusive proxy for the market portfolio than the S&P 500 (specifically, an index that includes human capital) and allow for the fact that beta changes over time, the performance of beta in explaining security returns is considerably enhanced (Jagannathan and Wang, 1996). We know that the CAPM is not a perfect model and that ultimately it will be greatly refined. Still, the logic of the model is compelling, and more sophisticated models of security pricing all rely on the key distinction between systematic and diversifiable risk. The CAPM therefore provides a useful framework for thinking rigorously about the relationship between security risk and return. This is as much as Copernicus had when he was shown the prepublication version of his book just before he passed away.

7.4 MULTIFACTOR MODELS AND THE CAPM

The index model allows us to decompose stock variance into systematic risk and firm-specific risk that can be diversified in large portfolios. In the index model, the return on the market portfolio summarized the aggregate impact of macro factors. In reality, however, systematic risk is not due to one source but instead derives from uncertainty in many economywide factors such as business-cycle risk, interest or inflation rate risk, energy price risk, and so on. It stands to reason that a more explicit representation of systematic risk, allowing stocks to exhibit different sensitivities to its various facets, would constitute a useful refinement of the single-factor model. We can expect that models that allow for several systematic factors—**multifactor models**—can provide better descriptions of security returns.

multifactor models

Models of security returns that respond to several systematic factors.

Let's illustrate with a two-factor model. Suppose the two most important macroeconomic sources of risk are the state of the business cycle reflected in returns on a broad market index such as the S&P 500 and unanticipated changes in interest rates captured by returns on a Treasury-bond portfolio. The return on any stock will respond both to sources of macro risk and to its own firm-specific influences. Therefore, we can expand the single-index model, Equation 7.3, describing the excess rate of return on stock i in some time period t as follows:

$$R_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{i\text{TB}} R_{\text{TB}t} + e_{it} \tag{7.5}$$

where $\beta_{i\text{TB}}$ is the sensitivity of the stock's excess return to that of the T-bond portfolio and $R_{\text{TB}t}$ is the excess return of the T-bond portfolio in month t.

How will the security market line of the CAPM generalize once we recognize the presence of multiple sources of systematic risk? Not surprisingly, a multifactor index model gives rise to a multifactor security market line in which the risk premium is determined by the exposure to *each* systematic risk factor and by a risk premium associated with each of those factors. Such a multifactor CAPM was first presented by Merton (1973).

In a two-factor economy of Equation 7.5, the expected rate of return on a security would be the sum of three terms:

1. The risk-free rate of return.
2. The sensitivity to the market index (i.e., the market beta, β_{iM}) times the risk premium of the index, $[E(r_M) - r_f]$.
3. The sensitivity to interest rate risk (i.e., the T-bond beta, $\beta_{i\text{TB}}$) times the risk premium of the T-bond portfolio, $[E(r_{\text{TB}}) - r_f]$.

This assertion is expressed mathematically as a two-factor security market line for security i:

$$E(r_i) = r_f + \beta_{iM}[E(r_M) - r_f] + \beta_{i\text{TB}}[E(r_{\text{TB}}) - r_f] \tag{7.6}$$

Equation 7.6 is an expansion of the simple security market line. Once we generalize the single-index SML to multiple risk sources, each with its own risk premium, the insights are similar.

EXAMPLE 7.8

A Two-Factor SML

Northeast Airlines has a market beta of 1.2 and a T-bond beta of .7. Suppose the risk premium of the market index is 6%, while that of the T-bond portfolio is 3%. Then the overall risk premium on Northeast stock is the sum of the risk premiums required as compensation for each source of systematic risk.

The risk premium attributable to market risk is the stock's exposure to that risk, 1.2, multiplied by the corresponding risk premium, 6%, or 1.2 × 6% = 7.2%. Similarly, the risk premium attributable to interest rate risk is .7 × 3% = 2.1%. The total risk premium is 7.2 + 2.1 = 9.3%. Therefore, if the risk-free rate is 4%, the expected return on the portfolio should be

4.0%	Risk-free rate
+ 7.2	+Risk premium for exposure to market risk
+ 2.1	+Risk premium for exposure to interest rate risk
13.3%	Total expected return

More concisely,

$$E(r) = 4\% + 1.2 \times 6\% + .7 \times 3\% = 13.3\%$$

Suppose the risk premiums in Example 7.8 were $E(r_M) - r_f = 4\%$ and $E(r_{TB}) - r_f = 2\%$. What would be the equilibrium expected rate of return on Northeast Airlines?

CONCEPT check 7.5

The multifactor model clearly gives us a richer way to think about risk exposures and compensation for those exposures than the single-index model or the CAPM. But what are the relevant additional systematic factors?

Three methodologies have been deployed to identify systematic factors in security returns, based on theory, regression analysis, or other statistical tools. The theory-based approach specifies potential extra-market risk factors on the basis of their potential impact on lifetime consumption and bequests. Broadly speaking, these variables fall into two groups: (1) prices of items that make up a substantial part of the lifetime consumption basket of many consumers, such as health care or housing, and (2) variables that affect future investment opportunities, such as interest rates or prices of inputs to major manufacturing and service industries. Investors are expected to respond to these sources of risk to their future consumption and investment opportunities by exhibiting excess demand for securities that can hedge those risks. This demand will drive up prices and drive down expected rates of return. So correlation with these sources of risk can induce its own risk premium. Variables that are important enough to affect security prices through a risk premium in such models are called *priced risk factors.* The theory therefore predicts a multi-index model in which portfolios that track each priced risk factor augment the market index in a multifactor version of the SML.

Some factors might help to explain returns but still might not carry a risk premium. For example, securities of firms in the same industry may be highly correlated. If we were to run a regression of the returns on one such security on the returns of the market index and a portfolio of the other securities in the industry, we would expect to find a significant coefficient on the industry portfolio. However, if this industry is a small part of the broad market, the industry risk can be diversified away. Thus, although an industry coefficient measures sensitivity to the industry factor, it does not necessarily represent exposure to systematic risk and will not result in a risk premium. We say that such factors are not priced, that is, they do not carry a risk premium.

The empirical content of a model of this type depends on the actual aggregate demand for these portfolios. So far, these models have not produced a clearly superior multi-index equation, suggesting that investors are not willing to pay significant premiums to hedge against these extra-market risk factors.

The regression-based approach seeks economic variables, or portfolios tracking those variables, that can significantly improve the explanatory power of the single-index equation. So far, one of these, the Fama-French factor model, has been most successful and will be discussed next.

The statistics-based approach deploys principle components and factor analysis procedures to identify systematic factors from only the return history of a security universe. This approach identifies a set of portfolios that explain returns well *within a given sample.* But in practice, the composition of these portfolios appears to change quickly over time and tends to perform poorly when applied to out-of-sample data. Consequently, this approach has largely been abandoned.

The Fama-French Three-Factor Model

Fama and French (1996) proposed a three-factor model that has become a standard tool for empirical studies of asset returns. They add to the market-index portfolios formed on the basis of firm size and book-to-market ratio to explain average returns. These additional factors are motivated by the observations that average returns on stocks of small firms and on stocks of firms with a high ratio of book value of equity to market value of equity have historically been higher than predicted by the security market line of the CAPM. This observation suggests that size or the book-to-market (B/M) ratio may be *proxies* for exposures to sources of systematic risk not captured by the CAPM beta, and thus result in return premiums. For example, Fama and French point out that firms with high ratios of book-to-market value are more likely to be in financial distress and that small stocks may be more sensitive to changes in business conditions. Thus, these variables may capture sensitivity to macroeconomic risk factors.

TABLE 7.3 Statistics for monthly rates of return (%), January 2006–December 2010

	Excess Return*		Total Return	
Security	**Average**	**Standard Deviation**	**Geometric Average**	**Cumulative Return**
T-bill	0	0	0.18	11.65
Market index†	0.26	5.44	0.30	19.51
SMB	0.34	2.46	0.31	20.70
HML	0.01	2.97	−0.03	−2.06
Google	0.94	10.40	0.60	43.17

*Total return for SMB and HML.

†Includes all NYSE, NASDAQ, and AMEX stocks.

While the high book-to-market group includes many firms in financial distress, which depresses market value relative to book value, for the most part this group includes relatively mature firms. The latter derive a larger share of their market value from assets already in place, rather than growth opportunities. This group often is called *value stocks.* In contrast, low-B/M companies are viewed as *growth firms* whose market values derive from anticipated future cash flows, rather than from assets already in place. Considerable evidence (which we will review in the following chapter) suggests that value stocks trade at lower prices than growth stocks (or, equivalently, have offered a higher average rate of return); the differential is known as the *value premium.*

While a value premium may be appropriate compensation for risk for a firm whose high B/M ratio reflects potential financial distress, it would seem paradoxical for firms whose high B/M ratio reflects maturity and thus more predictable future cash flows. It implies that, other things equal, the required rate for growth stocks is lower than that of more mature value firms. This is a puzzle; one explanation is that mature firms with large amounts of installed capital confront higher adjustment costs in adapting to shocks in the product markets in which they operate.

How can we make the Fama-French (FF) model operational? To illustrate, we will follow the same general approach that we applied for Google earlier, but now using the more general model.

Collecting and processing data To create portfolios that track the size and B/M factors, one can sort industrial firms by size (market capitalization or market "cap") and by B/M ratio. The size premium is constructed as the difference in returns between small and large firms and is denoted by SMB ("small minus big"). Similarly, the B/M premium is calculated as the difference in returns between firms with a high versus low B/M ratio and is denoted HML ("high minus low" ratio).

Taking the difference in returns between two portfolios has an economic interpretation. The SMB return, for example, equals the return from a long position in small stocks, financed with a short position in the large stocks. Note that this is a portfolio that entails no *net* investment.[11]

Summary statistics for these portfolios in our sample period are reported in Table 7.3. We use a broad market index, the value-weighted return on all stocks traded on U.S. national exchanges (NYSE, Amex, and NASDAQ) to compute the excess return on the market portfolio.

The "returns" of the SMB and HML portfolios require careful interpretation. As noted above, these portfolios do not by themselves represent investment portfolios, as they entail

[11]Interpreting the returns on the SMB and HML portfolios is a bit subtle because both portfolios are zero net investments, and therefore one cannot compute profit per dollar invested. For example, in the SMB portfolio, for every dollar held in small capitalization stocks, there is an offsetting short position in large capitalization stocks. The "return" for this portfolio is actually the profit on the overall position per dollar invested in the small-cap firms (or equivalently, per dollar shorted in the large-cap firms).

TABLE 7.4 Regression statistics for alternative specifications:
1.A Single index with S&P 500 as market proxy
1.B Single index with broad market index (NYSE + NASDAQ + Amex)
2. Fama-French three-factor model (broad market + SMB + HML)

Monthly returns January 2006–December 2010

Estimate	Single Index Specification: S&P 500	Single Index Specification: Broad Market Index	FF 3-Factor Specification with Broad Market Index
Correlation coefficient	0.59	0.61	0.70
Adjusted *R*-Square	0.34	0.36	0.47
Residual SD = Regression SE (%)	8.46	8.33	7.61
Alpha = Intercept (%)	0.88 (1.09)	0.64 (1.08)	0.62 (0.99)
Market beta	1.20 (0.21)	1.16 (0.20)	1.51 (0.21)
SMB (size) beta	–	–	−0.20 (0.44)
HML (book to market) beta	–	–	−1.33 (0.37)

Note: Standard errors in parenthesis.

zero net investment. Rather, they may be interpreted as side bets on whether one type of stock will beat another (e.g., large versus small ones for SMB).

To apply the FF three-factor portfolio to Google, we need to estimate Google's beta on each factor. To do so, we generalize the regression Equation 7.3 of the single-index model and fit a multivariate regression:[12]

$$r_G - r_f = \alpha_G + \beta_M (r_M - r_f) + \beta_{HML} r_{HML} + \beta_{SMB} r_{SMB} + e_G \tag{7.7}$$

To the extent that returns on the size (SMB) and book-to-market (HML) portfolios proxy for risk that is not fully captured by the market index, the beta coefficients on these portfolios represent exposure to systematic risks beyond the market-index beta.[13]

Estimation results Both the single-index model (alternatively employing the S&P 500 Index and the broad market index) and the FF three-factor model are summarized in Table 7.4. The broad market index includes more than 4,000 stocks, while the S&P 500 includes only 500 of the largest U.S. stocks, in which list Google ranked fourteenth in January 2012.[14]

In this sample, the broad market index tracks Google's returns better than the S&P 500, and the three-factor model is a better specification than the one-factor model. This is reflected in three aspects of a successful specification: a higher adjusted R-square, a lower residual SD, and a smaller value of alpha. This outcome turns out to be typical, which makes a broader market index the choice of researchers and the FF model the current first-line empirical model of security returns.[15]

[12]These data are available from Kenneth French's website: **mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.**

[13]When we estimate Equation 7.7, we subtract the risk-free return from the market portfolio but not from the returns on the SMB or HML portfolios. The total rate of return on the market index represents compensation for *both* the time value of money (the risk-free rate) and investment risk. Therefore, only the excess of its return above the risk-free rate represents a premium or reward for bearing risk. In contrast, as noted in footnote 11, the SMB or HML portfolios are zero-net-investment positions. As a result, there is no compensation required for time value, only for risk, and the total "return" therefore may be interpreted as a risk premium.

[14]You may ask, "Why switch to another market index?" In Table 7.2 we were concerned with typical industry practice. When using the more sophisticated FF model, it is important to use a more representative index than the S&P 500, specifically one with greater representation of smaller and younger firms.

[15]The FF model is often augmented by an additional factor, usually *momentum,* which classifies stocks according to which ones have recently increased or recently decreased in price. Liquidity is also increasingly used as yet another additional factor.

Google's market beta estimate is very different in the three-factor model (1.51 versus 1.20 or 1.16 in one-factor models). Moreover, this coefficient value implies high cyclicality and is significantly greater than 1: It exceeds 1 by 2.43 standard errors. The SMB beta is negative (−.20), as you would expect for a firm as large as Google, yet it is not significantly different from zero (standard error = .44). Google still exhibits a negative and significant book-to-market beta (coefficient = 1.33, standard error = .37), however, indicating that it is still a growth stock.

What we learn from this regression While the FF three-factor model offers a richer and more accurate description of asset returns, applying this model requires two more forecasts of future returns, namely, for the SMB and HML portfolios. We have so far in this section been using a T-bill rate of 2.75% and a market risk premium of 5.5%. If we add to these values a forecast of 2.5% for the SMB premium and 4% for HML, the required rate for an investment with the same risk as Google's equity would be

$$E(r_G) = r_f + \beta_M[E(r_M) - r_f] + \beta_{SMB}E(r_{SMB}) + \beta_{HML}E(r_{HML})$$
$$2.75 + (1.51 \times 5.5) + (-.20 \times 2.5) + (-1.33 \times 4) = 5.24\%$$

which is considerably lower than the rate derived from cyclical considerations alone (i.e., single-beta models). Notice from this example that to obtain expected rates of return, the FF model requires, in addition to a forecast of the market-index return, a forecast of the returns of the SMB and HML portfolios, making the model more difficult to apply. This can be a critical issue. If such forecasts are difficult to devise, the single-factor model may be preferred even if it is less successful in explaining *past* returns.[16]

Another reason a multi-index model is more difficult to implement is that currently it would be difficult to hold the prescribed optimal portfolio. As of yet, there are no vehicles (index funds or ETFs) to directly invest in SMB and HML.

Passive investors would have to invest in a suitable small-stock portfolio and short a large-stock portfolio to substitute for SMB. Similarly, they would have to buy value stocks and short growth stocks to substitute for HML. This is no small feat. Even for professional managers, investing in SMB and HML would be challenging. It is no wonder that while the FF model (and its variants with even additional factors) has largely superseded the single-index CAPM for the purpose of benchmarking investment performance, the single-index model still dominates the investments industry.

Multifactor Models and the Validity of the CAPM

The single-index CAPM fails empirical tests because its empirical representation, the single-index model, inadequately explains returns on too many securities. In short, too many statistically significant values of alpha (which should be zero) show up in single-index regressions. Despite this failure, it is still widely used in the industry.

Multifactor models such as the FF model may also be tested by the prevalence of significant alpha values. The three-factor model shows a material improvement over the single-index model in that regard. But the use of multi-index models comes at a price: They require forecasts of the additional factor returns. If forecasts of those additional factors are themselves subject to forecast error, these models will be less accurate than the theoretically inferior single-index model. Nevertheless, multifactor models have a definite appeal, since it is clear that real-world risk is multifaceted.

Merton (1973) first showed that the CAPM could be extended to allow for multiple sources of systematic risk. His model results in a multifactor security market line like that of Equation 7.8 but with risk factors that relate to the extra-market sources of risk that investors

[16]This is a fairly common outcome: Theoretically inferior models with fewer explanatory variables often describe out-of-sample outcomes more accurately than models employing more explanatory variables. This reflects in part the tendency of some researchers to "data mine," that is, to search too aggressively for variables that help describe a sample but have no staying power out of sample. In addition, each explanatory variable of a model must be forecast to make a prediction, and each of those forecasts adds some uncertainty to the prediction.

wish to hedge. In this light, a reasonable correct interpretation of multivariate index models such as FF is that they constitute an application of the multifactor CAPM, rather than a rejection of the underlying logic of the simple model.

7.5 ARBITRAGE PRICING THEORY

One reason for skepticism about the validity of the CAPM is the unrealistic nature of the assumptions needed to derive it. Most unappealing are assumptions 2.A-C, namely, that all investors are identical in every way but wealth and risk aversion. For this reason, as well as for its economic insights, the arbitrage pricing theory (APT) is of great interest. To understand this theory we begin with the concept of *arbitrage.*

Arbitrage is the act of exploiting mispricing of two or more securities to achieve risk-free profits. As a trivial example, consider a security that is priced differently in two markets. A long position in the cheaper market financed by a short position in the expensive market will lead to a sure profit. As investors avidly pursue this strategy, prices are forced back into alignment, so arbitrage opportunities vanish almost as quickly as they materialize.

arbitrage
Creation of riskless profits made possible by relative mispricing among securities.

The first to apply this concept to equilibrium security returns was Ross (1976), who developed the **arbitrage pricing theory (APT).** The APT depends on the observation that well-functioning capital markets preclude arbitrage opportunities. A violation of the APT's pricing relationships will cause extremely strong pressure to restore them even if only a limited number of investors become aware of the disequilibrium. Ross's accomplishment is to derive the equilibrium rates of return that would prevail in a market where prices are aligned to eliminate arbitrage opportunities. The APT thus avoids the most objectionable assumptions of the CAPM.

arbitrage pricing theory (APT)
A theory of risk-return relationships derived from no-arbitrage considerations in large capital markets.

Well-Diversified Portfolios and Arbitrage Pricing Theory

To illustrate how the APT works, we will begin with a single-index market; generalization to multi-factor markets is straightforward. The *excess* rate of return on any security, *S,* is then $R_S = \alpha_S + \beta_S R_M + e_S$, using an observed benchmark *M.*

Suppose a portfolio, *P*, is believed to have a positive alpha. We can use the benchmark portfolio (with a beta of 1) to hedge away or "purge" the systematic risk of *P* and convert it to a zero-beta portfolio. We can go even further and turn the positive-alpha, zero-beta portfolio into a zero-net-investment position by adding an appropriate position in the risk-free asset. In all, we combine the positive-alpha *P* with both the benchmark and T-bills to create a costless, zero-beta portfolio, *A*, with a positive alpha. Table 7.5 shows how.

Table 7.5 shows that portfolio *A* with excess return $\alpha_P + e_P$ is still risky as long as the residual variance, σ_e^2, is positive. This shows that a zero-investment, zero-beta, positive-alpha portfolio is not necessarily an arbitrage opportunity; true arbitrage implies no risk. However, if *P* were highly diversified, its residual risk would be small. A portfolio with practically negligible residual risk is called a **well-diversified portfolio.** The difference in the scatter diagrams of any asset versus that of a well-diversified portfolio with the same beta is shown in Figure 7.5.

well-diversified portfolio
A portfolio sufficiently diversified that nonsystematic risk is negligible.

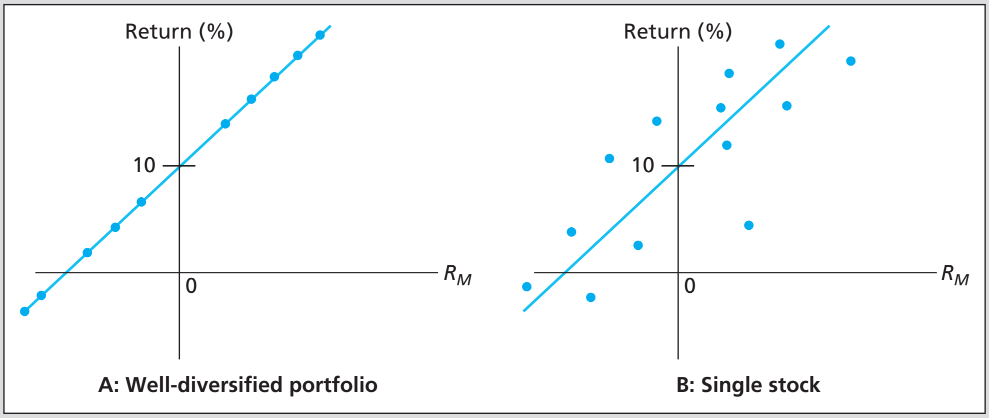
TABLE 7.5 Steps to convert a well-diversified portfolio into an arbitrage portfolio

Portfolio Weight*	In Asset	Contribution to Excess Return
$w_P = 1$	Portfolio *P*	$w_P(\alpha_P + \beta_P R_M + e_P) = \alpha_P + \beta_P R_M + e_P$
$w_M = -\beta_P$	Benchmark	$w_M R_M = -\beta_P R_M$
$w_f = \beta_P - 1$	Risk-free asset	$w_f \cdot 0 = 0$
$\sum w = 0$	Portfolio *A*	$\alpha_P + e_P$

*When alpha is negative, you would reverse the signs of each portfolio weight to achieve a portfolio *A* with positive alpha and no net investment.

FIGURE 7.5

Security characteristic lines



arbitrage portfolio

A zero-net-investment, risk-free portfolio with a positive return.

Portfolio *A*, when constructed from a *well-diversified portfolio*, is an **arbitrage portfolio.** An arbitrage portfolio is a money machine: It can generate risk-free profits with zero net investment. Therefore, investors who succeed in constructing one will scale it up as much as they can, financing with as much leverage and/or as many short positions as available.

EXAMPLE 7.9

Constructing an Arbitrage Portfolio

Suppose the benchmark, *M*, is the observed, broad market index that includes over 4,000 stocks (NYSE + NASDAQ + Amex). Imagine that on December 31, 2005, a portfolio manager possessed the following five-year predictions based on security and macro analyses:

1. The cumulative risk-free rate from rolling over T-bills over the next five years is estimated at 11.5%, an annual rate of 2.2%.
2. The cumulative return on the benchmark is estimated at 20%, an annual rate of 3.71%.
3. The S&P 500, which we will treat as portfolio *P*, is composed of large-capitalization stocks and is believed to be *overpriced.* Its expected cumulative return is forecast at 12%, an annual rate of 2.29%.
4. The S&P 500 beta against the benchmark is estimated at .95, which leads to the following calculation for its alpha: 2.29% = 2.2% + α + .95(3.71 − 2.2); α = −1.34% per year.

Because alpha is negative, we reverse the weights in Table 7.5 and set $w_P = -1$, $w_M = .95$, $w_f = .05$. The alpha on *A* is then positive: α_A = 1.34%.

Example 7.9 shows how to construct an arbitrage portfolio from a mispriced well-diversified portfolio. But it leaves us with an important question: While the S&P 500 is highly diversified, is even this index sufficiently diversified to make *A* a risk-free arbitrage portfolio? Table 7.6 shows the weight of the 10 largest stocks in the S&P 500. While these firms are only 2% of the firms in the index, they account for almost 20% of the market capitalization, and their weights in the index are far from negligible.

TABLE 7.6 Ten largest capitalization stocks in the S&P 500 portfolio and their weights (Dec. 31, 2009)

ExxonMobil	3.26	IBM	1.73
Microsoft	2.37	AT&T	1.67
Apple	1.91	JPMorgan Chase	1.65
Johnson & Johnson	1.79	GE	1.62
Procter & Gamble	1.78	Chevron	1.56
Total for 10 largest firms			19.34

TABLE 7.7 Regression statistics of the S&P 500 portfolio on the benchmark portfolio, January 2006–December 2010

Linear Regression

Regression Statistics		
R	0.9933	
R-square	0.9866	
Adjusted *R*-square	0.9864	**Annualized**
Regression SE	0.5968	2.067
Total number of observations	60	

S&P 500 = −0.1909 + 0.9337* Benchmark

	Coefficients	**Standard Error**	***t*-stat**	***p*-level**
Intercept	−0.1909	0.0771	−2.4752	0.0163
Benchmark	0.9337	0.0143	65.3434	0.0000

TABLE 7.8 Annual standard deviation of the real, inflation, and nominal rates

Period	Real Rate	Inflation Rate	Nominal Rate
1/1/2006–12/31/2010	1.46	1.46	0.61
1/1/1996–12/31/2000	0.57	0.54	0.17
1/1/1986–12/31/1990	0.86	0.83	0.37

With hindsight, we can estimate the residual risk of the S&P 500 from a regression of its monthly returns against the benchmark over the prediction period (January 1, 2006, to December 31, 2010). The essential regression output is displayed in Table 7.7. Notice that both alpha (−.19% per month or −2.27% per year) and beta (.93) are close to the 2005 prediction of 2.33% and .95. Most important is the (annualized) standard deviation of the regression residuals, called the *standard error of the regression,* which was 2.07%. Is this residual SD small enough for us to deem the S&P 500 “well-diversified”?

To answer the question, we recognize that investors who consider a zero-investment arbitrage portfolio must in any event invest their existing wealth *somewhere.* The risk of their alternative portfolio is therefore relevant to the discussion. Obviously, the lowest-risk investment would be to roll over T-bills. A measure of the risk of this strategy is the uncertainty of its real rate of return over the prediction period.[17] Table 7.8 suggests that the annual SD of the real rate from rolling over bills is in the range of .5%–1.5% per year depending on the sample period.

Our question then comes down to this: What would be the marginal increase in risk from adding an arbitrage portfolio with an SD of about 2% per year to a portfolio with an SD of .5%–1.5% per year? Since the two rates are uncorrelated, the variance of the portfolio will be the sum of the variances. The SD of this complete portfolio minus the SD of the T-bill portfolio is the *marginal* risk of the S&P 500 in its use as an arbitrage portfolio. The following table shows some examples of the marginal risk of the arbitrage portfolio, first treating T-bills as the initial portfolio to which the arbitrage portfolio is added (with three assumptions for the SD of its real return) and then treating the benchmark risky portfolio as the initial position, with an assumed SD = 20%.

[17]Obviously, five-year TIPS would carry a practically zero risk to the real rate. But we deal here with active portfolio managers who continuously rebalance their portfolios and must maintain considerable liquidity. For very short holding periods, TIPS would not be practical for these investors.

SD of Real Rate on Initial Portfolio	SD of Total Portfolio	Marginal Risk
0.5% (T-bills)	$(.005^2 + .0207^2)^{1/2} = 2.13\%$	1.63%
1.0 (T-bills)	2.30	1.30
1.5 (T-bills)	2.56	1.06
20.0 (benchmark)	20.11	0.11

There is no widely accepted threshold for the acceptable marginal risk of an arbitrage portfolio in a practical application. Nevertheless, the marginal risk in the first three lines of the table, which is just about the same as the SD of the real rate on bills, may well be above the appropriate threshold. Moreover, an alpha of 2% per year in this example is not even decisively statistically significant. We learn from this exercise that well-diversified portfolios are not easy to construct, and arbitrage opportunities are likely few and far between. On the other hand, when the arbitrage portfolio is added to the risky benchmark portfolio, the marginal increase in overall standard deviation is minimal.

We are now ready to derive the APT. Our argument follows from Example 7.9. Investors, however few, will invest large amounts in any arbitrage portfolio they can identify. This will entail large-scale purchases of positive-alpha portfolios or large-scale shorting of negative-alpha portfolios. These actions will move the prices of component securities until alpha disappears. In the end, when alphas of all well-diversified portfolios are driven to zero, their return equations become

$$r_P = r_f + \beta_P(r_M - r_f) + e_P \tag{7.8}$$

Taking the expectations in Equation 7.8 results in the familiar CAPM mean–beta equation:

$$E(r_P) = r_f + \beta_P[E(r_M) - r_f] \tag{7.9}$$

For portfolios such as the S&P 500 that shed *most* residual risk, we can still expect buying and selling pressure to drive their alpha close to zero. If alphas of portfolios with very small residual risk are near zero, then even less diversified portfolios will tend to have small alpha values. Thus, the APT implies a hierarchy of certainty about alphas of portfolios, based on the degree of diversification.

The APT and the CAPM

Why did we need so many restrictive assumptions to derive the CAPM when the APT seems to arrive at the expected return–beta relationship with seemingly fewer and less objectionable assumptions? The answer is simple: Strictly speaking, the APT applies only to well-diversified portfolios. Absence of riskless arbitrage alone cannot guarantee that, in equilibrium, the expected return–beta relationship will hold for any and all assets.

With additional effort, however, one can use the APT to show that the relationship must hold approximately even for individual assets. The essence of the proof is that if the expected return–beta relationship were violated by many individual securities, it would be virtually impossible for all well-diversified portfolios to satisfy the relationship. So the relationship must *almost* surely hold true for individual securities.

We say "almost" because, according to the APT, there is no guarantee that all individual assets will lie on the SML. If only a few securities violated the SML, their effect on well-diversified portfolios could conceivably be negligible. In this sense, it is possible that the SML relationship is violated for some securities. If many securities violate the expected return–beta relationship, however, the relationship will no longer hold for well-diversified portfolios comprising these securities, and arbitrage opportunities will be available.

The APT serves many of the same functions as the CAPM. It gives us a benchmark for fair rates of return that can be used for capital budgeting, security valuation, or performance evaluation of managed portfolios. Moreover, the APT highlights the crucial distinction between nondiversifiable risk (systematic or factor risk) that requires a reward in the form of a risk premium and diversifiable risk that does not.

The bottom line is that neither of these theories dominates the other. The APT is more general in that it gets us to the expected return–beta relationship without requiring many of the unrealistic assumptions of the CAPM, particularly the reliance on the market portfolio. The latter improves the prospects for testing the APT. But the CAPM is more general in that it applies to all assets without reservation. The good news is that both theories agree on the expected return–beta relationship.

It is worth noting that because past tests of the mean–beta relationship examined the rates of return on highly diversified portfolios, they actually came closer to testing the APT than the CAPM. Thus, it appears that econometric concerns, too, favor the APT.

Multifactor Generalization of the APT and CAPM

So far, we've examined the APT in a one-factor world. In reality, there are several sources of systematic risk such as uncertainty in the business cycle, interest rates, energy prices, and so on. Presumably, exposure to any of these factors will affect a stock's appropriate expected return. We can use a multifactor version of the APT to accommodate these multiple sources of risk.

Expanding the single-factor model of Equation 7.8 to a two-factor model:

$$R_i = \alpha_i + \beta_{i1} R_{M1} + \beta_{i2} R_{M2} + e_i \qquad \textbf{(7.10)}$$

where R_{M1} and R_{M2} are the excess returns on portfolios that represent the two systematic factors. Factor 1 might be, for example, unanticipated changes in industrial production, while factor 2 might represent unanticipated changes in short-term interest rates. We assume again that there are many securities available with any combination of betas. This implies that we can form well-diversified **factor portfolios** with a beta of 1 on one factor and zero on all others. Thus, a factor portfolio with a beta of 1 on the first factor will have a rate of return of R_{M1}; a factor portfolio with a beta of 1 on the second factor will have a rate of return of R_{M2}; and so on. Factor portfolios can serve as the benchmark portfolios for a multifactor generalization of the security market line relationship.

factor portfolio

A well-diversified portfolio constructed to have a beta of 1 on one factor and a beta of zero on any other factor.

EXAMPLE 7.10

Multifactor SML

Suppose the two-factor portfolios, here called portfolios 1 and 2, have expected returns $E(r_1) = 10\%$ and $E(r_2) = 12\%$. Suppose further that the risk-free rate is 4%. The risk premium on the first factor portfolio is therefore 6%, while that on the second factor portfolio is 8%.

Now consider an arbitrary well-diversified portfolio (P), with beta on the first factor, $\beta_{P1} = .5$, and on the second factor, $\beta_{P2} = .75$. The multifactor APT states that the portfolio risk premium must equal the sum of the risk premiums required as compensation to investors for each source of systematic risk. The risk premium attributable to risk factor 1 is the portfolio's exposure to factor 1, β_{P1}, times the risk premium earned on the first factor portfolio, $E(r_1) - r_f$. Therefore, the portion of portfolio P's risk premium that is compensation for its exposure to the first risk factor is $\beta_{P1}[E(r_1) - r_f] = .5(10\% - 4\%) = 3\%$, while the risk premium attributable to risk factor 2 is $\beta_{P2}[E(r_2) - r_f] = .75(12\% - 4\%) = 6\%$. The total risk premium on the portfolio, therefore, should be $3 + 6 = 9\%$, and the total return on the portfolio should be 13%.

4%	Risk-free rate
+ 3%	Risk premium for exposure to factor 1
+ 6%	Risk premium for exposure to factor 2
13%	Total expected return

EXCEL APPLICATIONS

Estimating the Index Model

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The spreadsheet below contains monthly returns for a small sample of stocks. A related workbook (also available at **www.mhhe.com/bkm**) contains spreadsheets that show raw returns, risk premiums, and beta coefficients for the stocks in the Dow Jones Industrial Average. The security characteristic lines are estimated with five years of monthly returns.

	A	B	C	D	E	F
1	Rates of Return					
2	**Month**	**Ford**	**Honda**	**Toyota**	**S&P 500**	**T-bills**
3						
4	Dec-08	-18.34	23.02	-2.95	-8.31	0.09
5	Nov-08	-14.87	-25.44	3.71	0.97	0.02
6	Oct-08	22.83	-26.04	-17.07	-7.04	0.08
7	Sep-08	-57.88	-13.77	-11.32	-16.67	0.15
8	Aug-08	16.59	-29.61	-4.23	-9.54	0.12
9	Jul-08	-7.08	-4.92	4.11	1.40	0.15
10	Jun-08	-0.21	-11.01	-8.46	-1.07	0.17

Excel Questions

1. What were the betas of Ford, Toyota, and Honda?
2. In light of each firm's exposure to the financial crisis in 2008-2009, does the value for Ford compared to Honda and Toyota make sense to you?

Suppose portfolio P of Example 7.10 actually has an expected excess return of 11% and therefore a positive alpha of 2%. We can generalize the methodology of Table 7.5 to construct an arbitrage portfolio for this two-factor problem. Table 7.9 shows how. Because P is well diversified, e_P must be small, and the excess return on the zero-investment, zero-beta portfolio A is just $\alpha_P = 2\%$.

Here, too, extensive trade by arbitrageurs will eliminate completely alphas of well-diversified portfolios. We conclude that, in general, the APT hierarchy of possible alpha values, declining with the extent of portfolio diversification, applies to any multifactor market. In the absence of private information from security and macro analyses, investors and corporate officers must use the multifactor SML equation (with zero alpha) to determine the expected rates on securities and the required rates of return on the firm's projects.

CONCEPT check 7.6

Using the factor portfolios of Example 7.10, find the fair rate of return on a security with $\beta_1 = .2$ and $\beta_2 = 1.4$.

TABLE 7.9 Constructing an arbitrage portfolio with two systematic factors

Portfolio Weight	In Asset	Contribution to Excess Return
1	Portfolio P	$\alpha_P + \beta_{P1}R_1 + \beta_{P2}R_2 + e_P = 11\% + e_P$
$-\beta_{P1} = -0.5$	Factor portfolio 1	$\beta_{P1}R_1 = -.5 \times 6\% = -3\%$
$-\beta_{P2} = -0.75$	Factor portfolio 2	$\beta_{P2}R_2 = -.75 \times 8\% = -6\%$
$\beta_{P1} + \beta_{P1} - 1 = 0.25$	Risk-free asset	0
Total $= 1$	Portfolio A	$\alpha_P + e_P = 2\% + e_P$

SUMMARY

- The CAPM assumes investors are rational, single-period planners who agree on a common input list from security analysis and seek mean-variance optimal portfolios.
- The CAPM assumes ideal security markets in the sense that (*a*) markets are large and investors are price takers, (*b*) there are no taxes or transaction costs, (*c*) all risky assets are publicly traded, and (*d*) any amount can be borrowed and lent at a fixed, risk-free rate. These assumptions mean that all investors will hold identical risky portfolios. The CAPM implies that, in equilibrium, the market portfolio is the unique mean-variance efficient tangency portfolio, which indicates that a passive strategy is efficient.
- The market portfolio is a value-weighted portfolio. Each security is held in a proportion equal to its market value divided by the total market value of all securities. The risk premium on the market portfolio is proportional to its variance, σ_M^2, and to the risk aversion of the average investor.
- The CAPM implies that the risk premium on any individual asset or portfolio is the product of the risk premium of the market portfolio and the asset's beta. The security market line shows the return demanded by investors as a function of the beta of their investment. This expected return is a benchmark for evaluating investment performance.
- In a single-index security market, once an index is specified, a security beta can be estimated from a regression of the security's excess return on the index's excess return. This regression line is called the security characteristic line (SCL). The intercept of the SCL, called alpha, represents the average excess return on the security when the index excess return is zero. The CAPM implies that alphas should be zero.
- The CAPM and the security market line can be used to establish benchmarks for evaluation of investment performance or to determine appropriate discount rates for capital budgeting applications. They are also used in regulatory proceedings concerning the "fair" rate of return for regulated industries.
- The CAPM is usually implemented as a single-factor model, with all systematic risk summarized by the return on a broad market index. However, multifactor generalizations of the basic model may be specified to accommodate multiple sources of systematic risk. In such multifactor extensions of the CAPM, the risk premium of any security is determined by its sensitivity to each systematic risk factor as well as the risk premium associated with that source of risk.
- There are two general approaches to finding extra-market systematic risk factors. One is characteristics-based and looks for factors that are empirically associated with high average returns and so may be proxies for relevant measures of systematic risk. The other focuses on factors that are plausibly important sources of risk to wide segments of investors and may thus command risk premiums.
- An arbitrage opportunity arises when the disparity between two or more security prices enables investors to construct a zero net investment portfolio that will yield a sure profit. The presence of arbitrage opportunities and the resulting volume of trades will create pressure on security prices that will persist until prices reach levels that preclude arbitrage. Only a few investors need to become aware of arbitrage opportunities to trigger this process because of the large volume of trades in which they will engage.
- When securities are priced so that there are no arbitrage opportunities, the market satisfies the no-arbitrage condition. Price relationships that satisfy the no-arbitrage condition are important because we expect them to hold in real-world markets.
- Portfolios are called *well diversified* if they include a large number of securities in such proportions that the residual or diversifiable risk of the portfolio is negligible.
- In a single-factor security market, all well-diversified portfolios must satisfy the expected return–beta relationship of the SML in order to satisfy the no-arbitrage condition. If all well-diversified portfolios satisfy the expected return–beta relationship, then all but a small number of securities also must satisfy this relationship.
- The APT implies the same expected return–beta relationship as the CAPM yet does not require that all investors be mean-variance optimizers. The price of this generality is that the APT does not guarantee this relationship for all securities at all times.
- A multifactor APT generalizes the single-factor model to accommodate several sources of systematic risk.

KEY TERMS

alpha, 200
arbitrage, 217
arbitrage portfolio, 218
arbitrage pricing theory (APT), 217
capital asset pricing model (CAPM), 194
expected return–beta relationship, 198
factor portfolio, 221
market portfolio (M), 195
multifactor models, 212
mutual fund theorem, 196
security characteristic line (SCL), 204
security market line (SML), 199
well-diversified portfolio, 217

KEY FORMULAS

CAPM: Market portfolio risk premium is proportional to average risk aversion and market risk:

$$E(r_M) - r_f = \bar{A}\sigma_M^2$$

SML: Expected return as a function of systematic risk:

$$E(r_i) = r_f + \beta_i[E(r_M) - r_f]$$

The index model in realized returns:

$$r_{it} - r_{ft} = \alpha_i + \beta_i\,(r_{Mt} - r_{ft}) + e_{it}$$

The two-index model in realized excess returns:

$$R_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{iTB} R_{TBt} + e_{it}$$

The two-factor SML (where TB is the second factor):

$$E(r_i) = r_f + \beta_{iM}[E(r_M) - r_f] + \beta_{iTB}[E(r_{TB}) - r_f]$$

The Fama-French three-factor model in realized returns:

$$r_i - r_f = \alpha_i + \beta_M\,(r_M - r_f) + \beta_{HML}\, r_{HML} + \beta_{SMB}\, r_{SMB} + e_i$$

Instructions to construct arbitrage portfolios for single- and two-factor markets are shown in Tables 7.5 and 7.9.

PROBLEM SETS

Select problems are available in McGraw-Hill's *Connect Finance.* Please see the Supplements section of the book's frontmatter for more information.

Basic

1. Suppose investors believe that the standard deviation of the market-index portfolio has increased by 50%. What does the CAPM imply about the effect of this change on the required rate of return on Google's investment projects? *(LO 7-1)*
2. Consider the statement: "If we can identify a portfolio that beats the S&P 500 Index portfolio, then we should reject the single-index CAPM." Do you agree or disagree? Explain. *(LO 7-1)*
3. Are the following true or false? Explain. *(LO 7-5)*
   a. Stocks with a beta of zero offer an expected rate of return of zero.
   b. The CAPM implies that investors require a higher return to hold highly volatile securities.
   c. You can construct a portfolio with beta of .75 by investing .75 of the investment budget in T-bills and the remainder in the market portfolio.
4. Here are data on two companies. The T-bill rate is 4% and the market risk premium is 6%.

Company	$1 Discount Store	Everything $5
Forecast return	12%	11%
Standard deviation of returns	8%	10%
Beta	1.5	1.0

What would be the fair return for each company, according to the capital asset pricing model (CAPM)? *(LO 7-1)*

5. Characterize each company in the previous problem as underpriced, overpriced, or properly priced. *(LO 7-2)*
6. What is the expected rate of return for a stock that has a beta of 1 if the expected return on the market is 15%? *(LO 7-2)*
   a. 15%.
   b. More than 15%.
   c. Cannot be determined without the risk-free rate.
7. Kaskin, Inc., stock has a beta of 1.2 and Quinn, Inc., stock has a beta of .6. Which of the following statements is *most* accurate? *(LO 7-1)*
   a. The expected rate of return will be higher for the stock of Kaskin, Inc., than that of Quinn, Inc.
   b. The stock of Kaskin, Inc., has more total risk than Quinn, Inc.
   c. The stock of Quinn, Inc., has more systematic risk than that of Kaskin, Inc.
8. Which of the following statements is *true?* Explain. *(LO 7-1)*
   a. It is possible that the APT is valid and the CAPM is not.
   b. It is possible that the CAPM is valid and the APT is not.

Intermediate

9. What must be the beta of a portfolio with $E(r_P) = 20\%$, if $r_f = 5\%$ and $E(r_M) = 15\%$? *(LO 7-2)*
10. The market price of a security is $40. Its expected rate of return is 13%. The risk-free rate is 7%, and the market risk premium is 8%. What will the market price of the security be if its beta doubles (and all other variables remain unchanged)? Assume the stock is expected to pay a constant dividend in perpetuity. *(LO 7-2)*
11. You are a consultant to a large manufacturing corporation considering a project with the following net after-tax cash flows (in millions of dollars):

Years from Now	After-Tax CF
0	−20
1–9	10
10	20

The project's beta is 1.7. Assuming $r_f = 9\%$ and $E(r_M) = 19\%$, what is the net present value of the project? What is the highest possible beta estimate for the project before its NPV becomes negative? *(LO 7-2)*

12. Consider the following table, which gives a security analyst's expected return on two stocks for two particular market returns: *(LO 7-2)*

Market Return	Aggressive Stock	Defensive Stock
5%	2%	3.5%
20	32	14

   a. What are the betas of the two stocks?
   b. What is the expected rate of return on each stock if the market return is equally likely to be 5% or 20%?
   c. If the T-bill rate is 8%, and the market return is equally likely to be 5% or 20%, draw the SML for this economy.
   d. Plot the two securities on the SML graph. What are the alphas of each?
   e. What hurdle rate should be used by the management of the aggressive firm for a project with the risk characteristics of the defensive firm's stock?

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If the simple CAPM is valid, which of the situations in Problems 13–19 below are possible? Explain. Consider each situation independently.

13.

Portfolio	Expected Return	Beta
A	20%	1.4
B	25	1.2

(LO 7-1)

14.

Portfolio	Expected Return	Standard Deviation
A	30%	35%
B	40	25

(LO 7-1)

15.

Portfolio	Expected Return	Standard Deviation
Risk-free	10%	0%
Market	18	24
A	16	12

(LO 7-1)

16.

Portfolio	Expected Return	Standard Deviation
Risk-free	10%	0%
Market	18	24
A	20	22

(LO 7-1)

17.

Portfolio	Expected Return	Beta
Risk-free	10%	0
Market	18	1.0
A	16	1.5

(LO 7-1)

18.

Portfolio	Expected Return	Beta
Risk-free	10%	0
Market	18	1.0
A	16	.9

(LO 7-1)

19.

Portfolio	Expected Return	Standard Deviation
Risk-free	10%	0%
Market	18	24
A	16	22

(LO 7-1)

20. Go to **www.mhhe.com/bkm** and link to Chapter 7 materials, where you will find a spreadsheet with monthly returns for GM, Ford, and Toyota, the S&P 500, and Treasury bills. *(LO 7-1)*
   a. Estimate the index model for each firm over the full five-year period. Compare the betas of each firm.
   b. Now estimate the betas for each firm using only the first two years of the sample and then using only the last two years. How stable are the beta estimates obtained from these shorter subperiods?

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In Problems 21–23 below, assume the risk-free rate is 8% and the expected rate of return on the market is 18%.

21. A share of stock is now selling for $100. It will pay a dividend of $9 per share at the end of the year. Its beta is 1. What do investors expect the stock to sell for at the end of the year? *(LO 7-2)*
22. I am buying a firm with an expected perpetual cash flow of $1,000 but am unsure of its risk. If I think the beta of the firm is zero, when the beta is really 1, how much *more* will I offer for the firm than it is truly worth? *(LO 7-2)*
23. A stock has an expected return of 6%. What is its beta? *(LO 7-2)*
24. Two investment advisers are comparing performance. One averaged a 19% return and the other a 16% return. However, the beta of the first adviser was 1.5, while that of the second was 1. *(LO 7-2)*
   a. Can you tell which adviser was a better selector of individual stocks (aside from the issue of general movements in the market)?
   b. If the T-bill rate were 6% and the market return during the period were 14%, which adviser would be the superior stock selector?
   c. What if the T-bill rate were 3% and the market return 15%?
25. Suppose the yield on short-term government securities (perceived to be risk-free) is about 4%. Suppose also that the expected return required by the market for a portfolio with a beta of 1 is 12%. According to the capital asset pricing model: *(LO 7-2)*
   a. What is the expected return on the market portfolio?
   b. What would be the expected return on a zero-beta stock?
   c. Suppose you consider buying a share of stock at a price of $40. The stock is expected to pay a dividend of $3 next year and to sell then for $41. The stock risk has been evaluated at $\beta = -.5$. Is the stock overpriced or underpriced?
26. Based on current dividend yields and expected capital gains, the expected rates of return on portfolios *A* and *B* are 11% and 14%, respectively. The beta of *A* is .8 while that of *B* is 1.5. The T-bill rate is currently 6%, while the expected rate of return of the S&P 500 Index is 12%. The standard deviation of portfolio *A* is 10% annually, while that of *B* is 31%, and that of the index is 20%. *(LO 7-2)*
   a. If you currently hold a market-index portfolio, would you choose to add either of these portfolios to your holdings? Explain.
   b. If instead you could invest *only* in bills and one of these portfolios, which would you choose?
27. Consider the following data for a one-factor economy. All portfolios are well diversified.

Portfolio	*E*(*r*)	Beta
A	10%	1.0
F	4	0

   Suppose another portfolio *E* is well diversified with a beta of 2/3 and expected return of 9%. Would an arbitrage opportunity exist? If so, what would the arbitrage strategy be? *(LO 7-4)*

28. Assume both portfolios A and B are well diversified, that $E(r_A) = 14\%$ and $E(r_B) = 14.8\%$. If the economy has only one factor, and $\beta_A = 1$ while $\beta_B = 1.1$, what must be the risk-free rate? *(LO 7-4)*
29. Assume a market index represents the common factor and all stocks in the economy have a beta of 1. Firm-specific returns all have a standard deviation of 30%.

    Suppose an analyst studies 20 stocks and finds that one-half have an alpha of 3%, and one-half have an alpha of −3%. The analyst then buys $1 million of an equally weighted portfolio of the positive-alpha stocks and sells short $1 million of an equally weighted portfolio of the negative-alpha stocks. *(LO 7-4)*
    a. What is the expected profit (in dollars), and what is the standard deviation of the analyst's profit?
    b. How does your answer change if the analyst examines 50 stocks instead of 20? 100 stocks?
30. If the APT is to be a useful theory, the number of systematic factors in the economy must be small. Why? *(LO 7-4)*
31. The APT itself does not provide information on the factors that one might expect to determine risk premiums. How should researchers decide which factors to investigate? Is industrial production a reasonable factor to test for a risk premium? Why or why not? *(LO 7-3)*
32. Suppose two factors are identified for the U.S. economy: the growth rate of industrial production, IP, and the inflation rate, IR. IP is expected to be 4% and IR 6%. A stock with a beta of 1 on IP and .4 on IR currently is expected to provide a rate of return of 14%. If industrial production actually grows by 5%, while the inflation rate turns out to be 7%, what is your best guess for the rate of return on the stock? *(LO 7-3)*
33. Suppose there are two independent economic factors, M_1 and M_2. The risk-free rate is 7%, and all stocks have independent firm-specific components with a standard deviation of 50%. Portfolios A and B are both well diversified.

Portfolio	Beta on M_1	Beta on M_2	Expected Return (%)
A	1.8	2.1	40
B	2.0	−0.5	10

What is the expected return–beta relationship in this economy? *(LO 7-5)*

Challenge

34. As a finance intern at Pork Products, Jennifer Wainwright's assignment is to come up with fresh insights concerning the firm's cost of capital. She decides that this would be a good opportunity to try out the new material on the APT that she learned last semester. As such, she decides that three promising factors would be (i) the return on a broad-based index such as the S&P 500; (ii) the level of interest rates, as represented by the yield to maturity on 10-year Treasury bonds; and (iii) the price of hogs, which are particularly important to her firm. Her plan is to find the beta of Pork Products against each of these factors and to estimate the risk premium associated with exposure to each factor. Comment on Jennifer's choice of factors. Which are most promising with respect to the likely impact on her firm's cost of capital? Can you suggest improvements to her specification? *(LO 7-3)*
35. Suppose the market can be described by the following three sources of systematic risk. Each factor in the following table has a mean value of zero (so factor values represent realized surprises relative to prior expectations), and the risk premiums associated with each source of systematic risk are given in the last column.

Systematic Factor	Risk Premium
Industrial production, IP	6%
Interest rates, INT	2
Credit risk, CRED	4

The excess return, R, on a particular stock is described by the following equation that relates realized returns to surprises in the three systematic factors:

$$R = 6\% + 1.0\ \text{IP} + .5\ \text{INT} + .75\ \text{CRED} + e$$

Find the equilibrium expected excess return on this stock using the APT. Is the stock overpriced or underpriced? *(LO 7-3)*

CFA Problems

CFA® PROBLEMS

1. Which of the following statements about the security market line (SML) are *true?* *(LO 7-2)*
   a. The SML provides a benchmark for evaluating expected investment performance.
   b. The SML leads all investors to invest in the same portfolio of risky assets.
   c. The SML is a graphic representation of the relationship between expected return and beta.
   d. Properly valued assets plot exactly on the SML.
2. Karen Kay, a portfolio manager at Collins Asset Management, is using the capital asset pricing model for making recommendations to her clients. Her research department has developed the information shown in the following exhibit. *(LO 7-2)*

Forecasted Returns, Standard Deviations, and Betas			
	Forecasted Return	Standard Deviation	Beta
Stock X	14.0%	36%	0.8
Stock Y	17.0	25	1.5
Market index	14.0	15	1.0
Risk-free rate	5.0		

   a. Calculate expected return and alpha for each stock.
   b. Identify and justify which stock would be more appropriate for an investor who wants to:
      i. Add this stock to a well-diversified equity portfolio.
      ii. Hold this stock as a single-stock portfolio.
3. Joan McKay is a portfolio manager for a bank trust department. McKay meets with two clients, Kevin Murray and Lisa York, to review their investment objectives. Each client expresses an interest in changing his or her individual investment objectives. Both clients currently hold well-diversified portfolios of risky assets. *(LO 7-1)*
   a. Murray wants to increase the expected return of his portfolio. State what action McKay should take to achieve Murray's objective. Justify your response in the context of the capital market line.
   b. York wants to reduce the risk exposure of her portfolio but does not want to engage in borrowing or lending activities to do so. State what action McKay should take to achieve York's objective. Justify your response in the context of the security market line.

4. Jeffrey Bruner, CFA, uses the capital asset pricing model (CAPM) to help identify mispriced securities. A consultant suggests Bruner use arbitrage pricing theory (APT) instead. In comparing CAPM and APT, the consultant made the following arguments:
   a. Both the CAPM and APT require a mean-variance efficient market portfolio.
   b. The CAPM assumes that one specific factor explains security returns but APT does not.

   State whether each of the consultant's arguments is correct or incorrect. Indicate, for each incorrect argument, why the argument is incorrect. *(LO 7-5)*
5. The security market line depicts: *(LO 7-2)*
   a. A security's expected return as a function of its systematic risk.
   b. The market portfolio as the optimal portfolio of risky securities.
   c. The relationship between a security's return and the return on an index.
   d. The complete portfolio as a combination of the market portfolio and the risk-free asset.
6. According to CAPM, the expected rate of return of a portfolio with a beta of 1 and an alpha of 0 is: *(LO 7-2)*
   a. Between r_M and r_f.
   b. The risk-free rate, r_f.
   c. $\beta (r_M - r_f)$.
   d. The expected return on the market, r_M.

The following table (for CFA Problems 7 and 8) shows risk and return measures for two portfolios.

Portfolio	Average Annual Rate of Return	Standard Deviation	Beta
R	11%	10%	0.5
S&P 500	14%	12%	1.0

7. When plotting portfolio *R* on the preceding table relative to the SML, portfolio *R* lies: *(LO 7-2)*
   a. On the SML.
   b. Below the SML.
   c. Above the SML.
   d. Insufficient data given.
8. When plotting portfolio *R* relative to the capital market line, portfolio *R* lies: *(LO 7-2)*
   a. On the CML.
   b. Below the CML.
   c. Above the CML.
   d. Insufficient data given.
9. Briefly explain whether investors should expect a higher return from holding portfolio *A* versus portfolio *B* under capital asset pricing theory (CAPM). Assume that both portfolios are fully diversified. *(LO 7-2)*

	Portfolio A	Portfolio B
Systematic risk (beta)	1.0	1.0
Specific risk for each individual security	High	Low

10. Assume that both *X* and *Y* are well-diversified portfolios and the risk-free rate is 8%.

Portfolio	Expected Return	Beta
X	16%	1.00
Y	12%	0.25

In this situation you could conclude that portfolios *X* and *Y*: (LO 7-4)
a. Are in equilibrium.
b. Offer an arbitrage opportunity.
c. Are both underpriced.
d. Are both fairly priced.

11. According to the theory of arbitrage: (LO 7-4)
a. High-beta stocks are consistently overpriced.
b. Low-beta stocks are consistently overpriced.
c. Positive alpha investment opportunities will quickly disappear.
d. Rational investors will pursue arbitrage consistent with their risk tolerance.

12. A zero-investment well-diversified portfolio with a positive alpha could arise if: (LO 7-5)
a. The expected return of the portfolio equals zero.
b. The capital market line is tangent to the opportunity set.
c. The law of one price remains unviolated.
d. A risk-free arbitrage opportunity exists.

13. An investor takes as large a position as possible when an equilibrium price relationship is violated. This is an example of: (LO 7-4)
a. A dominance argument.
b. The mean-variance efficient frontier.
c. Arbitrage activity.
d. The capital asset pricing model.

14. In contrast to the capital asset pricing model, arbitrage pricing theory: (LO 7-4)
a. Requires that markets be in equilibrium.
b. Uses risk premiums based on micro variables.
c. Specifies the number and identifies specific factors that determine expected returns.
d. Does not require the restrictive assumptions concerning the market portfolio.

WEB *master*

1. A firm's beta can be estimated from the slope of the characteristic line. The first step is to plot the return on the firm's stock (*y*-axis) versus the return on a broad market index (*x*-axis). Next, a regression line is estimated to find the slope.
   a. Go to **finance.yahoo.com,** enter the symbol for Alcoa, and click on *Get Quotes*. On the left-side menu, click on *Historical Prices;* then enter starting and ending dates that correspond to the most recent two years. Select the *Daily* option. Save the data to a spreadsheet.
   b. Repeat the process to get comparable data for the S&P 500 Index (symbol ^GSPC). Download the data and copy it into the same spreadsheet as Alcoa with dates aligned.
   c. Sort the data from earliest to latest. Calculate the excess return on the stock and the return on the index for each day using the adjusted closing prices. (You can use four-week T-bill rates to calculate excess returns from the Federal Reserve website at **www.federalreserve.gov/releases/h15/data.htm.**)
   d. Prepare an *xy* scatter plot with no line inserted. Be sure that the firm's excess returns represent the *y*-variable and the market's excess returns represent the *x*-variable.
   e. Select one of the data points by pointing to it and clicking the left mouse button. While the point is selected, right-click to pull up a shortcut menu. Select *Add*

Trendline, choose the linear type, then click on the *Options* tab and select *Display Equation on Chart*. When you click on OK, the trendline and the equation appear. The trendline represents the regression equation. What is Alcoa's beta?

2. In the previous question, you used 60 months of data to calculate the beta of Alcoa. Now compute the alpha of Alcoa in two consecutive periods. Estimate the index-model regression using the first 30 months of data. Now repeat the process using the second half of the sample. This will give you the alpha (intercept) and beta (slope) estimates for two consecutive time periods. How do the two alphas compare to the risk-free rate and to each other? Select 11 other firms and repeat the regressions to find the alphas for the first two-year period and the last two-year period.
3. Given your results for Question 2, investigate the extent to which beta in one period predicts beta in future periods and whether alpha in one period predicts alpha in future periods. Regress the beta of each firm in the second period (Y) against the beta in the first period (X). (If you estimated regressions for a dozen firms in Question 2, you will have 12 observations in this regression.) Do the same for the alphas of each firm. Use the coefficients you found to forecast the betas of the 12 firms for the next two-year period.
4. Our expectation is that beta in the first period predicts beta in the next period but that alpha in the first period has no power to predict alpha in the next period. (In other words, the regression coefficient on first-period beta will be statistically significant in explaining second-period beta, but the coefficient on alpha will not be.) Why does this prediction make sense? Is it borne out by the data?
5. *a.* Which of the stocks would you classify as defensive? Which would be classified as aggressive?

   b. Do the beta coefficients for the low-beta firms make sense given the industries in which these firms operate? Briefly explain.

SOLUTIONS TO CONCEPT checks

7.1 The CML would still represent efficient investments. We can characterize the entire population by two representative investors. One is the "uninformed" investor, who does not engage in security analysis and holds the market portfolio, while the other optimizes using the Markowitz algorithm with input from security analysis. The uninformed investor does not know what input the informed investor uses to make portfolio purchases. The uninformed investor knows, however, that if the other investor is informed, the market portfolio proportions will be optimal. Therefore, to depart from these proportions would constitute an uninformed bet, which will, on average, reduce the efficiency of diversification with no compensating improvement in expected returns.

7.2 Substituting the historical mean and standard deviation in Equation 7.1 yields a coefficient of risk aversion of

$$\bar{A} = \frac{E(r_M) - r_f}{\sigma_M^2} = \frac{.075}{.20^2} = 1.88$$

This relationship also tells us that for the historical standard deviation and a coefficient of risk aversion of 3.5, the risk premium would be

$$E(r_M) - r_f = \bar{A}\sigma_M^2 = 3.5 \times .20^2 = .14 = 14\%$$

7.3 $\beta_{\text{Dell}} = 1.25$, $\beta_{\text{GE}} = 1.15$. Therefore, given the investment proportions, the portfolio beta is

$$\beta_P = w_{\text{Dell}}\beta_{\text{Dell}} + w_{\text{GE}}\beta_{\text{GE}} = (.75 \times 1.25) + (.25 \times 1.15) = 1.225$$

and the risk premium of the portfolio will be

$$E(r_P) - r_f = \beta_P[E(r_M) - r_f] = 1.225 \times 8\% = 9.8\%$$

7.4 *a.* The alpha of a stock is its expected return in excess of that required by the CAPM.

$$\alpha = E(r) - \{r_f + \beta[E(r_M) - r_f]\}$$
$$\alpha_{XYZ} = 12 - [5 + 1.0(11 - 5)] = 1$$
$$\alpha_{ABC} = 13 - [5 + 1.5(11 - 5)] = -1\%$$

b. The project-specific required rate of return is determined by the project beta coupled with the market risk premium and the risk-free rate. The CAPM tells us that an acceptable expected rate of return for the project is

$$r_f + \beta[E(r_M) - r_f] = 8 + 1.3(16 - 8) = 18.4\%$$

which becomes the project's hurdle rate. If the IRR of the project is 19%, then it is desirable. Any project (of similar beta) with an IRR less than 18.4% should be rejected.

7.5 $E(r) = 4\% + 1.2 \times 4\% + .7 \times 2\% = 10.2\%$

7.6 Using Equation 7.11, the expected return is

$$4 + (0.2 \times 6) + (1.4 \times 8) = 16.4\%$$

Chapter 8

The Efficient Market Hypothesis

Learning Objectives:

LO8-1 Demonstrate why security price changes should be essentially unpredictable in an efficient market.

LO8-2 Cite evidence that supports and contradicts the efficient market hypothesis.

LO8-3 Provide interpretations of various stock market "anomalies."

LO8-4 Formulate investment strategies that make sense in informationally efficient markets.

One of the early applications of computers in economics in the 1950s was to analyze economic time series. Business-cycle theorists felt that tracing the evolution of several economic variables over time would clarify and predict the progress of the economy through boom and bust periods. A natural candidate for analysis was the behavior of stock market prices over time. On the assumption that stock prices reflect the prospects of the firm, recurrent patterns of peaks and troughs in economic performance ought to show up in those prices.

When Maurice Kendall (1953) examined this proposition, however, he found to his great surprise that he could identify no predictable patterns in stock prices. Prices seemed to evolve randomly. They were as likely to go up as they were to go down on any particular day, regardless of past performance. The data provided no way to predict price movements.

At first blush, Kendall's results were disturbing to some financial economists. They seemed to imply that the stock market is dominated by erratic market psychology, or "animal spirits"—that it follows no logical rules. In short, the results appeared to confirm the irrationality of the market. On further reflection, however, economists came to reverse their interpretation of Kendall's study.

It soon became apparent that random price movements indicated a well-functioning or efficient market, not an irrational one. In this chapter we explore the reasoning behind what may seem a surprising conclusion. We show how competition among analysts leads naturally to market efficiency, and we examine the implications of the efficient market hypothesis for investment policy. We also consider empirical evidence that supports and contradicts the notion of market efficiency.

Related websites for this chapter are available at www.mhhe.com/bkm.

8.1 RANDOM WALKS AND THE EFFICIENT MARKET HYPOTHESIS

Suppose Kendall had discovered that changes in stock prices can be reliably predicted. What a gold mine this would have been. If they could use Kendall's equations to predict stock prices, investors would reap unending profits simply by purchasing stocks that the computer model implied were about to increase in price and by selling those stocks about to fall in price.

A moment's reflection should be enough to convince yourself that this situation could not persist for long. For example, suppose that the model predicts with great confidence that XYZ stock price, currently at $100 per share, will rise dramatically in three days to $110. What would all investors with access to the model's prediction do today? Obviously, they would place a great wave of immediate buy orders to cash in on the forthcoming increase in stock price. No one holding XYZ, however, would be willing to sell. The net effect would be an *immediate* jump in the stock price to $110. The forecast of a future price increase will lead instead to an immediate price increase. In other words, the stock price will immediately reflect the "good news" implicit in the model's forecast.

This simple example illustrates why Kendall's attempt to find recurrent patterns in stock price movements was likely to fail. A forecast about favorable *future* performance leads instead to favorable *current* performance, as market participants all try to get in on the action before the price increase.

More generally, one might say that any information that could be used to predict stock performance should already be reflected in stock prices. As soon as there is any information indicating that a stock is underpriced and therefore offers a profit opportunity, investors flock to buy the stock and immediately bid up its price to a fair level, where only ordinary rates of return can be expected. These "ordinary rates" are simply rates of return commensurate with the risk of the stock.

However, if prices are bid immediately to fair levels, given all available information, it must be that they increase or decrease only in response to new information. New information, by definition, must be unpredictable; if it could be predicted, then the prediction would be part of today's information. Thus stock prices that change in response to new (unpredictable) information also must move unpredictably.

This is the essence of the argument that stock prices should follow a **random walk,** that is, that price changes should be random and unpredictable. Far from a proof of market irrationality, randomly evolving stock prices would be the necessary consequence of intelligent investors competing to discover relevant information on which to buy or sell stocks before the rest of the market becomes aware of that information.

random walk

The notion that stock price changes are random and unpredictable.

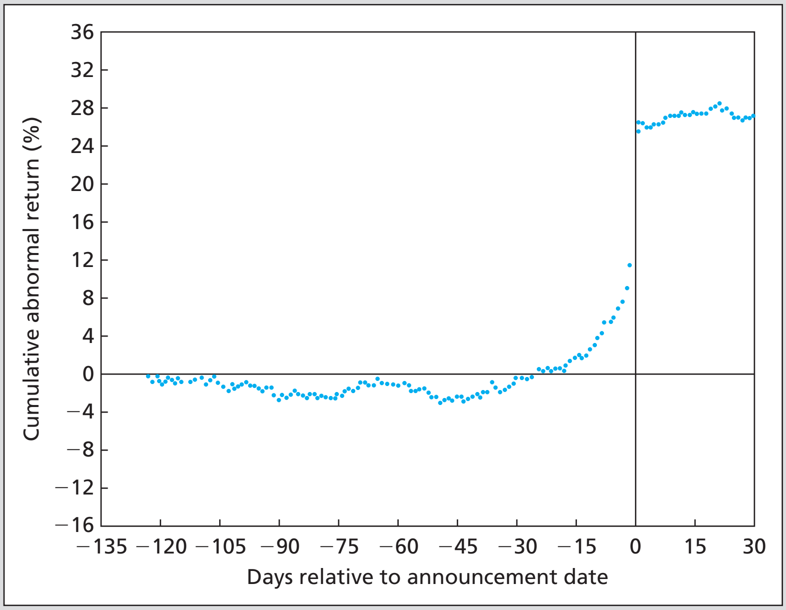
Don't confuse randomness in price *changes* with irrationality in the *level* of prices. If prices are determined rationally, then only new information will cause them to change. Therefore, a random walk would be the natural result of prices that always reflect all current knowledge. Indeed, if stock price movements were predictable, that would be damning evidence of stock market inefficiency, because the ability to predict prices would indicate that all available information was not already reflected in stock prices. Therefore, the notion that stocks already reflect all available information is referred to as the **efficient market hypothesis** (EMH).[1]

efficient market hypothesis

The hypothesis that prices of securities fully reflect available information about securities.

[1]Market efficiency should not be confused with the idea of efficient portfolios introduced in Chapter 6. An informationally efficient *market* is one in which information is rapidly disseminated and reflected in prices. An efficient *portfolio* is one with the highest expected return for a given level of risk.

FIGURE 8.1



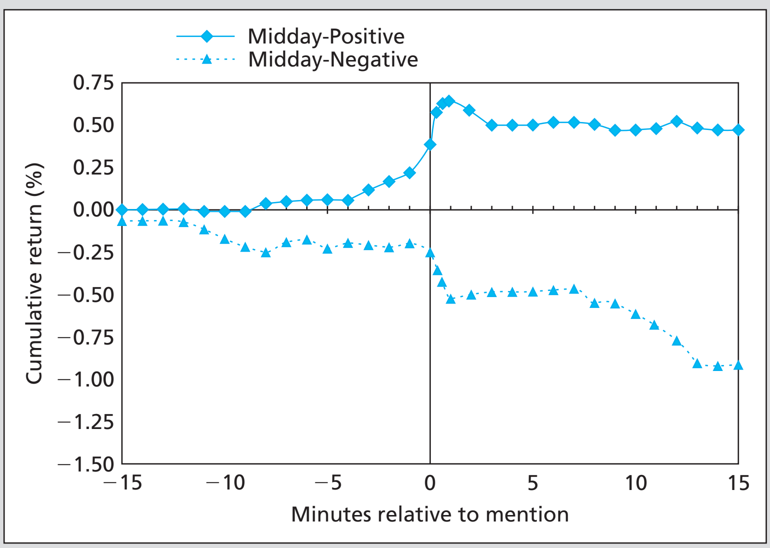
Cumulative abnormal returns before takeover attempts: Target companies

Source: Arthur Keown and John Pinkerton, "Merger Announcements and Insider Trading Activity," *Journal of Finance* 36 (September 1981), pp. 855–869. Used with permission of John Wiley and Sons, via Copyright Clearance Center. Updates courtesy of Jinghua Yan.

Figure 8.1 illustrates the response of stock prices to new information in an efficient market. The graph plots the price response of a sample of 194 firms that were targets of takeover attempts. In most takeovers, the acquiring firm pays a substantial premium over current market prices. Therefore, announcement of a takeover attempt should cause the stock price to jump. The figure shows that stock prices jump dramatically on the day the news becomes public. However, there is no further drift in prices *after* the announcement date, suggesting that prices reflect the new information, including the likely magnitude of the takeover premium, by the end of the trading day.

Even more dramatic evidence of rapid response to new information may be found in intraday prices. For example, Patel and Wolfson (1984) show that most of the stock price response to corporate dividend or earnings announcements occurs within 10 minutes of the announcement. A nice illustration of such rapid adjustment is provided in a study by Busse and Green (2002), who track minute-by-minute stock prices of firms that are featured on CNBC's "Morning" or "Midday Call" segments.[2] Minute 0 in Figure 8.2 is the

FIGURE 8.2



Stock price reaction to CNBC reports. The figure shows the reaction of stock prices to on-air stock reports during the "Midday Call" segment on CNBC. The chart plots cumulative returns beginning 15 minutes before the stock report.

Source: Reprinted from J. A. Busse and T. C. Green, "Market Efficiency in Real Time," *Journal of Financial Economics* 65 (2002), p. 422. Copyright 2002 with permission from Elsevier Science.

[2]You can find an intraday movie version of this figure at **www.bus.emory.edu/cgreen/docs/cnbc/cnbc.html.**

time at which the stock is mentioned on the midday show. The top line is the average price movement of stocks that receive positive reports, while the bottom line reports returns on stocks with negative reports. Notice that the top line levels off, indicating that the market has fully digested the news, within 5 minutes of the report. The bottom line levels off within about 12 minutes.

Competition as the Source of Efficiency

Why should we expect stock prices to reflect "all available information"? After all, if you are willing to spend time and money on gathering information, it might seem reasonable that you could turn up something that has been overlooked by the rest of the investment community. When information is costly to uncover and analyze, one would expect investment analysis calling for such expenditures to result in an increased expected return.

This point has been stressed by Grossman and Stiglitz (1980). They argued that investors will have an incentive to spend time and resources to analyze and uncover new information only if such activity is likely to generate higher investment returns. Thus, in market equilibrium, efficient information-gathering activity should be fruitful. Moreover, it would not be surprising to find that the degree of efficiency differs across various markets. For example, emerging markets that are less intensively analyzed than U.S. markets or in which accounting disclosure requirements are less rigorous may be less efficient than U.S. markets. Small stocks which receive relatively little coverage by Wall Street analysts may be less efficiently priced than large ones. Therefore, while we would not go so far as to say that you absolutely cannot come up with new information, it makes sense to consider and respect your competition.

EXAMPLE 8.1

Rewards for Incremental Performance

Consider an investment management fund currently managing a $5 billion portfolio. Suppose that the fund manager can devise a research program that could increase the portfolio rate of return by one-tenth of 1% per year, a seemingly modest amount. This program would increase the dollar return to the portfolio by $5 billion × .001, or $5 million. Therefore, the fund would be willing to spend up to $5 million per year on research to increase stock returns by a mere tenth of 1% per year. With such large rewards for such small increases in investment performance, it should not be surprising that professional portfolio managers are willing to spend large sums on industry analysts, computer support, and research effort, and therefore that price changes are, generally speaking, difficult to predict.

With so many well-backed analysts willing to spend considerable resources on research, easy pickings in the market will be rare. Moreover, the incremental rates of return on research activity may be so small that only managers of the largest portfolios will find them worth pursuing.

Although it may not literally be true that "all" relevant information will be uncovered, it is virtually certain that there are many investigators hot on the trail of most leads that seem likely to improve investment performance. Competition among these many well-backed, highly paid, aggressive analysts ensures that, as a general rule, stock prices ought to reflect available information regarding their proper levels.

It is often said that the most precious commodity on Wall Street is information, and the competition for it is intense. Sometimes the quest for a competitive advantage can tip over into a search for illegal inside information. In 2011, Raj Rajaratnam, the head of the Galleon Group hedge fund, which once managed $6.5 billion, was convicted on insider trading charges for soliciting tips from a network of corporate insiders and traders. Rajaratnam's case was only one of several major insider trading cases working their way through the courts in 2011. While Galleon's practices were egregious, it often can be difficult to draw a clear line separating legitimate and prohibited sources of information. For example, a large industry of *expert-network* firms has emerged in the last decade to connect (for a fee) investors to industry experts who can provide unique perspective on a company. As the nearby box discusses, this sort of arrangement can easily cross the line into insider trading.

On the MARKET FRONT

MATCHMAKERS FOR THE INFORMATION AGE

The most precious commodity on Wall Street is information, and informed players can charge handsomely for providing it. An industry of so-called *expert-network providers* has emerged to sell access to experts with unique insights about a wide variety of firms and industries to investors who need that information to make decisions. These firms have been dubbed "matchmakers for the information age." Experts can range from doctors who help predict the release of blockbuster drugs to meteorologists who forecast weather that can affect commodity prices to business executives who can provide specialized insight about companies and industries.

But it's turned out that some of those experts have peddled prohibited inside information. In 2011, Winifred Jiau, a consultant for Primary Global Research, was convicted of selling information about Nvidia and Marvell Technologies to the hedge fund SAC Capital Advisors. Several other employees of Primary Global also have been charged with insider trading.

Expert firms are supposed to provide only public information, along with the expert's insights and perspective. But the temptation to hire experts with inside information and charge handsomely for access to them is obvious. The SEC has raised concerns about the boundary between legitimate and illegal services, and several hedge funds in 2011 shut down after raids searching for evidence of such illicit activity.

In the wake of increased scrutiny, compliance efforts of both buyers and sellers of expert information have mushroomed. The largest network firm is Gerson Lehrman Group, with a stable of 300,000 experts. It now maintains down-to-the-minute records of which of its experts talks to whom and the topics they have discussed.[3] These records could be turned over to authorities in the event of an insider trading investigation. And for their part, some hedge funds have simply ceased working with expert-network firms or have promulgated clearer rules for when their employees may talk with consultants.

Even with these safeguards, however, there remains room for trouble. For example, an investor may meet an expert through a legitimate network, and then the two may establish a consulting relationship on their own. The legal matchmaking becomes the precursor to the illegal selling of insider tips. Where there is a will to cheat, there usually will be a way.

Versions of the Efficient Market Hypothesis

It is common to distinguish among three versions of the EMH: the weak, semistrong, and strong forms of the hypothesis. These versions differ by their notions of what is meant by the term "all available information."

weak-form EMH

The assertion that stock prices already reflect all information contained in the history of past trading.

The **weak-form** hypothesis asserts that stock prices already reflect all information that can be derived by examining market trading data such as the history of past prices, trading volume, or short interest. This version of the hypothesis implies that trend analysis is fruitless. Past stock price data are publicly available and virtually costless to obtain. The weak-form hypothesis holds that if such data ever conveyed reliable signals about future performance, all investors already would have learned to exploit the signals. Ultimately, the signals lose their value as they become widely known because a buy signal, for instance, would result in an immediate price increase.

semistrong-form EMH

The assertion that stock prices already reflect all publicly available information.

The **semistrong-form** hypothesis states that all publicly available information regarding the prospects of a firm already must be reflected in the stock price. Such information includes, in addition to past prices, fundamental data on the firm's product line, quality of management, balance sheet composition, patents held, earning forecasts, and accounting practices. Again, if investors have access to such information from publicly available sources, one would expect it to be reflected in stock prices.

strong-form EMH

The assertion that stock prices reflect all relevant information, including inside information.

Finally, the **strong-form** version of the efficient market hypothesis states that stock prices reflect all information relevant to the firm, even including information available only to company insiders. This version of the hypothesis is quite extreme. Few would argue with the proposition that corporate officers have access to pertinent information long enough before public release to enable them to profit from trading on that information. Indeed, much of the activity of the Securities and Exchange Commission is directed toward preventing insiders from profiting by exploiting their privileged situation. Rule 10b-5 of the Security Exchange Act of 1934 sets limits on trading by corporate officers, directors, and substantial owners, requiring them to report trades to the SEC. These insiders, their relatives, and any associates who trade on information supplied by insiders are considered in violation of the law.

[3]"Expert Networks Are the Matchmakers for the Information Age," *The Economist,* June 16, 2011.

Defining insider trading is not always easy, however. After all, stock analysts are in the business of uncovering information not already widely known to market participants. As we saw in Chapter 3 and in the nearby box, the distinction between private and inside information is sometimes murky.

Notice one thing that all versions of the EMH have in common: They all assert that prices should reflect *available* information. We do not expect traders to be superhuman or market prices to never turn out to be wrong. We will always like more information about a company's prospects than will be available. Sometimes market prices will turn out in retrospect to have been outrageously high; at other times, absurdly low. The EMH asserts only that at the given time, using current information, we cannot be sure if today's prices will ultimately prove themselves to have been too high or too low. If markets are rational, however, we can expect them to be correct on average.

CONCEPT check 8.1

a. Suppose you observed that high-level managers make superior returns on investments in their company's stock. Would this be a violation of weak-form market efficiency? Would it be a violation of strong-form market efficiency?

b. If the weak form of the efficient market hypothesis is valid, must the strong form also hold? Conversely, does strong-form efficiency imply weak-form efficiency?

8.2 IMPLICATIONS OF THE EMH

Technical Analysis

Technical analysis is essentially the search for recurrent and predictable patterns in stock prices. Although technicians recognize the value of information regarding future economic prospects of the firm, they believe that such information is not necessary for a successful trading strategy. This is because whatever the fundamental reason for a change in stock price, if the stock price responds slowly enough, the analyst will be able to identify a trend that can be exploited during the adjustment period. The key to successful technical analysis is a sluggish response of stock prices to fundamental supply-and-demand factors. This prerequisite, of course, is diametrically opposed to the notion of an efficient market.

technical analysis

Research on recurrent and predictable stock price patterns and on proxies for buy or sell pressure in the market.

Technical analysts are sometimes called *chartists* because they study records or charts of past stock prices, hoping to find patterns they can exploit to make a profit. As an example of technical analysis, consider the *relative strength* approach. The chartist compares stock performance over a recent period to performance of the market or other stocks in the same industry. A simple version of relative strength takes the ratio of the stock price to a market indicator such as the S&P 500 Index. If the ratio increases over time, the stock is said to exhibit relative strength because its price performance is better than that of the broad market. Such strength presumably may continue for a long enough period of time to offer profit opportunities.

resistance level

A price level above which it is supposedly unlikely for a stock or stock index to rise.

One of the most commonly heard components of technical analysis is the notion of **resistance levels** or **support levels.** These values are said to be price levels above which it is difficult for stock prices to rise or below which it is unlikely for them to fall, and they are believed to be levels determined by market psychology.

support level

A price level below which it is supposedly unlikely for a stock or stock index to fall.

EXAMPLE 8.2

Resistance Levels

Consider stock XYZ, which traded for several months at a price of $72 and then declined to $65. If the stock eventually begins to increase in price, $72 is considered a resistance level (according to this theory) because investors who bought originally at $72 will be eager to sell their shares as soon as they can break even on their investment. Therefore, at prices near $72 a wave of selling pressure would exist. Such activity imparts a type of "memory" to the market that allows past price history to influence current stock prospects.

The efficient market hypothesis implies that technical analysis is without merit. The past history of prices and trading volume is publicly available at minimal cost. Therefore, any information that was ever available from analyzing past prices has already been reflected in stock prices. As investors compete to exploit their common knowledge of a stock's price history, they necessarily drive stock prices to levels where expected rates of return are exactly commensurate with risk. At those levels one cannot expect abnormal returns.

As an example of how this process works, consider what would happen if the market believed that a level of $72 truly were a resistance level for stock XYZ in Example 8.2. No one would be willing to purchase the stock at a price of $71.50, because it would have almost no room to increase in price but ample room to fall. However, if no one would buy it at $71.50, then $71.50 would become a resistance level. But then, using a similar analysis, no one would buy it at $71, or $70, and so on. The notion of a resistance level is a logical conundrum. Its simple resolution is the recognition that if the stock is ever to sell at $71.50, investors *must* believe that the price can as easily increase as fall. The fact that investors are willing to purchase (or even hold) the stock at $71.50 is evidence of their belief that they can earn a fair expected rate of return at that price.

CONCEPT check 8.2

If everyone in the market believes in resistance levels, why do these beliefs not become self-fulfilling prophecies?

An interesting question is whether a technical rule that seems to work will continue to work in the future once it becomes widely recognized. A clever analyst may occasionally uncover a profitable trading rule, but the real test of efficient markets is whether the rule itself becomes reflected in stock prices once its value is discovered. Once a useful technical rule (or price pattern) is discovered, it ought to be invalidated when the mass of traders attempts to exploit it. In this sense, price patterns ought to be *self-destructing.*

Thus the market dynamic is one of a continual search for profitable trading rules, followed by destruction by overuse of those rules found to be successful, followed by more search for yet-undiscovered rules. We return to the rationale for technical analysis as well as some of its methods in the next chapter.

Fundamental Analysis

fundamental analysis

Research on determinants of stock value, such as earnings and dividend prospects, expectations for future interest rates, and risk of the firm.

Fundamental analysis uses earnings and dividend prospects of the firm, expectations of future interest rates, and risk evaluation of the firm to determine proper stock prices. Ultimately, it represents an attempt to determine the present discounted value of all the payments a stockholder will receive from each share of stock. If that value exceeds the stock price, the fundamental analyst would recommend purchasing the stock.

Fundamental analysts usually start with a study of past earnings and an examination of company financial statements. They supplement this analysis with further detailed economic analysis, ordinarily including an evaluation of the quality of the firm's management, the firm's standing within its industry, and the prospects for the industry as a whole. The hope is to attain insight into future performance of the firm that is not yet recognized by the rest of the market. Chapters 12 through 14 provide a detailed discussion of the types of analyses that underlie fundamental analysis.

Once again, the efficient market hypothesis predicts that *most* fundamental analysis also is doomed to failure. If the analyst relies on publicly available earnings and industry information, his or her evaluation of the firm's prospects is not likely to be significantly more accurate than those of rival analysts. There are many well-informed, well-financed firms conducting such market research, and in the face of such competition it will be difficult to uncover data not also available to other analysts. Only analysts with a unique insight will be rewarded.

Fundamental analysis is much more difficult than merely identifying well-run firms with good prospects. Discovery of good firms does an investor no good in and of itself if the rest of the market also knows those firms are good. If the knowledge is already public, the investor will be forced to pay a high price for those firms and will not realize a superior rate of return.

The trick is not to identify firms that are good but to find firms that are *better* than everyone else's estimate. Similarly, poorly run firms can be great bargains if they are not quite as bad as their stock prices suggest.

This is why fundamental analysis is difficult. It is not enough to do a good analysis of a firm; you can make money only if your analysis is better than that of your competitors because the market price will already reflect all commonly available information.

Active versus Passive Portfolio Management

By now it is apparent that casual efforts to pick stocks are not likely to pay off. Competition among investors ensures that any easily implemented stock evaluation technique will be used widely enough so that any insights derived will be reflected in stock prices. Only serious analysis and uncommon techniques are likely to generate the *differential* insight necessary to yield trading profits.

Moreover, these techniques are economically feasible only for managers of large portfolios. If you have only $100,000 to invest, even a 1%-per-year improvement in performance generates only $1,000 per year, hardly enough to justify herculean efforts. The billion-dollar manager, however, reaps extra income of $10 million annually from the same 1% increment.

If small investors are not in a favored position to conduct active portfolio management, what are their choices? The small investor probably is better off investing in mutual funds. By pooling resources in this way, small investors can gain from economies of scale.

More difficult decisions remain, though. Can investors be sure that even large mutual funds have the ability or resources to uncover mispriced stocks? Furthermore, will any mispricing be sufficiently large to repay the costs entailed in active portfolio management?

Proponents of the efficient market hypothesis believe that active management is largely wasted effort and unlikely to justify the expenses incurred. Therefore, they advocate a **passive investment strategy** that makes no attempt to outsmart the market. A passive strategy aims only at establishing a well-diversified portfolio of securities without attempting to find under- or overvalued stocks. Passive management is usually characterized by a buy-and-hold strategy. Because the efficient market theory indicates that stock prices are at fair levels, given all available information, it makes no sense to buy and sell securities frequently, which generates large brokerage fees without increasing expected performance.

passive investment strategy

Buying a well-diversified portfolio without attempting to search out mispriced securities.

One common strategy for passive management is to create an **index fund,** which is a fund designed to replicate the performance of a broad-based index of stocks. For example, Vanguard's 500 Index Fund holds stocks in direct proportion to their weight in the Standard & Poor's 500 stock price index. The performance of the 500 Index Fund therefore replicates the performance of the S&P 500. Investors in this fund obtain broad diversification with relatively low management fees. The fees can be kept to a minimum because Vanguard does not need to pay analysts to assess stock prospects and does not incur transaction costs from high portfolio turnover. Indeed, while the typical annual expense ratio for an actively managed equity fund is around 1% of assets, the expense ratio of the 500 Index Fund is only .17%. Today, Vanguard's 500 Index Fund is among the largest equity mutual funds with over $100 billion of assets in mid-2011. At the end of 2011, about 15% of assets in equity mutual funds were indexed.

index fund

A mutual fund holding shares in proportion to their representation in a market index such as the S&P 500.

Indexing need not be limited to the S&P 500, however. For example, some of the funds offered by the Vanguard Group track the Wilshire 5000 Index, the Barclays Capital U.S. Aggregate Bond Index, the MSCI index of small-capitalization U.S. companies, the European equity market, and the Pacific Basin equity market. Several other mutual fund complexes offer indexed portfolios, but Vanguard dominates the retail market for indexed products.

Exchange-traded funds, or ETFs, are a close (and usually lower-expense) alternative to indexed mutual funds. As noted in Chapter 4, these are shares in diversified portfolios that can be bought or sold just like shares of individual stock. ETFs matching several broad stock market indexes such as the S&P 500 or Wilshire 5000 indexes and dozens of international and industry stock indexes are available to investors who want to hold a diversified sector of a market without attempting active security selection.

CONCEPT check 8.3

What would happen to market efficiency if *all* investors attempted to follow a passive strategy?

The Role of Portfolio Management in an Efficient Market

If the market is efficient, why not throw darts at *The Wall Street Journal* instead of trying rationally to choose a stock portfolio? This is a tempting conclusion to draw from the notion that security prices are fairly set, but it is far too facile. There is a role for rational portfolio management, even in perfectly efficient markets.

You have learned that a basic principle in portfolio selection is diversification. Even if all stocks are priced fairly, each still poses firm-specific risk that can be eliminated through diversification. Therefore, rational security selection, even in an efficient market, calls for the selection of a well-diversified portfolio providing the systematic risk level that the investor wants.

Rational investment policy also requires that tax considerations be reflected in security choice. High-tax-bracket investors generally will not want the same securities that low-bracket investors find favorable. At an obvious level, high-bracket investors find it advantageous to buy tax-exempt municipal bonds despite their relatively low pretax yields, whereas those same bonds are unattractive to low-tax-bracket investors. At a more subtle level, high-bracket investors might want to tilt their portfolios in the direction of capital gains as opposed to interest income, because capital gains are taxed less heavily and because the option to defer the realization of capital gains income is more valuable the higher the current tax bracket. Hence these investors may prefer stocks that yield low dividends yet offer greater expected capital gains income. They also will be more attracted to investment opportunities for which returns are sensitive to tax benefits, such as real estate ventures.

A third argument for rational portfolio management relates to the particular risk profile of the investor. For example, a Toyota executive whose annual bonus depends on Toyota's profits generally should not invest additional amounts in auto stocks. To the extent that his or her compensation already depends on Toyota's well-being, the executive is already overinvested in Toyota and should not exacerbate the lack of diversification. This lesson was learned with considerable pain in September 2008 by Lehman Brothers employees who were famously invested in their own firm when the company failed. Roughly 30% of the shares in the firm were owned by its 24,000 employees, and their losses on those shares were around $10 billion.

Investors of varying ages also might warrant different portfolio policies with regard to risk bearing. For example, older investors who are essentially living off savings might choose to avoid long-term bonds whose market values fluctuate dramatically with changes in interest rates (discussed in Part Four). Because these investors are living off accumulated savings, they require conservation of principal. In contrast, younger investors might be more inclined toward long-term inflation-indexed bonds. The steady flow of real income over long periods of time that is locked in with these bonds can be more important than preservation of principal to those with long life expectancies.

In conclusion, there is a role for portfolio management even in an efficient market. Investors' optimal positions will vary according to factors such as age, tax bracket, risk aversion, and employment. The role of the portfolio manager in an efficient market is to tailor the portfolio to these needs, rather than to beat the market.

Resource Allocation

We've focused so far on the investments implications of the efficient market hypothesis. Deviations from efficiency may offer profit opportunities to better-informed traders at the expense of less informed traders.

However, deviations from informational efficiency would also result in a large cost that will be borne by all citizens, namely, inefficient resource allocation. Recall that in a capitalist economy, investments in *real* assets such as plant, equipment, and know-how are guided in large part by the prices of financial assets. For example, if the value of telecommunication capacity reflected in stock market prices exceeds the cost of installing such capacity, managers might justifiably conclude that telecom investments seem to have positive net present value. In this manner, capital market prices guide allocation of real resources.

If markets were inefficient and securities commonly mispriced, then resources would be systematically misallocated. Corporations with overpriced securities will be able to obtain capital too cheaply, and corporations with undervalued securities might forgo investment opportunities because the cost of raising capital will be too high. Therefore, inefficient capital markets would diminish one of the most potent benefits of a market economy. As an example of what can go wrong, consider the dot-com bubble of the late 1990s, which sent a strong but, as it turned out, wildly overoptimistic signal about prospects in Internet and telecommunication firms and ultimately led to substantial overinvestment in those industries.

Before writing off markets as a means to guide resource allocation, however, one has to be reasonable about what can be expected from market forecasts. In particular, you shouldn't confuse an efficient market, where all available information is reflected in prices, with a perfect-foresight market. Even "all available information" is still far from complete information, and generally rational market forecasts will sometimes be wrong; sometimes, in fact, they will be very wrong.

8.3 ARE MARKETS EFFICIENT?

The Issues

Not surprisingly, the efficient market hypothesis does not exactly arouse enthusiasm in the community of professional portfolio managers. It implies that a great deal of the activity of portfolio managers—the search for undervalued securities—is at best wasted effort, and quite probably harmful to clients because it costs money and leads to imperfectly diversified portfolios. Consequently, the EMH has never been widely accepted on Wall Street, and debate continues today on the degree to which security analysis can improve investment performance. Before discussing empirical tests of the hypothesis, we want to note three factors that together imply that the debate probably never will be settled: the *magnitude issue,* the *selection bias issue,* and the *lucky event issue.*

The magnitude issue We noted that an investment manager overseeing a $5 billion portfolio who can improve performance by only .1% per year will increase investment earnings by $.001 \times \$5$ billion $=$ $5 million annually. This manager clearly would be worth her salary! Yet can we, as observers, statistically measure her contribution? Probably not: A .1% contribution would be swamped by the yearly volatility of the market. Remember, the annual standard deviation of the well-diversified S&P 500 Index has been around 20%. Against these fluctuations a small increase in performance would be hard to detect.

All might agree that stock prices are very close to fair values and that only managers of large portfolios can earn enough trading profits to make the exploitation of minor mispricing worth the effort. According to this view, the actions of intelligent investment managers are the driving force behind the constant evolution of market prices to fair levels. Rather than ask the qualitative question, "Are markets efficient?" we should instead ask a more quantitative question: "How efficient are markets?"

The selection bias issue Suppose that you discover an investment scheme that could really make money. You have two choices: either publish your technique in *The Wall Street Journal* to win fleeting fame, or keep your technique secret and use it to earn millions of dollars. Most investors would choose the latter option, which presents us with a conundrum. Only investors who find that an investment scheme cannot generate abnormal returns will be willing to report their findings to the whole world. Hence opponents of the efficient markets

On the MARKET FRONT

HOW TO GUARANTEE A SUCCESSFUL MARKET NEWSLETTER

Suppose you want to make your fortune publishing a market newsletter. You need first to convince potential subscribers that you have talent worth paying for. But what if you have no talent? The solution is simple: Start eight newsletters.

In year 1, let four of your newsletters predict an up-market and four a down-market. In year 2, let half of the originally optimistic group of newsletters continue to predict an up-market and the other half a down-market. Do the same for the originally pessimistic group. Continue in this manner to obtain the pattern of predictions in the table that follows (U = prediction of an up-market, D = prediction of a down-market).

After three years, no matter what has happened to the market, one of the newsletters would have had a perfect prediction record. This is because after three years there are $2^3 = 8$ outcomes for the market, and we have covered all eight possibilities with the eight newsletters. Now, we simply slough off the seven unsuccessful newsletters, and market the eighth newsletter based on its perfect track record. If we want to establish a newsletter with a perfect track record over a four-year period, we need $2^4 = 16$ newsletters. A five-year period requires 32 newsletters, and so on.

After the fact, the one newsletter that was always right will attract attention for your uncanny foresight and investors will rush to pay large fees for its advice. Your fortune is made, and you have never even researched the market!

WARNING: This scheme is illegal! The point, however, is that with hundreds of market newsletters, you can find one that has stumbled onto an apparently remarkable string of successful predictions without any real degree of skill. After the fact, *someone's* prediction history can seem to imply great forecasting skill. This person is the one we will read about in *The Wall Street Journal;* the others will be forgotten.

Newsletter Predictions

Year	1	2	3	4	5	6	7	8
1	U	U	U	U	D	D	D	D
2	U	U	D	D	U	U	D	D
3	U	D	U	D	U	D	U	D

view of the world always can use evidence that various techniques do not provide investment rewards as proof that the techniques that do work simply are not being reported to the public. This is a problem in *selection bias;* the outcomes we are able to observe have been preselected in favor of failed attempts. Therefore, we cannot fairly evaluate the true ability of portfolio managers to generate winning stock market strategies.

The lucky event issue In virtually any month it seems we read an article about some investor or investment company with a fantastic investment performance over the recent past. Surely the superior records of such investors disprove the efficient market hypothesis.

Yet this conclusion is far from obvious. As an analogy to the investment game, consider a contest to flip the most number of heads out of 50 trials using a fair coin. The expected outcome for any person is, of course, 50% heads and 50% tails. If 10,000 people, however, compete in this contest, it would not be surprising if at least one or two contestants flipped more than 75% heads. In fact, elementary statistics tells us that the expected number of contestants flipping 75% or more heads would be two. It would be silly, though, to crown these people the "head-flipping champions of the world." Obviously, they are simply the contestants who happened to get lucky on the day of the event. (See the nearby box.)

The analogy to efficient markets is clear. Under the hypothesis that any stock is fairly priced given all available information, any bet on a stock is simply a coin toss. There is equal likelihood of winning or losing the bet. However, if many investors using a variety of schemes make fair bets, statistically speaking, *some* of those investors will be lucky and win a great majority of the bets. For every big winner, there may be many big losers, but we never hear of these managers. The winners, though, turn up in *The Wall Street Journal* as the latest stock market gurus; then they can make a fortune publishing market newsletters.

Our point is that after the fact there will have been at least one successful investment scheme. A doubter will call the results luck; the successful investor will call it skill. The proper test would be to see whether the successful investors can repeat their performance in another period, yet this approach is rarely taken.

With these caveats in mind, we turn now to some of the empirical tests of the efficient market hypothesis.

CONCEPT check 8.4

Legg Mason's Value Trust, managed by Bill Miller, outperformed the S&P 500 in each of the 15 years ending in 2005. Is Miller's performance sufficient to dissuade you from a belief in efficient markets? If not, would *any* performance record be sufficient to dissuade you? Now consider that in the next 3 years, the fund dramatically underperformed the S&P 500; by the end of 2008, its cumulative 18-year performance was barely different from the index. Does this affect your opinion?

Weak-Form Tests: Patterns in Stock Returns

Returns over short horizons Early tests of efficient markets were tests of the weak form. Could speculators find trends in past prices that would enable them to earn abnormal profits? This is essentially a test of the efficacy of technical analysis.

One way of discerning trends in stock prices is by measuring the *serial correlation* of stock market returns. Serial correlation refers to the tendency for stock returns to be related to past returns. Positive serial correlation means that positive returns tend to follow positive returns (a momentum type of property). Negative serial correlation means that positive returns tend to be followed by negative returns (a reversal or "correction" property). Both Conrad and Kaul (1988) and Lo and MacKinlay (1988) examine weekly returns of NYSE stocks and find positive serial correlation over short horizons. However, the correlation coefficients of weekly returns tend to be fairly small, at least for large stocks for which price data are the most reliably up to date. Thus, while these studies demonstrate weak price trends over short periods,[4] the evidence does not clearly suggest the existence of trading opportunities.

While broad market indexes demonstrate only weak serial correlation, there appears to be stronger momentum in performance across market sectors exhibiting the best and worst recent returns. In an investigation of intermediate-horizon stock price behavior (using 3- to 12-month holding periods), Jegadeesh and Titman (1993) found a **momentum effect** in which good or bad recent performance of particular stocks continues over time. They conclude that while the performance of individual stocks is highly unpredictable, *portfolios* of the best-performing stocks in the recent past appear to outperform other stocks with enough reliability to offer profit opportunities. Thus, it appears that there is evidence of short- to intermediate-horizon price momentum in both the aggregate market and cross-sectionally (i.e., across particular stocks).

momentum effect

The tendency of poorly performing stocks and well-performing stocks in one period to continue that abnormal performance in following periods.

Returns over long horizons Although short- to intermediate-horizon returns suggest momentum in stock market prices, studies of long-horizon returns (i.e., returns over multiyear periods) by Fama and French (1988) and Poterba and Summers (1988) indicate pronounced *negative* long-term serial correlation in the performance of the aggregate market. The latter result has given rise to a "fads hypothesis," which asserts that the stock market might overreact to relevant news. Such overreaction leads to positive serial correlation (momentum) over short time horizons. Subsequent correction of the overreaction leads to poor performance following good performance and vice versa. The corrections mean that a run of positive returns eventually will tend to be followed by negative returns, leading to negative serial correlation over longer horizons. These episodes of apparent overshooting followed by correction give the stock market the appearance of fluctuating around its fair value.

These long-horizon results are dramatic, but the studies offer far from conclusive evidence regarding efficient markets. First, the study results need not be interpreted as evidence for stock market fads. An alternative interpretation of these results holds that they indicate only that the market risk premium varies over time. For example, when the risk premium and the required

[4]On the other hand, there is evidence that share prices of individual securities (as opposed to broad market indexes) are more prone to reversals than continuations at very short horizons. See, for example, B. Lehmann, "Fads, Martingales and Market Efficiency," *Quarterly Journal of Economics* 105 (February 1990), pp. 1–28; and N. Jegadeesh, "Evidence of Predictable Behavior of Security Returns," *Journal of Finance* 45 (September 1990), pp. 881–898. However, as Lehmann notes, this is probably best interpreted as due to liquidity problems after big movements in stock prices as market makers adjust their positions in the stock.

return on the market rises, stock prices will fall. When the market then rises (on average) at this higher rate of return, the data convey the impression of a stock price recovery. The apparent overshooting and correction is in fact no more than a rational response of market prices to changes in discount rates.

In addition to studies suggestive of overreaction in overall stock market returns over long horizons, many other studies suggest that over long horizons, extreme performance in particular securities also tends to reverse itself: The stocks that have performed best in the recent past seem to underperform the rest of the market in following periods, while the worst past performers tend to offer above-average future performance. De Bondt and Thaler (1985) and Chopra, Lakonishok, and Ritter (1992) find strong tendencies for poorly performing stocks in one period to experience sizable reversals over the subsequent period, while the best-performing stocks in a given period tend to follow with poor performance in the following period.

reversal effect

The tendency of poorly performing stocks and well-performing stocks in one period to experience reversals in the following period.

For example, the De Bondt and Thaler study found that if one were to rank-order the performance of stocks over a five-year period and then group stocks into portfolios based on investment performance, the base-period "loser" portfolio (defined as the 35 stocks with the worst investment performance) outperformed the "winner" portfolio (the top 35 stocks) by an average of 25% (cumulative return) in the following three-year period. This **reversal effect,** in which losers rebound and winners fade back, suggests that the stock market overreacts to relevant news. After the overreaction is recognized, extreme investment performance is reversed. This phenomenon would imply that a *contrarian* investment strategy—investing in recent losers and avoiding recent winners—should be profitable. Moreover, these returns seem pronounced enough to be exploited profitably.

Thus it appears that there may be short-run momentum but long-run reversal patterns in price behavior both for the market as a whole and across sectors of the market. One interpretation of this pattern is that short-run overreaction (which causes momentum in prices) may lead to long-term reversals (when the market recognizes its past error).

Predictors of Broad Market Returns

Several studies have documented the ability of easily observed variables to predict market returns. For example, Fama and French (1988) showed that the return on the aggregate stock market tends to be higher when the dividend/price ratio, the dividend yield, is high. Campbell and Shiller (1988) found that the earnings yield can predict market returns. Keim and Stambaugh (1986) showed that bond market data such as the spread between yields on high- and low-grade corporate bonds also help predict broad market returns.

Again, the interpretation of these results is difficult. On the one hand, they may imply that stock returns can be predicted, in violation of the efficient market hypothesis. More probably, however, these variables are proxying for variation in the market risk premium. For example, given a level of dividends or earnings, stock prices will be lower and dividend and earnings yields will be higher when the risk premium (and therefore the expected market return) is higher. Thus a high dividend or earnings yield will be associated with higher market returns. This does not indicate a violation of market efficiency. The predictability of market returns is due to predictability in the risk premium, not in risk-adjusted abnormal returns.

Fama and French (1989) showed that the yield spread between high- and low-grade bonds has greater predictive power for returns on low-grade bonds than for returns on high-grade bonds, and greater predictive power for stock returns than for bond returns, suggesting that the predictability in returns is in fact a risk premium rather than evidence of market inefficiency. Similarly, the fact that the dividend yield on stocks helps to predict bond market returns suggests that the yield captures a risk premium common to both markets rather than mispricing in the equity market.

Semistrong Tests: Market Anomalies

Fundamental analysis uses a much wider range of information to create portfolios than does technical analysis. Investigations of the efficacy of fundamental analysis ask whether publicly available information beyond the trading history of a security can be used to improve investment performance, and therefore they are tests of semistrong-form market efficiency. Surprisingly, several easily accessible statistics, for example a stock's price–earnings ratio or its market capitalization, seem to predict abnormal risk-adjusted returns. Findings such as these, which we will review in the following pages, are difficult to reconcile with the efficient market hypothesis and therefore are often referred to as efficient market **anomalies.**

anomalies
Patterns of returns that seem to contradict the efficient market hypothesis.

A difficulty in interpreting these tests is that we usually need to adjust for portfolio risk before evaluating the success of an investment strategy. Many tests, for example, have used the CAPM to adjust for risk. However, we know that even if beta is a relevant descriptor of stock risk, the empirically measured quantitative trade-off between risk as measured by beta and expected return differs from the predictions of the CAPM. If we use the CAPM to adjust portfolio returns for risk, inappropriate adjustments may lead to the conclusion that various portfolio strategies can generate superior returns, when in fact it simply is the risk adjustment procedure that has failed.

Another way to put this is to note that tests of risk-adjusted returns are *joint tests* of the efficient market hypothesis *and* the risk adjustment procedure. If it appears that a portfolio strategy can generate superior returns, we must then choose between rejecting the EMH and rejecting the risk adjustment technique. Usually, the risk adjustment technique is based on more-questionable assumptions than is the EMH; by opting to reject the procedure, we are left with no conclusion about market efficiency.

An example of this issue is the discovery by Basu (1977, 1983) that portfolios of low price–earnings (P/E) ratio stocks have higher returns than do high P/E portfolios. The **P/E effect** holds up even if returns are adjusted for portfolio beta. Is this a confirmation that the market systematically misprices stocks according to P/E ratio? This would be an extremely surprising and, to us, disturbing conclusion, because analysis of P/E ratios is such a simple procedure. Although it may be possible to earn superior returns by using hard work and much insight, it hardly seems plausible that such a simplistic technique is enough to generate abnormal returns.

P/E effect
Portfolios of low P/E stocks have exhibited higher average risk-adjusted returns than high P/E stocks.

Another interpretation of these results is that returns are not properly adjusted for risk. If two firms have the same expected earnings, the riskier stock will sell at a lower price and lower P/E ratio. Because of its higher risk, the low P/E stock also will have higher expected returns. Therefore, unless the CAPM beta fully adjusts for risk, P/E will act as a useful additional descriptor of risk and will be associated with abnormal returns if the CAPM is used to establish benchmark performance.

The small-firm-in-January effect The so-called size or **small-firm effect,** originally documented by Banz (1981), is illustrated in Figure 8.3. It shows the historical performance of portfolios formed by dividing the NYSE stocks into 10 portfolios each year according to firm size (i.e., the total value of outstanding equity). Average annual returns between 1926 and 2010 are consistently higher on the small-firm portfolios. The difference in average annual return between portfolio 10 (with the largest firms) and portfolio 1 (with the smallest firms) is 8.8%. Of course, the smaller-firm portfolios tend to be riskier. But even when returns are adjusted for risk using the CAPM, there is still a consistent premium for the smaller-sized portfolios.

small-firm effect
Stocks of small firms have earned abnormal returns, primarily in the month of January.

Imagine earning a premium of this size on a billion-dollar portfolio. Yet it is remarkable that following a simple (even simplistic) rule such as "invest in low-capitalization stocks" should enable an investor to earn excess returns. After all, any investor can measure firm size at little cost. One would not expect such minimal effort to yield such large rewards.

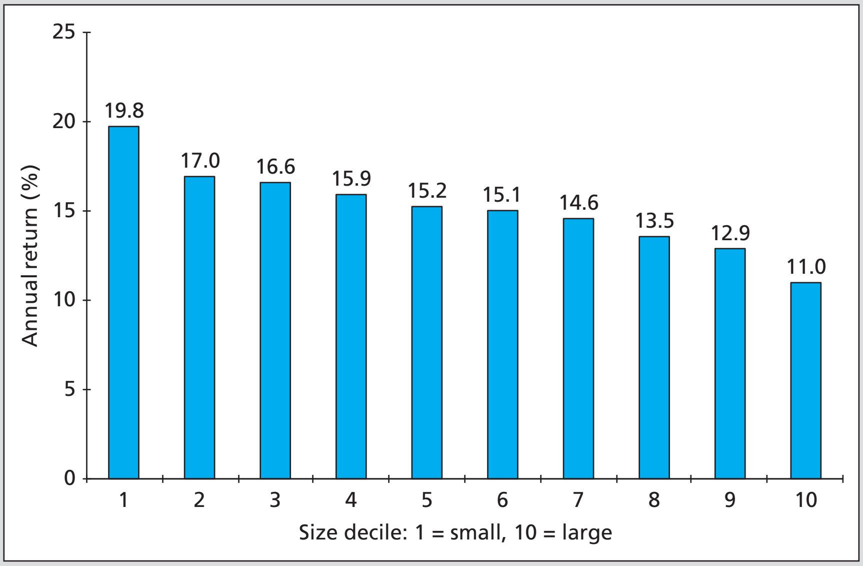
Later studies (Keim, 1983; Reinganum, 1983; and Blume and Stambaugh, 1983) showed that the small-firm effect occurs virtually entirely in January, in fact, in the first two weeks of January. The size effect is in fact a "small-firm-in-January" effect.

The neglected-firm and liquidity effects Arbel and Strebel (1983) gave another interpretation of the small-firm-in-January effect. Because they tend to be neglected by large institutional traders, information about smaller firms is less available. This information deficiency makes smaller firms riskier investments that command higher returns. "Brand-name" firms, after all, are subject to considerable monitoring from institutional investors, which promises high-quality information, and presumably investors do not purchase "generic" stocks without the prospect of greater returns.

FIGURE 8.3

Average annual return for 10 size-based portfolios, 1926–2010

Source: Authors' calculations using data obtained from Prof. Kenneth French's data library, **http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.**



neglected-firm effect

The tendency of investments in stock of less well-known firms to generate abnormal returns.

As evidence for the **neglected-firm effect,** Arbel (1985) divided firms into highly researched, moderately researched, and neglected groups based on the number of institutions holding the stock. The January effect was in fact largest for the neglected firms. An article by Merton (1987) shows that neglected firms might be expected to earn higher equilibrium returns as compensation for the risk associated with limited information. In this sense the neglected-firm premium is not strictly a market inefficiency but is a type of risk premium.

Work by Amihud and Mendelson (1986, 1991) on the effect of liquidity on stock returns might be related to both the small-firm and neglected-firm effects. They argue that investors will demand a rate-of-return premium to invest in less liquid stocks that entail higher trading costs. In accord with their hypothesis, Amihud and Mendelson showed that these stocks show a strong tendency to exhibit abnormally high risk-adjusted rates of return. Because small and less-analyzed stocks as a rule are less liquid, the liquidity effect might be a partial explanation of their abnormal returns. However, this theory does not explain why the abnormal returns of small firms should be concentrated in January. In any case, exploiting these effects can be more difficult than it would appear. The high trading costs on small stocks can easily wipe out any apparent abnormal profit opportunity.

Book-to-market ratios Fama and French (1992) showed that a powerful predictor of returns across securities is the ratio of the book value of the firm's equity to the market value of equity. Fama and French stratified firms into 10 groups according to book-to-market ratios and examined the average rate of return of each of the 10 groups. Figure 8.4 is an updated version of their results. The decile with the highest book-to-market ratio had an average annual return of 17.3%, while the lowest-ratio decile averaged only 11%. The dramatic dependence of returns on book-to-market ratio is independent of beta, suggesting either that high book-to-market ratio firms are relatively underpriced or that the book-to-market ratio is serving as a proxy for a risk factor that affects equilibrium expected returns.

book-to-market effect

The tendency for investments in shares of firms with high ratios of book value to market value to generate abnormal returns.

In fact, Fama and French found that after controlling for the size and **book-to-market effects,** beta seemed to have no power to explain average security returns.[5] This finding is an important challenge to the notion of rational markets, since it seems to imply that a factor

[5]However, a study by S. P. Kothari, Jay Shanken, and Richard G. Sloan (1995) finds that when betas are estimated using annual rather than monthly returns, securities with high beta values do in fact have higher average returns. Moreover, the authors find a book-to-market effect that is attenuated compared to the results in Fama and French and furthermore is inconsistent across different samples of securities. They conclude that the empirical case for the importance of the book-to-market ratio may be somewhat weaker than the Fama and French study would suggest.

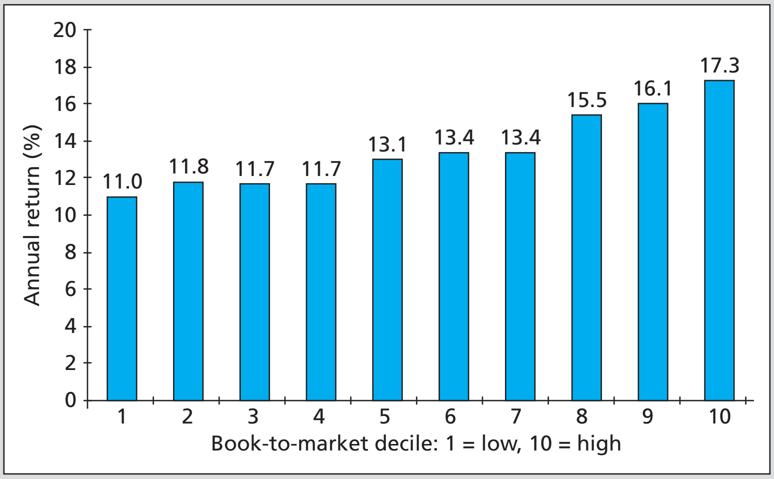


FIGURE 8.4

Average annual return as a function of the book-to-market ratio, 1926–2010

Source: Website of Prof. Kenneth French, **http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.**

that should affect returns—systematic risk—seems not to matter, while a factor that should not matter—the book-to-market ratio—seems capable of predicting future returns. We will return to the interpretation of this anomaly.

Post-earnings-announcement price drift A fundamental principle of efficient markets is that any new information ought to be reflected in stock prices very rapidly. When good news is made public, for example, the stock price should jump immediately. A puzzling anomaly, therefore, is the apparently sluggish response of stock prices to firms' earnings announcements, as uncovered by Ball and Brown (1968). Their results were later confirmed and extended in many other papers.[6]

The "news content" of an earnings announcement can be evaluated by comparing the announcement of actual earnings to the value previously expected by market participants. The difference is the "earnings surprise." (Market expectations of earnings can be roughly measured by averaging the published earnings forecasts of Wall Street analysts or by applying trend analysis to past earnings.) Rendleman, Jones, and Latané (1982) provide an influential study of sluggish price response to earnings announcements. They calculate earnings surprises for a large sample of firms, rank the magnitude of the surprise, divide firms into 10 deciles based on the size of the surprise, and calculate abnormal returns for each decile. The abnormal return of each portfolio is the return adjusting for both the market return in that period and the portfolio beta. It measures return over and above what would be expected given market conditions in that period. Figure 8.5 plots cumulative abnormal returns by decile.

Their results are dramatic. The correlation between ranking by earnings surprise and abnormal returns across deciles is as predicted. There is a large abnormal return (a jump in cumulative abnormal return) on the earnings announcement day (time 0). The abnormal return is positive for positive-surprise firms and negative for negative-surprise firms.

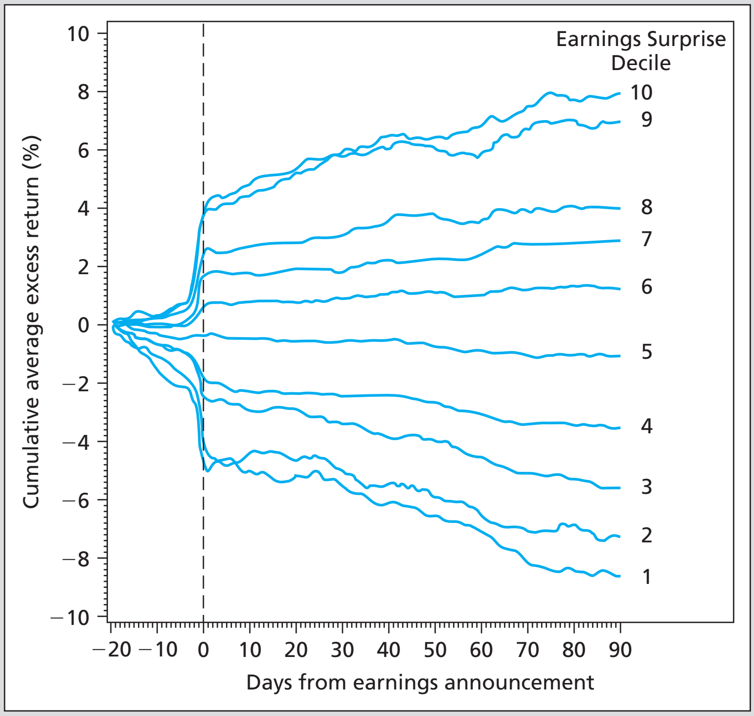
The more remarkable, and interesting, result of the study concerns stock price movement *after* the announcement date. The cumulative abnormal returns of positive-surprise stocks continue to rise—in other words, exhibit momentum—even after the earnings information becomes public, while the negative-surprise firms continue to suffer negative abnormal returns. The market appears to adjust to the earnings information only gradually, resulting in a sustained period of abnormal returns.

[6]There is a voluminous literature on this phenomenon, often referred to as post-earnings-announcement price drift. For more recent papers that focus on why such drift may be observed, see V. Bernard and J. Thomas, "Evidence That Stock Prices Do Not Fully Reflect the Implications of Current Earnings for Future Earnings," *Journal of Accounting and Economics* 13 (1990), pp. 305–340, or R. H. Battalio and R. Mendenhall, "Earnings Expectation, Investor Trade Size, and Anomalous Returns around Earnings Announcements," *Journal of Financial Economics* 77 (2005), pp. 289–319.

FIGURE 8.5

Cumulative abnormal returns in response to earnings announcements

Source: Reprinted from R. J. Rendleman Jr., C. P. Jones, and H. A. Latané, "Empirical Anomalies Based on Unexpected Earnings and the Importance of Risk Adjustments," *Journal of Financial Economics* 10 (1982), pp. 269–287. Copyright 1982 with permission from Elsevier Science.



Evidently, one could have earned abnormal profits simply by waiting for earnings announcements and purchasing a stock portfolio of positive-earnings-surprise companies. These are precisely the types of predictable continuing trends that ought to be impossible in an efficient market.

Bubbles and market efficiency Every so often, it seems (at least in retrospect) that asset prices lose their grounding in reality. For example, in the tulip mania in seventeenth-century Holland, tulip prices peaked at several times the annual income of a skilled worker. This episode has become the symbol of a speculative "bubble" in which prices appear to depart from any semblance of intrinsic value. Less than a century later, the South Sea bubble in England became almost as famous. In this episode, the share price of the South Sea Company rose from £128 in January 1720 to £550 in May and peaked at around £1,000 in August—just before the bubble burst and the share price collapsed to £150 in September, leading to widespread bankruptcies among those who had borrowed to buy shares on credit. In fact, the company was a major lender of money to investors willing to buy (and thus bid up) its shares. This sequence may sound familiar to anyone who lived through the dot-com boom and bust of 1995–2002[7] or, more recently, the financial turmoil of 2008, with origins widely attributed to a collapsing housing price bubble (see Chapter 1).

It is hard to defend the position that security prices in these instances represented rational, unbiased assessments of intrinsic value. And, in fact, some economists, most notably Hyman Minsky, have suggested that bubbles arise naturally. During periods of stability and rising prices, investors extrapolate that stability into the future and become more willing to take on risk. Risk premiums shrink, leading to further increases in asset prices, and expectations become even more optimistic in a self-fulfilling cycle. But, in the end, pricing and risk taking become excessive and the bubble bursts. Ironically, the initial period of stability fosters behavior that ultimately results in instability.

[7]The dot-com boom gave rise to the term *irrational exuberance.* In this vein consider that one company, going public in the investment boom of 1720, described itself simply as "a company for carrying out an undertaking of great advantage, but nobody to know what it is."

But beware of jumping to the conclusion that asset prices may generally be thought of as arbitrary and obvious trading opportunities abundant. First, most bubbles become "obvious" only *after* they have burst. At the time, there is often a seemingly defensible rationale for the price run-up. In the dot-com boom, for example, many contemporary observers rationalized stock price gains as justified by the prospect of a new and more profitable economy, driven by technological advances. Even the irrationality of the tulip mania may have been overblown in its later retelling.[8] In addition, security valuation is intrinsically difficult. Given the considerable imprecision of estimates of intrinsic value, large bets on perceived mispricing may entail hubris.

Moreover, even if you suspect that prices are in fact "wrong," it can be difficult to take advantage of them. We explore these issues in more detail in the following chapter. For now, we can simply point out some impediments to making aggressive bets against an asset: the costs of short-selling overpriced securities as well as potential problems obtaining the securities to sell short and the possibility that, even if you are ultimately correct, the market may disagree and prices still can move dramatically against you in the short term, thus wiping out your portfolio.

Strong-Form Tests: Inside Information

It would not be surprising if insiders were able to make superior profits trading in their firm's stock. In other words, we do not expect markets to be strong-form efficient; we regulate and limit trades based on inside information. The ability of insiders to trade profitably in their own stock has been documented in studies by Jaffe (1974), Seyhun (1986), Givoly and Palmon (1985), and others. Jaffe's was one of the earlier studies that documented the tendency for stock prices to rise after insiders intensively bought shares and to fall after intensive insider sales.

Can other investors benefit by following insiders' trades? The Securities and Exchange Commission requires all insiders to register their trading activity, and it publishes these trades in an *Official Summary of Security Transactions and Holdings.* Since 2002, insiders must report large trades to the SEC within two business days. Once the *Official Summary* is published, the trades become public information. At that point, if markets are efficient, fully and immediately processing the information released in the *Official Summary* of trading, an investor should no longer be able to profit from following the pattern of those trades. Several Internet sites contain information on insider trading.

The study by Seyhun, which carefully tracked the public release dates of the *Official Summary,* found that following insider transactions would be to no avail. Although there is some tendency for stock prices to increase even after the *Official Summary* reports insider buying, the abnormal returns are not of sufficient magnitude to overcome transaction costs.

Interpreting the Anomalies

How should we interpret the ever-growing anomalies literature? Does it imply that markets are grossly inefficient, allowing for simplistic trading rules to offer large profit opportunities? Or are there other, more-subtle interpretations?

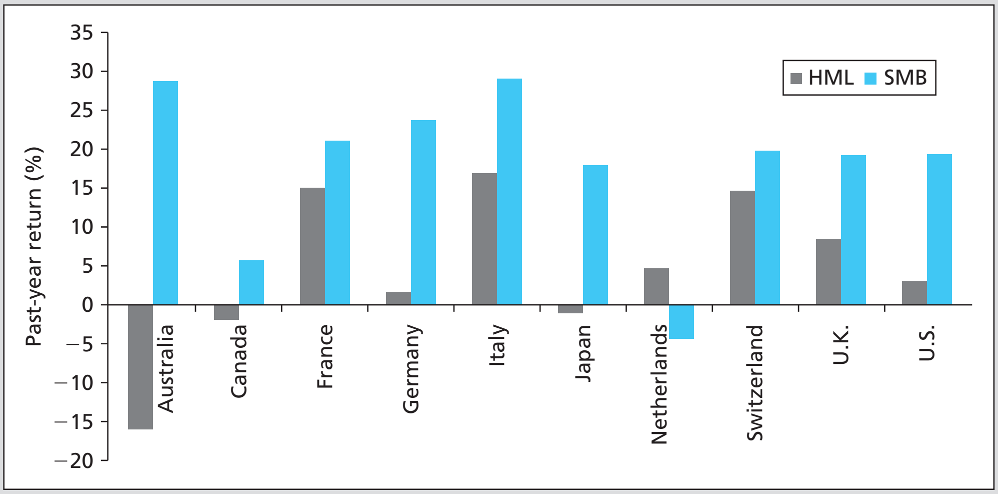
Risk premiums or inefficiencies? The price–earnings, small-firm, market-to-book, momentum, and long-term reversal effects are currently among the most puzzling phenomena in empirical finance. There are several interpretations of these effects. First note that to some extent, some of these phenomena may be related. The feature that small firms, low-market-to-book firms, and recent "losers" seem to have in common is a stock price that has fallen considerably in recent months or years. Indeed, a firm can become a small firm or a low-market-to-book firm by suffering a sharp drop in price. These groups therefore may contain a relatively high proportion of distressed firms that have suffered recent difficulties.

[8]For interesting discussions of this possibility, see Peter Garber, *Famous First Bubbles: The Fundamentals of Early Manias* (Cambridge: MIT Press, 2000), and Anne Goldgar, *Tulipmania: Money, Honor, and Knowledge in the Dutch Golden Age* (Chicago: University of Chicago Press, 2007).

FIGURE 8.6

Return to style portfolio as a predictor of GDP growth. Average difference in the return on the style portfolio in years before good GDP growth versus in years before bad GDP growth. Positive value means the style portfolio does better in years prior to good macroeconomic performance. HML = high minus low portfolio, sorted on ratio of book-to-market value. SMB = small minus big portfolio, sorted on firm size.

Source: Reprinted from J. Liew and M. Vassalou, "Can Book-to-Market, Size, and Momentum Be Risk Factors That Predict Economic Growth?" *Journal of Financial Economics* 57 (2000), pp. 221–245. Copyright 2000 with permission from Elsevier Science.



Fama and French (1993) argue that these effects can be explained as manifestations of risk premiums. Using their three-factor model, they show that stocks with higher "betas" (also known as factor loadings) on size or market-to-book factors have higher average returns; they interpret these returns as evidence of a risk premium associated with the factor. This model does a much better job than the one-factor CAPM in explaining security returns. While size or book-to-market ratios per se are obviously not risk factors, they perhaps might act as proxies for more fundamental determinants of risk. Fama and French argue that these patterns of returns may therefore be consistent with an efficient market in which expected returns are consistent with risk. In this regard, it is worth noting that returns to "style portfolios," for example, the return on portfolios constructed based on the ratio of book-to-market value (specifically, the Fama-French high minus low book-to-market portfolio) or firm size (the return on the small minus big firm portfolio) do indeed seem to predict business cycles in many countries. Figure 8.6 shows that returns on these portfolios tend to have positive returns in years prior to rapid growth in gross domestic product.

The opposite interpretation is offered by Lakonishok, Shleifer, and Vishny (1995), who argue that these phenomena are evidence of inefficient markets, more specifically, of systematic errors in the forecasts of stock analysts. They believe that analysts extrapolate past performance too far into the future and therefore overprice firms with recent good performance and underprice firms with recent poor performance. Ultimately, when market participants recognize their errors, prices reverse. This explanation is consistent with the reversal effect and also, to a degree, is consistent with the small-firm and book-to-market effects because firms with sharp price drops may tend to be small or have high book-to-market ratios.

If Lakonishok, Shleifer, and Vishney are correct, we ought to find that analysts systematically err when forecasting returns of recent "winner" versus "loser" firms. A study by La Porta (1996) is consistent with this pattern. He finds that shares of firms for which analysts predict low growth rates of earnings actually perform better than those with high expected earnings growth. Analysts seem overly pessimistic about firms with low growth prospects and overly optimistic about firms with high growth prospects. When these too-extreme expectations are "corrected," the low-expected-growth firms outperform high-expected-growth firms.

Anomalies or data mining? We have covered many of the so-called anomalies cited in the literature, but our list could go on and on. Some wonder whether these anomalies are really unexplained puzzles in financial markets or whether they instead are an artifact of data mining. After all, if one reruns the computer database of past returns over and over and examines stock returns along enough dimensions, simple chance will cause some criteria to *appear* to predict returns.

In this regard, it is noteworthy that some anomalies have not shown much staying power after being reported in the academic literature. For example, after the small-firm effect was published in the early 1980s, it promptly disappeared for much of the rest of the decade. Similarly, the book-to-market strategy, which commanded considerable attention in the early 1990s, was ineffective for the rest of that decade.

Still, even acknowledging the potential for data mining, a common thread seems to run through many of the anomalies we have considered, lending support to the notion that there is a real puzzle to explain. Value stocks—defined by low P/E ratio, high book-to-market ratio, or depressed prices relative to historic levels—seem to have provided higher average returns than "glamour" or growth stocks.

One way to address the problem of data mining is to find a data set that has not already been researched and see whether the relationship in question shows up in the new data. Such studies have revealed size, momentum, and book-to-market effects in other security markets around the world. While these phenomena may be a manifestation of a systematic risk premium, the precise nature of that risk is not fully understood.

8.4 MUTUAL FUND AND ANALYST PERFORMANCE

We have documented some of the apparent chinks in the armor of efficient market proponents. For investors, the issue of market efficiency boils down to whether skilled investors can make consistent abnormal trading profits. The best test is to look at the performance of market professionals to see if they can generate performance superior to that of a passive index fund that buys and holds the market. We will look at two facets of professional performance: that of stock market analysts who recommend investment positions and that of mutual fund managers who actually manage portfolios.

Stock Market Analysts

Stock market analysts historically have worked for brokerage firms, which presents an immediate problem in interpreting the value of their advice: Analysts have tended to be overwhelmingly positive in their assessment of the prospects of firms.[9] For example, Barber, Lehavy, McNichols, and Trueman (2001) find that on a scale of 1 (strong buy) to 5 (strong sell), the average recommendation for 5,628 covered firms in 1996 was 2.04. As a result, one cannot take positive recommendations (e.g., to buy) at face value. Instead, we must look at either the relative enthusiasm of analyst recommendations compared to those for other firms or at the change in consensus recommendations.

Womack (1996) focuses on changes in analysts' recommendations and finds that positive changes are associated with increased stock prices of about 5% and negative changes result in average price decreases of 11%. One might wonder whether these price changes reflect the market's recognition of analysts' superior information or insight about firms or, instead, simply result from new buy or sell pressure brought on by the recommendations themselves. Womack argues that price impact seems to be permanent and, therefore, consistent with the hypothesis that analysts do in fact reveal new information. Jegadeesh, Kim, Krische, and Lee (2004) also find that changes in consensus recommendations are associated with price changes, but that the *level* of consensus recommendations is an inconsistent predictor of future stock performance.

[9]This problem may be less severe in the future; as noted in Chapter 3, one recent reform intended to mitigate the conflict of interest in having brokerage firms that sell stocks also provide investment advice is to separate analyst coverage from the other activities of the firm.

Barber, Lehavy, McNichols, and Trueman (2001) focus on the level of consensus recommendations and show that firms with the most favorable recommendations outperform those with the least favorable recommendations. While their results seem impressive, the authors note that portfolio strategies based on analyst consensus recommendations would result in extremely heavy trading activity with associated costs that probably would wipe out the potential profits from the strategy.

In sum, the literature suggests some value is added by analysts but some ambiguity remains. Are superior returns following analyst upgrades due to revelation of new information or due to changes in investor demand in response to the changed outlook? Also, are these results exploitable by investors who necessarily incur trading costs?

Mutual Fund Managers

As we pointed out in Chapter 4, casual evidence does not support the claim that professionally managed portfolios can consistently beat the market. Figure 4.4 in that chapter demonstrated that between 1972 and 2010 the returns of a passive portfolio indexed to the Wilshire 5000 typically would have been better than those of the average equity fund. On the other hand, there was some (admittedly inconsistent) evidence of persistence in performance, meaning that the better managers in one period tended to be better managers in following periods. Such a pattern would suggest that the better managers can with some consistency outperform their competitors, and it would be inconsistent with the notion that market prices already reflect all relevant information.

The analyses cited in Chapter 4 were based on total returns; they did not properly adjust returns for exposure to systematic risk factors. In this section we revisit the question of mutual fund performance, paying more attention to the benchmark against which performance ought to be evaluated.

As a first pass, we can examine the risk-adjusted returns (i.e., the alpha, or return in excess of required return based on beta and the market return in each period) of a large sample of mutual funds. But the market index may not be an adequate benchmark against which to evaluate mutual fund returns. Because mutual funds tend to maintain considerable holdings in equity of small firms, whereas the S&P 500 exclusively comprises large firms, mutual funds as a whole will tend to outperform the S&P when small firms outperform large ones and underperform when small firms fare worse. Thus a better benchmark for the performance of funds would be an index that incorporates the stock market performance of smaller firms.

The importance of the benchmark can be illustrated by examining the returns on small stocks in various subperiods.[10] In the 20-year period between 1945 and 1964, for example, a small-stock index underperformed the S&P 500 by about 4% per year (i.e., the alpha of the small-stock index after adjusting for systematic risk was −4%). In the following 20-year period, between 1965 and 1984, small stocks outperformed the S&P 500 Index by 10%. Thus if one were to examine mutual fund returns in the earlier period, they would tend to look poor, not necessarily because fund managers were poor stock pickers but simply because mutual funds as a group tended to hold more small stocks than were represented in the S&P 500. In the later period, funds would look better on a risk-adjusted basis relative to the S&P 500 because small stocks performed better. The "style choice," that is, the exposure to small stocks (which is an asset allocation decision) would dominate the evaluation of performance even though it has little to do with managers' stock-picking ability.[11]

The conventional performance benchmark today is a four-factor model, which employs the three Fama-French factors (the return on the market index, and returns to portfolios based on size and book-to-market ratio) augmented by a momentum factor (a portfolio constructed based on prior-year stock return). Alphas constructed using an expanded index

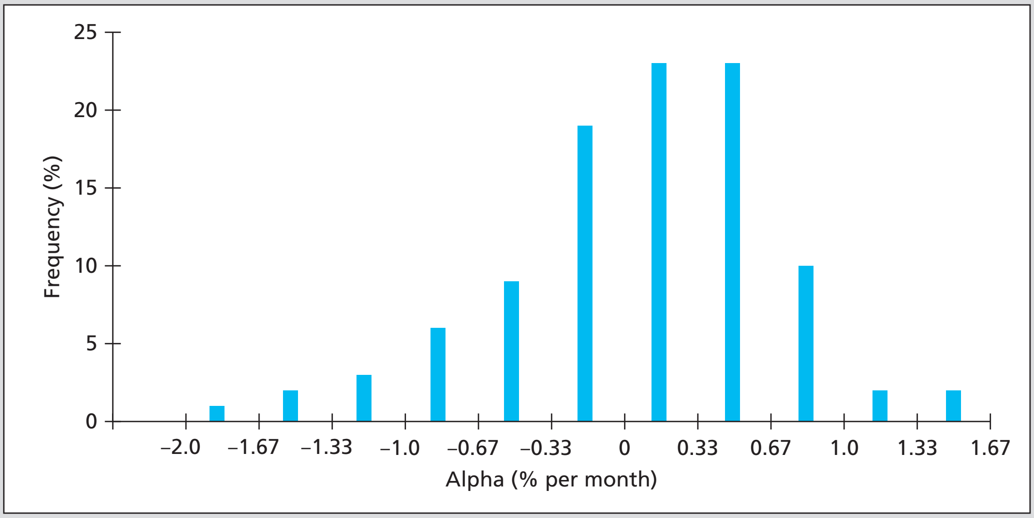
[10] This illustration and the statistics cited are based on E. J. Elton, M. J. Gruber, S. Das, and M. Hlavka, "Efficiency with Costly Information: A Reinterpretation of Evidence from Managed Portfolios," *Review of Financial Studies* 6 (1993), pp. 1–22.

[11] Remember that the asset allocation decision is usually in the hands of the individual investor. Investors allocate their investment portfolios to funds in asset classes they desire to hold, and they can reasonably expect only that mutual fund portfolio managers will choose stocks advantageously *within* those asset classes.

FIGURE 8.7

Mutual fund alphas computed using a four-factor model of expected return, 1993–2007. (The best and worst 2.5% of observations are excluded from this distribution.)

Source: Professor Richard Evans, University of Virginia, Darden School of Business. Used with permission.



model using these four factors control for a wide range of mutual fund–style choices that may affect average returns, for example, an inclination to growth versus value or small-versus large-capitalization stocks. Figure 8.7 shows a frequency distribution of four-factor alphas for U.S. domestic equity funds.[12] The results show that the distribution of alpha is roughly bell-shaped, with a slightly negative mean. On average, it does not appear that these funds outperform their style-adjusted benchmarks.

Consistent with Figure 8.7, Fama and French (2010) use the four-factor model to assess the performance of equity mutual funds and show that while they may exhibit positive alphas *before* fees, after the fees charged to their customers, alphas were negative. Likewise, Wermers (2000), who uses both style portfolios as well as the characteristics of the stocks held by mutual funds to control for performance, also finds positive gross alphas but negative net alphas after controlling for fees and risk.

Carhart (1997) reexamines the issue of consistency in mutual fund performance to see whether better performers in one period continue to outperform in later periods. He uses the four-factor extension described above and finds that after controlling for these factors, there is only minor persistence in relative performance across managers. Moreover, much of that persistence seems due to expenses and transactions costs rather than gross investment returns.

Even allowing for expenses and turnover, some amount of performance persistence seems to be due to differences in investment strategy. Carhart finds, however, that the evidence of persistence is concentrated at the two extremes. Figure 8.8, from his study, documents performance persistence. Equity funds are ranked into 1 of 10 groups by performance in the formation year, and the performance of each group in the following years is plotted. It is clear that except for the best-performing top-decile group and the worst-performing 10th-decile group, performance in future periods is almost independent of earlier-year returns. Carhart's results suggest that there may be a small group of exceptional managers who can with some consistency outperform a passive strategy, but that for the majority of managers over- or underperformance in any period is largely a matter of chance.

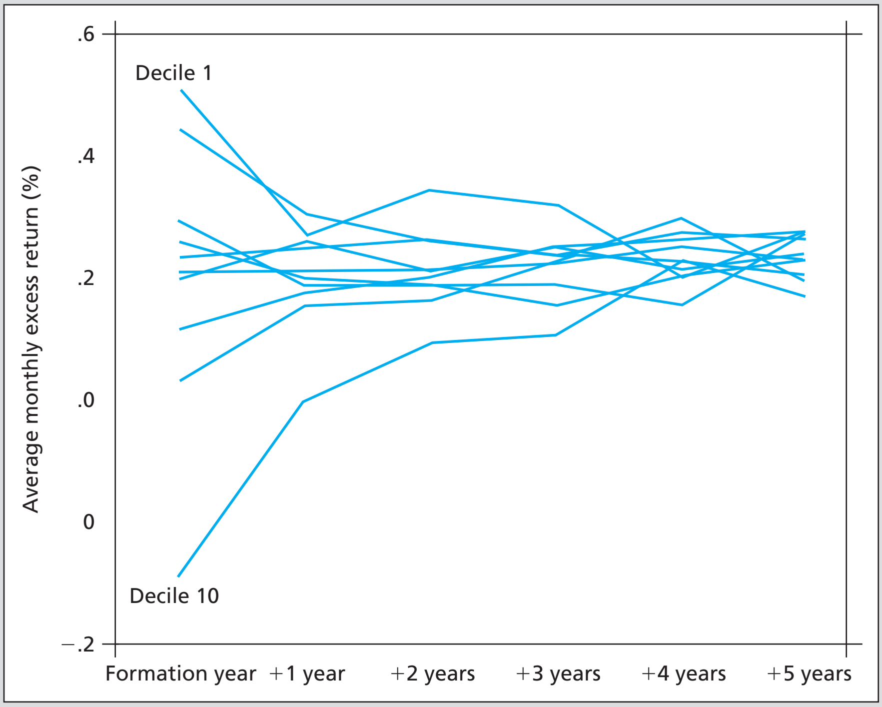
Bollen and Busse (2004) find more evidence of performance persistence, at least over short horizons. They rank mutual fund performance using the four-factor model over a base

[12]We are grateful to Professor Richard Evans for this data.

FIGURE 8.8

Persistence of mutual fund performance. Performance over time of mutual fund groups ranked by initial-year performance

Source: Mark M. Carhart, "On Persistence in Mutual Fund Performance," *Journal of Finance* 52 (March 1997), pp. 57–82. Used with permission of John Wiley and Sons, via Copyright Clearance Center.



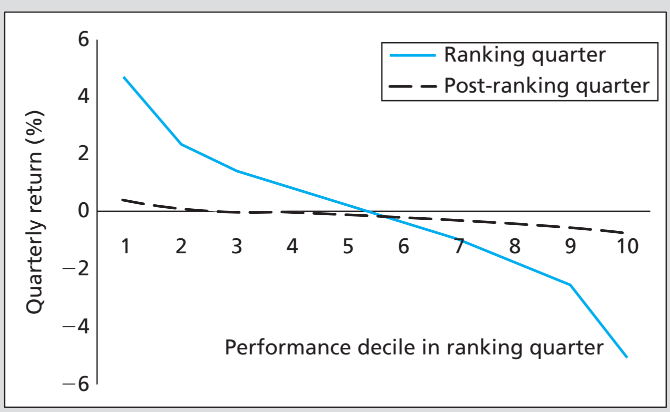
quarter, assign funds into one of 10 deciles according to base-period alpha, and then look at performance in the following quarter. Figure 8.9 illustrates their results. The solid line is the average alpha of funds within each of the deciles in the base period (expressed on a quarterly basis). The steepness of that line reflects the considerable dispersion in performance in the ranking period. The dashed line is the average performance of the funds in each decile in the following quarter. The shallowness of this line indicates that most of the original performance differential disappears. Nevertheless, the plot is still clearly downward-sloping, so it appears that at least over a short horizon such as one quarter, there is some performance consistency. However, that persistence is probably too small a fraction of the original performance differential to justify performance chasing by mutual fund customers.

This pattern is actually consistent with the prediction of an influential paper by Berk and Green (2004). They argue that skilled mutual fund managers with abnormal performance will attract new funds until the additional costs and complexity of managing those extra funds

FIGURE 8.9

Risk-adjusted performance in ranking quarter and following quarter

Source: Nicolas P. B. Bollen and Jeffrey A. Busse, "Short-Term Persistence in Mutual Fund Performance," *Review of Financial Studies* 19 (2004), pp. 569–597, by permission of Oxford University Press.



drive alphas down to zero. Thus, skill will show up not in superior returns but rather in the amount of funds under management. Therefore, even if managers are skilled, alphas will be short-lived, as they seem to be in Figure 8.9.

In contrast to the extensive studies of equity fund managers, there have been few studies of the performance of bond fund managers. Blake, Elton, and Gruber (1993) examined the performance of fixed-income mutual funds. They found that, on average, bond funds underperform passive fixed-income indexes by an amount roughly equal to expenses and that there is no evidence that past performance can predict future performance. More recently, Chen, Ferson, and Peters (2010) find that, on average, bond mutual funds outperform passive bond indexes in terms of gross returns but underperform once the fees they charge their investors are subtracted, a result similar to those others have found for equity funds.

Thus the evidence on the risk-adjusted performance of professional managers is mixed at best. We conclude that the performance of professional managers is broadly consistent with market efficiency. The amounts by which professional managers as a group beat or are beaten by the market fall within the margin of statistical uncertainty. In any event, it is quite clear that performance superior to passive strategies is far from routine. Studies show either that most managers cannot outperform passive strategies or that if there is a margin of superiority, it is small.

On the other hand, a small number of investment superstars—Peter Lynch (formerly of Fidelity's Magellan Fund), Warren Buffett (of Berkshire Hathaway), John Templeton (of Templeton Funds), and Mario Gabelli (of GAMCO), among them—have compiled career records that show a consistency of superior performance hard to reconcile with absolutely efficient markets. In a careful statistical analysis of mutual fund "stars," Kosowski, Timmerman, Wermers, and White (2006) conclude that the stock-picking ability of a minority of managers is sufficient to cover their costs and that their superior performance tends to persist over time. However, Nobel Prize–winner Paul Samuelson (1989) points out that the records of the vast majority of professional money managers offer convincing evidence that there are no easy strategies to guarantee success in the securities markets.

So, Are Markets Efficient?

There is a telling joke about two economists walking down the street. They spot a $20 bill on the sidewalk. One starts to pick it up, but the other one says, "Don't bother; if the bill were real someone would have picked it up already."

The lesson is clear. An overly doctrinaire belief in efficient markets can paralyze the investor and make it appear that no research effort can be justified. This extreme view is probably unwarranted. There are enough anomalies in the empirical evidence to justify the search for underpriced securities that clearly goes on.

The bulk of the evidence, however, suggests that any supposedly superior investment strategy should be taken with many grains of salt. The market is competitive *enough* that only differentially superior information or insight will earn money; the easy pickings have been picked. In the end it is likely that the margin of superiority that any professional manager can add is so slight that the statistician will not easily be able to detect it.

We conclude that markets are very efficient, but that rewards to the especially diligent, intelligent, or creative may in fact be waiting.

SUMMARY

- Statistical research has shown that to a close approximation stock prices seem to follow a random walk with no discernible predictable patterns that investors can exploit. Such findings are now taken to be evidence of market efficiency, that is, evidence that market prices reflect all currently available information. Only new information will move stock prices, and this information is equally likely to be good news or bad news.
- Market participants distinguish among three forms of the efficient market hypothesis. The weak form asserts that all information to be derived from past trading data already is reflected in stock prices. The semistrong form claims that all publicly available information is already reflected. The strong form, which generally is acknowledged to be extreme, asserts that all information, including insider information, is reflected in prices.

- Technical analysis focuses on stock price patterns and on proxies for buy or sell pressure in the market. Fundamental analysis focuses on the determinants of the underlying value of the firm, such as current profitability and growth prospects. Because both types of analysis are based on public information, neither should generate excess profits if markets are operating efficiently.
- Proponents of the efficient market hypothesis often advocate passive as opposed to active investment strategies. The policy of passive investors is to buy and hold a broad-based market index. They expend resources neither on market research nor on frequent purchase and sale of stocks. Passive strategies may be tailored to meet individual investor requirements.
- Empirical studies of technical analysis do not generally support the hypothesis that such analysis can generate superior trading profits. One notable exception to this conclusion is the apparent success of momentum-based strategies over intermediate-term horizons.
- Several anomalies regarding fundamental analysis have been uncovered. These include the P/E effect, the small-firm-in-January effect, the neglected-firm effect, post-earnings-announcement price drift, and the book-to-market effect. Whether these anomalies represent market inefficiency or poorly understood risk premiums is still a matter of debate.
- By and large, the performance record of professionally managed funds lends little credence to claims that most professionals can consistently beat the market.

KEY TERMS

anomalies, 247
book-to-market effect, 248
efficient market hypothesis, 235
fundamental analysis, 240
index fund, 241
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PROBLEM SETS

Select problems are available in McGraw-Hill's *Connect Finance.* Please see the Supplements section of the book's frontmatter for more information.

Basic

1. If markets are efficient, what should be the correlation coefficient between stock returns for two nonoverlapping time periods? *(LO 8-1)*
2. "If all securities are fairly priced, all must offer equal expected rates of return." Comment. *(LO 8-1)*
3. If prices are as likely to increase as decrease, why do investors earn positive returns from the market on average? *(LO 8-1)*
4. A successful firm like Microsoft has consistently generated large profits for years. Is this a violation of the EMH? *(LO 8-2)*
5. At a cocktail party, your co-worker tells you that he has beaten the market for each of the last three years. Suppose you believe him. Does this shake your belief in efficient markets? *(LO 8-2)*
6. Which of the following statements are *true* if the efficient market hypothesis holds? *(LO 8-1)*
   a. It implies that future events can be forecast with perfect accuracy.
   b. It implies that prices reflect all available information.
   c. It implies that security prices change for no discernible reason.
   d. It implies that prices do not fluctuate.
7. In an efficient market, professional portfolio management can offer all of the following benefits *except* which of the following? *(LO 8-4)*
   a. Low-cost diversification.
   b. A targeted risk level.

c. Low-cost record keeping.
d. A superior risk-return trade-off.

8. Which version of the efficient market hypothesis (weak, semistrong, or strong-form) focuses on the most inclusive set of information? *(LO 8-1)*
9. "Highly variable stock prices suggest that the market does not know how to price stocks." Respond. *(LO 8-1)*
10. Which of the following sources of market inefficiency would be most easily exploited? *(LO 8-4)*
    a. A stock price drops suddenly due to a large block sale by an institution.
    b. A stock is overpriced because traders are restricted from short sales.
    c. Stocks are overvalued because investors are exuberant over increased productivity in the economy.

KAPLAN SCHWESER

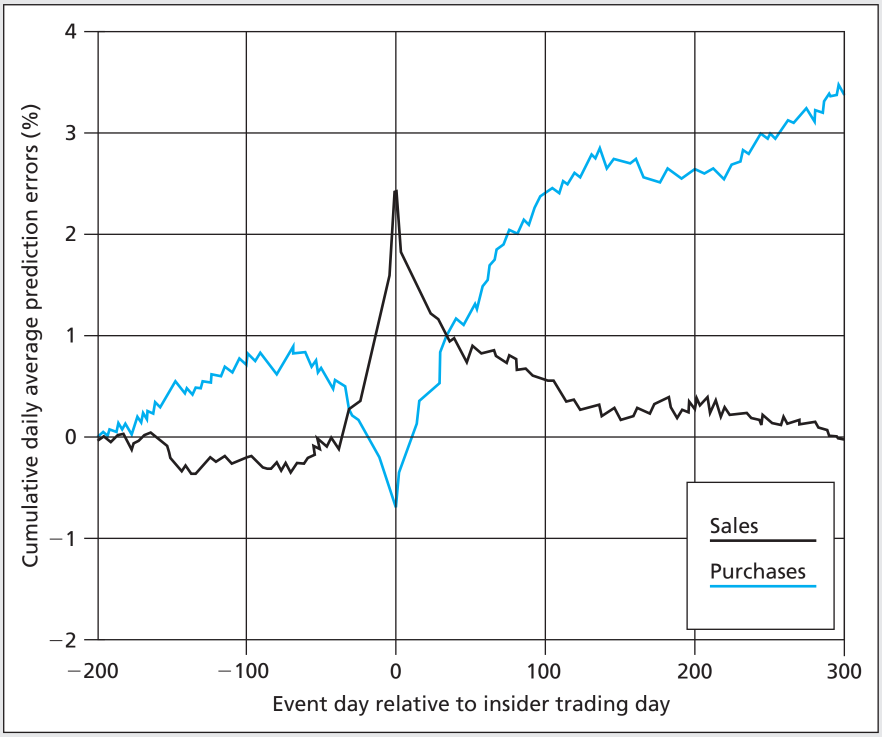
Intermediate

11. Which of the following most appears to contradict the proposition that the stock market is *weakly* efficient? Explain. *(LO 8-3)*
    a. Over 25% of mutual funds outperform the market on average.
    b. Insiders earn abnormal trading profits.
    c. Every January, the stock market earns abnormal returns.
12. Suppose that, after conducting an analysis of past stock prices, you come up with the following observations. Which would appear to *contradict* the *weak form* of the efficient market hypothesis? Explain. *(LO 8-3)*
    a. The average rate of return is significantly greater than zero.
    b. The correlation between the return during a given week and the return during the following week is zero.
    c. One could have made superior returns by buying stock after a 10% rise in price and selling after a 10% fall.
    d. One could have made higher-than-average capital gains by holding stocks with low dividend yields.
13. Which of the following observations would provide evidence *against* the *semistrong form* of the efficient market theory? Explain. *(LO 8-3)*
    a. Mutual fund managers do not on average make superior returns.
    b. You cannot make superior profits by buying (or selling) stocks after the announcement of an abnormal rise in dividends.
    c. Low P/E stocks tend to have positive abnormal returns.
    d. In any year approximately 50% of pension funds outperform the market.
14. Steady Growth Industries has never missed a dividend payment in its 94-year history. Does this make it more attractive to you as a possible purchase for your stock portfolio? *(LO 8-4)*
15. Suppose you find that prices of stocks before large dividend increases show on average consistently positive abnormal returns. Is this a violation of the EMH? *(LO 8-3)*
16. "If the business cycle is predictable, and a stock has a positive beta, the stock's returns also must be predictable." Respond. *(LO 8-1)*
17. Which of the following phenomena would be either consistent with or a violation of the efficient market hypothesis? Explain briefly. *(LO 8-3)*
    a. Nearly half of all professionally managed mutual funds are able to outperform the S&P 500 in a typical year.
    b. Money managers that outperform the market (on a risk-adjusted basis) in one year are likely to outperform in the following year.
    c. Stock prices tend to be predictably more volatile in January than in other months.
    d. Stock prices of companies that announce increased earnings in January tend to outperform the market in February.
    e. Stocks that perform well in one week perform poorly in the following week.

18. Why are the following "effects" considered efficient market anomalies? Are there rational explanations for these effects? **(LO 8-2)**
    a. P/E effect
    b. Book-to-market effect
    c. Momentum effect
    d. Small-firm effect
19. Dollar-cost averaging means that you buy equal dollar amounts of a stock every period, for example, $500 per month. The strategy is based on the idea that when the stock price is low, your fixed monthly purchase will buy more shares, and when the price is high, fewer shares. Averaging over time, you will end up buying more shares when the stock is cheaper and fewer when it is relatively expensive. Therefore, by design, you will exhibit good market timing. Evaluate this strategy. **(LO 8-4)**
20. We know that the market should respond positively to good news and that good-news events such as the coming end of a recession can be predicted with at least some accuracy. Why, then, can we not predict that the market will go up as the economy recovers? **(LO 8-1)**
21. You know that firm XYZ is very poorly run. On a scale of 1 (worst) to 10 (best), you would give it a score of 3. The market consensus evaluation is that the management score is only 2. Should you buy or sell the stock? **(LO 8-4)**
22. Good News, Inc., just announced an increase in its annual earnings, yet its stock price fell. Is there a rational explanation for this phenomenon? **(LO 8-1)**
23. Shares of small firms with thinly traded stocks tend to show positive CAPM alphas. Is this a violation of the efficient market hypothesis? **(LO 8-3)**

Challenge

24. Examine the accompanying figure, which presents cumulative abnormal returns both before and after dates on which insiders buy or sell shares in their firms. How do you interpret this figure? What are we to make of the pattern of CARs before and after the event date? **(LO 8-3)**



Source: Reprinted from Nejat H. Seyhun, "Insiders, Profits, Costs of Trading and Market Efficiency," *Journal of Financial Economics* 16 pp. 189–212, copyright June 1986, with permission from Elsevier.

25. Suppose that as the economy moves through a business cycle, risk premiums also change. For example, in a recession when people are concerned about their jobs, risk tolerance might be lower and risk premiums might be higher. In a booming economy, tolerance for risk might be higher and risk premiums lower. *(LO 8-3)*
    a. Would a predictably shifting risk premium such as described here be a violation of the efficient market hypothesis?
    b. How might a cycle of increasing and decreasing risk premiums create an appearance that stock prices "overreact," first falling excessively and then seeming to recover?

CFA Problems

CFA® PROBLEMS

1. The semistrong form of the efficient market hypothesis asserts that stock prices: *(LO 8-1)*
    a. Fully reflect all historical price information.
    b. Fully reflect all publicly available information.
    c. Fully reflect all relevant information including insider information.
    d. May be predictable.
2. Assume that a company announces an unexpectedly large cash dividend to its shareholders. In an efficient market *without* information leakage, one might expect: *(LO 8-1)*
    a. An abnormal price change at the announcement.
    b. An abnormal price increase before the announcement.
    c. An abnormal price decrease after the announcement.
    d. No abnormal price change before or after the announcement.
3. Which one of the following would provide evidence *against* the *semistrong form* of the efficient market theory? *(LO 8-3)*
    a. About 50% of pension funds outperform the market in any year.
    b. You cannot make abnormal profits by buying stocks after an announcement of strong earnings.
    c. Trend analysis is worthless in forecasting stock prices.
    d. Low P/E stocks tend to have positive abnormal returns over the long run.
4. According to the efficient market hypothesis: *(LO 8-3)*
    a. High-beta stocks are consistently overpriced.
    b. Low-beta stocks are consistently overpriced.
    c. Positive alphas on stocks will quickly disappear.
    d. Negative-alpha stocks consistently yield low returns for arbitrageurs.
5. A "random walk" occurs when: *(LO 8-1)*
    a. Stock price changes are random but predictable.
    b. Stock prices respond slowly to both new and old information.
    c. Future price changes are uncorrelated with past price changes.
    d. Past information is useful in predicting future prices.
6. A market anomaly refers to: *(LO 8-3)*
    a. An exogenous shock to the market that is sharp but not persistent.
    b. A price or volume event that is inconsistent with historical price or volume trends.
    c. A trading or pricing structure that interferes with efficient buying and selling of securities.
    d. Price behavior that differs from the behavior predicted by the efficient market hypothesis.
7. Some scholars contend that professional managers are incapable of outperforming the market. Others come to an opposite conclusion. Compare and contrast the assumptions about the stock market that support (*a*) passive portfolio management and (*b*) active portfolio management. *(LO 8-2)*

8. You are a portfolio manager meeting a client. During the conversation that follows your formal review of her account, your client asks the following question: *(LO 8-2)*

   My grandson, who is studying investments, tells me that one of the best ways to make money in the stock market is to buy the stocks of small-capitalization firms late in December and to sell the stocks one month later. What is he talking about?

   a. Identify the apparent market anomalies that would justify the proposed strategy.
   b. Explain why you believe such a strategy might or might not work in the future.
9. *a.* Briefly explain the concept of the efficient market hypothesis (EMH) and each of its three forms—weak, semistrong, and strong—and briefly discuss the degree to which existing empirical evidence supports each of the three forms of the EMH. *(LO 8-2)*
   b. Briefly discuss the implications of the efficient market hypothesis for investment policy as it applies to: *(LO 8-4)*
      i. Technical analysis in the form of charting.
      ii. Fundamental analysis.
   c. Briefly explain the roles or responsibilities of portfolio managers in an efficient market environment. *(LO 8-4)*
10. Growth and value can be defined in several ways. *Growth* usually conveys the idea of a portfolio emphasizing or including only companies believed to possess above-average future rates of per-share earnings growth. Low current yield, high price-to-book ratios, and high price-to-earnings ratios are typical characteristics of such portfolios. *Value* usually conveys the idea of portfolios emphasizing or including only issues currently showing low price-to-book ratios, low price-to-earnings ratios, above-average levels of dividend yield, and market prices believed to be below the issues' intrinsic values. *(LO 8-3)*
   a. Identify and provide reasons why, over an extended period of time, value-stock investing might outperform growth-stock investing.
   b. Explain why the outcome suggested in (*a*) should not be possible in a market widely regarded as being highly efficient.
11. Your investment client asks for information concerning the benefits of active portfolio management. She is particularly interested in the question of whether active managers can be expected to consistently exploit inefficiencies in the capital markets to produce above-average returns without assuming higher risk.

    The semistrong form of the efficient market hypothesis asserts that all publicly available information is rapidly and correctly reflected in securities prices. This implies that investors cannot expect to derive above-average profits from purchases made after information has become public because security prices already reflect the information's full effects. *(LO 8-2)*
   a. Identify and explain two examples of empirical evidence that tend to support the EMH implication stated above.
   b. Identify and explain two examples of empirical evidence that tend to refute the EMH implication stated above.
   c. Discuss reasons why an investor might choose not to index even if the markets were, in fact, semistrong-form efficient.

WEB master

1. Use data from **finance.yahoo.com** to answer the following questions.
   a. Collect the following data for 25 firms of your choosing.
      i. Book-to-market ratio.
      ii. Price–earnings ratio.
      iii. Market capitalization (size).
      iv. Price–cash flow ratio (i.e, market capitalization/operating cash flow).
      v. Another criterion that interests you.

You can find this information by choosing a company and then clicking on *Key Statistics.* Rank the firms based on each of the criteria separately, and divide the firms into five groups based on their ranking for each criterion. Calculate the average rate of return for each group of firms.

Do you confirm or reject any of the anomalies cited in this chapter? Can you uncover a new anomaly? Note: For your test to be valid, you must form your portfolios based on criteria observed at the *beginning* of the period when you form the stock groups. Why?

b. Use the price history from the *Historical Prices* tab to calculate the beta of each of the firms in part (*a*). Use this beta, the T-bill rate, and the return on the S&P 500 to calculate the risk-adjusted abnormal return of each stock group. Does any anomaly uncovered in the previous question persist after controlling for risk?

c. Now form stock groups that use two criteria simultaneously. For example, form a portfolio of stocks that are both in the lowest quintile of price–earnings ratio and in the highest quintile of book-to-market ratio. Does selecting stocks based on more than one characteristic improve your ability to devise portfolios with abnormal returns? Repeat the analysis by forming groups that meet three criteria simultaneously. Does this yield any further improvement in abnormal returns?

2. Several websites list information on earnings surprises. Much of the information supplied is from Zacks.com. Each day the largest positive and negative surprises are listed. Go to **www.zacks.com/research/earnings/today_eps.php** and identify the top positive and the top negative earnings surprises for the day. The table will list the time and date of the announcement.

a. Do you notice any difference between the times of day that positive announcements tend to be made versus negative announcements?

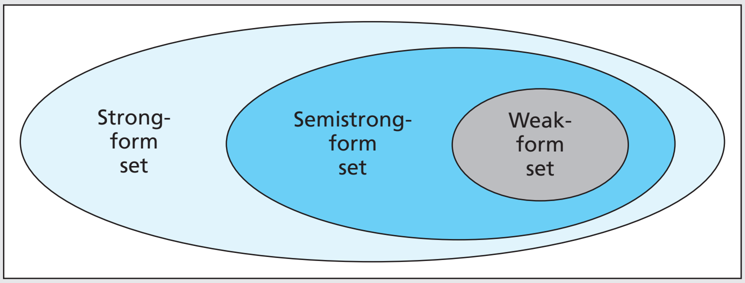
b. Identify the tickers for the top three positive surprises. Once you have identified the top surprises, go to **finance.yahoo.com.** Enter the ticker symbols and obtain quotes for these securities. Examine the five-day charts for each of the companies. Is the information incorporated into price quickly? Is there any evidence of prior knowledge or anticipation of the disclosure in advance of the trading?

c. Choose one of the stocks listed and click on its symbol to follow the link for more information. Click on the link for *Interactive Java Charting* that appears under the graph. In the *Graph Control* dialog box choose a period of five years and select the box that says "EPS Surprise." The resulting chart will show positive earnings surprises as green bars and negative surprises as red bars. You can move the cursor over various parts of the graph to investigate what happened to the price and trading volume of the stock around each of the surprise events. Do you notice any patterns?

SOLUTIONS TO CONCEPT checks

8.1 *a.* A high-level manager might well have private information about the firm. Her ability to trade profitably on that information is not surprising. This ability does not violate weak-form efficiency: The abnormal profits are not derived from an analysis of past price and trading data. If they were, this would indicate that there is valuable information that can be gleaned from such analysis. But this ability does violate strong-form efficiency. Apparently, there is some private information that is not already reflected in stock prices.

b. The information sets that pertain to the weak, semistrong, and strong form of the EMH can be described by the following illustration:



The weak-form information set includes only the history of prices and volumes. The semistrong-form set includes the weak form set *plus* all other publicly available information. In turn, the strong-form set includes the semistrong set *plus* insiders' information. It is illegal to act on this incremental information (insiders' private information). The direction of *valid* implication is

$$\text{Strong-form EMH} \Rightarrow \text{Semistrong-form EMH} \Rightarrow \text{Weak-form EMH}$$

The reverse direction implication is *not* valid. For example, stock prices may reflect all past price data (weak-form efficiency) but may not reflect relevant fundamental data (semistrong-form inefficiency).

8.2 The point we made in the preceding discussion is that the very fact that we observe stock prices near so-called resistance levels belies the assumption that the price can be a resistance level. If a stock is observed to sell *at any price,* then investors must believe that a fair rate of return can be earned if the stock is purchased at that price. It is logically impossible for a stock to have a resistance level *and* offer a fair rate of return at prices just below the resistance level. If we accept that prices are appropriate, we must reject any presumption concerning resistance levels.

8.3 If *everyone* follows a passive strategy, sooner or later prices will fail to reflect new information. At this point there are profit opportunities for active investors who uncover mispriced securities. As they buy and sell these assets, prices again will be driven to fair levels.

8.4 The answer depends on your prior beliefs about market efficiency. Miller's initial record was incredibly strong. On the other hand, with so many funds in existence, it is less surprising that *some* fund would appear to be consistently superior after the fact. Exceptional past performance of a small number of managers is possible by chance even in an efficient market. A better test is provided in "continuation studies." Are better performers in one period more likely to repeat that performance in later periods? Miller's record in the last three years fails the continuation or consistency criterion.

Behavioral Finance and Technical Analysis

Chapter 9

Learning Objectives:

LO9-1 Describe several behavioral biases, and explain how they could lead to anomalies in stock market prices and returns.

LO9-2 Explain why limits to arbitrage might allow anomalies due to behavioral biases to persist over time.

LO9-3 Identify reasons why technical analysis may be profitable.

LO9-4 Use indicators such as volume, put/call ratios, breadth, short interest, or confidence indexes to measure the "technical conditions" of the market.

The efficient market hypothesis makes two important predictions. First, it implies that security prices properly reflect whatever information is available to investors. A second implication follows immediately: Active traders will find it difficult to outperform passive strategies such as holding market indexes. To do so would require differential insight; this in a highly competitive market is very hard to come by.

Unfortunately, it is hard to devise measures of the "true" or intrinsic value of a security, and correspondingly difficult to test directly whether prices match those values. Therefore, most tests of market efficiency have focused on the performance of active trading strategies. These tests have been of two kinds. The anomalies literature has examined strategies that apparently *would* have provided superior risk-adjusted returns (e.g., investing in stocks with momentum or in value rather than glamour stocks). Other tests have looked at the results of *actual* investments by asking whether professional managers have been able to beat the market.

Neither class of tests has proven fully conclusive. The anomalies literature suggests that several strategies would have provided superior returns. But there are questions as to whether some of these apparent anomalies reflect risk premiums not captured by simple models of risk and return, or even if they merely reflect data mining. Moreover, the apparent inability of the typical money manager to turn these anomalies into superior returns on actual portfolios casts additional doubt on their "reality."

A relatively new school of thought dubbed *behavioral finance* argues that the sprawling literature on trading strategies has missed a larger and more important point by overlooking the first implication of efficient markets—the correctness of security prices. This may be the more important implication, since market economies rely on prices to allocate resources efficiently. The behavioral school argues that even if security prices are wrong, it still can be difficult to exploit them, and, therefore, that the failure to uncover obviously successful trading rules or traders cannot be taken as proof of market efficiency.

Whereas conventional theories presume that investors are rational, behavioral finance starts with the assumption that they are not. We will examine some of the information-processing and behavioral irrationalities uncovered by psychologists in other contexts and show how these tendencies applied to financial markets might result in some of the anomalies discussed in the previous chapter. We then consider the limitations of strategies designed to take advantage of behaviorally induced mispricing. If the limits to such arbitrage activity are severe, mispricing can survive even if some rational investors attempt to exploit it. We turn next to technical analysis and show how behavioral models give some support to techniques that clearly would be useless in efficient markets. We close the chapter with a brief survey of some of these technical strategies.

Related websites for this chapter are available at www.mhhe.com/bkm.

9.1 THE BEHAVIORAL CRITIQUE

behavioral finance

Models of financial markets that emphasize potential implications of psychological factors affecting investor behavior.

The premise of **behavioral finance** is that conventional financial theory ignores how real people make decisions and that people make a difference.[1] A growing number of economists have come to interpret the anomalies literature as consistent with several "irrationalities" that seem to characterize individuals making complicated decisions. These irrationalities fall into two broad categories: first, that investors do not always process information correctly and therefore infer incorrect probability distributions about future rates of return; and second, that even given a probability distribution of returns, they often make inconsistent or systematically suboptimal decisions.

Of course, the existence of irrational investors would not by itself be sufficient to render capital markets inefficient. If such irrationalities did affect prices, then sharp-eyed arbitrageurs taking advantage of profit opportunities might be expected to push prices back to their proper values. Thus, the second leg of the behavioral critique is that in practice the actions of such arbitrageurs are limited and therefore insufficient to force prices to match intrinsic value.

This leg of the argument is important. Virtually everyone agrees that if prices are right (i.e., price = intrinsic value), then there are no easy profit opportunities. But the converse is not necessarily true. If behaviorists are correct about limits to arbitrage activity, then the absence of profit opportunities does not necessarily imply that markets are efficient. We've noted that most tests of the efficient market hypothesis have focused on the existence of profit opportunities, often as reflected in the performance of money managers. But their failure to systematically outperform passive investment strategies need not imply that markets are in fact efficient.

We will start our summary of the behavioral critique with the first leg of the argument, surveying a sample of the informational processing errors uncovered by psychologists in

[1]The discussion in this section is based on an excellent survey article: Nicholas Barberis and Richard Thaler, "A Survey of Behavioral Finance," in the *Handbook of the Economics of Finance,* eds. G. M. Constantinides, M. Harris, and R. Stulz (Amsterdam: Elsevier, 2003).

other areas. We next examine a few of the behavioral irrationalities that seem to characterize decision makers. Finally, we look at limits to arbitrage activity and conclude with a tentative assessment of the import of the behavioral debate.

Information Processing

Errors in information processing can lead investors to misestimate the true probabilities of possible events or associated rates of return. Several such biases have been uncovered. Here are four of the more important ones.

Forecasting errors A series of experiments by Kahneman and Tversky (1972, 1973) indicates that people give too much weight to recent experience compared to prior beliefs when making forecasts (sometimes dubbed a *memory bias*) and tend to make forecasts that are too extreme given the uncertainty inherent in their information. De Bondt and Thaler (1990) argue that the P/E effect can be explained by earnings expectations that are too extreme. In this view, when forecasts of a firm's future earnings are high, perhaps due to favorable recent performance, they tend to be *too* high relative to the objective prospects of the firm. This results in a high initial P/E (due to the optimism built into the stock price) and poor subsequent performance when investors recognize their error. Thus, high P/E firms tend to be poor investments.

Overconfidence People tend to overestimate the precision of their beliefs or forecasts, and they tend to overestimate their abilities. In one famous survey, 90% of drivers in Sweden ranked themselves as better-than-average drivers. Such overconfidence may be responsible for the prevalence of active versus passive investment management—itself an anomaly to adherents of the efficient market hypothesis. Despite the growing popularity of indexing, only about 15% of the equity in the mutual fund industry is held in indexed accounts. The dominance of active management in the face of the typical underperformance of such strategies (consider the generally disappointing performance of actively managed mutual funds reviewed in Chapter 4 as well as in the previous chapter) is consistent with a tendency to overestimate ability.

An interesting example of overconfidence in financial markets is provided by Barber and Odean (2001), who compare trading activity and average returns in brokerage accounts of men and women. They find that men (in particular, single men) trade far more actively than women, consistent with the generally greater overconfidence among men well-documented in the psychology literature. They also find that trading activity is highly predictive of poor investment performance. The top 20% of accounts ranked by portfolio turnover had average returns seven percentage points lower than the 20% of the accounts with the lowest turnover rates. As they conclude, "Trading [and by implication, overconfidence] is hazardous to your wealth."

Overconfidence appears to be a widespread phenomenon, also showing up in many corporate finance contexts. For example, overconfident CEOs are more likely to overpay for target firms when making corporate acquisitions (Malmedier and Tate, 2008). Just as overconfidence can degrade portfolio investments, it also can lead such firms to make poor investments in real assets.

Conservatism A **conservatism bias** means that investors are too slow (too conservative) in updating their beliefs in response to new evidence. This means that they might initially underreact to news about a firm, so that prices will fully reflect new information only gradually. Such a bias would give rise to momentum in stock market returns.

conservatism bias

Investors are too slow (too conservative) in updating their beliefs in response to recent evidence.

Sample-size neglect and representativeness The notion of **representativeness bias** holds that people commonly do not take into account the size of a sample, acting as if a small sample is just as representative of a population as a large one. They may therefore infer a pattern too quickly based on a small sample and extrapolate apparent trends too far into the future. It is easy to see how such a pattern would be consistent with overreaction and correction anomalies. A short-lived run of good earnings reports or high stock returns would lead such investors to revise their assessments of likely future performance and

representativeness bias

People are too prone to believe that a small sample is representative of a broad population and infer patterns too quickly.

thus generate buying pressure that exaggerates the price run-up. Eventually, the gap between price and intrinsic value becomes glaring and the market corrects its initial error. Interestingly, stocks with the best recent performance suffer reversals precisely in the few days surrounding earnings announcements, suggesting that the correction occurs just as investors learn that their initial beliefs were too extreme (Chopra, Lakonishok, and Ritter, 1992).

CONCEPT check 9.1

We saw in the previous chapter that stocks seem to exhibit a pattern of short- to middle-term momentum, along with long-term reversals. How might this pattern arise from an interplay between the conservatism and representativeness biases?

Behavioral Biases

Even if information processing were perfect, many studies conclude that individuals would tend to make less-than-fully rational decisions using that information. These behavioral biases largely affect how investors frame questions of risk versus return, and therefore make risk-return trade-offs.

framing

Decisions are affected by how choices are posed, for example, as gains relative to a low baseline level or losses relative to a higher baseline.

Framing Decisions seem to be affected by how choices are **framed.** For example, an individual may reject a bet when it is posed in terms of the risk surrounding possible gains but may accept that same bet when described in terms of the risk surrounding potential losses. In other words, individuals may act risk averse in terms of gains but risk seeking in terms of losses. But in many cases, the choice of how to frame a risky venture—as involving gains or losses—can be arbitrary.

EXAMPLE 9.1

Framing

Consider a coin toss with a payoff of $50 for tails. Now consider a gift of $50 that is bundled with a bet that imposes a loss of $50 if that coin toss comes up heads. In both cases, you end up with zero for heads and $50 for tails. But the former description frames the coin toss as posing a risky gain while the latter frames the coin toss in terms of risky losses. The difference in framing can lead to different attitudes toward the bet.

mental accounting

A specific form of framing in which people segregate certain decisions.

Mental accounting **Mental accounting** is a specific form of framing in which people segregate certain decisions. For example, an investor may take a lot of risk with one investment account but establish a very conservative position with another account that is dedicated to her child's education. Rationally, it might be better to view both accounts as part of the investor's overall portfolio with the risk-return profiles of each integrated into a unified framework. Statman (1997) argues that mental accounting is consistent with some investors' irrational preference for stocks with high cash dividends (they feel free to spend dividend income, but would not "dip into capital" by selling a few shares of another stock with the same total rate of return) and with a tendency to ride losing stock positions for too long (since "behavioral investors" are reluctant to realize losses). In fact, investors are more likely to sell stocks with gains than those with losses, precisely contrary to a tax-minimization strategy (Shefrin and Statman, 1985; Odean, 1998).

Mental accounting effects also can help explain momentum in stock prices. The *house money effect* refers to gamblers' greater willingness to accept new bets if they currently are ahead. They think of (i.e., frame) the bet as being made with their "winnings account," that is, with the casino's and not with their own money, and thus are more willing to accept risk. Analogously, after a stock market run-up, individuals may view investments as largely funded out of a "capital gains account," become more tolerant of risk, discount future cash flows at a lower rate, and thus further push up prices.

Regret avoidance Psychologists have found that individuals who make decisions that turn out badly have more regret (blame themselves more) when that decision was more unconventional. For example, buying a blue-chip portfolio that turns down is not as painful as experiencing the same losses on an unknown start-up firm. Any losses on the blue-chip stocks can be more easily attributed to bad luck rather than bad decision making and cause less regret. De Bondt and Thaler (1987) argue that such **regret avoidance** is consistent with both the size and book-to-market effect. Higher-book-to-market firms tend to have depressed stock prices. These firms are "out of favor" and more likely to be in a financially precarious position. Similarly, smaller, less well-known firms are also less conventional investments. Such firms require more "courage" on the part of the investor, which increases the required rate of return. Mental accounting can add to this effect. If investors focus on the gains or losses of individual stocks, rather than on broad portfolios, they can become more risk averse concerning stocks with recent poor performance, discount their cash flows at a higher rate, and thereby create a value-stock risk premium.

regret avoidance

People blame themselves more for unconventional choices that turn out badly so they avoid regret by making conventional decisions.

CONCEPT check 9.2

How might the P/E effect (discussed in the previous chapter) also be explained as a consequence of regret avoidance?

Prospect theory **Prospect theory** modifies the analytic description of rational risk-averse investors found in standard financial theory.[2] Figure 9.1, Panel A, illustrates the conventional description of a risk-averse investor. Higher wealth provides higher satisfaction or "utility," but at a diminishing rate (the curve flattens as the individual becomes wealthier). This gives rise to risk aversion: A gain of $1,000 increases utility by less than a loss of $1,000 reduces it; therefore, investors will reject risky prospects that don't offer a risk premium.

prospect theory

Behavioral theory that investor utility depends on gains or losses from investors' starting position, rather than on their levels of wealth.

Figure 9.1, Panel B, shows a competing description of preferences characterized by "loss aversion." Utility depends not on the *level* of wealth, as in Panel A, but on *changes* in wealth from current levels. Moreover, to the left of zero (zero denotes no change from current wealth), the curve is convex rather than concave. This has several implications. Whereas many conventional utility functions imply that investors may become less risk averse as wealth increases, the function in Panel B always recenters on current wealth, thereby ruling out such decreases in risk aversion and possibly helping to explain high average historical equity risk premiums. Moreover, the convex curvature to the left of the origin in Panel B will induce investors to be risk seeking rather than risk averse when it comes to losses. Consistent with loss aversion, traders in the T-bond futures contract have been observed to assume significantly greater risk in afternoon sessions following morning sessions in which they have lost money (Coval and Shumway, 2005).

These are only a sample of many behavioral biases uncovered in the literature. Many have implications for investor behavior. The nearby box offers some good examples.

Limits to Arbitrage

Behavioral biases would not matter for stock pricing if rational arbitrageurs could fully exploit the mistakes of behavioral investors. Trades of profit-seeking investors would correct any misalignment of prices. However, behavioral advocates argue that in practice, several factors limit the ability to profit from mispricing.[3]

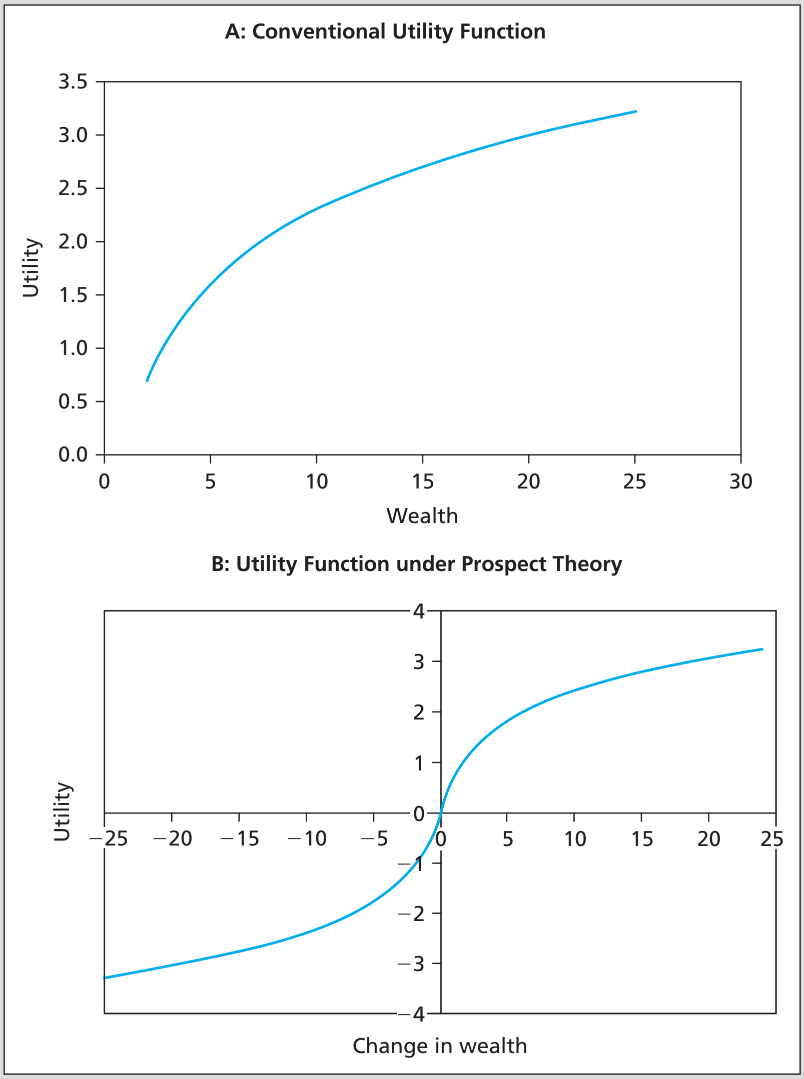
Fundamental risk Suppose that a share of IBM is underpriced. Buying it may present a profit opportunity, but it is hardly risk-free, since the presumed market underpricing can get worse. While price eventually should converge to intrinsic value, this may not happen until after the trader's investment horizon. For example, the investor may be a mutual fund manager who may lose clients (not to mention a job!) if short-term performance is poor or a trader who

[2] Prospect theory originated with a highly influential paper about decision making under uncertainty by D. Kahneman and A. Tversky, "Prospect Theory: An Analysis of Decision under Risk," *Econometrica* 47 (1979), pp. 263–291.

[3] Some of the more influential references on limits to arbitrage are J. B. DeLong, A. Schleifer, L. Summers, and R. Waldmann, "Noise Trader Risk in Financial Markets," *Journal of Political Economy* 98 (August 1990), pp. 704–738; and A. Schleifer and R. Vishny, "The Limits of Arbitrage," *Journal of Finance* 52 (March 1997), pp. 35–55.

FIGURE 9.1

Prospect theory
Panel A: A conventional utility function is defined in terms of wealth and is concave, resulting in risk aversion.
Panel B: Under loss aversion, the utility function is defined in terms of changes from current wealth. It is also convex to the left of the origin, giving rise to risk-seeking behavior in terms of losses.



may run through her capital if the market turns against her, even temporarily. A comment often attributed to the famous economist John Maynard Keynes is that "markets can remain irrational longer than you can remain solvent." The *fundamental risk* incurred in exploiting apparent profit opportunities presumably will limit the activity of traders.

EXAMPLE 9.2

Fundamental Risk

In the first part of 2011, the NASDAQ index fluctuated at a level around 2,700. From that perspective, the value the index had reached 10 years earlier, around 5,000, seemed obviously crazy. Surely some investors living through the Internet "bubble" of the late 1990s must have identified the index as grossly overvalued, suggesting a good selling opportunity. But this hardly would have been a riskless arbitrage opportunity. Consider that NASDAQ may also have been overvalued in 1999 when it first crossed above 3,500 (30% above its value in 2011). An investor in 1999 who believed (as it turns out, quite correctly) that NASDAQ was overvalued at 3,500 and decided to sell it short would have suffered enormous losses as the index increased by another 1,500 points before finally peaking at 5,000. While the investor might have derived considerable satisfaction at eventually being proven right about the overpricing, by entering a year before the market "corrected," he might also have gone broke.

WHY IT'S SO TOUGH TO FIX YOUR PORTFOLIO

If your portfolio is out of whack, you could ask an investment adviser for help. But you might have better luck with your therapist.

It's a common dilemma: You know you have the wrong mix of investments, but you cannot bring yourself to fix the mess. Why is it so difficult to change? At issue are three mental mistakes.

CHASING WINNERS

Looking to lighten up on bonds and get back into stocks? Sure, you know stocks are a long-term investment and, sure, you know they are best bought when cheap.

Yet it's a lot easier to pull the trigger and buy stocks if the market has lately been scoring gains. "People are influenced by what has happened most recently, and then they extrapolate from that," says Meir Statman, a finance professor at Santa Clara University in California. "But often, they end up being optimistic and pessimistic at just the wrong time."

Consider some results from the UBS Index of Investor Optimism, a monthly poll conducted by UBS and the Gallup Organization. Each month, the poll asks investors what gain they expect from their portfolio during the next 12 months. Result? You guessed it: The answers rise and fall with the stock market.

For instance, during the bruising bear market, investors grew increasingly pessimistic, and at the market bottom they were looking for median portfolio gains of just 5%. But true to form, last year's rally brightened investors' spirits and by January they were expecting 10% returns.

GETTING EVEN

This year's choppy stock market hasn't scared off just bond investors. It has also made it difficult for stock investors to rejigger their portfolios.

Blame it on the old "get even, then get out" syndrome. With stocks treading water, many investors are reluctant to sell, because they are a long way from recovering their bear-market losses. To be sure, investors who bought near the peak are underwater, whether they sell or not. But selling losers is still agonizing, because it means admitting you made a mistake.

"If you're rational and you have a loss, you sell, take the tax loss and move on," Prof. Statman says. "But if you're a normal person, selling at a loss tears your heart out."

MUSTERING COURAGE

Whether you need to buy stocks or buy bonds, it takes confidence to act. And right now, investors just aren't confident. "There's this status-quo bias," says John Nofsinger, a finance professor at Washington State University in Pullman, Washington. "We're afraid to do anything, because we're afraid we'll regret it."

Once again, it's driven by recent market action. When markets are flying high, folks attribute their portfolio's gains to their own brilliance. That gives them the confidence to trade more and to take greater risks. Overreacting to short-term market results is, of course, a great way to lose a truckload of money. But with any luck, if you are aware of this pitfall, maybe you will avoid it.

Or maybe [this is] too optimistic. "You can tell somebody that investors have all these behavioral biases," says Terrance Odean, a finance professor at the University of California at Berkeley. "So what happens? The investor thinks, 'Oh, that sounds like my husband. I don't think many investors say, 'Oh, that sounds like me.'"

SOURCE: Jonathan Clements, *The Wall Street Journal Online,* June 23, 2004.

Implementation costs Exploiting overpricing can be particularly difficult. Short-selling a security entails costs; short-sellers may have to return the borrowed security on little notice, rendering the horizon of the short sale uncertain; other investors such as many pension or mutual fund managers face strict limits on their discretion to short securities. This can limit the ability of arbitrage activity to force prices to fair value.

Model risk One always has to worry that an apparent profit opportunity is more apparent than real. Perhaps you are using a faulty model to value the security, and the price actually is right. Mispricing may make a position a good bet, but it is still a risky one, which limits the extent to which it will be pursued.

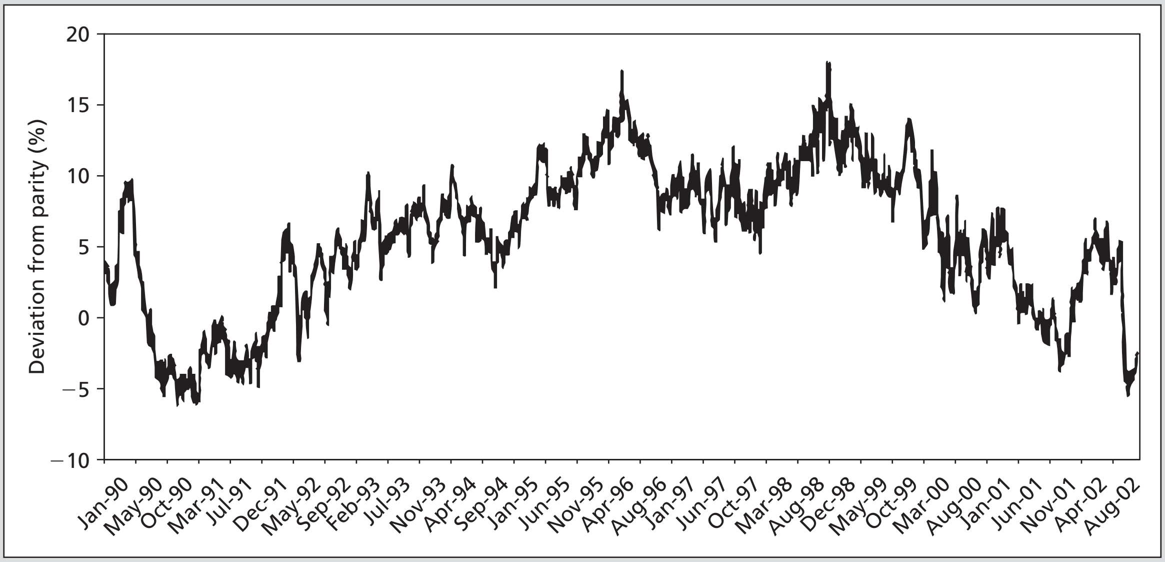
Limits to Arbitrage and the Law of One Price

While one can debate the implications of much of the anomalies literature, surely the Law of One Price (positing that effectively identical assets should have identical prices) should be satisfied in rational markets. Yet there are several instances where the law seems to have been violated. These instances are good case studies of the limits to arbitrage.

FIGURE 9.2

Pricing of Royal Dutch relative to Shell (*deviation from parity*)

Source: O. A. Lamont and R. H. Thaler, "Anomalies: The Law of One Price in Financial Markets," *Journal of Economic Perspectives* 17 (Fall 2003), pp. 191–202. Used with permission of American Economic Association.



"Siamese twin" companies[4] In 1907, Royal Dutch Petroleum and Shell Transport merged their operations into one firm. The two original companies, which continued to trade separately, agreed to split all profits from the joint company on a 60/40 basis. Shareholders of Royal Dutch receive 60% of the cash flow, and those of Shell receive 40%. One would therefore expect that Royal Dutch should sell for exactly 60/40 = 1.5 times the price of Shell. But this is not the case. Figure 9.2 shows that the relative value of the two firms has departed considerably from this "parity" ratio for extended periods of time.

Doesn't this mispricing give rise to an arbitrage opportunity? If Royal Dutch sells for more than 1.5 times Shell, why not buy relatively underpriced Shell and short-sell overpriced Royal? This seems like a reasonable strategy, but if you had followed it in February 1993 when Royal sold for about 10% more than its parity value, Figure 9.2 shows that you would have lost a lot of money as the premium widened to about 17% before finally reversing after 1999. As in Example 9.2, this opportunity posed fundamental risk.

Equity carve-outs Several equity carve-outs also have violated the Law of One Price.[5] To illustrate, consider the case of 3Com, which in 1999 decided to spin off its Palm division. It first sold 5% of its stake in Palm in an IPO, announcing that it would distribute the remaining 95% of its Palm shares to 3Com shareholders six months later in a spinoff. Each 3Com shareholder would receive 1.5 shares of Palm in the spinoff.

Once Palm shares began trading, but prior to the spinoff, the share price of 3Com should have been *at least* 1.5 times that of Palm. After all, each share of 3Com entitled its owner to 1.5 shares of Palm *plus* an ownership stake in a profitable company. Instead, Palm shares at the IPO actually sold for *more* than the 3Com shares. The *stub value* of 3Com (i.e., the value of each 3Com share net of the value of the claim to Palm represented by that share) could be computed as the price of 3Com minus 1.5 times the price of Palm. This calculation, however, implies that 3Com's stub value was negative, this despite the fact that it was a profitable company with cash assets alone of about $10 per share.

[4]This discussion is based on K. A. Froot and E. M. Dabora, "How Are Stock Prices Affected by the Location of Trade?" *Journal of Financial Economics* 53 (1999), pp. 189–216.

[5]O. A. Lamont and R. H. Thaler, "Can the Market Add and Subtract? Mispricing in Tech Carve-Outs," *Journal of Political Economy* 111 (2003), pp. 227–268.

Again, an arbitrage strategy seems obvious. Why not buy 3Com and sell Palm? The limit to arbitrage in this case was the inability of investors to sell Palm short. Virtually all available shares in Palm were already borrowed and sold short, and the negative stub values persisted for more than two months.

Closed-end funds We noted in Chapter 4 that closed-end funds often sell for substantial discounts or premiums from net asset value. This is "nearly" a violation of the Law of One Price, since one would expect the value of the fund to equal the value of the shares it holds. We say nearly, because in practice, there are a few wedges between the value of the closed-end fund and its underlying assets. One is expenses. The fund incurs expenses that ultimately are paid for by investors, and these will reduce share price. On the other hand, if managers can invest fund assets to generate positive risk-adjusted returns, share price might exceed net asset value.

Lee, Shleifer, and Thaler (1991) argue that the patterns of discounts and premiums on closed-end funds are driven by changes in investor sentiment. They note that discounts on various funds move together and are correlated with the return on small stocks, suggesting that all are affected by common variation in sentiment. One might consider buying funds selling at a discount from net asset value and selling those trading at a premium, but discounts and premiums can widen, subjecting this strategy too to fundamental risk. Pontiff (1996) demonstrates that deviations of price from net asset value in closed-end funds tend to be higher in funds that are more difficult to arbitrage, for example, those with more idiosyncratic volatility.

CONCEPT check 9.3

Fundamental risk may be limited by a "deadline" that forces a convergence between price and intrinsic value. What do you think would happen to a closed-end fund's discount if the fund announced that it plans to liquidate in six months, at which time it will distribute NAV to its shareholders?

Closed-end fund discounts are a good example of apparent anomalies that also may have rational explanations. Ross (2002) demonstrates that they can be reconciled with rational investors even if expenses or fund abnormal returns are modest. He shows that if a fund has a dividend yield of δ, an alpha (risk-adjusted abnormal return) of α, and expense ratio of ε, then using the constant-growth dividend discount model (see Chapter 13), the premium of the fund over its net asset value will be

$$\frac{\text{Price} - \text{NAV}}{\text{NAV}} = \frac{\alpha - \varepsilon}{\delta + \varepsilon - \alpha}$$

If the fund manager's performance more than compensates for expenses (i.e., if $\alpha > \varepsilon$), the fund will sell at a premium to NAV; otherwise it will sell at a discount. For example, suppose $\alpha = .015$, the expense ratio is $\varepsilon = .0125$, and the dividend yield is $\delta = .02$. Then the premium will be .14, or 14%. But if the market turns sour on the manager and revises its estimate of α downward to .005, that premium quickly turns into a discount of 43%.

This analysis might explain why the public is willing to purchase closed-end funds at a premium; if investors do not expect α to exceed ε, they won't purchase shares in the fund. But the fact that most premiums eventually turn into discounts indicates how difficult it is for management to fulfill these expectations.[6]

Bubbles and Behavioral Economics

In Example 9.2, we pointed out that the stock market run-up of the late 1990s, and even more spectacularly, the run-up of the technology-heavy NASDAQ market, seems in retrospect to have been an obvious bubble. In a six-year period beginning in 1995, the NASDAQ index

[6]We might ask why this logic of discounts and premiums does not apply to open-end mutual funds since they incur similar expense ratios. Because investors in these funds can redeem shares for NAV, the shares cannot sell at a discount to NAV. Expenses in open-end funds reduce returns in each period rather than being capitalized into price and inducing a discount.

increased by a factor of more than 6. Former Fed Chairman Alan Greenspan famously characterized the dot-com boom as an example of "irrational exuberance," and his assessment turned out to be correct: By October 2002, the index fell to less than one-fourth the peak value it had reached only two and a half years earlier. This episode seems to be a case in point for advocates of the behavioral school, exemplifying a market moved by irrational investor sentiment. Moreover, in accord with behavioral patterns, as the dot-com boom developed, it seemed to feed on itself, with investors increasingly confident of their investment prowess (overconfidence bias) and apparently willing to extrapolate short-term patterns into the distant future (representativeness bias).

Only five years later, another bubble, this time in housing prices, was underway. As in the dot-com bubble, expectations of continued price increases fueled speculative demand by purchasers. Shortly thereafter, of course, housing prices stalled and then fell. The bursting bubble set off the worst financial crisis in 75 years.

On the other hand, bubbles are a lot easier to identify as such once they are over. While they are going on, it is not as clear that prices are irrationally exuberant, and, indeed, many financial commentators at the time justified the dot-com boom as consistent with glowing forecasts for the "new economy." A simple example shows how hard it can be to tie down the fair value of stock investments.[7]

EXAMPLE 9.3

A Stock Market Bubble?

In 2000, near the peak of the dot-com boom, the dividends paid by the firms included in the S&P 500 totaled \$154.6 million. If the discount rate for the index was 9.2% and the expected dividend growth rate was 8%, the value of these shares according to the constant-growth dividend discount model (see Chapter 13 for more on this model) would be

$$\text{Value} = \frac{\text{Dividend}}{\text{Discount rate} - \text{Growth rate}} = \frac{\$154.6}{.092 - .08} = \$12{,}883 \text{ million}$$

This was quite close to the actual total value of those firms at the time. But the estimate is highly sensitive to the input values, and even a small reassessment of their prospects would result in a big revision of price. Suppose the expected dividend growth rate fell to 7.4%. This would reduce the value of the index to

$$\text{Value} = \frac{\text{Dividend}}{\text{Discount rate} - \text{Growth rate}} = \frac{\$154.6}{.092 - .074} = \$8{,}589 \text{ million}$$

which was about the value to which the S&P 500 firms had fallen by October 2002. In light of this example, the run-up and crash of the 1990s seems easier to reconcile with rational behavior.

Still, other evidence seems to tag the dot-com boom as at least partially irrational. Consider, for example, the results of a study by Rau, Dimitrov, and Cooper (2001) documenting that firms adding ".com" to the end of their names during this period enjoyed a meaningful stock price increase. That doesn't sound like rational valuation.

Evaluating the Behavioral Critique

As investors, we are concerned with the existence of profit opportunities. The behavioral explanations of efficient market anomalies do not give guidance as to how to exploit any irrationality. For investors, the question is still whether there is money to be made from mispricing, and the behavioral literature is largely silent on this point.

However, as we have emphasized above, one of the important implications of the efficient market hypothesis is that security prices serve as reliable guides to the allocation of real assets. If prices are distorted, then capital markets will give misleading signals (and incentives) as to where the economy may best allocate resources. In this crucial dimension, the behavioral

[7]The following example is taken from R. A. Brealey, S. C. Myers, and F. Allen, *Principles of Corporate Finance,* 9th ed. (Burr Ridge, IL: McGraw-Hill/Irwin, 2008).

critique of the efficient market hypothesis is certainly important irrespective of any implication for investment strategies.

There is considerable debate among financial economists concerning the strength of the behavioral critique. Many believe that the behavioral approach is too unstructured, in effect allowing virtually any anomaly to be explained by some combination of irrationalities chosen from a laundry list of behavioral biases. While it is easy to "reverse engineer" a behavioral explanation for any particular anomaly, these critics would like to see a consistent or unified behavioral theory that can explain a *range* of anomalies.

More fundamentally, others are not convinced that the anomalies literature as a whole is a convincing indictment of the efficient market hypothesis. Fama (1998) reviews the anomalies literature and mounts a counterchallenge to the behavioral school. He notes that the anomalies are inconsistent in terms of their support for one type of irrationality versus another. For example, some papers document long-term corrections (consistent with overreaction), while others document long-term continuations of abnormal returns (consistent with underreaction). Moreover, the statistical significance of many of these results is hard to assess. Even small errors in choosing a benchmark against which to compare returns can cumulate to large apparent abnormalities in long-term returns. Therefore, many of the results in these studies are sensitive to small benchmarking errors, and Fama argues that seemingly minor changes in methodology can have big impacts on conclusions.

The behavioral critique of full rationality in investor decision making is well taken, but the extent to which limited rationality affects asset pricing remains controversial. Whether or not investor irrationality affects asset prices, however, behavioral finance already makes important points about portfolio management. Investors who are aware of the potential pitfalls in information processing and decision making that seem to characterize their peers should be better able to avoid such errors. Ironically, the insights of behavioral finance may lead to some of the same policy conclusions embraced by efficient market advocates. For example, an easy way to avoid some behavioral minefields is to pursue passive, largely indexed portfolio strategies. It seems that only rare individuals can consistently beat passive strategies; this conclusion may hold true whether your fellow investors are behavioral or rational.

9.2 TECHNICAL ANALYSIS AND BEHAVIORAL FINANCE

Technical analysis attempts to exploit recurring and predictable patterns in stock prices to generate superior investment performance. Technicians do not deny the value of fundamental information but believe that prices only gradually close in on intrinsic value. As fundamentals shift, astute traders can exploit the adjustment to a new equilibrium.

For example, one of the best-documented behavioral tendencies is the *disposition effect,* which refers to the tendency of investors to hold on to losing investments. Behavioral investors seem reluctant to realize losses. Grinblatt and Han (2005) show that the disposition effect can lead to momentum in stock prices even if fundamental values follow a random walk. The fact that the demand of "disposition investors" for a company's shares depends on the price history of those shares means that prices close in on fundamental values only over time, consistent with the central motivation of technical analysis.

Behavioral biases may also be consistent with technical analysts' use of volume data. An important behavioral trait noted above is overconfidence, a systematic tendency to overestimate one's abilities. As traders become overconfident, they may trade more, inducing an association between trading volume and market returns (Gervais and Odean, 2001). Technical analysis thus uses volume data as well as price history to direct trading strategy.

Finally, technicians believe that market fundamentals can be perturbed by irrational or behavioral factors, sometimes labeled "sentiment variables." More or less random price fluctuations will accompany any underlying price trend, creating opportunities to exploit corrections as these fluctuations dissipate.

Trends and Corrections

Much of technical analysis seeks to uncover trends in market prices. This is in effect a search for momentum. Momentum can be absolute, in which case one searches for upward price trends, or relative, in which case the analyst looks to invest in one sector over another (or even take on a long-short position in the two sectors). Relative strength statistics (see page 280) are designed to uncover these cross-sector potential opportunities.

Momentum and moving averages While we all would like to buy shares in firms whose prices are trending upward, this begs the question of how to identify the underlying direction of prices, if in fact such trends actually exist. A primary tool for this purpose is the moving average.

The moving average of a stock price is the average price over a given interval, where that interval is updated as time passes. For example, a 50-day moving average traces the average price over the previous 50 days. The average is recomputed each day by dropping the oldest observation and adding the newest. Figure 9.3 is a moving-average chart for Intel. Notice that the moving average (the blue curve) is a "smoothed" version of the original data series (the jagged red curve).

After a period in which prices have been falling, the moving average will be above the current price (because the moving average continues to "average in" the older and higher prices until they leave the sample period). In contrast, when prices have been rising, the moving average will be below the current price.

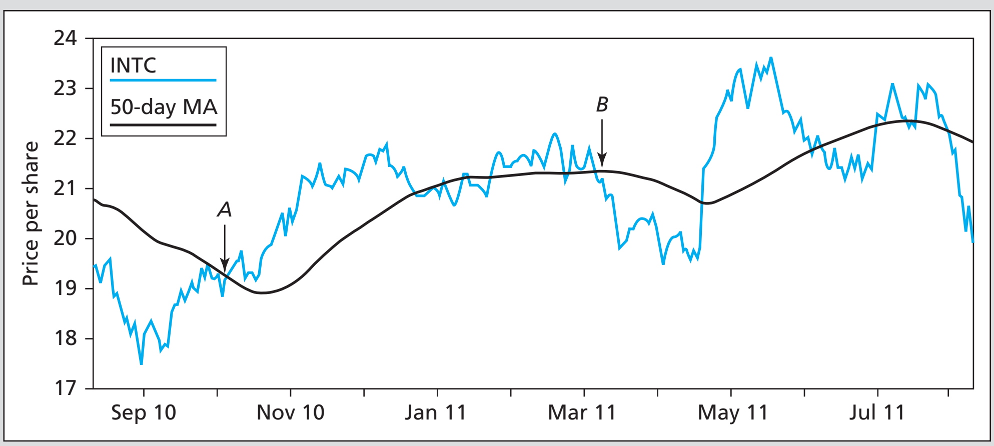
Breaking through the moving average from below, as at point *A* in Figure 9.3, is taken as a bullish signal, because it signifies a shift from a falling trend (with prices below the moving average) to a rising trend (with prices above the moving average). Conversely, when prices drop below the moving average, as at point *B,* analysts might conclude that market momentum has become negative.

Other techniques also are used to uncover potential momentum in stock prices. Two of the more famous ones are Elliott wave theory and Kondratieff waves. Both posit the existence of long-term trends in stock market prices that may be disturbed by shorter-term trends as well as daily fluctuations of little importance. Elliott wave theory superimposes long-term and short-term wave cycles in an attempt to describe the complicated pattern of actual price movements. Once the longer-term waves are identified, investors presumably can buy when the

FIGURE 9.3

Share price and 50-day moving average for Intel

Source: Yahoo! Finance, **finance.yahoo.com,** August 11, 2011.



long-term direction of the market is positive. While there is considerable noise in the actual evolution of stock prices, by properly interpreting the wave cycles, one can, according to the theory, predict broad movements. Similarly, Kondratieff waves are named after a Russian economist who asserted that the macroeconomy (and therefore the stock market) moves in broad waves lasting between 48 and 60 years. Kondratieff's assertion is hard to evaluate empirically, however, because cycles that last about 50 years provide only two independent data points per century, which is hardly enough data to test the predictive power of the theory.

EXAMPLE 9.4

Moving Averages

Consider the price data in the following table. Each observation represents the closing level of the Dow Jones Industrial Average (DJIA) on the last trading day of the week. The five-week moving average for each week is the average of the DJIA over the previous five weeks. For example, the first entry, for week 5, is the average of the index value between weeks 1 and 5: 12,290, 12,380, 12,399, 12,379, and 12,450. The next entry is the average of the index values between weeks 2 and 6, and so on.

Figure 9.4 plots the level of the index and the five-week moving average. Notice that while the index itself moves up and down rather abruptly, the moving average is a relatively smooth series, since the impact of each week's price movement is averaged with that of the previous weeks. Week 16 is a bearish point according to the moving-average rule. The price series crosses from above the moving average to below it, signifying the beginning of a downward trend in stock prices.

Week	DJIA	5-Week Moving Average	Week	DJIA	5-Week Moving Average
1	12,290		11	12,590	12,555
2	12,380		12	12,652	12,586
3	12,399		13	12,625	12,598
4	12,379		14	12,657	12,624
5	12,450	12,380	15	12,699	12,645
6	12,513	12,424	16	12,647	12,656
7	12,500	12,448	17	12,610	12,648
8	12,565	12,481	18	12,595	12,642
9	12,524	12,510	19	12,499	12,610
10	12,597	12,540	20	12,466	12,563

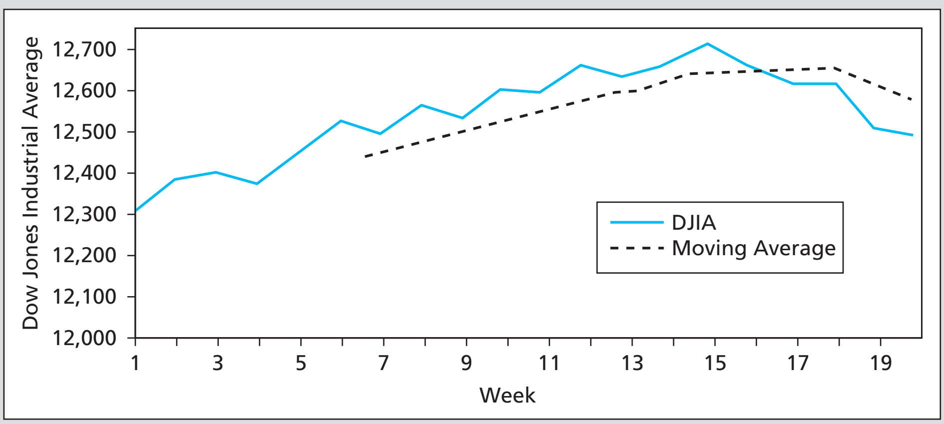
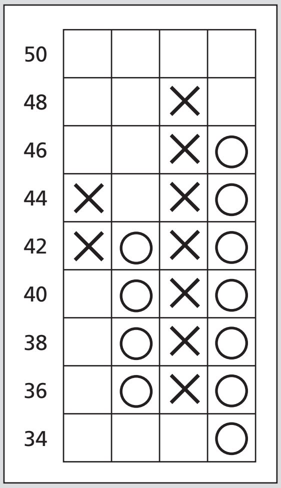


FIGURE 9.4

Moving averages

FIGURE 9.5

Point and figure chart for Table 9.1



Point and figure charts A variant on pure trend analysis is the *point and figure chart* depicted in Figure 9.5. This figure has no time dimension. It simply traces significant upward or downward movements in stock prices without regard to their timing. The data for Figure 9.5 come from Table 9.1.

Suppose, as in Table 9.1, that a stock's price is currently $40. If the price rises by at least $2, you put an X in the first column at $42 in Figure 9.5. Another increase of at least $2 calls for placement of another X in the first column, this time at the $44 level. If the stock then falls by at least $2, you start a new column and put an O next to $42. Each subsequent $2 price fall results in another O in the second column. When prices reverse yet again and head upward, you begin the third column with an X denoting each consecutive $2 price increase.

The single asterisks in Table 9.1 mark an event resulting in the placement of a new X or O in the chart. The daggers denote price movements that result in the start of a new column of Xs or Os.

Sell signals are generated when the stock price *penetrates* previous lows, and buy signals occur when previous high prices are penetrated. A *congestion area* is a horizontal band of Xs and Os created by several price reversals. These regions correspond to support and resistance levels and are indicated in Figure 9.6, which is an actual chart for Atlantic Richfield.

TABLE 9.1 Stock price history

Date	Price	Date	Price
January 2	$40	February 1	$40*
January 3	40.50	February 2	41
January 4	41	February 5	40.50
January 5	42*	February 6	42*
January 8	41.50	February 7	45*
January 9	42.50	February 8	44.50
January 10	43	February 9	46*
January 11	43.75	February 12	47
January 12	44*	February 13	48*
January 15	45	February 14	47.50
January 16	44	February 15	46†
January 17	41.50†	February 16	45
January 18	41	February 19	44*
January 19	40*	February 20	42*
January 22	39	February 21	41
January 23	39.50	February 22	40*
January 24	39.75	February 23	41
January 25	38*	February 26	40.50
January 26	35*	February 27	38*
January 29	36†	February 28	39
January 30	37	March 1	36*
January 31	39*	March 2	34*

*Indicates an event that has resulted in a stock price increase or decrease of at least $2.

†Denotes a price movement that has resulted in either an upward or a downward reversal in the stock price.

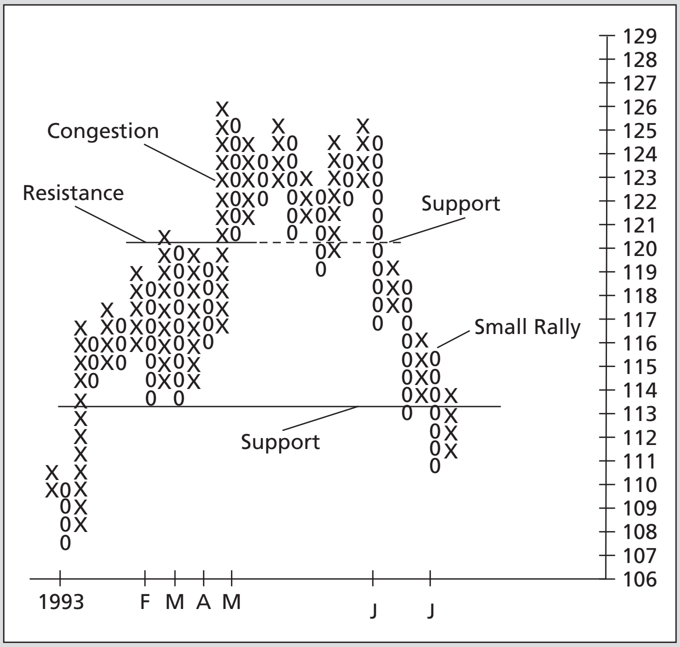


FIGURE 9.6

Point and figure chart for Atlantic Richfield

One can devise point and figure charts using price increments other than $2, but it is customary in setting up a chart to require reasonably substantial price changes before marking pluses or minuses.

CONCEPT check 9.4

Draw a point and figure chart using the history in Table 9.1 with price increments of $3.

Breadth The **breadth** of the market is a measure of the extent to which movement in a market index is reflected widely in the price movements of all the stocks in the market. The most common measure of breadth is the spread between the number of stocks that advance and decline in price. If advances outnumber declines by a wide margin, then the market is viewed as being stronger because the rally is widespread. These numbers are reported daily in *The Wall Street Journal* (see Figure 9.7).

breadth

The extent to which movements in broad market indexes are reflected widely in movements of individual stock prices.

Markets Diary

Issues	**NYSE**	**Nasdaq**	**Amex**
Advancing	2,787	2,119	308
Declining	270	441	129
Unchanged	33	64	18
Total	3,090	2,624	455
Issues at			
New 52 week high	7	6	1
New 52 week low	125	141	16
Share Volume			
Total	4,937,076,320	2,096,877,765	113,647,520
Advancing	4,681,742,414	2,010,365,013	75,275,155
Declining	231,468,687	79,349,368	36,759,430
Unchanged	23,865,219	7,163,384	1,612,935

FIGURE 9.7

Market diary

Source: *The Wall Street Journal Online,* August 11, 2011. Reprinted by permission of *The Wall Street Journal,*

TABLE 9.2 Breadth

Day	Advances	Declines	Net Advances	Cumulative Breadth
1	1,802	1,748	54	54
2	1,917	1,640	277	331
3	1,703	1,772	−69	262
4	1,512	2,122	−610	−348
5	1,633	2,004	−371	−719

Note: The sum of advances plus declines varies across days because some stock prices are unchanged.

Some analysts cumulate breadth data each day as in Table 9.2. The cumulative breadth for each day is obtained by adding that day's net advances (or declines) to the previous day's total. The direction of the cumulated series is then used to discern broad market trends. Analysts might use a moving average of cumulative breadth to gauge broad trends.

relative strength

Recent performance of a given stock or industry compared to that of a broader market index.

Relative strength **Relative strength** measures the extent to which a security has outperformed or underperformed either the market as a whole or its particular industry. Relative strength is computed by calculating the ratio of the price of the security to a price index for the industry. For example, the relative strength of Toyota versus the auto industry would be measured by movements in the ratio of the price of Toyota divided by the level of an auto industry index. A rising ratio implies Toyota has been outperforming the rest of the industry. If relative strength can be assumed to persist over time, then this would be a signal to buy Toyota.

Similarly, the relative strength of an industry relative to the whole market can be computed by tracking the ratio of the industry price index to the market price index.

Sentiment Indicators

Trin statistic Market volume is sometimes used to measure the strength of a market rise or fall. Increased investor participation in a market advance or retreat is viewed as a measure of the significance of the movement. Technicians consider market advances to be a more favorable omen of continued price increases when they are associated with increased trading volume. Similarly, market reversals are considered more bearish when associated with higher volume. The **trin statistic** is defined as

trin statistic

The ratio of average volume in declining issues to average volume in advancing issues.

$$\text{Trin} = \frac{\text{Volume declining/Number declining}}{\text{Volume advancing/Number advancing}}$$

Therefore, trin is the ratio of average trading volume in declining issues to average volume in advancing issues. Ratios above 1 are considered bearish because the falling stocks would then have higher average volume than the advancing stocks, indicating net selling pressure.

The Wall Street Journal Online provides the data necessary to compute trin in its Markets Diary section. Using the data in Figure 9.7, trin for the NYSE on this day was:

$$\text{Trin} = \frac{\$231{,}468{,}687/270}{\$4{,}681{,}742{,}414/2{,}787} = .51$$

Note, however, that for every buyer, there must be a seller of stock. Rising volume in a rising market should not necessarily indicate a larger imbalance of buyers versus sellers.

For example, a trin statistic above 1, which is considered bearish, could equally well be interpreted as indicating that there is more *buying* activity in declining issues.

Confidence index *Barron's* computes a confidence index using data from the bond market. The presumption is that actions of bond traders reveal trends that will emerge soon in the stock market.

confidence index

Ratio of the yield of top-rated corporate bonds to the yield on intermediate-grade bonds.

The **confidence index** is the ratio of the average yield on 10 top-rated corporate bonds divided by the average yield on 10 intermediate-grade corporate bonds. The ratio will always be below 100% because higher-rated bonds will offer lower promised yields to maturity. When bond traders are optimistic about the economy, however, they might require smaller default premiums on lower-rated debt. Hence, the yield spread will narrow, and the confidence index will approach 100%. Therefore, higher values of the confidence index are bullish signals.

CONCEPT check 9.5

Yields on lower-rated debt typically rise along with fears of recession. This reduces the confidence index. When these yields increase, should the stock market be expected to fall, or will it already have fallen?

short interest

The total number of shares currently sold short in the market.

Short interest **Short interest** is the total number of shares of stock currently sold short in the market. Some technicians interpret high levels of short interest as bullish, some as bearish. The bullish perspective is that, because all short sales must be covered (i.e., short-sellers eventually must purchase shares to return the ones they have borrowed), short interest represents latent future demand for the stocks. As short sales are covered, the demand created by the share purchase will force prices up.

The bearish interpretation of short interest is based on the fact that short-sellers tend to be larger, more sophisticated investors. Accordingly, increased short interest reflects bearish sentiment by those investors "in the know," which would be a negative signal of the market's prospects.

Put/call ratio Call options give investors the right to buy a stock at a fixed "exercise" price and therefore are a way of betting on stock price increases. Put options give the right to sell a stock at a fixed price and therefore are a way of betting on stock price decreases.[8] The ratio of outstanding put options to outstanding call options is called the **put/call ratio.** Because put options do well in falling markets while call options do well in rising markets, deviations of the ratio from historical norms are considered to be a signal of market sentiment and therefore predictive of market movements.

put/call ratio

Ratio of put options to call options outstanding on a stock.

Interestingly, however, a change in the ratio can be given a bullish or a bearish interpretation. Many technicians see an increase in the ratio as bearish, as it indicates growing interest in put options as a hedge against market declines. Thus, a rising ratio is taken as a sign of broad investor pessimism and a coming market decline. Contrarian investors, however, believe that a good time to buy is when the rest of the market is bearish because stock prices are then unduly depressed. Therefore, they would take an increase in the put/call ratio as a signal of a buy opportunity.

A Warning

The search for patterns in stock market prices is nearly irresistible, and the ability of the human eye to discern apparent patterns is remarkable. Unfortunately, it is possible to perceive

[8]Puts and calls were defined in Chapter 2, Section 2.5. They are discussed more fully in Chapter 15.

patterns that really don't exist. Consider Figure 9.8, which presents simulated and actual values of the Dow Jones Industrial Average during 1956 taken from a famous study by Harry Roberts (1959). In Figure 9.8B, it appears as though the market presents a classic head-and-shoulders pattern where the middle hump (the head) is flanked by two shoulders. When the price index "pierces the right shoulder"—a technical trigger point—it is believed to be heading lower, and it is time to sell your stocks. Figure 9.8A also looks like a "typical" stock market pattern.

Can you tell which of the two graphs is constructed from the real value of the Dow and which from the simulated data? Figure 9.8A is based on the real data. The graph in Panel B was generated using "returns" created by a random-number generator. These returns *by construction* were patternless, but the simulated price path that is plotted appears to follow a pattern much like that of Panel A.

Figure 9.9 shows the weekly price *changes* behind the two panels in Figure 9.8. Here the randomness in both series—the stock price as well as the simulated sequence—is obvious.

A problem related to the tendency to perceive patterns where they don't exist is data mining. After the fact, you can always find patterns and trading rules that would have generated enormous profits. If you test enough rules, some will have worked in the past. Unfortunately, picking a theory that would have worked after the fact carries no guarantee of future success.

In evaluating trading rules, you should always ask whether the rule would have seemed reasonable *before* you looked at the data. If not, you might be buying into the one arbitrary rule among many that happened to have worked in the recent past. The hard but crucial question is whether there is reason to believe that what worked in the past should continue to work in the future.

FIGURE 9.8

Actual and simulated levels for stock market prices of 52 weeks

Note: Friday closing levels, December 30, 1955–December 28, 1956, Dow Jones Industrial Average.
Source: Harry Roberts, "Stock Market 'Patterns' and Financial Analysis: Methodological Suggestions," *Journal of Finance* 14 (March 1959), pp. 1–10. Used with permission of John Wiley and Sons, via Copyright Clearance Center.

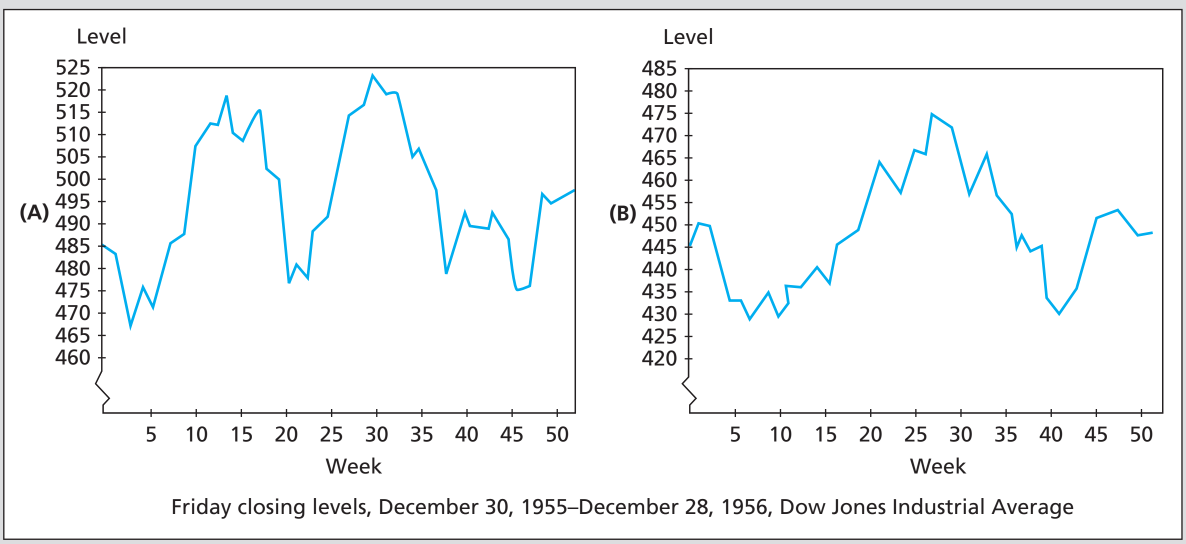
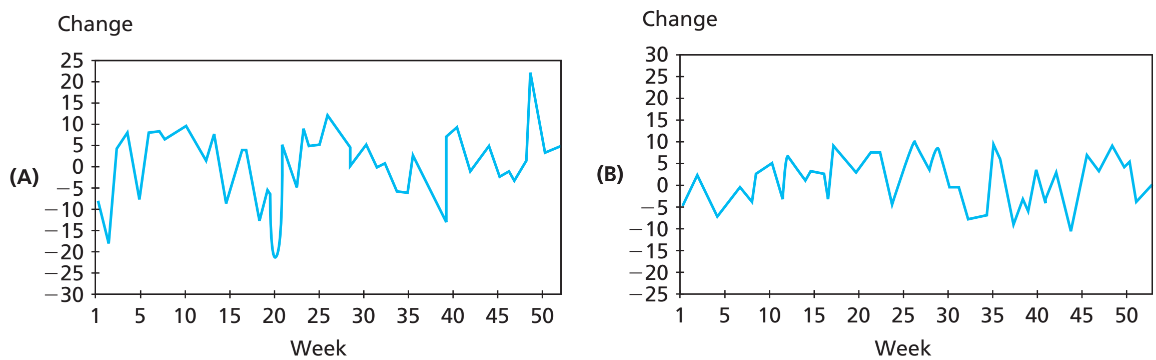


FIGURE 9.9

Actual and simulated changes in weekly stock prices for 52 weeks

Note: Changes from Friday to Friday (closing) January 6, 1956–December 28, 1956, Dow Jones Industrial Average.
Source: Harry Roberts, "Stock Market 'Patterns' and Financial Analysis: Methodological Suggestions," *Journal of Finance* 14 (March 1959), pp. 1–10. Used with permission of John Wiley and Sons, via Copyright Clearance Center.



Changes from Friday to Friday (closing) January 6, 1956–December 28, 1956, Dow Jones Industrial Average

SUMMARY

- Behavioral finance focuses on systematic irrationalities that characterize investor decision making. These "behavioral shortcomings" may be consistent with several efficient market anomalies.
- Among the information processing errors uncovered in the psychology literature are memory bias, overconfidence, conservatism, and representativeness. Behavioral tendencies include framing, mental accounting, regret avoidance, and loss aversion.
- Limits to arbitrage activity impede the ability of rational investors to exploit pricing errors induced by behavioral investors. For example, fundamental risk means that even if a security is mispriced, it still can be risky to attempt to exploit the mispricing. This limits the actions of arbitrageurs who take positions in mispriced securities. Other limits to arbitrage are implementation costs, model risk, and costs to short-selling. Occasional failures of the Law of One Price suggest that limits to arbitrage are sometimes severe.
- The various limits to arbitrage mean that even if prices do not equal intrinsic value, it still may be difficult to exploit the mispricing. As a result, the failure of traders to beat the market may not be proof that markets are in fact efficient, with prices equal to intrinsic value.
- Technical analysis also uses volume data and sentiment indicators. These are broadly consistent with several behavioral models of investor activity. Technical analysis is the search for recurring and predictable patterns in stock prices. It is based on the premise that prices only gradually close in on intrinsic value. As fundamentals shift, astute traders can exploit the adjustment to a new equilibrium.
- Technical analysts try to uncover trends in stock prices and anticipate reversals of those trends. Moving averages, relative strength, and breadth are used in various trend-based strategies.
- Some sentiment indicators are the trin statistic, the confidence index, and the put/call ratio.

KEY TERMS

behavioral finance, 266
breadth, 279
confidence index, 281
conservatism bias, 267
framing, 268
mental accounting, 268
prospect theory, 269
put/call ratio, 281
regret avoidance, 269
relative strength, 280
representativeness bias, 267
short interest, 281
trin statistic, 280

PROBLEM SETS

Basic

1. Match each example to one of the following behavioral characteristics. *(LO 9-1)*

a. Investors are slow to update their beliefs when given new evidence.	i. Disposition effect
b. Investors are reluctant to bear losses due to their unconventional decisions.	ii. Representativeness bias
c. Investors exhibit less risk tolerance in their retirement accounts versus their other stock accounts.	iii. Regret avoidance
d. Investors are reluctant to sell stocks with "paper" losses.	iv. Conservatism bias
e. Investors disregard sample size when forming views about the future from the past.	v. Mental accounting

2. After reading about three successful investors in *The Wall Street Journal* you decide that active investing will also provide you with superior trading results. What sort of behavioral tendency are you exhibiting? *(LO 9-1)*
3. What do we mean by fundamental risk, and why may such risk allow behavioral biases to persist for long periods of time? *(LO 9-2)*
4. What are the strong points of the behavioral critique of the efficient market hypothesis? What are some problems with the critique? *(LO 9-2)*
5. What are some possible investment implications of the behavioral critique? *(LO 9-1)*

KAPLAN SCHWESER

6. Jill Davis tells her broker that she does not want to sell her stocks that are below the price she paid for them. She believes that if she just holds on to them a little longer, they will recover, at which time she will sell them. What behavioral characteristic does Davis have as the basis for her decision making? *(LO 9-1)*
   a. Loss aversion
   b. Conservatism
   c. Representativeness

KAPLAN SCHWESER

7. After Polly Shrum sells a stock, she avoids following it in the media. She is afraid that it may subsequently increase in price. What behavioral characteristic does Shrum have as the basis for her decision making? *(LO 9-1)*
   a. Fear of regret
   b. Representativeness
   c. Mental accounting

KAPLAN SCHWESER

8. All of the following actions are consistent with feelings of regret *except:* *(LO 9-1)*
   a. Selling losers quickly.
   b. Hiring a full-service broker.
   c. Holding on to losers too long.

9. Which one of the following would be a bullish signal to a technical analyst using moving average rules? *(LO 9-4)*
   a. A stock price crosses above its 52-week moving average.
   b. A stock price crosses below its 52-week moving average.
   c. The stock's moving average is increasing.
   d. The stock's moving average is decreasing.

Intermediate

10. What is meant by data mining, and why must technical analysts be careful not to engage in it? *(LO 9-3)*
11. Even if prices follow a random walk, they still may not be informationally efficient. Explain why this may be true, and why it matters for the efficient allocation of capital in our economy. *(LO 9-2)*
12. What is meant by "limits to arbitrage"? Give some examples of such limits. *(LO 9-2)*
13. Following a shock to a firm's intrinsic value, the share price will slowly but surely approach that new intrinsic value. Is this view characteristic of a technical analyst or a believer in efficient markets? Explain. *(LO 9-3)*
14. Use the data from *The Wall Street Journal* in Figure 9.7 to verify the trin ratio for the NYSE. Is the trin ratio bullish or bearish? *(LO 9-4)*
15. Calculate breadth for the NYSE using the data in Figure 9.7. Is the signal bullish or bearish? *(LO 9-4)*
16. Collect data on the DJIA for a period covering a few months. Try to identify primary trends. Can you tell whether the market currently is in an upward or downward trend? *(LO 9-4)*
17. Suppose Baa-rated bonds currently yield 7%, while Aa-rated bonds yield 5%. Now suppose that due to an increase in the expected inflation rate, the yields on both bonds increase by 1%. What would happen to the confidence index? Would this be interpreted as bullish or bearish by a technical analyst? Does this make sense to you? *(LO 9-4)*
18. Table 9.3 presents price data for Computers, Inc., and a computer industry index. Does Computers, Inc., show relative strength over this period? *(LO 9-4)*
19. Use the data in Table 9.3 to compute a five-day moving average for Computers, Inc. Can you identify any buy or sell signals? *(LO 9-4)*
20. Construct a point and figure chart for Computers, Inc., using again the data in Table 9.3. Use $2 increments for your chart. Do the buy or sell signals derived from your chart correspond to those derived from the moving-average rule (see the previous problem)? *(LO 9-4)*
21. Yesterday, the Dow Jones industrials gained 54 points. However, 1,704 issues declined in price while 1,367 advanced. Why might a technical analyst be concerned even though the market index rose on this day? *(LO 9-4)*
22. Table 9.4 contains data on market advances and declines. Calculate cumulative breadth and decide whether this technical signal is bullish or bearish. *(LO 9-4)*
23. If the trading volume in advancing shares on day 1 in the previous problem was 1.1 billion shares, while the volume in declining issues was .9 billion shares, what was the trin statistic for that day? Was trin bullish or bearish? *(LO 9-4)*
24. Given the following data on bond yields, is the confidence index rising or falling? What might explain the pattern of yield changes? *(LO 9-4)*

	This Year	Last Year
Yield on top-rated corporate bonds	8%	8.5%
Yield on intermediate-grade corporate bonds	10.5	10

TABLE 9.3 Computers, Inc., stock price history

Trading Day	Computers, Inc.	Industry Index	Trading Day	Computers, Inc.	Industry Index
1	19.63	50.0	21	19.63	54.1
2	20	50.1	22	21.50	54.0
3	20.50	50.5	23	22	53.9
4	22	50.4	24	23.13	53.7
5	21.13	51.0	25	24	54.8
6	22	50.7	26	25.25	54.5
7	21.88	50.5	27	26.25	54.6
8	22.50	51.1	28	27	54.1
9	23.13	51.5	29	27.50	54.2
10	23.88	51.7	30	28	54.8
11	24.50	51.4	31	28.50	54.2
12	23.25	51.7	32	28	54.8
13	22.13	52.2	33	27.50	54.9
14	22	52.0	34	29	55.2
15	20.63	53.1	35	29.25	55.7
16	20.25	53.5	36	29.50	56.1
17	19.75	53.9	37	30	56.7
18	18.75	53.6	38	28.50	56.7
19	17.50	52.9	39	27.75	56.5
20	19	53.4	40	28	56.1

TABLE 9.4 Market advances and declines

Day	Advances	Declines	Day	Advances	Declines
1	906	704	6	970	702
2	653	986	7	1002	609
3	721	789	8	903	722
4	503	968	9	850	748
5	497	1095	10	766	766

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25. Go to **www.mhhe.com/bkm** and link to the material for Chapter 9, where you will find five years of weekly returns for the S&P 500. *(LO 9-4)*
    a. Set up a spreadsheet to calculate the 26-week moving average of the index. Set the value of the index at the beginning of the sample period equal to 100. The index value in each week is then updated by multiplying the previous week's level by (1 + rate of return over previous week).
    b. Identify every instance in which the index crosses through its moving average from below. In how many of the weeks following a cross-through does the index increase? Decrease?
    c. Identify every instance in which the index crosses through its moving average from above. In how many of the weeks following a cross-through does the index increase? Decrease?
    d. How well does the moving-average rule perform in identifying buy or sell opportunities?

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26. Go to **www.mhhe.com/bkm** and link to the material for Chapter 9, where you will find five years of weekly returns for the S&P 500 and Fidelity's Select Banking Fund (ticker FSRBX). *(LO 9-4)*

a. Set up a spreadsheet to calculate the relative strength of the banking sector compared to the broad market. (*Hint:* As in the previous problem, set the initial value of the sector index and the S&P 500 Index equal to 100, and use each week's rate of return to update the level of each index.)

b. Identify every instance in which the relative strength ratio increases by at least 5% from its value five weeks earlier. In how many of the weeks immediately following a substantial increase in relative strength does the banking sector outperform the S&P 500? In how many of those weeks does the banking sector underperform the S&P 500?

c. Identify every instance in which the relative strength ratio decreases by at least 5% from its value five weeks earlier. In how many of the weeks immediately following a substantial decrease in relative strength does the banking sector underperform the S&P 500? In how many of those weeks does the banking sector outperform the S&P 500?

d. How well does the relative strength rule perform in identifying buy or sell opportunities?

Challenge

27. One apparent violation of the Law of One Price is the pervasive discrepancy between the prices and net asset values of closed-end mutual funds. Would you expect to observe greater discrepancies on diversified or less diversified funds? Why? *(LO 9-2)*

CFA Problems

CFA® PROBLEMS

1. Don Sampson begins a meeting with his financial adviser by outlining his investment philosophy as shown below:

Statement Number	Statement
1	Investments should offer strong return potential but with very limited risk. I prefer to be conservative and to minimize losses, even if I miss out on substantial growth opportunities.
2	All nongovernmental investments should be in industry-leading and financially strong companies.
3	Income needs should be met entirely through interest income and cash dividends. All equity securities held should pay cash dividends.
4	Investment decisions should be based primarily on consensus forecasts of general economic conditions and company-specific growth.
5	If an investment falls below the purchase price, that security should be retained until it returns to its original cost. Conversely, I prefer to take quick profits on successful investments.
6	I will direct the purchase of investments, including derivative securities, periodically. These aggressive investments result from personal research and may not prove consistent with my investment policy. I have not kept records on the performance of similar past investments, but I have had some "big winners."

Select the statement from the table above that best illustrates each of the following behavioral finance concepts. Justify your selection. *(LO 9-1)*

i. Mental accounting.
ii. Overconfidence (illusion of control).
iii. Reference dependence (framing).

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2. Monty Frost's tax-deferred retirement account is invested entirely in equity securities. Because the international portion of his portfolio has performed poorly in the past, he has reduced his international equity exposure to 2%. Frost's investment adviser has recommended an increased international equity exposure. Frost responds with the following comments:
   a. Based on past poor performance, I want to sell all my remaining international equity securities once their market prices rise to equal their original cost.
   b. Most diversified international portfolios have had disappointing results over the past five years. During that time, however, the market in country XYZ has outperformed all other markets, even our own. If I do increase my international equity exposure, I would prefer that the entire exposure consist of securities from country XYZ.
   c. International investments are inherently more risky. Therefore, I prefer to purchase any international equity securities in my "speculative" account, my best chance at becoming rich. I do not want them in my retirement account, which has to protect me from poverty in my old age.

   Frost's adviser is familiar with behavioral finance concepts but prefers a traditional or standard finance approach (modern portfolio theory) to investments.

   Indicate the behavioral finance concept that Frost most directly exhibits in each of his three comments. Explain how each of Frost's comments can be countered by using an argument from standard finance. ***(LO 9-1)***
3. Louise and Christopher Maclin live in London, United Kingdom, and currently rent an apartment in the metropolitan area. During an initial discussion of the Maclins' financial plans, Christopher Maclin makes the following statements to the Maclins' financial adviser, Grant Webb:
   a. "I have used the Internet extensively to research the outlook for the housing market over the next five years, and I believe now is the best time to buy a house."
   b. "I do not want to sell any bond in my portfolio for a lower price than I paid for the bond."
   c. "I will not sell any of my company stock because I know my company and I believe it has excellent prospects for the future."

   For each statement (*a*)–(*c*) identify the behavioral finance concept most directly exhibited. Explain how each behavioral finance concept is affecting the Maclins' investment decision making. ***(LO 9-1)***
4. During an interview with her investment adviser, a retired investor made the following two statements:
   a. "I have been very pleased with the returns I've earned on Petrie stock over the past two years, and I am certain that it will be a superior performer in the future."
   b. "I am pleased with the returns from the Petrie stock because I have specific uses for that money. For that reason, I certainly want my retirement fund to continue owning the Petrie stock."

   Identify which principle of behavioral finance is most consistent with each of the investor's two statements. ***(LO 9-1)***
5. Claire Pierce comments on her life circumstances and investment outlook:

   I must support my parents who live overseas on Pogo Island. The Pogo Island economy has grown rapidly over the past two years with minimal inflation, and consensus forecasts call for a continuation of these favorable trends for the foreseeable future. Economic growth has resulted from the export of a natural resource used in an exciting new technology application.

   I want to invest 10% of my portfolio in Pogo Island government bonds. I plan to purchase long-term bonds because my parents are likely to live more than 10 years. Experts uniformly do not foresee a resurgence of inflation on Pogo Island, so I am certain that the total returns produced by the bonds will cover my parents' spending needs for many years to come. There should be no exchange rate risk because the bonds are denominated in local currency. I want to buy the Pogo Island bonds but am not willing to distort my portfolio's long-term asset allocation to do so. The overall mix of stocks, bonds, and other investments should not change. Therefore, I am considering selling one of my U.S. bond funds to raise cash to buy the Pogo Island bonds. One possibility is my High Yield Bond Fund, which has declined 5% in value year to date. I am not excited about this fund's prospects; in fact I think it is likely to decline more, but there is a small probability that it could recover very quickly. So I have decided instead to sell my Core Bond Fund that has appreciated 5% this year. I expect this investment to continue to deliver attractive returns, but there is a small chance this year's gains might disappear quickly.

Once that shift is accomplished, my investments will be in great shape. The sole exception is my Small Company Fund, which has performed poorly. I plan to sell this investment as soon as the price increases to my original cost.

Identify three behavioral finance concepts illustrated in Pierce's comments and describe each of the three concepts. Discuss how an investor practicing standard or traditional finance would challenge each of the three concepts. *(LO 9-1)*

WEB master

1. Log on to **finance.yahoo.com** to find the monthly dividend-adjusted closing prices for the most recent four years for Abercrombie & Fitch (ANF). Also collect the closing level of the S&P 500 Index over the same period.
   a. Calculate the four-month moving average of both the stock and the S&P 500 over time. For each series, use Excel to plot the moving average against the actual level of the stock price or index. Examine the instances where the moving average and price series cross. Is the stock more or less likely to increase when the price crosses through the moving average? Does it matter whether the price crosses the moving average from above or below? How reliable would an investment rule based on moving averages be? Perform your analysis for both the stock price and the S&P 500.
   b. Calculate and plot the relative strength of the stock compared to the S&P 500 over the sample period. Find all instances in which relative strength of the stock increases by more than 10 percentage points (e.g., an increase in the relative strength index from .93 to 1.03) and all those instances in which relative strength of the stock decreases by more than 10 percentage points. Is the stock more or less likely to outperform the S&P in the following two months when relative strength has increased or to underperform when relative strength has decreased? In other words, does relative strength continue? How reliable would an investment rule based on relative strength be?
2. The Yahoo! Finance charting function allows you to specify comparisons between companies by choosing the *Technical Analysis* tab. Short interest ratios are found in the *Key Statistics* table. Prepare charts of moving averages and obtain short interest ratios for GE and SWY. Prepare a one-year chart of the 50- and 200-day average price of GE, SWY, and the S&P 500 Index.
   a. Which, if either, of the companies is priced above its 50- and 200-day averages?
   b. Would you consider their charts as bullish or bearish? Why?
   c. What are the short interest ratios for the two companies?

SOLUTIONS TO CONCEPT checks

9.1 Conservatism implies that investors will at first respond too slowly to new information, leading to trends in prices. Representativeness can lead them to extrapolate trends too far into the future and overshoot intrinsic value. Eventually, when the pricing error is corrected, we observe a reversal.

9.2 Out-of-favor stocks will exhibit low prices relative to various proxies for intrinsic value such as earnings. Because of regret avoidance, these stocks will need to offer a more attractive rate of return to induce investors to hold them. Thus, low P/E stocks might on average offer higher rates of return.

Active Investment Management

PART

6

Passive investment, or indexing, is the preferred strategy for investors who believe markets are essentially efficient. While administration of passive portfolios requires efficient organization and trading structure, there is no need for security analysis or portfolio strategy. In contrast, active managers believe that markets are not efficient and that bargains can be found in security markets by application of asset valuation and portfolio theory.

Earlier in the text we asked how to effectively exploit security mispricing—how to balance superior expected returns with the increased risk from reduced diversification. In a market economy, we look to incentives: How will portfolio managers behave when evaluated using standard performance measures? They will choose portfolios that are expected to provide them with the best evaluation. Superior results, in turn, will lead to increased assets under management and higher profits. Accordingly, Chapter 18 discusses performance evaluation and techniques to achieve superior performance.

Investing across borders is conceptually a simple extension of efficient diversification. This pursuit confronts the effects of political risk and uncertain exchange rates on future performance, however. These issues, unique to international investing, are addressed in Chapter 19.

Chapter 20 covers hedge funds, probably the most active of active managers. It also focuses on some of the special problems encountered in evaluation of hedge fund performance.

Investments originate with a savings plan that diverts funds from consumption to investment. Taxes and inflation complicate the relationship between how much you save and what you will be able to achieve with your accumulating investment fund. Chapter 21 introduces a framework to formulate an effective household savings/investment plan.

Professional management of active investment begins with a contractual relationship between a client and a portfolio manager. The economic needs of clients must be articulated and their objectives translated into an operational financial plan. For this purpose, the CFA Institute has laid out a broad framework for active investment management. Chapter 22 familiarizes you with this framework.

Chapters in This Part:

Portfolio Performance Evaluation

Chapter 18

Learning Objectives:

LO18-1 Compute risk-adjusted rates of return, and use them to evaluate investment performance.

LO18-2 Determine which risk-adjusted performance measure is appropriate in a variety of investment contexts.

LO18-3 Apply style analysis to assess portfolio strategy.

LO18-4 Decompose portfolio returns into components attributable to asset allocation choices versus security selection choices.

LO18-5 Assess the presence and value of market-timing ability.

In previous chapters, we derived predictions for expected return as a function of risk. In this chapter, we ask how we can evaluate the performance of a portfolio manager accounting for portfolio risk. Adjusting average returns for risk presents a host of issues because the proper measure of risk may not be obvious and risk levels may change along with portfolio composition.

We begin with conventional approaches to risk adjustment. These use the risk measures developed earlier to rank investment results. We show the problems with these approaches when you try to apply them in a real and complex world. Finally, we examine evaluation procedures used in the field. We show how overall investment results are decomposed and attributed to the underlying asset allocation and security selection decisions of the portfolio manager.

We finally turn to two specific forms of active management: market timing based solely on macroeconomic factors, and security selection based on microeconomic forecasting.

Related websites for this chapter are available at www.mhhe.com/bkm.

18.1 RISK-ADJUSTED RETURNS

Investment Clients, Service Providers, and Objectives of Performance Evaluation

Individual households as well as institutional money managers must decide whether to use passive or active management. **Passive management** involves (1) capital allocation between **cash** (almost-risk-free vehicles such as money market funds) and the chosen risky portfolio constructed from one or more index funds or ETFs.

passive management

Holding a well-diversified portfolio without attempting to search out security mispricing.

cash

Shorthand for virtually risk-free money market securities.

Still, the concept of passive management is not completely unambiguous. At one extreme, passive investors will commit to capital allocation with a fixed risky portfolio and change their allocations only infrequently in response to significant changes in circumstances or risk tolerance. At the other extreme, they will adjust portfolio weights based on estimates of risk derived, for example, from VIX (the volatility indexes, discussed in Chapter 16) or from other sources.

Alternatively, households and institutional endowments may choose **active management,** in which case they usually become clients of professional portfolio managers.[1] The dividing line between passive and active management is the forecasting of future rates of return on asset classes and/or individual assets. Such forecasting is more difficult than estimation of risk by an order of magnitude. The reason for this is quite subtle and is lost on many professional as well as novice investors. Competition among the vast number of investors means that security prices generally reflect publicly available information. Thus, successful forecasting of future prices and rates of return requires differential *private* information. To estimate risk, on the other hand, investors can freely and quite easily use publicly available information, making these estimates a commodity. Accordingly, we call active managers those who forecast returns in conjunction with risk to construct optimal portfolios. A few professionals restrict their activity to *market timing* (switching between risky portfolios and cash), some concentrate on asset allocation only, and most engage in both asset allocation and security selection.[2]

active management

Attempts to achieve returns higher than commensurate with risk by forecasting broad markets and/or by identifying mispriced securities.

Both clients and professionals are interested in performance evaluation. Clients need to know whether their chosen professionals produce adequate net-of-fee returns. Professionals need to shore up their methodology and maintain qualified staff with adequate compensation to compete in the market for these services. Lapses in performance can cost them dearly as evidence shows that funds under management flow quickly from underachievers to superperformers.

Performance evaluation of a portfolio is difficult because of the great volatility of asset returns. A portfolio's average return over an evaluation period is inadequate to measure performance. To begin with, the average return realized over any particular period may not represent the *expected* return. Surely, luck (good *or* bad) should not be allowed to dominate the evaluation process. Even when the average return does approximate expected return, it still would be invalid as a measure of performance because it ignores risk—we expect higher-risk investments to outperform lower-risk ones in average to boom markets and to underperform in bear markets. Hence, we must estimate portfolio risk to determine the adequacy of the average return. Since volatility generates statistical errors in estimates of both expected return and risk, we must remain skeptical of the evaluation process.

Comparison Groups

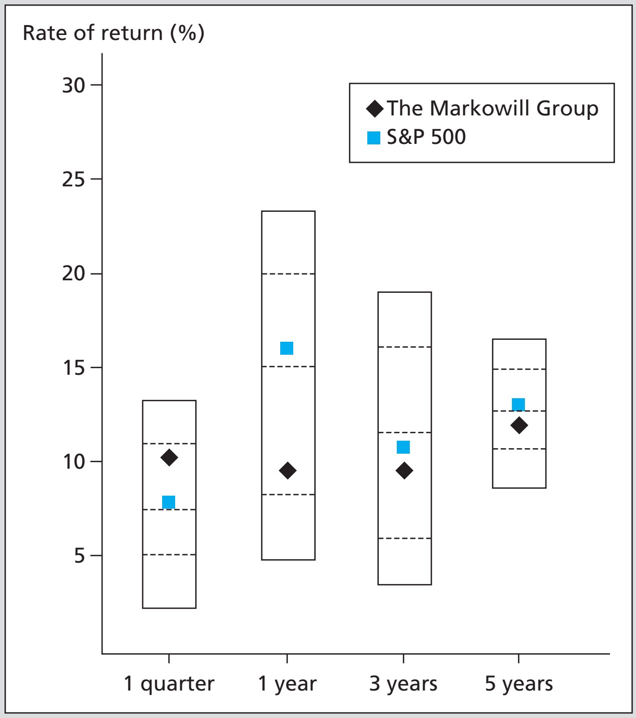
The simplest and most popular way to adjust returns for portfolio risk is to compare rates of return with those of other investment funds with similar risk characteristics. For example, high-yield bond portfolios are grouped into one "universe," growth stock equity funds are grouped into another universe, and so on. Then the average returns of each fund within the

[1] Households and institutional endowments that conduct active management in-house become their own clients. The adage that a lawyer who represents himself has a fool for a client doesn't necessarily apply here.

[2] Many professional managers are prohibited from extensive market timing by a prospectus or contract that fixes a range of allowed weights in cash instruments.

FIGURE 18.1

Universe comparison: Periods ending December 31, 2012



comparison universe

The set of portfolio managers with similar investment styles that is used to assess relative performance.

universe are ordered, and each portfolio manager receives a percentile ranking depending on relative performance within the **comparison universe,** the collection of funds to which performance is compared. For example, the manager with the ninth-best performance in a universe of 100 funds would be the 90th percentile manager: Her performance was better than 90% of all competing funds over the evaluation period.

These relative rankings usually are displayed in a chart like Figure 18.1. The chart summarizes performance rankings over four periods: one quarter, one year, three years, and five years. The top and bottom lines of each box are drawn at the rate of return of the 95th and 5th percentile managers. The three dotted lines correspond to the rates of return of the 75th, 50th (median), and 25th percentile managers. The diamond is drawn at the average return of a particular fund, the Markowill Group, and the square is drawn at the average return of a benchmark index such as the S&P 500. This format provides an easy-to-read representation of the performance of the fund relative to the comparison universe.

This comparison with other managers of similar investment groups is a useful first step in evaluating performance. Even so, such rankings can be misleading. Consider that within a particular universe some managers may concentrate on particular subgroups, so that portfolio characteristics are not truly comparable. For example, within the equity universe, one manager may concentrate on high-beta stocks. Similarly, within fixed-income universes, interest rate risk can vary across managers. These considerations suggest that we need more precise risk adjustment.

Basic Performance-Evaluation Statistics

Performance evaluation relies on the index model discussed in Sections 6.5 and 7.2 and on the CAPM of Section 7.1. The single-index model equation applied to a portfolio P is

$$R_{Pt} = \beta_P R_{Mt} + \alpha_P + e_{Pt} \tag{18.1}$$

where $R_{Pt} = r_{Pt} - r_{ft}$ is portfolio P's excess return over cash equivalents during period t, r_{ft} is the return on cash, and R_{Mt} is the excess return on the market index. β_P is the portfolio's sensitivity to the market index, hence its measure of systematic risk, and $\beta_P R_{Mt}$ is the component of return that is driven by the market. The extra-market or nonsystematic component, $\alpha_P + e_{Pt}$, includes the portfolio alpha plus zero-mean noise, e, called the *residual,* which is uncorrelated with R_M. Thus, the expected excess return of the portfolio for some evaluation period is

$$E(R_P) = \beta_P E(R_M) + \alpha_P \tag{18.2}$$

We measure *expected* returns over the period (unfortunately, with sampling error) by *average* return.

The CAPM hypothesis is that the market portfolio is mean-variance efficient. The index model uses an index portfolio, M, to proxy for the theoretical market portfolio, and hence it is the benchmark passive strategy against which competing portfolios are measured. The CAPM hypothesis is that the alpha of all securities and competing portfolios is zero. A professional who claims to outperform the index must produce a positive alpha; the validity of the CAPM doesn't preclude some professionals from doing so, as long as the totality of investments that exhibit positive alpha is not large relative to aggregate wealth in the economy.

What about portfolio risk? As noted above, beta measures *systematic* risk since the variance of the market-driven return component is

$$\text{Var}(\beta_P R_{Mt}) = \beta_P^2 \sigma_M^2 \tag{18.3}$$

and the term σ_M^2 is the same for all portfolios. The extra-market component of return contributes the quantity $\text{Var}(e_P)$ to portfolio variance. The standard deviation of the residual return e, which we will denote here as σ_e, is called *residual risk* or *residual SD*. The variance of the return on P is thus the sum of the variances (since the systematic and residual components are uncorrelated):

$$\sigma_P^2 = \beta_P^2 \sigma_M^2 + \sigma_e^2 \tag{18.4}$$

We may now prepare the statistics that are used for performance evaluation of a portfolio P from a sample of observations over an interval of T periods (usually months). The procedure includes the following steps:

1. Obtain the time series of R_{Pt} for portfolio P, and R_{Mt} for the benchmark M.
2. Compute the arithmetic averages of the series $\overline{R}_P$, $\overline{R}_M$. These are taken as estimates of the expected returns of portfolios P and M for the evaluation period.
3. Compute the standard deviations of returns for portfolios P and M, σ_P and σ_M. These serve as estimates of the total risk of P and M.
4. Run a regression of R_{Pt} on R_{Mt} to obtain estimates of P's beta, alpha, residual SD, and correlation with the benchmark. Check the significance statistics to see that the sample is reasonable. In particular, if the beta coefficient estimate is not significant, the sample may be insufficient for the performance-evaluation statistics discussed below.
5. Recall from Equation 18.2 that the regression intercept is P's alpha, $\alpha_P = \overline{R}_P - \beta_P \overline{R}_M$.
6. Recall from Equation 18.4 that the standard error, or residual standard deviation, of the regression is $\sigma_e = \text{SQRT}(\sigma_P^2 - \beta_P^2 \sigma_M^2)$.

Table 18.1 presents performance-evaluation statistics for two professionally managed portfolios, P and Q, the benchmark, M, and cash. Notice that P is aggressive with a beta of 1.25. Q might be a hedge fund, not completely market-neutral (which would entail a beta of zero), but still with a defensive beta of .5. Thus, most of the volatility of Q is due to its residual SD.

Performance Evaluation of Entire-Wealth Portfolios Using the Sharpe Ratio and M-Square

Consider The Diabetes Foundation, a small charity, whose board has decided to invest its endowment in one of the three portfolios of Table 18.1. In this case, the total risk of the chosen portfolio will also be the endowment's risk. Accordingly, the familiar **Sharpe ratio,**

Sharpe ratio

Reward-to-volatility ratio; ratio of portfolio excess return to standard deviation.

$$S = \frac{\overline{R}}{\sigma} \tag{18.5}$$

TABLE 18.1 Performance of two managed portfolios, *P* and *Q*, the benchmark portfolio, *M*, and cash equivalents

	Portfolio *P*	Portfolio *Q*	Benchmark	Cash
Average return	13.6	9.5	10.4	4
Average excess return (%)	9.60	5.50	6.37	0
Standard deviation (%)	24.1	18.0	18.5	0
Beta (pure number)	1.25	0.50	1.0	0
Alpha (%)	1.6	2.3	0	0
Residual SD (%)	6.79	15.44	0	0
Correlation with benchmark	0.96	0.51	1	0
Sharpe ratio	0.398	0.306	0.344	0
M-square (%)	1.00	−0.72	0	0
Treynor measure	7.68	11.00	6	0
Information ratio	0.24	0.15	0	0

which measures risk by total volatility (SD), must determine the choice. Table 18.1 shows that the Sharpe ratio of portfolio P (.398) is highest; hence P would be the charity's choice. Notice that P's average return is sufficiently large to compensate for the fact that it is the highest SD portfolio; conversely, although Q is the least volatile, its Sharpe ratio is the lowest.

The Sharpe ratio has a clear interpretation, namely, the incremental return an investor may expect for every increase of 1% of standard deviation. It is the slope of the capital allocation line supported by that portfolio. But should investors consider the difference in Sharpe ratios between portfolio P and the benchmark portfolio M (.398 − .344 = .054) large? That is harder to interpret and leads us to a variant on the Sharpe ratio.

Imagine a portfolio with the same standard deviation as the benchmark, σ_M. Then the difference between the Sharpe ratios of the portfolio and the benchmark would be the difference in their risk premiums divided by that common standard deviation. Put differently, ranking portfolios with a common volatility by Sharpe ratio will be equivalent to ranking them very simply by risk premium—estimated from the sample excess returns, as in Equation 18.5. This makes comparison of portfolios with equal standard deviation easy to interpret.

Can we transform P to an equivalent portfolio with the same standard deviation as the benchmark, σ_M, without affecting its Sharpe ratio? Yes, we can: Recall that the slope of P's CAL is the Sharpe ratio of *all* portfolios on that line. Therefore, we just choose the portfolio on CAL_P that has standard deviation σ_M. All portfolios on CAL_P are mixtures of portfolio P with risk-free borrowing or lending. When we invest a weight w in P and $1 - w$ in the risk-free asset, we just slide up (when $w > 1$) or down (when $w < 1$) the CAL. Call P^* the portfolio created by mixing P with the risk-free asset in just the right proportion to make the standard deviation match that of the benchmark. In other words, portfolio P^* is portfolio P with just the right amount of leverage to make the standard deviation match that of the benchmark.

We form P^* by choosing $w = \sigma_M/\sigma_P$ because this makes the SD of P^* equal to $w\sigma_P = \sigma_M$. The risk premium of portfolio P^* therefore can be written in terms of the Sharpe ratio of P:

$$\overline{R}_{P*} = w\overline{R}_P = \frac{\sigma_M}{\sigma_P}\overline{R}_P = \sigma_M S_P$$

Similarly, the risk premium of the benchmark can be written in terms of its Sharpe ratio:

$$\overline{R}_M = \sigma_M \frac{\overline{R}_M}{\sigma_M} = \sigma_M S_M$$

M-square (M^2)

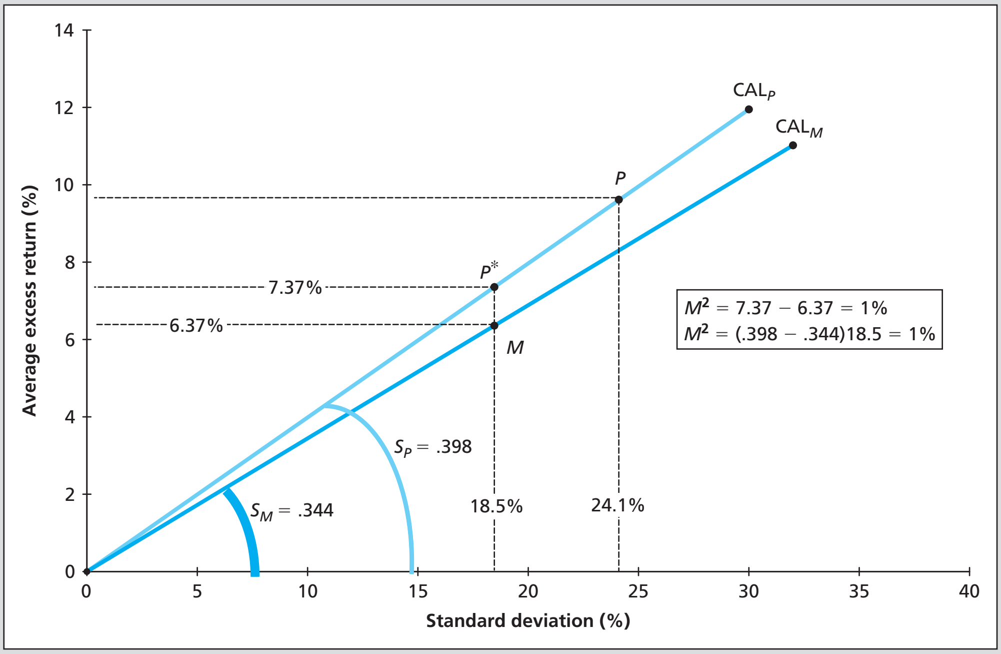
Return difference between a managed portfolio leveraged to match the volatility of a passive index and the return on that index.

The difference between the risk premium of P^*, the leverage-adjusted version of P, and the benchmark is known as **M-square** (after Leah and Franco Modigliani)[3] and is written M^2.

[3]The M-square measure was developed independently by Graham and Harvey (1997) and by Modigliani and Modigliani (1997).

FIGURE 18.2

M^2 of portfolio P



$$M^2 = \overline{R}_{P*} - \overline{R}_M = \sigma_M(S_P - S_M) \quad \textbf{(18.6)}$$

M-square is the rate-of-return differential between P^* and M, a legitimate and easy-to-interpret performance measure because the portfolios are volatility-matched. Equation 18.6 shows us that it also is a simple transformation of the difference between their Sharpe ratios. Table 18.1 and Figure 18.2 show this vividly.

Suppose that instead of investing all its funds in P, the endowment had invested only $\sigma_M/\sigma_P = 18.5/24.1 = .7676$ or 76.76% of its funds, with the remainder placed in risk-free assets. The average excess return on this P^* would have been $.7676 \times 9.6\% = 7.37\%$. Thus M-square of P is $7.37\% - 6.37\% = 1\%$. We also could find this measure directly from the difference in Sharpe ratios: $18.5\% \times (.398 - .344) = 1\%$. (For practice, verify that the M-square of portfolio Q is .72%.)

Performance Evaluation of Fund of Funds Using the Treynor Measure

We have used the Sharpe ratio, or its variant M-square, to choose between an actively managed portfolio competing with a passive benchmark as the sole risky position for an endowment. But some funds are so large that they engage *several* managers to run risky component portfolios. For example, CalPERS (the California Public Employee Retirement System) is a large pension fund with around $220 billion to invest in September 2011. Like many large plans, it uses a **fund of funds** approach, allocating the endowment among a number of professional managers (funds) based in part on performance. This requires a different performance measure.

fund of funds

Mutual funds or hedge funds that invest in other funds.

To see why, suppose CalPERS considers two managers, and it so happens that both establish portfolios with average returns, beta, and residual standard deviation equal to those of portfolio Q in Table 18.1. If Q were considered as the *sole* investment portfolio, the endowment fund would reject it because it has a negative M-square of −.72%. But if the residuals of the two Q-like portfolios are uncorrelated, the residual SD of an equally weighted portfolio of the two would be only $\sqrt{2 \times (\frac{1}{2} \times 15.44)^2} = 10.92\%$. In turn, the total portfolio SD would be $\sqrt{(\beta\sigma_M)^2 + \text{Var}(e)} = \sqrt{(.5 \times 18.5)^2 + 10.92^2} = 14.31\%$. With the same average return as Q, the Sharpe ratio of the combined portfolio is $5.5/14.31 = .384$, and its M-square is *positive*: $18.5(.384 - .344) = .74\%$. The improvement is due to the benefits of diversification that arise when we combine the two funds into an equally weighted portfolio. With residual risk lessened due to diversification, the trade-off of excess return to total volatility is enhanced.

This exercise suggests that for a fund of funds, where residual risk can be largely diversified away, we should compare average excess return to nondiversifiable or *systematic*, rather than total, risk. Since beta measures systematic risk, Treynor (1965) proposed the following measure, since named after him:

$$T = \frac{\overline{R}}{\beta} \tag{18.7}$$

Treynor measure

Ratio of portfolio excess return to beta.

As Table 18.1 demonstrates, the **Treynor measure** can differ from Sharpe's, suggesting that the proper performance measure depends on the role of the risky position in the investor's overall portfolio.

Performance Evaluation of a Portfolio Added to the Benchmark Using the Information Ratio

Now we consider yet another scenario, one in which an endowment considers adding a position in an actively managed portfolio to an already existing passive portfolio. The Central State University endowment has so far been a passive investor. Presented with a positive alpha achieved by the managers of P and Q, the board decides to add a position in just one of the portfolios. Which should it choose? To answer, we must choose the portfolio which, when combined with the benchmark, generates the higher Sharpe ratio. In Section 6.5 we saw that the key to this problem is the **information ratio.**

information ratio

Ratio of alpha to the standard deviation of diversifiable risk.

We can calculate the Sharpe ratio of the optimized portfolio from the Sharpe ratio of the benchmark M, and the information ratio of the added portfolio using Equation 6.17, which we repeat here:

$$S_O = \text{SQRT}\left[S_M^2 + \left(\frac{\alpha_P}{\sigma_P}\right)^2\right] \tag{18.8}$$

where $\frac{\alpha_P}{\sigma_P}$ is the information ratio of portfolio P. Table 18.1 indicates that the information ratio of P, .24, is higher than that of Q, .15. Equation 18.8 tells us that the Sharpe ratio of the optimized portfolio using P will be .42, but only .38 using Q—still better than the benchmark's .34. Once again, we see that the role of the evaluated portfolio in the investor's complete portfolio determines the choice of performance measure. A different measure can lead to different judgment of superiority. The following table summarizes our conclusions:

Performance Measure	Definition	Application
Sharpe	$\dfrac{\text{Excess return}}{\text{Standard deviation}}$	When choosing among portfolios competing for the overall risky portfolio
Treynor	$\dfrac{\text{Excess return}}{\text{Beta}}$	When ranking many portfolios that will be mixed to form the overall risky portfolio
Information ratio	$\dfrac{\text{Alpha}}{\text{Residual standard deviation}}$	When evaluating a portfolio to be mixed with the benchmark portfolio

The Relation of Alpha to Performance Measures

Jensen measure

The alpha of an investment.

Alpha, also known as the **Jensen measure** after Michael Jensen, who first proposed it, appears everywhere in performance evaluation; why then, did we not present it as a performance measure? To answer, we must see its relation in the three performance measures.

When short sales are allowed, a negative alpha is as good as, or even better than, a positive one when constructing the optimal portfolio. A negative, or short, position in a negative-alpha stock will turn the alpha positive. If the stock's beta is positive, a negative position in it will also reduce systematic and therefore overall risk. Thus, a negative-alpha, positive-beta stock serves double duty in optimizing a portfolio.[4] However, if short sales are prohibited, a negative-alpha stock is no better than a zero-alpha one because it must be ignored.

The foregoing not withstanding, when it comes to performance evaluation, we judge *ex-post* (after the fact) portfolio returns. We cannot directly observe the manager's *ex-ante* (before the fact) expectations. We know that the manager must have judged the alpha positive ex-ante. But forecasting errors or just bad luck could have driven an ex-ante positive-alpha portfolio into negative-alpha territory. Thus, in performance evaluation, a negative realized alpha must be taken to indicate below-average performance.

The relation of the Jensen measure to the Sharpe and Treynor measures can be gleaned from Equation 18.2. Substituting the right-hand side of the equation for the average excess return and employing some manipulation, we find that the Sharpe measure is

$$S_P = \frac{\overline{R}_P}{\sigma_P} = \frac{\beta_P \overline{R}_M}{\sigma_P} + \frac{\alpha_P}{\sigma_P}; \;\; \beta_P = \rho\frac{\sigma_P}{\sigma_M}$$

$$S_P = S_M \rho + \frac{\alpha_P}{\sigma_P} \tag{18.9}$$

$$S_P - S_M = S_M(\rho - 1) + \frac{\alpha_P}{\sigma_P}$$

where ρ is the correlation between the excess return of P and the benchmark. First, observe that alpha alone does not determine which portfolio has a larger Sharpe ratio. The standard deviation of P and its correlation with the benchmark are also important. Thus positive alpha is *not* a sufficient condition for a managed portfolio to offer a higher Sharpe measure than the passive benchmark.

While it is not sufficient, a positive alpha is *necessary* to obtain a higher Sharpe ratio than the benchmark's S_M, because $S_M(\rho - 1)$ is negative. Superior performance in this context is a stiff challenge because, to achieve a positive alpha, it is necessary to construct a portfolio that is different from the benchmark. But this, in turn, will increase residual risk (which lowers the correlation coefficient) and offset the improvement in alpha. Notice that portfolio Q has a larger alpha, 2.3%, than P, 1.6%. Moreover, its ratio of alpha to standard deviation ($2.3/18 = .128$) is far greater than P's ($1.6/24.1 = .066$). Despite all this, Q's Sharpe ratio is smaller because its correlation coefficient with M is low (.51) compared with P's (.96).

The Treynor measure, which measures performance of a portfolio within a fund of funds, also is related to the portfolio alpha via Equation 18.2 as follows:

$$T_P = \frac{\overline{R}_P}{\beta_P} = \frac{\beta_P \overline{R}_M + \alpha_P}{\beta_P} = \overline{R}_M + \frac{\alpha_P}{\beta_P}$$

$$\beta_M = 1 \quad T_M = \overline{R}_M \tag{18.10}$$

$$T_P - T_M = \frac{\alpha_P}{\beta_P}$$

[4]Since a negative-beta stock is a rarity, negative alpha is generally better than a positive one when short sales are allowed.

$\overline{R}_M$ is common to all portfolios; therefore, the relative rank of T_P is determined by the ratio $\frac{\alpha_P}{\beta_P}$. Thus here, too, a positive alpha is necessary but not sufficient to rank alternative active portfolios; we also need to know beta.

Finally, to complete the list, a positive alpha is needed to increase the Sharpe ratio of any portfolio to which the measured one is added. At the stage of constructing a portfolio, negative-alpha securities are useful because one can take a short position in them. But in performance evaluation we are asking whether the realized return on a manager's portfolio suggests we should employ the manager in the future to construct a piece of our overall portfolio. We are willing to do so only if the manager's forecasts have translated to realized returns with a positive alpha.

However, here, too, alpha alone cannot rank portfolios, since a portfolio with lower alpha but also lower residual risk still can be judged of better overall performance (Sharpe ratio). We can be sure, though, that a negative alpha indicates inferior performance by all performance measures.

CONCEPT check 18.1

Consider the following data for a particular sample period when returns were high:

	Portfolio *P*	Market *M*
Average return	35%	28%
Beta	1.2	1.0
Standard deviation	42%	30%

Calculate alpha and the three performance measures for portfolio *P* and the market. The T-bill rate during the period was 6%. By which measures did portfolio *P* outperform the market?

Alpha Capture and Alpha Transport

alpha capture

Construction of a positive-alpha portfolio with all systematic risk hedged away.

alpha transfer or alpha transport

Establishing alpha while using index products both to hedge market exposure and to establish exposure to desired sectors.

In the next chapter, we will see that many hedge funds seek positive alpha with zero beta. They wish to obtain abnormal returns without taking a stance on the direction of the broad market. Even if a portfolio is *relatively* underpriced, it may still suffer losses if it falls with the market. The solution is to hedge out the market exposure of the portfolio by selling either the stock index or stock-index futures. This long stock–short index strategy provides a *market neutral* position while maintaining the positive alpha and is therefore called **alpha capture.** With the captured alpha, you can establish any desired sensitivity to particular market sectors using index products such as ETFs.

This last procedure is called **alpha transfer** or **alpha transport,** because you transfer alpha from the sector where you find it to the market sector in which you seek exposure. Finding alpha requires skill. By contrast, beta, or market exposure, is a "commodity product" that can be supplied cheaply through index funds.

EXAMPLE 18.1

Alpha Capture and Transport

Zeta, a portfolio manager, established a positive-alpha portfolio *P* with a positive exposure to the market index: $\beta_{PM} = 1.3$. Now she wishes to transfer the alpha. Her objective is a portfolio that is market neutral but with positive exposure to the health care sector. In other words, she wants to "transport" her positive-alpha portfolio from a broad market exposure to a narrow health care exposure, a sector she believes will outperform. Her goal is a zero-net-investment position with a beta of zero on the market index but with a beta of .5 on a health care sector index.

We call Zeta's final portfolio *Z,* which will be constructed from positions in the original positive-alpha portfolio *P,* the market index portfolio *M,* the health care index portfolio *H,* and the risk-free asset *F.* Zeta will first isolate alpha by neutralizing *P*'s market beta. She will then use a health care sector index portfolio to establish her desired exposure to health care. In the end, she wants her final portfolio *Z* to have a zero beta on the broad market, $\beta_{ZM} = 0$, and a beta of .5 on health care, $\beta_{ZH} = .5$.

Zeta's statistical analysis implies that a health care exchange-traded fund, XLV, has a market beta, $\beta_{XLV} = .9$. Therefore, as she establishes exposure to the health care portfolio, she will also

(continued)

take on market exposure, and this too must be hedged away. Therefore, as Table 18.2 shows, she must take a position in the market index sufficiently large to offset the beta of portfolio P as well as the additional market exposure created by her position in the health care ETF. The hedging strategy that creates pure exposure to the health care sector is similar to the hedging of factor exposures that we encountered in the discussion of the arbitrage pricing theory (see Tables 7.5 and 7.9).[5]

EXAMPLE 18.1

Alpha Capture and Transport (concluded)

An important issue that is often lost when evaluating ex-post alpha is statistical significance. After all, even if the true alpha is zero, you expect to estimate a positive alpha in roughly 50% of the evaluated portfolios (and a negative alpha in the other 50%). Given capital market volatility, it is fair to expect that even truly nonzero alphas often would be statistically insignificant. We would be more inclined to believe a nonzero alpha of a portfolio manager is a real phenomenon if it persists over time. Take a look at Figure 8.8 of Chapter 8. Unfortunately, the graph suggests that persistence of alpha is mostly found in negative-alpha portfolios, and little is evident in portfolios of positive alpha.

Performance Evaluation with a Multi-Index Model

The Fama-French (FF) three-factor model discussed in Section 7.4 has almost completely replaced the single-index model in academic performance evaluation, and has been gaining "market share" in the investment services industry.[6] Evidence in favor of augmenting the market index with the size (SMB) and value (HML) portfolios is compelling. How should this affect performance evaluation?

Expanding Equation 18.1 to include the size and value factors, we have,[7]

$$R_{Pt} = \beta_P R_{Mt} + \beta_{SMB} r_{SMBt} + \beta_{HML} r_{HMLt} + \alpha_P + e_{Pt}$$
$$\overline{R}_{Pt} = \beta_P \overline{R}_{Mt} + \beta_{SMB} \overline{r}_{SMBt} + \beta_{HML} \overline{r}_{HMLt} + \alpha_P \tag{18.11}$$

Equation 18.11 states that expected return is determined by betas on three factors, not just by beta relative to the market index. Notice that the index portfolio, M, has zero alpha; if you regress R_M on the three right-hand-side portfolios, it will be completely explained by the first factor, R_M, and so it will have an intercept of zero. The same applies to SMB and HML, and thus any portfolio formed from one or more index portfolios will have zero alpha.

TABLE 18.2 Alpha capture and transfer to the health care sector

Portfolio Weight*	In Asset	Contribution to Excess Returns
$w_P = 1$	P	$w_P(\alpha_P + \beta_{PM} R_M + e_P) = \alpha_P + 1.3 R_M + e_P$
$w_{XLV} = .5$	XLV	$w_{XLV} R_{XLV} = .5(.9 R_M + e_{XLV}) = .45 R_M + .5e_{XLV}$
$w_M = -\beta_P - .5\beta_{XLV}$ $= -1.75$	M	$w_M R_M = -1.75 R_M$
$w_F = -1 - .5 + 1.75$	Risk-free	0
0	Portfolio Z	$\alpha_P + e_P + .5e_{XLV}$

*If P's alpha is negative, then reverse the sign of w_P and adjust the signs of w_M and w_F.

[5]However, in this application, portfolio Z is *not* an arbitrage portfolio; it is not likely to be even approximately well diversified. The idea is to hedge all systematic exposures except for that to health care specific risk.

[6]The three FF factors (market, SMB, and HML) sometimes are augmented by a momentum portfolio (long in recent losers and short in recent gainers) and/or by a liquidity portfolio (long in liquid and short in illiquid stocks).

[7]Notice that we replace uppercase R (which usually denotes an excess return relative to the risk-free rate) with lowercase r for the SMB and HML factors because these portfolios already are excess returns, for example, small-stock returns over large-stock returns. These are zero-net-investment portfolios (for example, long small stocks and short large stocks), and thus have an opportunity cost of zero rather than r_f.

EXCEL APPLICATIONS

Performance Measures

eXcel

Please visit us at www.mhhe.com/bkm

The Excel model "Performance Measures" calculates all of the performance measures discussed in this chapter. The model available on our website is built to allow you to compare eight different portfolios and to rank them on all measures discussed in this chapter.

	A	B	C	D	E	F	G	H	I	J	K
1	**Performance Measurement**										
2											
3											
4											
5		**Average**	**Standard**	**Beta**	**Unsystematic**	**Sharpe**	**Treynor**	**Jensen**	**M^2**	**T^2**	**Appraisal**
6	**Fund**	**Return**	**Deviation**	**Coefficient**	**Risk**	**Ratio**	**Measure**	**Alpha**	**Measure**	**Measure**	**Ratio**
7	Alpha	.2800	.2700	1.7000	.0500	0.8148	.1294	-.0180	-.0015	-.0106	-0.3600
8	Omega	.3100	.2600	1.6200	.0600	0.9615	.1543	.0232	.0235	.0143	0.3867
9	Omicron	.2200	.2100	0.8500	.0200	0.7619	.1882	.0410	-.0105	.0482	2.0500
10	Millennium	.4000	.3300	2.5000	.2700	1.0303	.1360	-.0100	.0352	-.0040	-0.0370
11	Big Value	.1500	.1300	0.9000	.0300	0.6923	.1000	-.0360	-.0223	-.0400	-1.2000
12	Momentum Watcher	.2900	.2400	1.4000	.1600	0.9583	.1643	.0340	.0229	-.0243	0.2125
13	Big Potential	.1500	.1100	0.5500	.0150	0.8182	.1636	.0130	-.0009	.0236	0.8667
14	S&P Index Return	.2000	.1700	1.0000	.0000	0.8235	.1400	.0000	.0000	.0000	0.0000
15	T-Bill Return	.06		0							
16											
17	**Ranking by Sharpe**										
18		**Return**	**S.D.**	**Beta**	**Unsy. Risk**	**Sharpe**	**Treynor**	**Jensen**	**M^2**	**T^2**	**Appraisal**
19	Millennium	.4000	.3300	2.5000	.2700	1.0303	.1360	-.0100	.0352	-.0040	-0.0370
20	Omega	.3000	.2600	1.6200	.0600	0.9615	.1543	.0232	.0235	.0143	0.3867
21	Momentum Watcher	.2900	.2400	1.4000	.1600	0.9583	.1643	.0340	.0229	.0243	0.2125
22	S&P Index Return	.2000	.1700	1.0000	.0000	0.8235	.1400	.0000	.0000	.0000	0.0000
23	Big Potential	.1500	.1100	0.5500	.0150	0.8182	.1636	.0130	-.0009	.0236	0.8667
24	Alpha	.2800	.2700	1.7000	.0500	0.8148	.1294	-.0180	-.0015	-.0106	-0.3600
25	Omicron	.2200	.2100	0.8500	.0200	0.7619	.1882	.0410	-.0105	.0482	2.0500
26	Big Value	.1500	.1300	0.9000	.0300	0.6923	.1000	-.0360	-.0223	-.0400	-1.2000
27											
28	**Ranking by Treynor**										
29		**Return**	**S.D.**	**Beta**	**Unsy. Risk**	**Sharpe**	**Treynor**	**Jensen**	**M^2**	**T^2**	**Appraisal**
30	Omicron	.2200	.2100	0.8500	.0200	0.7619	.1882	.0140	-.0105	.0482	2.0500
31	Momentum Watcher	.2900	.2400	1.4000	.1600	0.9583	.1643	.0340	.0229	.0243	0.2125
32	Big Potential	.1500	.1100	0.5500	.0150	0.8182	.1636	.0130	-.0009	.0236	0.8667
33	Omega	.3100	.2600	1.6200	.0600	0.9615	.1543	.0232	.0235	.0143	0.3867
34	S&P Index Return	.2000	.1700	1.0000	.0000	0.8235	.1400	.0000	.0000	.0000	0.0000
35	Millennium	.4000	.3300	2.5000	.2700	1.0303	.1360	-.0100	.0352	-.0040	-0.0370
36	Alpha	.2800	.2700	1.7000	.0500	0.8148	.1294	-.0180	-.0015	-.0106	-0.3600

Excel Questions

1. Examine the performance measures of the funds included in the spreadsheet. Rank the funds by the five performance measures. Are the rankings across funds consistent? What explains these results?
2. Which fund would you choose if you were considering investing the entire risky portion of your portfolio? What if you were considering adding a small position in one of these funds to a portfolio invested in the market index?

In a three-factor security market as described by Equation 18.11, the market index is no longer the single efficient portfolio. Instead, we can use as our benchmark (default) portfolio the combination of factor portfolios that maximizes the Sharpe ratio. Once this benchmark is identified, the Treynor-Black method can be deployed: First identify the optimal active portfolio based on alpha values from security analysis; then mix the active portfolio with the said benchmark to find the optimal risky portfolio. This implies that the information ratio from the multifactor equation is the appropriate performance measure for an active portfolio to be added to the multifactor benchmark.

Failure to recognize the multi-index equation (when valid) in favor of a misspecified single-index equation can lead one to overestimate performance. Apparent alpha values that reflect the impact of omitted factors will be mistaken for superior performance.

Recent research by Cremers, Petajisto, and Zitzewitz (2010) shows that indexes such as the S&P 500 and Russell 2000 demonstrate significant nonzero alphas when evaluated using the FF model even when a momentum factor is added. The problem in finding adequate passive benchmarks tells us that performance evaluation is really (after more than 40 years) still in its infancy and our inferences should elicit some healthy skepticism.

18.2 STYLE ANALYSIS

Style analysis was introduced by Nobel Laureate William Sharpe (1992). The popularity of the concept was aided by a widely cited study (Brinson et al., 1991) concluding that 91.5% of the variation in returns of 82 mutual funds could be explained by the funds' asset allocation to bills, bonds, and stocks. Later studies that considered asset allocation across a broader range of asset classes found that as much as 97% of fund returns can be explained by asset allocation alone.

Sharpe considered 12 asset class (style) portfolios. His idea was to regress fund returns on indexes representing a range of asset classes. The regression coefficient on each index would then measure the implicit allocation to that "style." Because funds are barred from short positions, the regression coefficients are constrained to be either zero or positive and to sum to 100%, so as to represent a complete asset allocation. The R-square of the regression would then measure the percentage of return variability due to style choice rather than security selection. Finally, in this regression there is no intercept, and residuals are not constrained to sum to zero. This sum equals the *total return from security selection.* This feature allows us to track the cumulative residual and observe how return from security selection evolves over time.

To illustrate the approach, consider Sharpe's study of the monthly returns on Fidelity's Magellan Fund over the period January 1985 through December 1989, shown in Table 18.3. While there are 12 asset classes, each one represented by a stock index, the regression coefficients are positive for only four of them. We can conclude that the fund returns are well explained by only four style portfolios. Moreover, these four style portfolios alone explain 97.3% of the variance of returns.

The proportion of return variability *not* explained by asset allocation can be attributed to security selection within asset classes. For Magellan, this was $100 - 97.3 = 2.7\%$. To evaluate the average contribution of stock selection to fund performance we track the residuals from the regression, displayed in Figure 18.3. The figure plots the cumulative effect of these residuals; the steady upward trend confirms Magellan's success at stock selection in this period. Notice that the plot in Figure 18.3 is far smoother than the plot in Figure 18.4, which shows Magellan's performance compared to a standard benchmark, the S&P 500. This reflects the fact that the regression-weighted index portfolio tracks Magellan's overall style much better than the S&P 500. The performance spread is much noisier using the S&P as the benchmark.

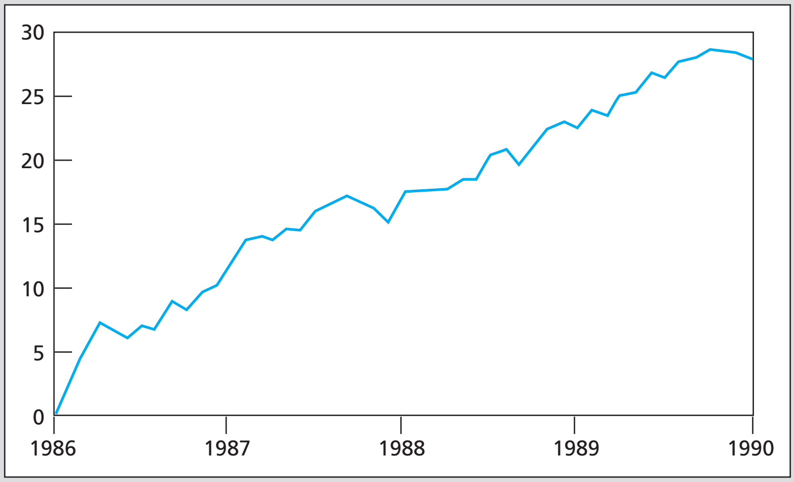
TABLE 18.3 Sharpe's style portfolios for the Magellan fund

	Regression Coefficient*
Bills	0
Intermediate bonds	0
Long-term bonds	0
Corporate bonds	0
Mortgages	0
Value stocks	0
Growth stocks	47
Medium-cap stocks	31
Small stocks	18
Foreign stocks	0
European stocks	4
Japanese stocks	0
Total	100
R-squared	97.3%

*Regressions are constrained to have nonnegative coefficients and to have coefficients that sum to 100%.

Source: William F. Sharpe, "Asset Allocation: Management Style and Performance Evaluation," *Journal of Portfolio Management,* Winter 1992, pp. 7–19. Used with permission of Institutional Investor, Inc., **www.iijournals.com.** All Rights Reserved.

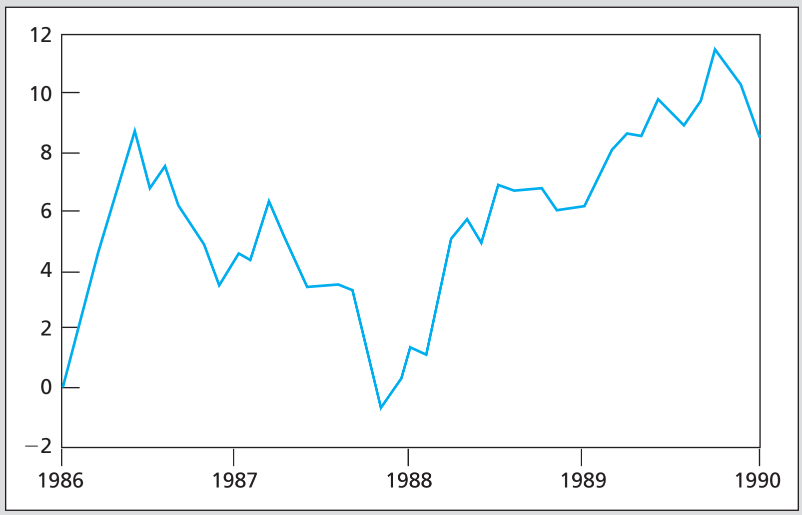
FIGURE 18.3



Fidelity Magellan Fund cumulative return difference: Fund versus style benchmark

Source: William F. Sharpe, "Asset Allocation: Management Style and Performance Evaluation," *Journal of Portfolio Management*, Winter 1992, pp. 7–19. Figure 17, p. 18. Used with permission of Institutional Investor, Inc., **www.iijournals.com.** All Rights Reserved.

FIGURE 18.4



Fidelity Magellan Fund cumulative return difference: Fund versus S&P 500

Source: William F. Sharpe, "Asset Allocation: Management Style and Performance Evaluation," *Journal of Portfolio Management*, Winter 1992, pp. 7–19. Figure 16, p. 17. Used with permission of Institutional Investor, Inc., **www.iijournals.com.** All Rights Reserved.

Of course, Magellan's consistently positive residual returns (reflected in the steadily increasing plot of cumulative return difference) is hardly common. Figure 18.5 shows the frequency distribution of average residuals across 636 mutual funds. The distribution has the familiar bell shape with a slightly negative mean of −.074% per month.

Style analysis has become very popular in the investment management industry and has spawned quite a few variations on Sharpe's methodology. Many portfolio managers utilize websites that help investors identify their style and stock selection performance. The nearby box shows that style analysis is at the heart of recent debates about the investment performance of hedge funds.

18.3 MORNINGSTAR'S RISK-ADJUSTED RATING

The commercial success of Morningstar, Inc., the premier source of information on mutual funds, has made its *Risk Adjusted Rating* (RAR) among the most widely used performance measures. The Morningstar five-star rating is coveted by the managers of the thousands of funds covered by the service.

On the MARKET FRONT

WHAT'S IT ALL ABOUT, ALPHA?

Too many notes. That's what Emperor Joseph II famously said to Mozart on seeing his opera "The Marriage of Figaro." But surely to think of a musical work as just a series of notes is to miss the magic.

Could the same be said about fund management? It is the fashion these days to separate beta (the systematic return delivered by the market) from alpha (the manager's skill). Investors are happy to pay high fees for the skill, but regard the market return as a commodity. Distinguishing the two is, however, sometimes difficult.

A fund manager might beat the market because of luck or recklessness, rather than skill, for example. Suppose he has packed his portfolio with oil stocks and then profits when the price of crude rises. More generally, alpha skeptics often attribute abnormal returns to "style bias," such as [the manager who favors stocks with an energy focus. Popular style biases are often based on factors that seem to have predicted past alpha, such as firm size.] But should the skeptics be biased against style bias? After all, the only portfolio utterly free of bias would be one that included the entire market.

Academics have entered this debate, trying to pin down the factors that drive a fund's performance. Bill Fung and Narayan Naik of London Business School have come up with a seven-factor model which, they say, can explain the bulk of hedge-fund performance. After allowing for these factors, the average fund of hedge funds has not produced any alpha in the past decade, except during the dot-com bubble. This approach suggests the whole idea of alpha might be an illusion.

However, it is also possible to take the opposite tack. This type of analysis gives managers no credit for choosing the systematic factors—the betas—that drive their portfolios. Yes, these betas could often have been bought for very low fees. But would an investor have been able to put them together in the right combination?

It is as if a diner in Gordon Ramsay's restaurants were brave enough to tell the irascible chef: "This meal was delicious. But chemical analysis shows it is 65% chicken, 20% carrot, 10% flour and 5% milk. I could have bought those ingredients for £1.50. Why should I pay £20?" The chef's reply, shorn of its expletives, might be: "The secret is in the mixing." This debate matters because people are now trying to replicate the performance of hedge funds with cloned portfolios.

There are two potential criticisms of the cloned approach. One is that it will simply reproduce all the systematic returns that hedge funds generate and none of their idiosyncratic magic. However, this "magic" is hard to pin down, and even if it does exist, it may be worth no more than the fees hedge funds charge.

The second criticism is that the clones will always be a step behind the smart money. You cannot clone a hedge fund until you know where it has been. But by then it may have moved on.

Mozart might have sympathized. His operas were more than the sum of his notes. But even if the great composer had no peers, he has had plenty of imitators.

SOURCE: Excerpted from *The Economist,* March 22, 2007.

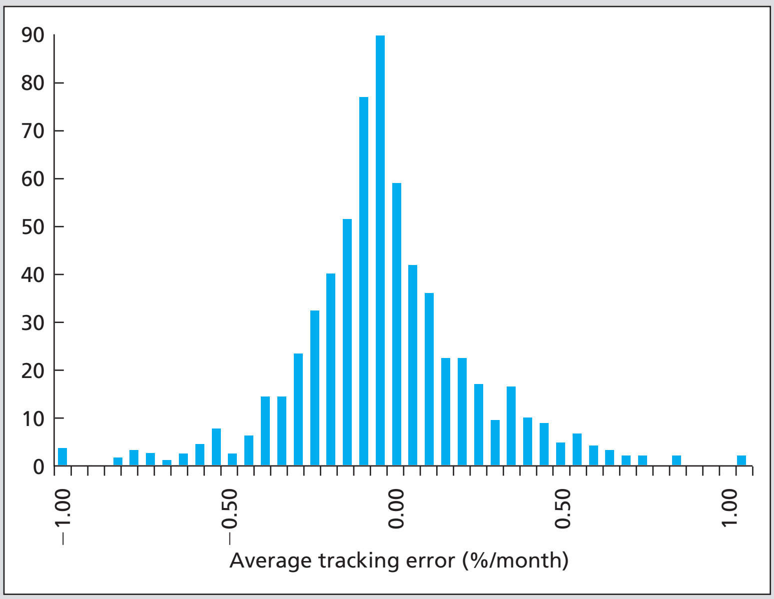


FIGURE 18.5

Average tracking error, 636 mutual funds, 1985–1989

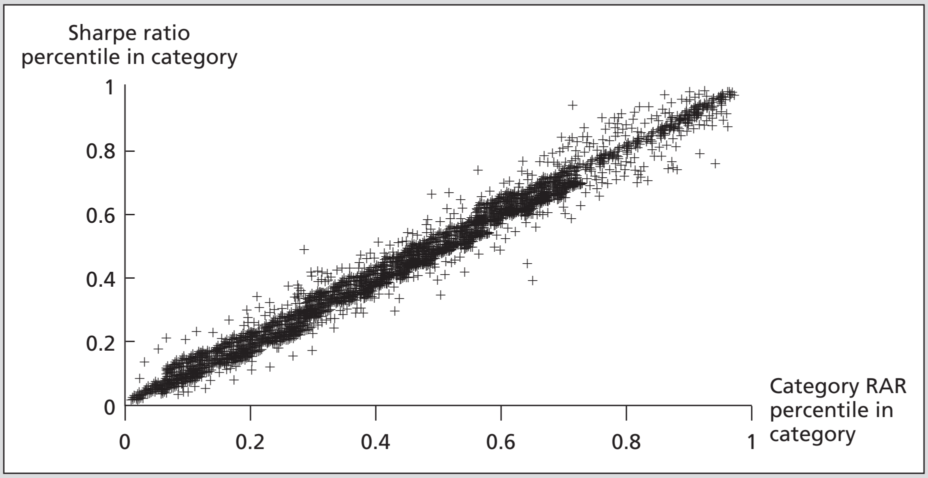
Source: William F. Sharpe, "Asset Allocation: Management Style and Performance Evaluation," *Journal of Portfolio Management,* Winter 1992, pp. 7–19. Figure 18, p. 18. Used with permission of Institutional Investor, Inc., **www.iijournals.com.** All Rights Reserved.

Morningstar calculates a number of RAR performance measures that are similar, although not identical, to the standard mean-variance measures (see Chapter 4 for a more detailed discussion). The most distinct measure, the Morningstar Star Rating, is based on comparison of each fund to a peer group. The peer group for each fund is selected on the basis of the fund's investment

FIGURE 18.6

Rankings based on Morningstar's category RARs and excess return Sharpe ratios

Source: William F. Sharpe (1997), "Morningstar Performance Measures," **www.stanford.edu/~ wfsharpe/art/stars/stars0.htm.** Used with permission.



universe (e.g., international, growth versus value, fixed-income) as well as portfolio characteristics such as average price-to-book value, price–earnings ratio, and market capitalization.

Morningstar computes fund returns (adjusted for loads) as well as a risk measure based on fund performance in its worst years. The risk-adjusted performance is ranked across funds in a style group, and stars are awarded based on the following table:

Percentile	Stars
0–10	1
10–32.5	2
32.5–67.5	3
67.5–90	4
90–100	5

The Morningstar RAR method produces results that are similar but not identical to that of the mean/variance-based Sharpe ratios. Figure 18.6 demonstrates the fit between ranking by RAR and by Sharpe ratios from the performance of 1,286 diversified equity funds over the period 1994–1996. Sharpe notes that this period is characterized by high returns that contribute to a good fit.

18.4 RISK ADJUSTMENTS WITH CHANGING PORTFOLIO COMPOSITION

One potential problem with risk-adjustment techniques is that they all assume that portfolio risk, whether it is measured by standard deviation or beta, is constant over the relevant time period. This isn't necessarily so. If a manager attempts to increase portfolio beta when she thinks the market is about to go up and to decrease beta when pessimistic, both the standard deviation and the beta of the portfolio will change over time. This can wreak havoc with our performance measures.

EXAMPLE 18.2

Risk Measurement with Changing Portfolio Composition

Suppose the Sharpe measure of the passive strategy (investing in a market-index fund) is .4. A portfolio manager is in search of a better, active strategy. Over an initial period of, say, four quarters, he executes a low-risk or defensive strategy with an annualized mean excess return of 1.5% and a standard deviation of 3.4%. This makes for a Sharpe measure of .44, which beats the passive strategy.

(continued)

EXAMPLE 18.2

Risk Measurement with Changing Portfolio Composition (concluded)

Over the next period of another four quarters, this manager finds that a high-risk strategy is optimal, with an annual mean excess return of 8.75% and standard deviation of 20%. Here again the Sharpe measure is .44. Over the two years, our manager maintains a better-than-passive Sharpe measure.

Figure 18.7 shows a pattern of (annualized) quarterly returns that is consistent with our description of the manager's strategy over two years. In the first four quarters, the excess returns are −3%, 5%, 1%, and 3%, consistent with the predicted mean and SD. In the next four quarters, the excess returns are −9%, 27%, 25%, and −8%, also consistent with predictions for the higher-volatility period. Thus, *each* year exhibits a Sharpe measure of .44.

But if we treat the eight-quarter sequence as a single measurement period instead of two independent periods, the portfolio's mean and standard deviation over the full period are 5.125% and 13.8% respectively, resulting in a Sharpe measure of only .37, apparently inferior to the passive strategy!

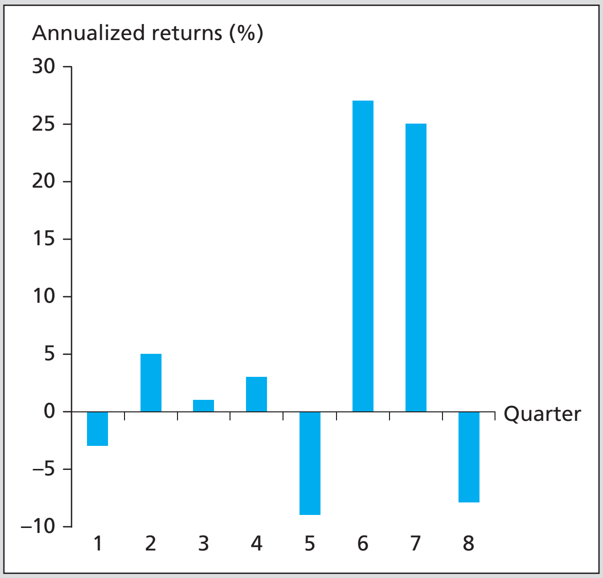


FIGURE 18.7

Portfolio returns. In the first four quarters, the firm follows a low-risk, low-return policy. In the next four quarters, it shifts to a high-risk, high-return policy.

What went wrong in Example 18.2? Sharpe's ratio does not recognize the shift in the mean from the first four quarters to the next as a result of a strategy change. Instead, the difference in mean returns in the two years adds to the *appearance* of volatility in portfolio returns. The change in mean returns across time periods contributed to the variability of returns over the same period. Unfortunately, an outside observer cannot tell that policy changes within the sample period are the source of some of the return variability. Therefore, the active strategy with shifting means appears riskier than it really is, which biases the estimate of the Sharpe measure downward.

When assessing the performance of actively managed portfolios, it is important to keep track of portfolio composition and changes in portfolio mean return and risk. We will see another example of this problem when we turn to market timing.

Another warning: When we address the performance of mutual funds selected *because* they have been successful, we need to be highly cautious in evaluating their track records. In particular, we need to recognize that even if all managers were equally skilled, a few "winners" would emerge by sheer chance each period. With thousands of funds in operation, the best-performing funds will have been wildly successful, even if these results reflect luck rather than skill. The nearby box addresses this issue. Another manifestation of selection bias arises when we limit a sample of funds to those for which returns are available over an entire sample period. This practice implies that we exclude from consideration all funds that were closed down over the sample period. The ensuing bias is called **survivorship bias.** It turns out that when even a small number of funds have failed, the upward bias in the performance of surviving funds can be substantial. Most mutual fund databases now include failed funds so that samples can be protected from survivorship bias.

survivorship bias

Upward bias in average fund performance due to the failure to account for failed funds over the sample period.

Performance Manipulation

Imagine a manager whose performance is measured over two-year return periods, as in Example 18.2, for which the Sharpe ratio in each year is .44. Now we are at the end of the first year, when the manager's portfolio has returned the aforementioned four annualized quarterly rates of −3%, 5%, 1%, and 3%, providing the assumed Sharpe ratio of .44.

At this point the manager identifies the high-risk but better-than-passive strategy of the example; but he recognizes that following this strategy, he can expect a losing Sharpe ratio of .37. But what if the portfolio is "de-levered" by shifting $^5/_6$ of its value into bills. With this shift to safety (and zero excess return), the four second-year returns would be −1.5%, 4.5%, 4.17%, and −1.33%. Because the Sharpe ratio is invariant to shifts between the risky portfolio and risk-free asset, the second-year Sharpe ratio is still .44. Despite this, the eight-quarter Sharpe ratio is now evaluated at .47.[8] So far, little damage has been wrought. But given that the prospectuses of many funds promise general investment strategies (for example, investing in equity), they may not allow significant investments in bills. In that event, managers may reduce risk by moving into low-beta stocks. This distorts their security selection decisions, and investors end up with less-than-optimal portfolios due to the manipulation of the Sharpe ratio.

This type of manipulation is only one in a menu that includes investments in derivatives. These strategies also can be used to manipulate the other performance measures discussed earlier in the chapter. A manipulation-free performance measure exists, but since it hasn't yet penetrated the industry, we leave it for future consideration.

18.5 PERFORMANCE ATTRIBUTION PROCEDURES

Rather than focus on risk-adjusted returns, practitioners often want simply to ascertain which decisions resulted in superior or inferior performance. Superior investment performance depends on an ability to be in the "right" securities at the right time. Timing and selection ability may be considered broadly, such as being in equities as opposed to fixed-income securities when the stock market is performing well. Or it may be defined at a more detailed level, such as choosing the relatively better-performing stocks within a particular industry.

Portfolio managers constantly make both broad-brush asset market allocation decisions as well as more detailed sector and security allocation decisions within markets. Performance attribution studies attempt to decompose overall performance into discrete components that may be identified with a particular level of the portfolio selection process.

Attribution analysis starts from the broadest asset allocation choices and progressively focuses on ever-finer details of portfolio choice. The difference between a managed portfolio's performance and that of a benchmark portfolio may be expressed as the sum of the contributions to performance of a series of decisions made at the various levels of the construction process. For example, one common attribution system decomposes performance into three components: (1) broad asset market allocation choices *across* equity, fixed-income, and money markets; (2) industry (sector) choice *within* each market; and (3) security choice within each sector.

To illustrate this method, consider the attribution results for a hypothetical portfolio. The portfolio invests in stocks, bonds, and money market securities. The portfolio return over the period is 5.34%. An attribution analysis appears in Tables 18.4 through 18.7.

bogey

The benchmark portfolio an investment manager is compared to for performance evaluation.

The first step is to establish a benchmark level of performance against which performance ought to be compared. This benchmark is called the **bogey.** It is the portfolio designed to fit the fund prospectus in the absence of any active choice, in other words, the fund's passive or default portfolio. A fund's prospectus likely dictates a "normal" range for investment weights, say 50%–70% in equities, 20%–40% in bonds, and 0%–10% in bills. We will use the middle of these ranges to form the bogey, with a neutral asset allocation of 60% in equities, 30% in bonds, and 10% in bills, as shown in Table 18.4.

The prospectus also likely specifies benchmarks for each asset class. For instance, the S&P 500 often serves as the benchmark for equity investments, and the Barclays Capital U.S. Aggregate Bond Index may be used for fixed income. Table 18.4 shows the returns on

[8]The two-year Sharpe ratio can be higher than that of each of the two individual years in part because of the bias correction to the SD (having to do with degrees of freedom). Instead of multiplying each annual estimate of SD by 4/3, we multiply the two-year estimate by "only" 8/7.

THE MAGELLAN FUND AND MARKET EFFICIENCY: ASSESSING THE PERFORMANCE OF MONEY MANAGERS

Fidelity's Magellan Fund outperformed the S&P 500 in eleven of the thirteen years ending in 1989. Is such performance consistent with the efficient market hypothesis? Casual statistical analysis would suggest not.

If outperforming the market were like flipping a fair coin, as would be the case if all securities were fairly priced, then the odds of an arbitrarily selected manager producing eleven out of thirteen winning years would be only about 0.95%, or 1 in 105. The Magellan Fund, however, is not a randomly selected fund. Instead, it is the fund that emerged after a thirteen-year "contest" as a clear winner. Given that we have chosen to focus on the winner of a money management contest, should we be surprised to find performance far above the mean? Clearly not.

Once we select a fund precisely because it has outperformed all other funds, the proper benchmark for predicted performance is no longer a standard index such as the S&P 500. The benchmark must be the expected performance of the best-performing fund out of a sample of randomly selected funds.

Consider as an analogy a coin flipping contest. If fifty contestants were to flip a coin thirteen times, and the winner were to flip eleven heads out of thirteen, we would not consider that evidence that the winner's coin was biased. Instead, we would recognize that with fifty contestants, the probability is greater than 40% that the individual who emerges as the winner would in fact flip heads eleven or more times. (In contrast, a coin chosen at random that resulted in eleven out of thirteen heads would be highly suspect!)

How then ought we evaluate the performance of those managers who show up in the financial press as (recently) superior performers. We know that after the fact some managers will have been lucky. When is the performance of a manager so good that even after accounting for selection bias—the selection of the ex post winner—we still cannot account for such performance by chance?

SELECTION BIAS AND PERFORMANCE BENCHMARKS

Consider this experiment. Allow fifty money managers to flip a coin thirteen times, and record the maximum number of heads realized by any of the contestants. (If markets are efficient, the coin will have the same probability of turning up heads as that of a money manager beating the market.) Now repeat the contest, and again record the winning number of heads. Repeat this experiment 10,000 times. When we are done, we can compute the frequency distribution of the winning number of heads over the 10,000 trials.

Table 1 (column 1) presents the results of such an experiment simulated on a computer. The table shows that in 9.2% of the contests, the winning number of heads was nine; in 47.4% of the trials ten heads would be enough to emerge as the best manager. Interestingly, in 43.3% of the trials, the winning number of heads was eleven or better out of thirteen.

TABLE 1

PROBABILITY DISTRIBUTION OF NUMBER OF SUCCESSFUL YEARS OUT OF THIRTEEN FOR THE BEST-PERFORMING MONEY MANAGER

	Managers in Contest			
Winning Years	**50**	**100**	**250**	**500**
8	0.1%	0	0	0
9	9.2	0.9	0	0
10	47.4	31.9	5.7	0.2
11	34.8	51.3	59.7	42.3
12	7.7	14.6	31.8	51.5
13	0.8	1.2	2.8	5.9
Mean winning years of best performer	10.43	10.83	11.32	11.63

Viewed in this context, the performance of Magellan is still impressive but somewhat less surprising. The simulation shows that out of a sample of 50 managers, chance alone would provide a 43.3% probability that *someone* would beat the market at least eleven out of thirteen years. Averaging over all 10,000 trials, the mean number of winning years necessary to emerge as most reliable manager over the thirteen-year contest was 10.43.

Therefore, once we recognize that Magellan is not a fund chosen at random, but a fund that came to our attention precisely because it turned out to perform so well, the frequency with which it beat the market is no longer high enough to be considered a violation of market efficiency. Indeed, using the conventional 5% confidence level, we could not reject the hypothesis that the consistency of its performance was due to chance.

The other columns in Table 1 present the frequency distributions of the winning number of successful coin flips (analogously, the number of years in which the best-performing manager beats an efficient market) for other possible sample sizes. Not surprisingly, as the pool of managers increases, the predicted best performance steadily gets better. By providing as a benchmark the probability distribution of the best performance, rather than the average performance, the table tells us how many grains of salt to add to reports of the latest investment guru.

SOURCE: Alan J. Marcus, "The Magellan Fund and Market Efficiency." *The Journal of Portfolio Management,* Fall 1990, pp. 85–86. Used with permission of Institutional Investor, Inc., **www.iijournals.com.**

TABLE 18.4 Performance of the managed portfolio

Bogey Performance and Excess Return

Component	Benchmark Weight	Return of Index during Month (%)
Equity (S&P 500)	.60	5.81
Bonds (U.S. Aggregate Index)	.30	1.45
Cash (money market)	.10	0.48
Bogey = (.60 × 5.81) + (.30 × 1.45) + (.10 × .48) = 3.97%		
Return of managed portfolio		5.34%
–Return of bogey portfolio		3.97
Excess return of managed portfolio		1.37%

these benchmarks for the relevant period. The neutral asset allocation, along with the returns on the benchmark indexes, generates the bogey return shown in Table 18.4, 3.97%. The table also records the actual portfolio return, 5.34%. The difference between actual and bogey returns, 1.37%, is the excess return of the managed portfolio. We next try to measure the relative contributions of asset allocation versus security selection decisions to this advantageous performance.

Asset Allocation Decisions

The managed portfolio is actually invested in the equity, fixed-income, and money markets with weights of 70%, 7%, and 23%, respectively. The portfolio's performance could be due to the departure of this weighting scheme from the benchmark 60/30/10 weights and/or to superior or inferior results *within* each of the three broad markets.

To isolate the effect of the manager's asset allocation choice, we measure the performance of a hypothetical portfolio that would have been invested in the *indexes* for each market with the actual weights of 70/7/23. This return measures the effect of the shift away from the benchmark 60/30/10 weights without allowing for any effects attributable to active management of the securities selected within each market.

Superior performance relative to the bogey is achieved by overweighting investments in markets that outperform the bogey and by underweighting poorly performing markets. The contribution of asset allocation to superior performance equals the sum over all markets of the excess weight in each market times the return of the index for each sector.

Table 18.5A demonstrates that asset allocation contributed 31 basis points to the portfolio's excess return of 137 basis points. The major contribution of asset allocation to superior performance in this period comes from the heavy weighting of the equity market when the equity market has an excellent return of 5.81%.

Sector and Security Selection Decisions

If .31% of the excess performance can be attributed to advantageous asset allocation across markets, the remaining 1.06% then must be attributable to sector selection and security selection within each market. Table 18.5B details the contribution of the managed portfolio's sector and security selection to total performance.

Panel B shows that the equity component of the managed portfolio has a return of 7.28% versus a return of 5.81% for the S&P 500. The fixed-income return is 1.89% versus 1.45% for the Aggregate Bond Index. The superior performance in both equity and fixed-income markets weighted by the portfolio proportions invested in each market sums to the 1.06% contribution to performance attributable to sector and security selection.

Table 18.6 documents the sources of the equity market performance by each sector within the market. The first three columns detail the allocation of funds within the equity market

TABLE 18.5 Performance attribution

A. Contribution of Asset Allocation to Performance

Market	(1) Actual Weight in Market	(2) Benchmark Weight in Market	(3) Excess Weight	(4) Index Return (%)	(5) = (3) × (4) Contribution to Performance (%)
Equity	.70	.60	.10	5.81	.5810
Fixed-income	.07	.30	−.23	1.45	−.3335
Cash	.23	.10	.13	0.48	.0624
Contribution of asset allocation					.3099

B. Contribution of Selection to Total Performance

Market	(1) Portfolio Performance (%)	(2) Index Performance (%)	(3) Excess Performance (%)	(4) Portfolio Weight	(5) = (3) × (4) Contribution (%)
Equity	7.28	5.81	1.47	.70	1.03
Fixed-income	1.89	1.45	0.44	.07	0.03
Contribution of selection within markets					1.06

TABLE 18.6 Sector allocation within the equity market

	(1)	(2)	(3)	(4)	(5) = (3) × (4)
	Beginning-of-Month Weights				
Sector	Portfolio	S&P 500	Difference in Weights	Sector Return (%)	Contribution of Sector Allocation (%)
Basic materials	0.0196	0.083	−.0634	6.9	−0.437
Business services	0.0784	0.041	.0374	7.0	0.262
Capital goods	0.0187	0.078	−.0593	4.1	−0.243
Consumer cyclical	0.0847	0.125	−.0403	8.8	−0.355
Consumer noncyclical	0.4037	0.204	.1997	10.0	1.997
Credit sensitive	0.2401	0.218	.0221	5.0	0.111
Energy	0.1353	0.142	−.0067	2.6	−0.017
Technology	0.0195	0.109	−.0895	0.3	−0.027
Total	1.0000	1.000	.0000		1.290

compared to their representation in the S&P 500. Column (4) shows the rate of return of each sector, and column (5) equals the product of the difference in the sector weight and the sector's performance.

Note that good performance derives from overweighting well-performing sectors such as consumer noncyclicals, as well as underweighting poorly performing sectors such as technology. The excess return of the equity component of the portfolio attributable to sector allocation alone is 1.29%. As the equity component of the portfolio outperformed the S&P 500 by 1.47%, we conclude that the effect of security selection within sectors must have contributed an additional 1.47 − 1.29, or .18%, to the performance of the equity component of the portfolio.

A similar sector analysis can be applied to the fixed-income portion of the portfolio, but we do not show those results here.

EXCEL APPLICATIONS

Performance Attribution

eXcel

Please visit us at www.mhhe.com/bkm

The Excel model "Performance Attribution" that is available on our website is built on the example that appears in Section 18.5. The model allows you to specify different allocations and to analyze the contribution sectors and weightings for different performances.

	A	B	C	D	E	F
1			**Chapter 18**			
2			**Performance Attribution**			
3	**Solution to Question**				**Contribution**	
4	**Bogey Portflio**		**Weight**	**Return on**	**to Portfolio**	
5	**Component**	**Index**	**Benchmark**	**Index**	**Return**	
6	Equity	S&P500	0.6	5.8100%	3.4860%	
7	Bonds	Aggregate Index	0.3	1.4500%	0.4350%	
8	Cash	Money Market	0.1	0.4800%	0.0480%	
9						
10	Return on Bogey				3.9690%	
11						
12				**Contribution**		
13	**Managed Portfolio**	**Portfolio**	**Actual**	**to Portfolio**		
14	**Component**	**Weight**	**Return**	**Return**		
15	Equity	.75	6.5000%	4.8750%		
16	Bonds	.12	1.2500%	0.1500%		
17	Cash	.13	0.4800%	0.0624%		
18						
19	Return on Managed			5.0874%		
20						
21	Express Return			1.1184%		
22						
23						
24			**Contribution of Asset Allocation**			
25		**Actual Weight**	**Benchmark**	**Excess**	**Market**	**Performance**
26	**Market**	**in Portfolio**	**Weight**	**Weight**	**Return**	**Contribution**
27	Equity	.75	.6	.15	5.8100%	.8715%
28	Fixed Income	.12	.3	-.18	1.4500%	-.2610%
29	Cash	.13	.1	.03	0.4800%	.0144%
30	Contribution of					
31	Asset Allocation					.6249%

Excel Questions

1. What would happen to the contribution of asset allocation to overall performance if the actual weights had been 70/17/13 in the three markets rather than 75/12/13? Explain your result.
2. Show what would happen to the contribution of security selection to performance if the actual return on the equity portfolio had been 7.5% instead of 6.5% and the return on the S&P 500 had been 6.81% instead of 5.81%. Explain your result.

Summing Up Component Contributions

In this attribution period, all facets of the portfolio selection process were successful. Table 18.7 details the contribution of each aspect of performance. Asset allocation across the major security markets contributes 31 basis points. Sector and security allocation within those markets contributes 106 basis points, for total excess portfolio performance of 137 basis points.

The sector and security allocation of 106 basis points can be partitioned further. Sector allocation within the equity market results in excess performance of 129 basis points, and security selection within sectors contributes 18 basis points. (The total equity excess performance of 147 basis points is multiplied by the 70% weight in equity to obtain the contribution to portfolio performance.) Similar partitioning could be done for the fixed-income sector.

CONCEPT check 18.2

a. Suppose the benchmark weights had been set at 70% equity, 25% fixed-income, and 5% cash equivalents. What then would be the contributions of the manager's asset allocation choices?

b. Suppose the S&P 500 return had been 5%. Recompute the contribution of the manager's security selection choices.

TABLE 18.7 Portfolio attribution: summary

		Contribution (basis points)
1. Asset allocation		31.0
2. Selection		
a. Equity excess return		
i. Sector allocation	129	
ii. Security selection	18	
	147 × .70 (portfolio weight) =	102.9
b. Fixed-income excess return	44 × .07 (portfolio weight) =	3.1
Total excess return of portfolio		137.0

18.6 MARKET TIMING

Pure **market timing** involves switching funds between the risky portfolio and cash in response to forecasts of relative performance. To evaluate the potential of a pure market-timing strategy, consider the fortunes of three families of investors who had $1 to invest on December 1, 1926. Their heirs counted their blessings 82 years later, in 2008. The investment history of the families included the Great Depression, a major bear market in 2008 (when the S&P 500 lost 39%), and seven other recessions in between. The families differed wildly in their investment strategy:

market timing

A strategy that moves funds between the risky portfolio and cash, based on forecasts of relative performance.

1. Family A invested solely in a money market or cash equivalents.
2. Family B invested solely in stocks (the S&P 500 portfolio), reinvesting all dividends.
3. Family C switched, *every month,* 100% of its funds between stocks and cash, based on its forecast of which sector would do better next month.

While the strategies of families A and B are straightforward, that of family C is worth pondering. Try asking friends: "What would it take to be a perfect market timer?" Many would venture that to accomplish perfect timing, the timer would need to be able to forecast the rate of return on stocks at the start of every month. But actually, you wouldn't need the precise rate of return: "All" the perfect timer would have to know is whether stocks will outperform cash! You might think that such elementary knowledge wouldn't be worth all that much. But examine Table 18.8, computed from the actual return history on cash and stocks.

The first panel of Table 18.8 provides the punch line: After 82 years, $1 returned $20 to the cash fund of family A, and most of those nominal profits were undone by inflation over the period. Despite the Great Depression and recessions of varying severity, the stock fund of family B outdid the cash fund by a factor of more than 80, ending up with $1,626. But the gains to the perfect-timing family C would have been otherworldly indeed (as was the family's power of prediction); the timing fund starting with $1 would have ended with $36.7 *billion.*

CONCEPT check 18.3

Use annual rates of return from the Online Learning Center (**www.mhhe.com/bkm**) to replicate Table 18.8 for the 1926–2008 period for a market timer who could perfectly forecast only once a year, rather than every month. Why is the performance of the annual timer not as good as that of the monthly timer?

These results have some lessons for us. The first has to do with the power of compounding. This effect is particularly important as ever more funds under management represent pension savings. The horizons of pension investments may not be as long as 82 years, but they are measured in decades, making compounding an important factor.

TABLE 18.8 Performance of cash, stocks, and perfect-timing strategies

	A. Cash	B. Stocks	C. Perfect Timing
I. Family fund as of the end of 2008			
Final proceeds	$20	$1,626	$36,699,302,473
II. Annualized monthly rate-of-return statistics (%)			
Geometric average	3.71	9.44	34.54
Arithmetic average	3.71	11.48	35.44
Minimum monthly rate*	−0.03	−28.73	−0.03
Maximum monthly rate†	1.52	41.65	41.65
Average excess return	0.00	7.77	31.73
Standard deviation	3.54	19.38	12.44

*Occurred in September 1931.

†Occurred in April 1933.

Both extreme values occurred during the Great Depression.

The second is a huge difference between the end value of the all-safe asset strategy ($20) and of the all-equity strategy ($1,626). Why would anyone invest in safe assets? By now you know the reason: risk. The annual standard deviation of the equity strategy was 19.38%. The high standard deviation of the return on the equity portfolio is commensurate with its significantly higher average return. The higher average excess return reflects the long-term risk premium.

Is the return premium on the perfect-timing strategy also a risk premium? It can't be: Because the perfect timer never does worse than either bills or the market, the extra return cannot be compensation for the possibility of poor returns; instead it is attributable to superior analysis. The value of superior information is reflected in the tremendous ending value of the portfolio. This value does not reflect compensation for risk.

Consider how you might choose between two hypothetical strategies. Strategy 1 offers a sure rate of return of 5%; strategy 2 offers an uncertain return that is given by 5% *plus* a random number that is equally likely to be either 0% or 5%. The results for each strategy are:

	Strategy 1 (%)	Strategy 2 (%)
Expected return	5	7.5
Standard deviation	0	2.5
Highest return	5	10.0
Lowest return	5	5.0

Clearly, strategy 2 dominates strategy 1, as its rate of return is *at least* equal to that of strategy 1 and sometimes greater. No matter how risk averse you are, you will always prefer strategy 2 to strategy 1, even though strategy 2 has a significant standard deviation. Compared to strategy 1, strategy 2 provides only good surprises, so the standard deviation in this case cannot be a measure of risk.

You can look at these strategies as analogous to the case of the perfect timer compared with either an all-equity or all-cash strategy. In every period, the perfect timer obtains at least as good a return, in some cases better. Therefore, the timer's standard deviation is a misleading measure of risk when you compare perfect timing to an all-equity or all-cash strategy.

Valuing Market Timing as an Option

Merton (1981) shows that perfect market timing can be viewed as a call option on the market index in this sense: Investing 100% in T-bills plus holding a call option on the equity portfolio will yield returns identical to those of the portfolio of the perfect timer who invests 100% in either the safe asset or the equity portfolio, whichever will yield the higher return. The perfect timer's return is shown in Figure 18.8. The rate of return is bounded from below by the risk-free rate, r_f.

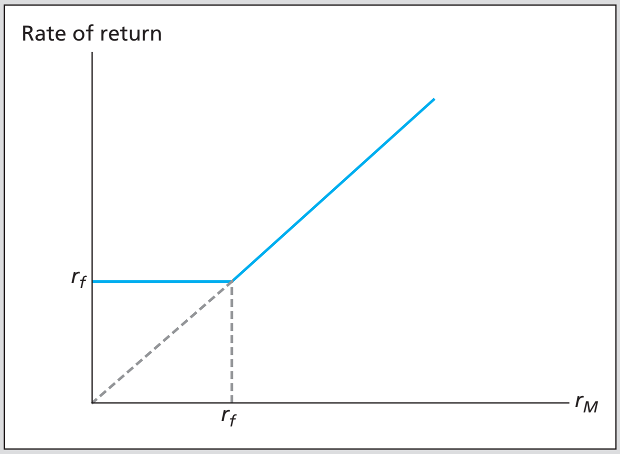


FIGURE 18.8

Rate of return of a perfect market timer

To see how timing ability can be treated as an option, suppose the market index currently is at S_0 and a call option on the index has exercise price of $X = S_0(1 + r_f)$. If the market outperforms bills over the coming period, S_T will exceed X; it will be less than X otherwise. Now look at the payoff to a portfolio consisting of this option plus S_0 dollars invested in bills.

	Payoff to Portfolio	
Outcome:	**$S_T \leq X$**	**$S_T > X$**
Bills	$S_0(1 + r_f)$	$S_0(1 + r_f)$
Option	0	$S_T - X$
Total	$S_0(1 + r_f)$	S_T

The portfolio returns the risk-free rate when the market return is less than the risk-free rate and pays the market return when the market beats bills. This represents perfect market timing. Consequently, the value of perfect-timing ability must equal the value of the call option.

Valuation of the call option embedded in market timing is relatively straightforward using the Black-Scholes formula. Set $S_0 = \$1$ (to find the value of the call per dollar invested in the market), use an exercise price of $X = (1 + r_f)$ (for example, the current risk-free rate is about .12%), and a volatility of $\sigma = 18\%$ (about the historical annual standard deviation of the S&P 500). For a once-a-month timer, $T = \frac{1}{12}$. According to the Black-Scholes formula, the call option conveyed by market-timing ability is worth 2.1% of assets, and this is the monthly fee one could presumably charge for such services. Annualized, that fee is about 28%, similar to the excess return of the market timer in Table 18.8. Less frequent timing would be worth less (see Concept Check 18.3). If one could time the market only on an annual basis, then $T = 1$ and the value of perfect timing would be about 7.2% per year.

The Value of Imperfect Forecasting

But managers are not perfect forecasters. While managers who are right most of the time presumably do very well, "right most of the time" does not mean merely the *percentage* of the time a manager is right. A Tucson, Arizona, weather forecaster who *always* predicts "no rain" may be right 90% of the time, but this "stopped clock" strategy does not require any forecasting ability.

Neither is the overall proportion of correct forecasts an appropriate measure of market forecasting ability. If the market is up two days out of three, and a forecaster always predicts a market advance, the two-thirds success rate is not a measure of forecasting ability. We need to examine the proportion of bull markets ($r_M > r_f$) correctly forecast *as well as* the proportion of bear markets ($r_M < r_f$) correctly forecast.

If we call P_1 the proportion of correct forecasts of bull markets and P_2 the proportion for bear markets, then $P_1 + P_2 - 1$ is the correct measure of timing ability. For example, a forecaster who always guesses correctly will have $P_1 = P_2 = 1$ and will show ability of 1 (100%). An analyst who always bets on a bear market will mispredict all bull markets ($P_1 = 0$), will correctly "predict" all bear markets ($P_2 = 1$), and will end up with timing ability of $P_1 + P_2 - 1 = 0$. If C denotes the (call option) value of a perfect market timer, then $(P_1 + P_2 - 1)C$ measures the value of imperfect forecasting ability.

The incredible potential payoff to accurate timing versus the relative scarcity of billionaires suggests that market timing is far from a trivial exercise and that very imperfect timing is the most that we can hope for.

CONCEPT check 18.4

What is the market-timing score of someone who flips a fair coin to predict the market?

Measurement of Market-Timing Performance

In its pure form, market timing involves shifting funds between a market-index portfolio and cash equivalents, such as T-bills or a money market fund, depending on whether the market as a whole is expected to outperform cash. In practice, most managers do not shift fully between cash and the market. How might we measure partial shifts into the market when it is expected to perform well?

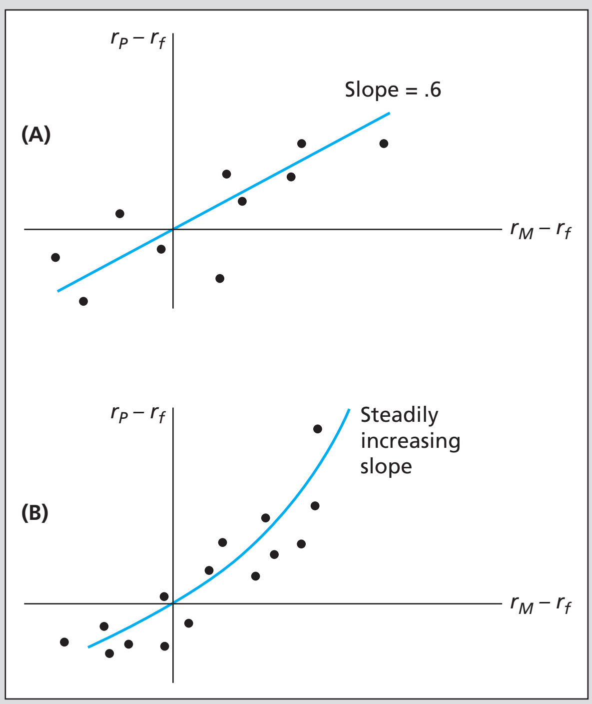
To simplify, suppose the investor holds only the market-index portfolio and T-bills. If the weight on the market were constant, say, .6, then the portfolio beta would also be constant, and the portfolio characteristic line would plot as a straight line with a slope .6, as in Figure 18.9A. If, however, the investor could correctly time the market and shift funds into it in periods when the market does well, the characteristic line would plot as in Figure 18.9B. The idea is that if the timer can predict bull and bear markets, more will be shifted into the market when the market is about to go up. The portfolio beta and the slope of the characteristic line will be higher when r_M is higher, resulting in the curved line that appears in Figure 18.9B.

FIGURE 18.9

Characteristic lines

A: No market timing, beta is constant

B: Market timing, beta increases with expected market excess return



Treynor and Mazuy (1966) tested to see whether portfolio betas did in fact increase prior to market advances, but they found little evidence of timing ability. A similar test was implemented by Henriksson (1984). Overall, 62% of the funds in his study had negative point estimates of timing ability.

In sum, empirical tests to date show little evidence of market-timing ability. Perhaps this should be expected; given the tremendous values to be reaped by a successful market timer, it would be surprising to uncover clear-cut evidence of such skills in nearly efficient markets.

SUMMARY

- The appropriate performance measure depends on the investment context. The Sharpe measure is most appropriate when the portfolio represents the entire investment fund. The Treynor measure is appropriate when the portfolio is to be mixed with several other assets, allowing for diversification of firm-specific risk outside each portfolio. The information ratio may be used when evaluating a portfolio to be mixed with the passive index portfolio.
- The shifting mean and variance of actively managed portfolios make it harder to assess performance. A typical example is the attempt of portfolio managers to time the market, resulting in ever-changing portfolio betas and standard deviations.
- Common attribution procedures partition performance improvements to asset allocation, sector selection, and security selection. Performance is assessed by calculating departures of portfolio composition from a benchmark or neutral portfolio.
- Active management has two components: market timing (or, more generally, asset allocation) and security analysis.
- The value of perfect market-timing ability is enormous. The rate of return to a perfect market timer will be uncertain, but the risk cannot be measured by standard deviation, because perfect timing dominates a passive strategy, providing only "good" surprises.
- Perfect-timing ability is equivalent to having a call option on the market portfolio. The value of that option can be determined using valuation techniques such as the Black-Scholes formula.
- The value of *imperfect* market timing depends on the sum of the probabilities of the true outcome conditional on the forecast: $P_1 + P_2 - 1$. Because the value of perfect timing equals that of the implicit call option C, imperfect timing can be valued by: $(P_1 + P_2 - 1)C$.

KEY TERMS

active management, 597
alpha capture, 604
alpha transfer or alpha transport, 604
bogey, 612
cash, 597
comparison universe, 598
fund of funds, 601
information ratio, 602
Jensen measure, 603
M-square (M^2), 600
market timing, 617
passive management, 597
Sharpe ratio, 599
survivorship bias, 611
Treynor measure, 602

KEY FORMULAS

Decomposition of the variance of a portfolio, P: $\sigma_P^2 = \beta_P^2\sigma_M^2 + \sigma_e^2$

Performance measures:

Sharpe ratio: $S = \dfrac{\overline{R}}{\sigma}$

M^2 of portfolio P relative to its Sharpe ratio: $M^2 = \overline{R}_{P*} - \overline{R}_M = \sigma_M(S_P - S_M)$

Treynor measure: $T = \dfrac{\overline{R}}{\beta}$

The information ratio of an incremental portfolio and the overall Sharpe ratio:

$$S_O = \text{SQRT}\left[S_M^2 + \left(\frac{\alpha_P}{\sigma_P}\right)^2\right]$$

The relation of alpha to other performance measures:

Sharpe ratio: $S_P = \dfrac{\overline{R}_P}{\sigma_P} = \dfrac{\beta_P R_M}{\sigma_P} + \dfrac{\alpha_P}{\sigma_P} \qquad \beta_P = \rho\dfrac{\sigma_P}{\sigma_M}$

$$S_P = S_M\rho + \frac{\alpha_P}{\sigma_P}$$

$$S_P - S_M = S_M(\rho - 1) + \frac{\alpha_P}{\sigma_P}$$

Treynor measure:

$$T_P = \frac{\overline{R}_P}{\beta_P} = \frac{\beta_P\overline{R}_M + \alpha_P}{\beta_P} = \overline{R}_M + \frac{\alpha_P}{\beta_P}$$

$$\beta_M = 1 \qquad T_M = \overline{R}_M$$

$$T_P - T_M = \frac{\alpha_P}{\beta_P}$$

Performance evaluation in a multi-index model:

$$R_{Pt} = \beta_P R_{Mt} + \beta_{SMB} r_{SMBt} + \beta_{HML} r_{HMLt} + \alpha_P + e_{Pt}$$

and

$$\overline{R}_{Pt} = \beta_P\overline{R}_{Mt} + \beta_{SMB}\bar{r}_{SMBt} + \beta_{HML}\bar{r}_{HMLt} + \alpha_P$$

PROBLEM SETS

Select problems are available in McGraw-Hill's *Connect Finance.* Please see the Supplements section of the book's frontmatter for more information.

Basic

1. The finance committee of an endowment has decided to shift part of its investment in an index fund to one of two professionally managed portfolios. Upon examination of past performance, a committee member proposes to choose the portfolio that achieved a greater alpha value. ***(LO 18-1)***
   a. Do you agree? Why or why not?
   b. Could a positive alpha be associated with inferior performance? Explain.
2. The board of a large pension fund noticed that the alpha value of the portfolio of one of its contract managers has recently increased. Should the fund increase the allocation to this portfolio? ***(LO 18-1)***
3. Could portfolio *A* show a higher Sharpe ratio than that of *B* and at the same time a lower M^2 measure? Explain. ***(LO 18-2)***
4. Two portfolio managers use different procedures to estimate alpha. One uses a single-index model regression, the other the Fama-French model. Other things equal, would you prefer the portfolio with the larger alpha based on the index model or the FF model? ***(LO 18-2)***

Intermediate

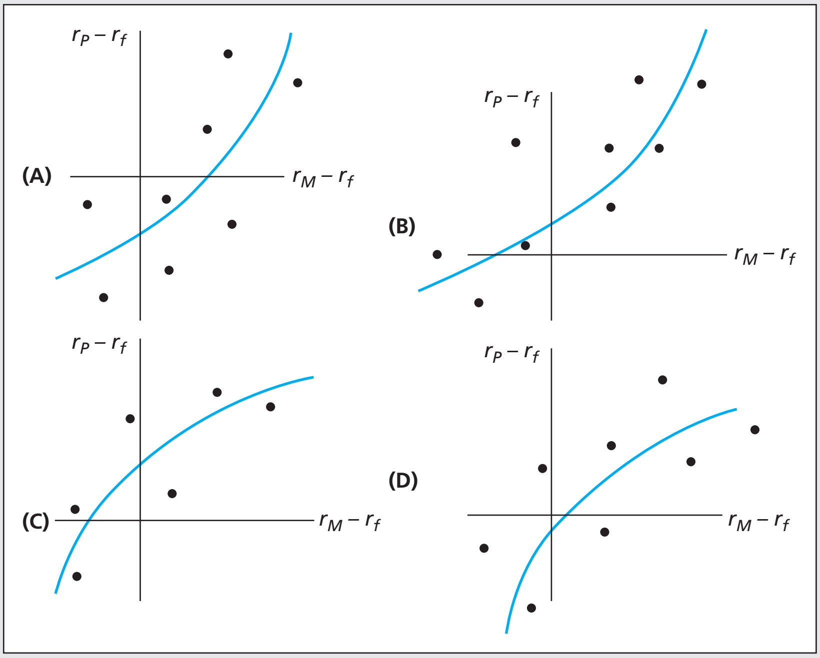
5. Based on current dividend yields and expected capital gains, the expected rates of return on portfolios *A* and *B* are 11% and 14%, respectively. The beta of *A* is .8, while that of *B* is 1.5. The T-bill rate is currently 6%, while the expected rate of return of the S&P 500

index is 12%. The standard deviation of portfolio *A* is 10% annually, while that of *B* is 31%, and that of the index is 20%. *(LO 18-2)*

a. If you currently hold a market-index portfolio, would you choose to add either of these portfolios to your holdings? Explain.

b. If instead you could invest *only* in bills and *one* of these portfolios, which would you choose?

6. Evaluate the timing and selection abilities of the four managers whose performances are plotted in the following four scatter diagrams. *(LO 18-5)*



7. Consider the following information regarding the performance of a money manager in a recent month. The table presents the actual return of each sector of the manager's portfolio in column (1), the fraction of the portfolio allocated to each sector in column (2), the benchmark or neutral sector allocations in column (3), and the returns of sector indexes in column (4). *(LO 18-4)*

	(1) Actual Return	(2) Actual Weight	(3) Benchmark Weight	(4) Index Return
Equity	2.0%	0.70	0.60	2.5% (S&P 500)
Bonds	1.0	0.20	0.30	1.2 (Aggregate Bond Index)
Cash	0.5	0.10	0.10	0.5

a. What was the manager's return in the month? What was her over- or underperformance?

b. What was the contribution of security selection to relative performance?

c. What was the contribution of asset allocation to relative performance? Confirm that the sum of selection and allocation contributions equals her total "excess" return relative to the bogey.

8. Conventional wisdom says one should measure a manager's investment performance over an entire market cycle. What arguments support this contention? What arguments contradict it? *(LO 18-1)*

9. Does the use of universes of managers with similar investment styles to evaluate relative investment performance overcome the statistical problems associated with instability of beta or total variability? *(LO 18-3)*
10. During a particular year, the T-bill rate was 6%, the market return was 14%, and a portfolio manager with beta of .5 realized a return of 10%. Evaluate the manager based on the portfolio alpha. *(LO 18-1)*
11. Bill Smith is evaluating the performance of four large-cap equity portfolios: funds *A, B, C,* and *D*. As part of his analysis, Smith computed the Sharpe ratio and the Treynor measure for all four funds. Based on his finding, the ranks assigned to the four funds are as follows:

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Fund	Treynor Measure Rank	Sharpe Ratio Rank
A	1	4
B	2	3
C	3	2
D	4	1

The difference in rankings for funds *A* and *D* is most likely due to: *(LO 18-2)*
a. A lack of diversification in fund *A* as compared to fund *D*.
b. Different benchmarks used to evaluate each fund's performance.
c. A difference in risk premiums.

Use the following information to answer Problems 12–16: Primo Management Co. is looking at how best to evaluate the performance of its managers. Primo has been hearing more and more about benchmark portfolios and is interested in trying this approach. As such, the company hired Sally Jones, CFA, as a consultant to educate the managers on the best methods for constructing a benchmark portfolio, how best to choose a benchmark, whether the style of the fund under management matters, and what they should do with their global funds in terms of benchmarking.

For the sake of discussion, Jones put together some comparative two-year performance numbers that relate to Primo's current domestic funds under management and a potential benchmark.

	Weight		Return	
Style Category	**Primo**	**Benchmark**	**Primo**	**Benchmark**
Large-cap growth	.60	.50	17%	16%
Mid-cap growth	.15	.40	24	26
Small-cap growth	.25	.10	20	18

As part of her analysis, Jones also takes a look at one of Primo's global funds. In this particular portfolio, Primo is invested 75% in Dutch stocks and 25% in British stocks. The benchmark invested 50% in each—Dutch and British stocks. On average, the British stocks outperformed the Dutch stocks. The euro appreciated 6% versus the U.S. dollar over the holding period, while the pound depreciated 2% versus the dollar. In terms of the local return, Primo outperformed the benchmark with the Dutch investments but underperformed the index with respect to the British stocks.

KAPLAN SCHWESER

12. What is the within-sector selection effect for each individual sector? *(LO 18-4)*

KAPLAN SCHWESER

13. Calculate the amount by which the Primo portfolio out- (or under-) performed the market over the period, as well as the contribution to performance of the pure sector allocation and security selection decisions. *(LO 18-4)*

14. If Primo decides to use return-based style analysis, will the R^2 of the regression equation of a passively managed fund be higher or lower than that of an actively managed fund? *(LO 18-3)*

KAPLAN SCHWESER

15. Which of the following statements about Primo's global fund is most correct? Primo appears to have a positive currency allocation effect as well as: *(LO 18-4)*
    a. A negative market allocation effect and a positive security allocation effect.
    b. A negative market allocation effect and a negative security allocation effect.
    c. A positive market allocation effect and a negative security allocation effect.

KAPLAN SCHWESER

16. Kelli Blakely is a portfolio manager for the Miranda Fund (Miranda), a core large-cap equity fund. The market proxy and benchmark for performance measurement purposes is the S&P 500. Although the Miranda portfolio generally mirrors the asset class and sector weightings of the S&P, Blakely is allowed a significant amount of leeway in managing the fund. Her portfolio holds only stocks found in the S&P 500 and cash.

    Blakely was able to produce exceptional returns last year (as outlined in the table below) through her market-timing and security selection skills. At the outset of the year, she became extremely concerned that the combination of a weak economy and geopolitical uncertainties would negatively impact the market. Taking a bold step, she changed her market allocation. For the entire year her asset class exposures averaged 50% in stocks and 50% in cash. The S&P's allocation between stocks and cash during the period was a constant 97% and 3%, respectively. The risk-free rate of return was 2%. *(LO 18-1)*

KAPLAN SCHWESER

One-Year Trailing Returns

	Miranda Fund	S&P 500
Return	10.2%	−22.5%
Standard deviation	37%	44%
Beta	1.10	1.00

    a. What are the Sharpe ratios for the Miranda Fund and the S&P 500?
    b. What are the M^2 measures for Miranda and the S&P 500?
    c. What is the Treynor measure for the Miranda Fund and the S&P 500?
    d. What is the Jensen measure for the Miranda Fund?

17. Go to **www.mhhe.com/bkm** and link to the material for Chapter 18, where you will find five years of monthly returns for two mutual funds, Vanguard's U.S. Growth Fund and U.S. Value Fund, as well as corresponding returns for the S&P 500 and the Treasury-bill rate. *(LO 18-2)*
    a. Set up a spreadsheet to calculate each fund's excess rate of return over T-bills in each month.
    b. Calculate the standard deviation of each fund over the five-year period.
    c. What was the beta of each fund over the five-year period? (You may wish to review the spreadsheets from Chapters 5 and 6 on the Index model.)
    d. What were the Sharpe, Jensen, and Treynor measures for each fund?

eXcel
Please visit us at www.mhhe.com/bkm

Challenge

18. Historical data suggest the standard deviation of an all-equity strategy is about 5.5% per month. Suppose the risk-free rate is now 1% per month and market volatility is at its historical level. What would be a fair monthly fee to a perfect market timer, according to the Black-Scholes formula? *(LO 18-5)*
19. A fund manager scrutinizing the record of two market timers comes up with this information: *(LO 18-5)*

Number of months that $r_M > r_f$		135
Correctly predicted by timer A	78	
Correctly predicted by timer B	86	
Number of months that $r_M < r_f$		92
Correctly predicted by timer A	57	
Correctly predicted by timer B	50	

a. What are the conditional probabilities, P_1 and P_2, and the total ability parameters for timers A and B?
b. Using the data given in this problem, and the historical data in the previous problem, what is a fair monthly fee for the two timers?

CFA® PROBLEMS

CFA Problems

1. A plan sponsor with a portfolio manager who invests in small-capitalization, high-growth stocks should have the plan sponsor's performance measured against which *one* of the following? *(LO 18-3)*
   a. S&P 500 Index.
   b. Wilshire 5000 Index.
   c. Dow Jones Industrial Average.
   d. Russell 2000 Index.
2. The chairman provides you with the following data, covering one year, concerning the portfolios of two of the fund's equity managers (manager A and manager B). Although the portfolios consist primarily of common stocks, cash reserves are included in the calculation of both portfolio betas and performance. By way of perspective, selected data for the financial markets are included in the following table. *(LO 18-1)*

	Total Return	Beta
Manager A	24.0%	1.0
Manager B	30.0	1.5
S&P 500	21.0	
Lehman Bond Index	31.0	
91-day Treasury bills	12.0	

   a. Calculate and compare the alpha of the two managers relative to each other and to the S&P 500.
   b. Explain *two* reasons the conclusions drawn from this calculation may be misleading.
3. Carl Karl, a portfolio manager for the Alpine Trust Company, has been responsible since 2015 for the City of Alpine's Employee Retirement Plan, a municipal pension fund. Alpine is a growing community, and city services and employee payrolls have expanded in each of the past 10 years. Contributions to the plan in fiscal 2020 exceeded benefit payments by a three-to-one ratio.

   The plan's board of trustees directed Karl five years ago to invest for total return over the long term. However, as trustees of this highly visible public fund, they cautioned him that volatile or erratic results could cause them embarrassment. They also noted a state statute that mandated that not more than 25% of the plan's assets (at cost) be invested in common stocks.

   At the annual meeting of the trustees in November 2020, Karl presented the following portfolio and performance report to the board.

ALPINE EMPLOYEE RETIREMENT PLAN

Asset Mix as of 9/30/20	At Cost (millions)		At Market (millions)	
Fixed-income assets:				
Short-term securities	$ 4.5	11.0%	$ 4.5	11.4%
Long-term bonds and mortgages	26.5	64.7	23.5	59.5
Common stocks	10.0	24.3	11.5	29.1
	$41.0	100.0%	$39.5	100.0%

INVESTMENT PERFORMANCE

	Annual Rates of Return for Periods Ending 9/30/20	
	5 Years	1 Year
Total Alpine Fund:		
Time-weighted	8.2%	5.2%
Dollar-weighted (Internal)	7.7%	4.8%
Assumed actuarial return	6.0%	6.0%
U.S. Treasury bills	7.5%	11.3%
Large sample of pension funds (average 60% equities, 40% fixed income)	10.1%	14.3%
Common stocks—Alpine Fund	13.3%	14.3%
Average portfolio beta coefficient	0.90	0.89
Standard & Poor's 500 Stock Index	13.8%	21.1%
Fixed-income securities—Alpine Fund	6.7%	1.0%
Salomon Brothers' Bond Index	4.0%	−11.4%

Karl was proud of his performance and was chagrined when a trustee made the following critical observations:

a. "Our one-year results were terrible, and it's what you've done for us lately that counts most."

b. "Our total fund performance was clearly inferior compared to the large sample of other pension funds for the last five years. What else could this reflect except poor management judgment?"

c. "Our common stock performance was especially poor for the five-year period."

d. "Why bother to compare your returns to the return from Treasury bills and the actuarial assumption rate? What your competition could have earned for us or how we would have fared if invested in a passive index (which doesn't charge a fee) are the only relevant measures of performance."

e. "Who cares about time-weighted return? If it can't pay pensions, what good is it!"

Appraise the merits of each of these statements and give counterarguments that Karl can use. *(LO 18-2)*

4. A portfolio manager summarizes the input from the macro and micro forecasts in the following table: *(LO 18-2)*

MICRO FORECASTS

Asset	Expected Return (%)	Beta	Residual Standard Deviation (%)
Stock A	20	1.3	58
Stock B	18	1.8	71
Stock C	17	0.7	60
Stock D	12	1.0	55

MACRO FORECASTS

Asset	Expected Return (%)	Standard Deviation (%)
T-bills	8	0
Passive equity portfolio	16	23

a. Calculate expected excess returns, alpha values, and residual variances for these stocks.
b. Construct the optimal risky portfolio.
c. What is Sharpe's measure for the optimal portfolio and how much of it is contributed by the active portfolio? What is the M^2?

WEB *master*

Morningstar has an extensive ranking system for mutual funds, including a screening program that allows you to select funds based on a number of factors. Open the Morningstar website at **www.morningstar.com** and click on the *Funds* link. Select the *Mutual Fund Quickrank* link from the right-side menu. Use the Quickrank screener to find a list of the funds with the highest five-year returns. Repeat the process to find the funds with the highest 10-year returns. How many funds appear on both lists?

Select three of the funds that appear on both lists. For each fund, click on the ticker symbol to get its Morningstar report and look in the Risk Measures section.

1. What is the fund's standard deviation?
2. What is the fund's Sharpe ratio?
3. What is the standard index? What is the best-fit index?
4. What are the beta and alpha coefficients using both the standard index and the best-fit index? How do these compare to the fund's parameters?

Look at the Management section of the report. Was the same manager in place for the entire 10-year period?

SOLUTIONS TO CONCEPT *checks*

18.1 Sharpe: $(\bar{r} - \bar{r}_f)/\sigma$

$S_P = (35 - 6)/42 = .69$

$S_M = (28 - 6)/30 = .733$

Jensen (or alpha): $\bar{r} - [\bar{r}_f + \beta(\bar{r}_M - \bar{r}_f)]$

$\alpha_P = 35 - [6 + 1.2(28 - 6)] = 2.6\%$

$\alpha_M = 0$

Treynor: $(\bar{r} - \bar{r}_f)/\beta$

$T_P = (35 - 6)/1.2 = 24.2$

$T_M = (28 - 6)/1.0 = 22$

18.2 Performance attribution

First compute the new bogey performance as

$$(.70 \times 5.81) + (.25 \times 1.45) + (.05 \times .48) = 4.45\%$$

a. Contribution of asset allocation to performance:

Market	(1) Actual Weight in Market	(2) Benchmark Weight in Market	(3) Excess Weight	(4) Index Return (%)	(5) = (3) × (4) Contribution to Performance (%)
Equity	.70	.70	.00	5.81	.000
Fixed-income	.07	.25	−.18	1.45	−.261
Cash	.23	.05	.18	0.48	.086
Contribution of asset allocation					−.175

b. Contribution of selection to total performance:

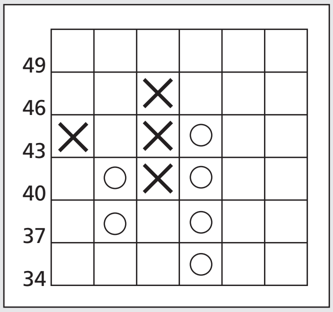
Market	(1) Portfolio Performance (%)	(2) Index Performance (%)	(3) Excess Performance (%)	(4) Portfolio Weight	(5) = (3) × (4) Contribution (%)
Equity	7.28	5.00	2.28	0.70	1.60
Fixed-income	1.89	1.45	0.44	0.07	0.03
Contribution of selection within markets					1.63

18.3 Import the series of annual returns on T-bills and large stocks (S&P 500).
- *a.* Compute the return to the perfect timer. You can use the Excel function = max (stock return, bill return) to select the greater of the two returns each year.
- *b.* Use Excel functions to estimate average and SD.
- *c.* Generate the wealth-index series. Set the wealth index at the end of 1925 to 1. Because the rates of return are expressed in percentages, the index value at the end of 1926 = 1 + rate(1926)/100. For the following years, index = previous index × (1 + this year's return/100).
- *d.* The wealth index for 2008 is the terminal value of the fund per $1 invested at the beginning of 1926.
- *e.* The geometric average equals: Terminal value^(1/82) − 1. Notice that this calculation results in a return expressed as a decimal, not percent.
- *f.* The performance of the annual timer is not as good as the monthly timer. The annual timer may switch funds between the market and T-bills only once per year. He cannot advantageously move funds between the market and bills across months *within* each year. Someone who can time perfectly will always be better off when allowed to make more frequent allocation choices.

18.4 The timer will guess bear or bull markets randomly. One-half of all bull markets will be preceded by a correct forecast, and, similarly, one-half of all bear markets will be preceded by a correct forecast. Hence, $P_1 + P_2 - 1 = ½ + ½ - 1 = 0$.

9.3 At liquidation, price will equal NAV. This puts a limit on fundamental risk. Investors need only carry the position for a few months to profit from the elimination of the discount. Moreover, as the liquidation date approaches, the discount should dissipate. This greatly limits the risk that the discount can move against the investor. At the announcement of impending liquidation, the discount should immediately disappear, or at least shrink considerably.

9.4



9.5 By the time the news of the recession affects bond yields, it also ought to affect stock prices. The market should fall *before* the confidence index signals that the time is ripe to sell.