UNIT 2   TACHEOMETRIC SURVEYING

Structure

2.1 INTRODUCTION

Generally, horizontal distances are measured by direct methods, i.e. laying of chains or tapes on ground. These methods are not always convenient if the ground is undulating, rough, difficult and inaccessible. Under these circumstances, indirect methods are used to obtain distances. One such method is "Tacheometry". Using tacheometric methods, elevations can also be determined. It is in fact a branch of angular surveying in which both the horizontal and vertical positions of points are determined from the instrumental observations, the chain surveys being entirely eliminated.

Objectives

After studying this unit, you should be able to

- explain the principle of tacheometry,

- describe various tacheometric methods,

- learn stadia principles,

- learn fixed hair and movable hair methods, and

- carry out tacheometrical surveying.

You will also be learning principles of anallactic lens which makes the calculations simple. Staff vertical and staff normal cases are also discussed. You will also understand tangential tacheometry and subtense bar methods including tacheometrical traversing.

2.2   PRINCIPLES OF TACHEOMETRY

2.2.1 Advantages of Tacheometry

Since both the quantities viz., horizontal distances and the difference of elevations are determined indirectly in tacheometric surveying, it has a number of advantages over the direct methods of measurement of these quantities.

In terrain where direct methods are not convenient, tacheometric methods can be used. Tacheometric methods are convenient for reconnaissance surveys of routes, for hydrographic surveying and for filling in details in a traverse. There is considerable saving in time and money with the use of tacheometric methods.

2.2.2 Tacheometer

A tacheometer is similar to an ordinary transit theodolite, generally a vernier theodolite itself, fitted with two stadia wires in addition to the central cross-hair. The stadia diaphragm has three horizontal hairs viz., a central horizontal hair and upper and lower stadia hairs. The upper and lower stadia hairs are equidistant from the central horizontal hair. Stadia hairs are sometimes called stadia lines.

For the purpose of tacheometry, even though an ordinary transit can be employed, accuracy and speed are increased if the instrument is specially designed for the work. The magnification of the telescope in tacheometer should be at least 20 to 30 diameters, with an aperture of at least 40 mm for a sufficiently bright image. The magnifying power of the eyepiece is also greater than for an ordinary transit to produce a clearer image of a staff held far away. Further, the altitude bubble is made more sensitive, since vertical angles form an important part of the data for calculation of elevation differences. Figure 2.1 shows a more commonly used pattern of stadia diaphragm.
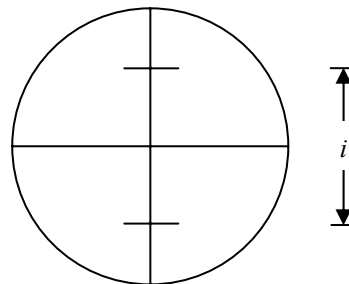

Figure 2.1 : Stadia Diaphragm

2.2.3 Stadia Rods

For short sights of about 100 m or less, an ordinary levelling staff may be used. For long sights, special staff called stadia rod is generally used. The graduations are in bold type (face about 50 mm to 150 mm wide and 15 mm to 60 mm thick) and the stadia rod is 3 m to 5 m long. To keep the staff or stadia rod vertical, a small circular spirit level is fitted on its backside. It is hinged to fold up.

2.2.4 Systems of Tacheometric Measurements

The underlying principle common to various systems of tacheometry is that the horizontal distance between an instrument station P and a point Q, as well as the elevation of Q relative to the instrument, can be deduced from

      (a)   the angle at P subtended by a known small distance at Q, and

      (b)   the vertical angle from P to Q.

This basic principle is applied in different ways in different tacheometric methods. There are basically three systems of tacheometric measurements such as stadia system, tangential system, and subtense bar system.

Stadia System

This is the more extensively used system of tacheometry particularly for detailed work, such as those required in engineering surveys. In this system, a tacheometer is first set up at a station, say *P*, and a staff is held at station *Q*, as shown in Figure 2.2. The difference of upper hair reading and lower hair reading is called staff intercept *s*. All the three hairs including central cross hair are read, and *s* is determined. Vertical angle, θ, corresponding to the central hair is also measured. These measurements enable determination of horizontal distance between *P* and *Q* and their difference in elevation. There are two different types of systems in stadia method. These are as follows :

Fixed Hair Method

In this method, the distance between the upper hair and lower hair, i.e. stadia interval *i*, on the diaphragm of the lens system is fixed. The staff intercept *s*, therefore, changes according to the distance *D* and vertical angle θ.

Movable Hair Method

In this method, the stadia interval '*i*' can be changed. The stadia hairs can be moved vertically up and down by using micrometer screws. The staff intercept *s*, in this case, is kept fixed. Two vanes (targets) are fixed on the staff at a fixed interval of 2 m or 3 m.

The fixed hair method is the one which is commonly used and, unless otherwise mentioned, stadia method means fixed hair method. Movable hair method is not in common use due to difficulties in determining the value of *i* accurately.
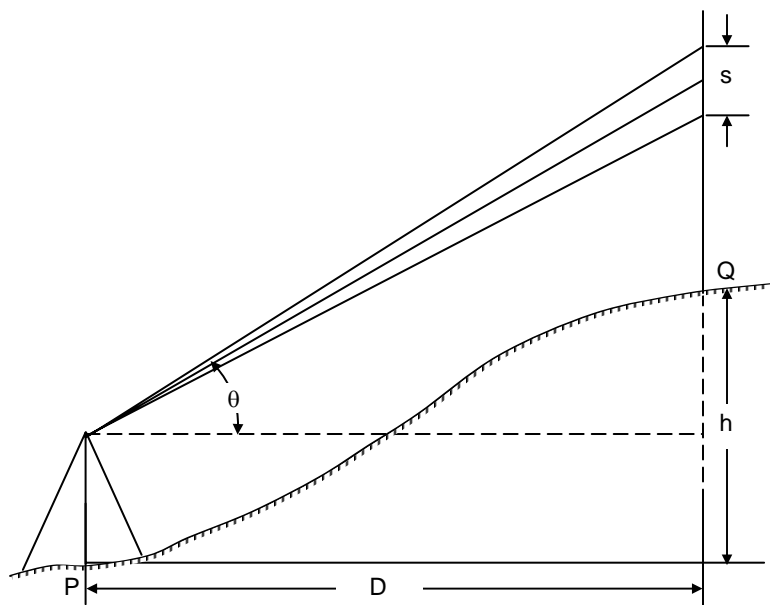

Figure 2.2 : The Stadia System

Tangential System

In this system, observations are not taken on stadia hairs. Instead vertical angles θ_1 and θ_2 to the two targets fixed on a staff are recorded (Figure 2.3). The targets are at a fixed distance s. Vertical angles θ_1, θ_2 and staff intercept s enable horizontal distance D and the difference of elevations to be determined.

In Figure 2.3, both the vertical angles θ_1 and θ_2 are the angles of elevations. There may be two more cases where either both the angles may be angles of depression or one of the angles is angle of elevation and another is angle of depression.
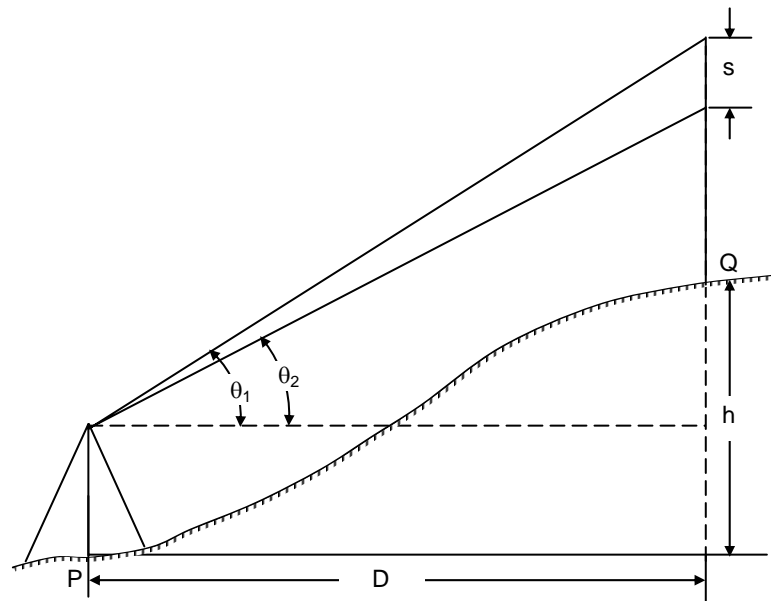
Figure 2.3 : Tangential System

Subtense Bar System

Subtense bar is a bar of fixed length generally 2 m fitted with two targets at the ends. The targets are at equal distance apart from the centre. The subtense bar can be fixed on a tripod stand and is kept horizontal.

As shown in Figure 2.4, angle α subtended by the two targets at station P is measured by a theodolite. The distance s between the targets and the angle α enable the distance D between station P and Q to be determined.
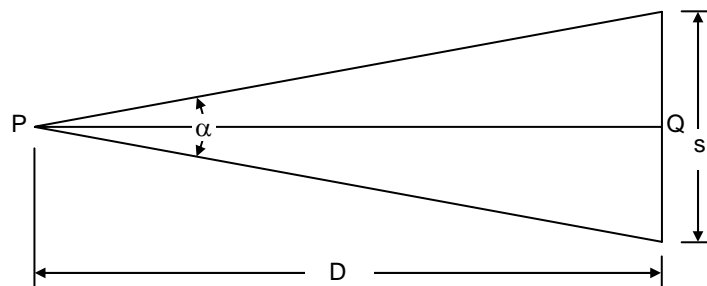
Figure 2.4 : Subtense Bar System (Shown in Plan)

2.2.5 Basic Principle of Stadia Method

We will derive distance and elevation formulae for fixed hair method assuming line of sight as horizontal and considering an external focusing type telescope. In Figure 2.5, O is the optical centre of the object glass. The three stadia hairs are a,

b and c and the corresponding readings on staff are A, B and C. Length of image of AB is ab. The other terms used in this figure are

f = focal length of the object glass,

i = stadia hair interval = ab,

s = staff intercept = AB,

c = distance from O to the vertical axis of the instrument,

d = distance from O to the staff,

d' = distance from O to the plane of the diaphragm, and

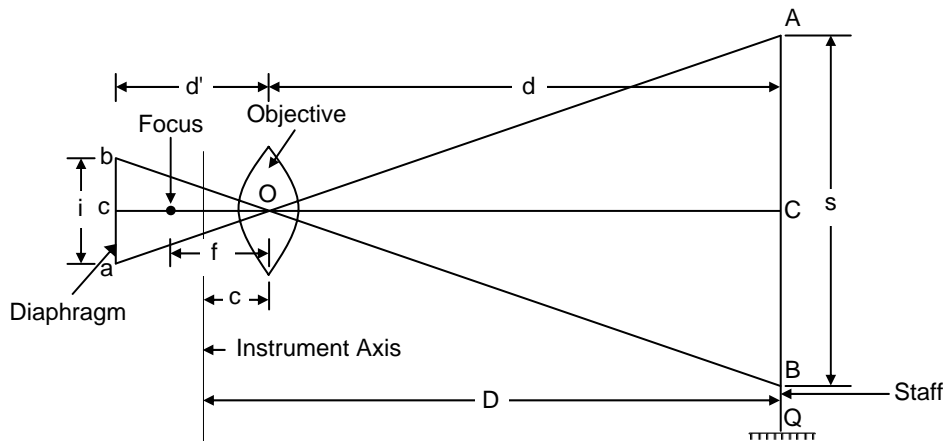D = horizontal distance from the vertical axis to the staff.

Figure 2.5 : Principle of Stadia Method

From similar Δ^s, AOB and aOb, we get

$$\frac{d}{d'} = \frac{s}{i}$$

And from lens formula,

$$\frac{1}{f} = \frac{1}{d'} + \frac{1}{d}$$

Combining the two equations, we get

$$d = \frac{fs}{i} + f$$

Adding c to both the sides

$$D = \frac{fs}{i} + (f + c) \qquad \qquad \ldots (2.1)$$

or $\qquad \qquad D = Ks + C, \qquad \qquad \ldots$ (2.2)

where the constant K is equal to (f/i). It is called **multiplying constant** of the tacheometer and is generally kept as 100. The constant C is equal to $(f + c)$. It is called **additive constant** whose value ranges from 30 cm to 50 cm for external focusing telescopes and 10 cm to 20 cm for internal focusing telescopes. For telescopes fitted with anallactic lens, C equals zero.

Elevation of the staff station Q can be determined as in simple levelling, using the central hair reading C and knowing the Reduced Level (RL) of the plane of collimation.

2.2.6 Movable Hair Method

In movable hair method, the staff intercept *s* is fixed, whereas the variable stadia-hair interval is adjusted by the use of micrometer screws. The upper and lower stadia hairs should simultaneously bisect the target ends on the staff. The interval *i* so measured enables determination of horizontal distance.

The expression

$$D = \frac{f \cdot s}{i} + (f + c) \text{ becomes}$$

$$D = \frac{f \cdot s}{n \cdot p} + (f + c),$$

where *p* = pitch of the screws, and

n = number of pitches between the stadia hairs or the number of turns necessary to move the stadia hairs until they subtend *s* on the staff.

This expression involves the two instrumental constants (*f/p*) and (*f* + *c*). A tacheometer with movable stadia hairs is called a *Subtense Theodolite*.

2.2.7 Anallactic Lens

The basic formula for determination of horizontal distance in stadia tacheometry is

$$D = \frac{f \cdot s}{i} + (f + c),$$

or $\qquad\qquad D = K \cdot s + C$

Due to the presence of the additive constant *C*, *D* is not directly proportional to *s*. This is accomplished by the introduction of an additional convex lens in the telescope, called an *anallactic lens*, placed between the eyepiece and object glass, and at a fixed distance from the latter.

The anallactic lens is provided in external focusing telescope. Its use simplifies the reduction of observations since the additive constant (*f* + *c*) is made zero and the multiplying constant *k* is made 100. However, there is objection to its use also as it increases the absorption of light in the telescope thereby causing reduction in brilliancy of the image. Anallactic lens is not fitted in internal focusing telescopes.

In internal focusing telescope, there is a concave lens between the objective and the eyepiece. This is not an anallactic lens. However, the additive constant *C* of an internal focusing telescope is extremely small. It is usually in the range of 5 cm to 15 cm. It is taken as zero in some of the modern tacheometers. Nevertheless, it may be assumed zero, unless specified.

2.3 METHODS OF TACHEOMETRY

2.3.1 Various Cases – Inclined Sights with Staff Vertical and Staff Normal to the Line of Sight

In stadia tacheometry with fixed hair method, horizontal sights may not always be available, particularly in rough and hilly terrain where differences of elevation between the instrument and staff stations are large. The measurements involve recording of vertical angle (elevation or depression) to the central stadia hair in

addition to the staff intercept. There may be two cases one in which staff is held vertical and another in which staff is held normal to the line of sight. Generally, the former is preferred.

Staff Vertical

In Figure 2.6, θ is the angle of elevation of the line of sight. Let us draw an intercept $A'B'$ through C perpendicular to OC. Now $ACA' = \theta$ and $AA'C$ may be taken practically equal to $90°$. $A'B'$ may, therefore, be expressed as $s.\cos\theta$ with small error, which can be neglected. Now, distance $OC = D$ can be expressed as

$$D = \frac{f.s}{i}\cos\theta + (f + c) \quad \text{Similar to the Eq. (2.1)}$$

and the required horizontal distance H will be

$$H = D\cos\theta,$$

or $$H = \frac{f.s}{i}\cos^2\theta + (f + c)\cos\theta \qquad \ldots (2.3)$$
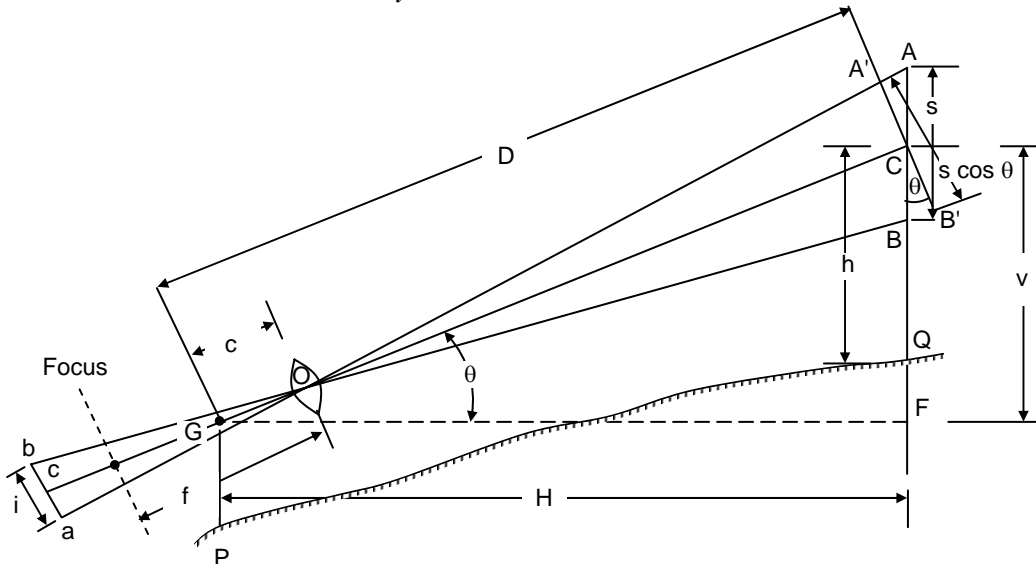
Figure 2.6 : Inclined Sights – Staff Vertical

For determination of elevation difference between P and Q, it is necessary to determine the value of V, i.e. FC, which is the difference of levels between the collimation plane and the point C (i.e. central hair reading) on the staff.

Now $$V = D.\sin\theta \text{ or } H.\tan\theta$$

or $$V = \frac{f.s}{i}.\frac{1}{2}\sin 2\theta + (f + c)\sin\theta \qquad \ldots (2.4)$$

Denoting QC, the central hair reading as h, the level difference between G and Q for an angle of elevation is given by

$$FQ = V - h$$

and if θ is angle of depression,

$$FQ = V + h$$

Now, if we express the level of collimation line above datum by Height of Instrument (HI), then

$$RL \text{ of } Q = HI + V - h$$

In case of a depressed sight

$$RL \text{ of } Q = HI - V - h$$

Staff Normal

Figure 2.7 shows the situation when the staff is held normal to the line of sight. Some sighting device is attached to the staff to enable the staffman to hold the staff normal to the line of sight. If no sighting device is attached, the inclination of the staff may be adjusted till the staff intercept is minimum.

In this case, $\qquad D = \dfrac{f \cdot s}{i} + (f + c)$

but $\qquad H = D \cos \theta + CC'$

$\qquad\qquad = D \cos \theta + h \sin \theta$

$$H = \frac{f \cdot s}{i} \cos \theta + (f + c) \cdot \cos \theta + h \cdot \sin \theta \qquad \dots 2.6 \text{ (a)}$$

In case of angle of depression, $h \sin \theta$ term will be subtractive, since the staff is held away from the instrument.

Now, $\qquad V = F'C = D \sin \theta$

or $\qquad V = \dfrac{f \cdot s}{i} \sin \theta + (f + c) \sin \theta \qquad\qquad \dots 2.6 \text{ (b)}$

For an angle of elevation,

$\qquad\qquad FQ = V - h \cdot \cos \theta,$

and $\qquad RL \text{ of } Q = HI + V - h \cos \theta$

And for an angle of depression,

$\qquad\qquad FQ = V + h \cos \theta,$

and $\qquad RL \text{ of } Q = HI - V - h \cos \theta$

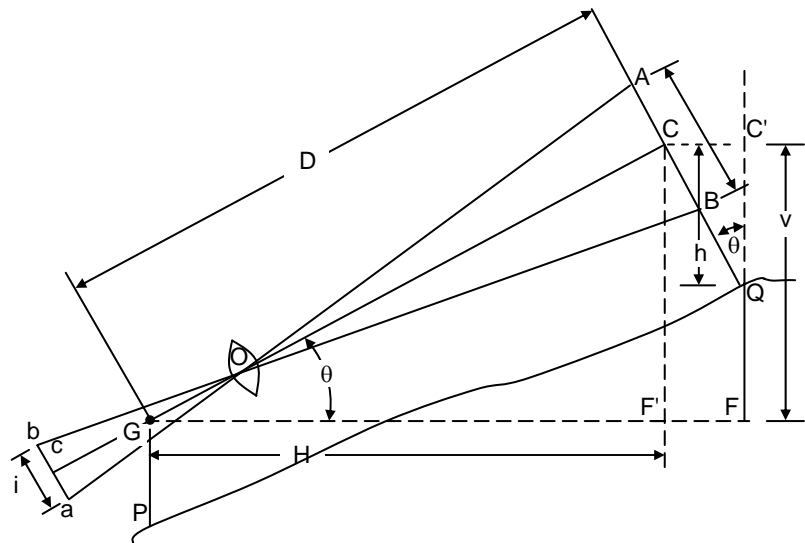where HI is height of instrument at P above datum

Figure 2.7 : Inclined Sight – Staff Normal

2.3.2 Tangential Method

The method of tangential tacheometry can be used when staff is held much away from the instrument making it difficult to read it. This method is useful when the diaphragm does not have stadia hairs. The staff used in this method is similar to the one employed in movable hair method. The distance between the target vanes may be 2 m or 3 m. Vertical angles θ_1 and θ_2 to the top and bottom targets are measured from the instrument station. The horizontal distance D and the vertical intercept V are computed from the values of s, θ_1 and θ_2. Depending upon the angles (i.e., angles of elevation or depression), there can be three case. These are described below.

Both the Angles being Angles of Elevation

Figure 2.8 shows the simplest case in which the ground is such that the horizontal line of sight is possible. The reading at B is observed by keeping the telescope level at A. The reading of A is noted and the angle θ_1 is also measured. By knowing the staff intercept, the horizontal distance D can be given as,

$$D = s. \cot \theta_1 \qquad \qquad \dots (2.7)$$
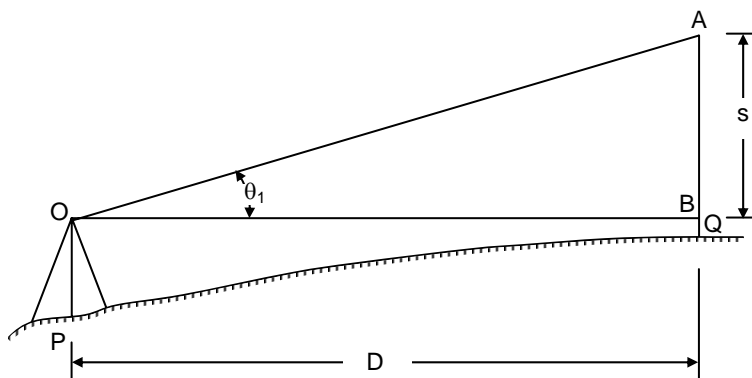
Figure 2.8 : Tangential System

The elevation of Q is determined from the reading obtained at B as in ordinary levelling.

When the ground does not permit a horizontal sight, two vertical angles θ_1 and θ_2 are measured as shown in Figure 2.9.

Figure 2.9 : Tangential System – Both Angles of Elevation

Now $\qquad \qquad AF = D \tan \theta_1$

and $\qquad \qquad BF = D \tan \theta_2$

$\therefore \qquad \qquad s = AF - BF = D (\tan \theta_1 - \tan \theta_2)$

$$\therefore \qquad D = \frac{s}{\tan \theta_1 - \tan \theta_2} \qquad \qquad \ldots (2.8)$$

and $\qquad V = D \cdot \tan \theta_2 = \dfrac{s \cdot \tan \theta_2}{\tan \theta_1 - \tan \theta_2} \qquad \qquad \ldots (2.9)$

Knowing *HI*, i.e., the height of the axis of the instrument above datum, the elevation of *Q* is given as,

RL of $\qquad Q = HI + FB - QB$

$$= HI + D. \tan \theta_2 - QB$$

Both the Angles being Angles of Depression

From Figure 2.10

$$s = BF - AF$$

$$\therefore \qquad D = \frac{s}{\tan \theta_2 - \tan \theta_1} \qquad \qquad \ldots (2.10)$$

and $\qquad V = D \cdot \tan \theta_1 = \dfrac{s \cdot \tan \theta_1}{\tan \theta_2 - \tan \theta_1} \qquad \qquad \ldots (2.11)$
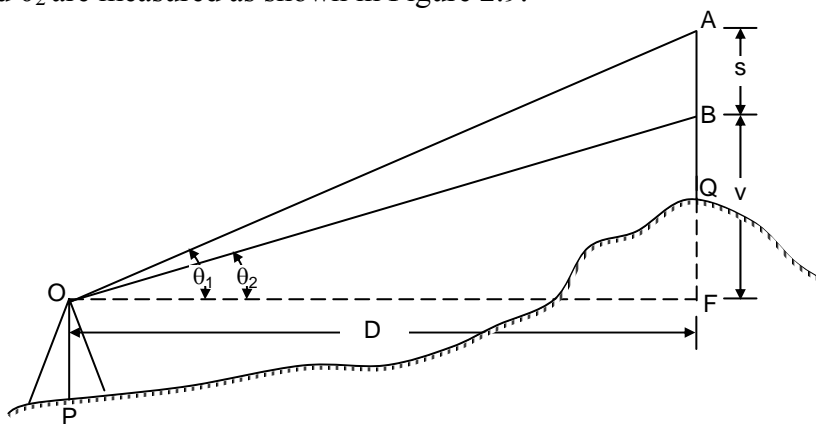
RL of $Q = HI - D \tan \theta_1 - s - QB$

Figure 2.10 : Tangential System – Both Angles of Depression

One Angle of Elevation and Another Angle of Depression

From Figure 2.11

$$s = AF + BF$$

$$= D \tan \theta_1 + D \tan \theta_2$$

and $\qquad V = D \tan \theta_1 \qquad \qquad \ldots (2.12)$

RL of $Q = HI - D. \tan \theta_2 - QB$

Figure 2.11 : Tangential System – One Angle of Elevation and Another Depression

2.3.3 Subtense Bar Method

In the subtense bar method, the horizontal angle subtended by two targets fixed on a horizontal bar at a known distance apart is measured at instrument station by theodolite. A subtense bar is as shown in Figure 2.12(a). The two targets are at a distance s apart, and each at $s/2$ from the centre, i.e. vertical axis. The bar can be mounted on a tripod stand and can be rotated about its vertical axis. The subtense bar should be kept perpendicular to the line of sight, which is set through a sight rule or a small telescope fitted at the centre of the subtense bar.

The horizontal angle α, as shown in Figure 2.12(b), is measured carefully by means of a theodolite. Method of repetition of horizontal angle measurement is used to measure angle α.
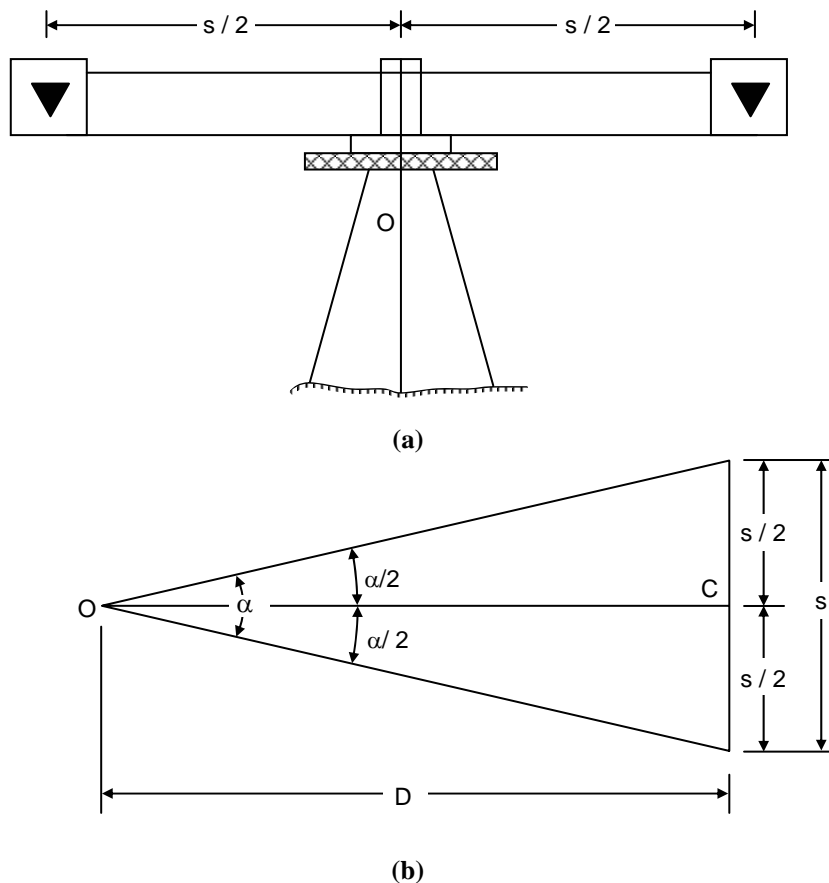
(a)

(b)
Figure 2.12 : Subtense Bar and its Theory

From the geometry,

$$D = \frac{1}{2} s \cdot \cot \frac{\alpha}{2} \qquad \ldots (2.13)$$

where, s = the distance between the targets of subtense bar, and

α = apex angle subtended by targets at O.

As α is small

$$\tan \frac{\alpha}{2} = \frac{\alpha}{2}$$

$$D = s/\alpha,$$

where α is in radians.

or $D = \dfrac{206265\, s}{\alpha''}$ (α in seconds)

If the vertical angle to the centre of the bar is measured, elevation differences can be determined. The subtense bar method can be used for measuring the length of traverse lines in rough country. This system can also be used for contouring.

2.3.4 Tacheometric Traversing

Stadia tacheometry may be used for control establishment of traversing. Stadia traverse is not as accurate as a theodolite traverse but both are somewhat similar. However, in stadia traverse, distances are measured indirectly by tacheometric methods and not by direct methods using tape etc. Stadia traverse is generally run to generate supplementary control between the stations already fixed by more accurate methods.

SAQ 1

(a) Derive an expression for the horizontal distance of a vertical staff from a tacheometer if the line of sight of the telescope is horizontal.

(b) What are the constants of a tacheometer and how are they determined?

(c) Derive expressions for the horizontal distance D and the vertical intercept V when the staff is (a) vertical, and (b) normal.

(d) Differentiate between the fixed-hair method and the movable-hair method. Discuss the advantages and disadvantages of each method.

(e) What is tangential method of tacheometry? What are its advantages and disadvantages over the stadia method?

(f) Discuss the subtense bar method of tacheometric surveying. What are its advantages?

(g) The following readings were taken with a tacheometer on to a vertical staff.

Horizontal Distance	Stadia Readings
46.20 m	0.780; 1.010; 1.240
51.20 m	1.860; 2.165; 2.470

Calculate the tacheometric constants.

(h) Stadia readings were taken with a theodolite on a vertical staff with the telescope inclined at an angle of depression of $3°30'$. The staff readings were 2.990, 2.055 and 1.120. The reduced level of the staff station is 100.000m, and the height of the instrument is 1.40 m. What is the reduced level of the ground at the instrument? Take constants as 100 and zero.

(i) A tacheometer is setup at an intermediate point on a traverse course PQ and the following observations are made on a staff held vertical.

Staff Station	Vertical Angle	Staff Intercept	Axial Hair Readings
P	+ 9°30′	2.250	2.105
Q	+ 6°00′	2.055	1.975

The constants are 100 and 0. Compute the length PQ and the reduced level of Q. RL of $P = 350.50$ m.

(j) Following observations were taken with a tacheometer fitted with an anallactic lens having value of constant as 100.

Inst. Station	Staff Station	Reduced Bearing	Vertical Angle	Staff Reading
O	P	N 37° W	4°12′	0.910, 1.510, 2.110
O	Q	N 23° E	5°42′	1.855, 2.705, 3.555

Calculate the horizontal distance between *P* and *Q*.

(k) The horizontal angle subtended at the theodolite station by a subtense bar with vanes 3 m apart is 0° 10 ′40′′. Calculate the horizontal distance between the theodolite and the subtense bar.

(l) The vertical angles to vanes fixed at 1 m and 3 m above the foot of the staff held vertically at a station *P* were – 1° 45′ and + 2° 30′, respectively. Find the horizontal distance and the reduced RL of *P* if the RL of the instrument axis is 110.00 m.

(m) The following notes refer to a traverse run by a tacheometer fitted with an anallactic lens, with constant 100 and staff held vertical.

Line	Bearing	Vertical Angle	Staff Intercept
PQ	30° 24′	+ 5° 06′	1.875
QR	300° 48′	+ 3° 48′	1.445
RS	226° 12′	– 2° 36′	1.725

Find the length and bearing of *SP*.

2.4 SUMMARY

In this unit, you have studied details of indirect methods of distance measurement including determination of elevation differences through tacheometric surveying methods. The horizontal and vertical distances can be obtained by optical means as opposed to the slower process of measurements by chain or tape. This method of tacheometric measurement is very rapid and convenient. The accuracy of tacheometry in general is not as good as with that of chaining.

2.5 ANSWERS TO SAQs

SAQ 1

The answers to theoretical questions may be found out in various sections of this unit. For numerical problems, the answers are as follows :

(g) 100, 0.20 m

(h) 112.050 m

(i) 422.13 m; 335.47 m

(j) 149.96 m (**Hint :** Use formula $a^2 = b^2 + c^2 - 2\,b\,c\cos A$)

(k) 966.87 m

(l)   26.95 m, 108.177

(m)   191.930 m, $126^o\,47\,'47''$or S $53^o\,12\,'13''$ E