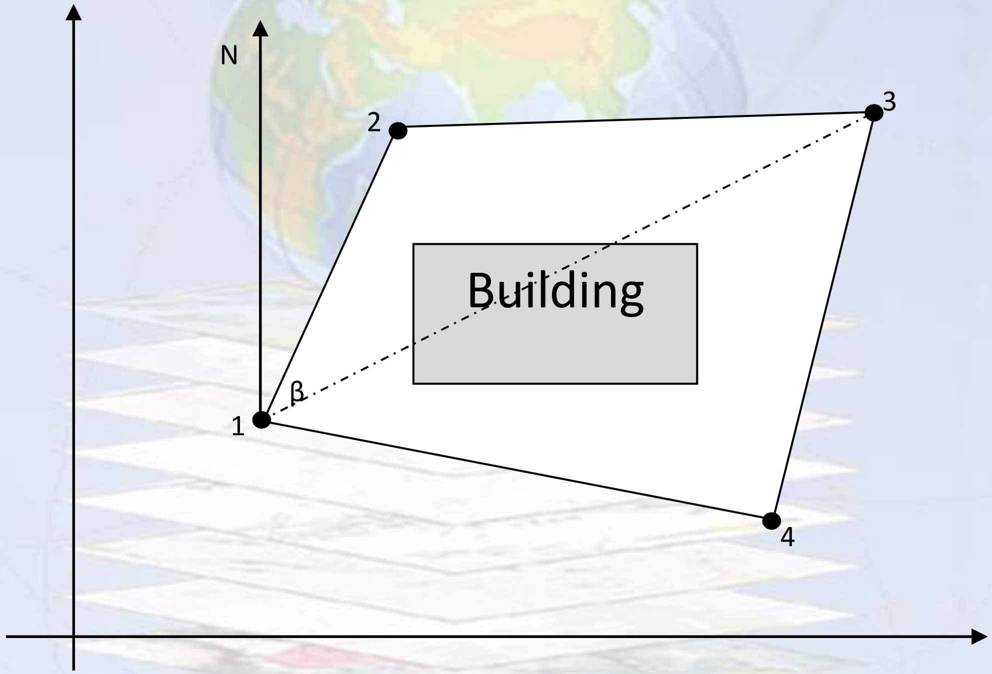
Surveying 1 / Dr. Najeh Tamim

CHAPTER 7

COORDINATE GEOMETRY AND TRAVERSE SURVEYING

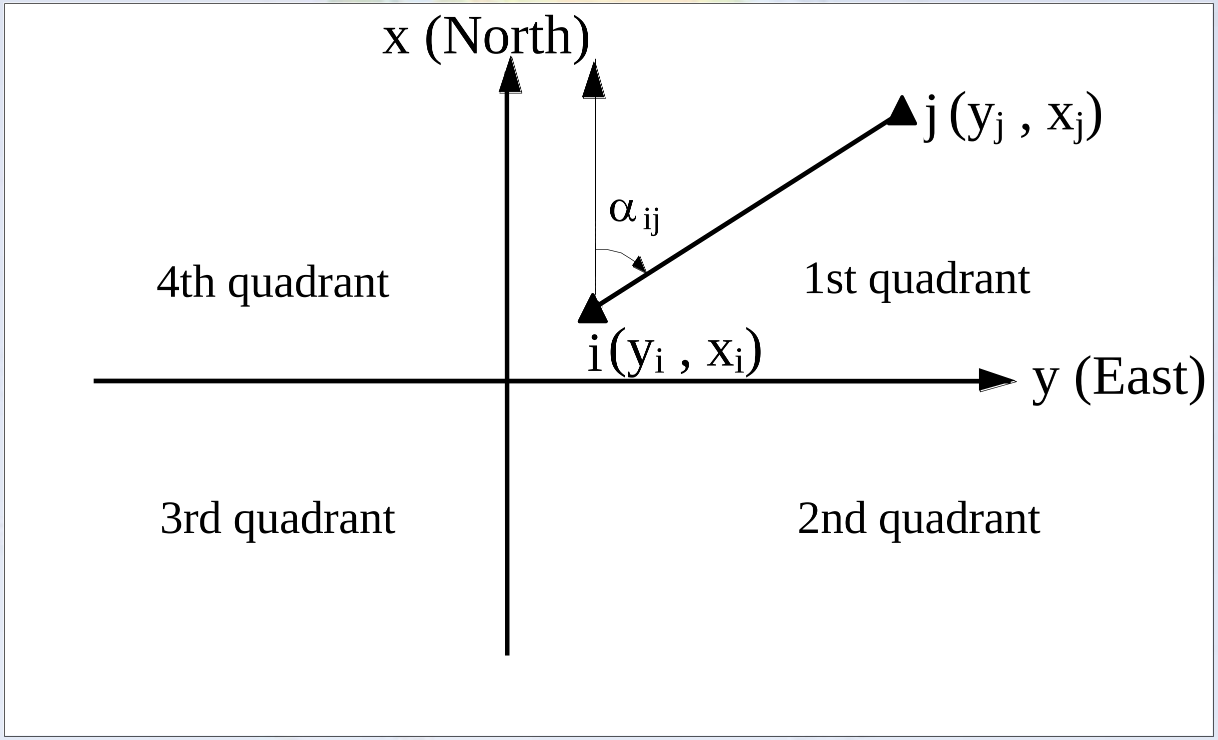
ADVANTAGES OF USING COORDINATES



1) Computation of distances
2) Computation of azimuths
3) Computation of internal angles
4) Computation of obstructed distances
5) Computation of areas
6) Drawing and plotting is easier (Autocad).

Coordinate System

THE INVERSE PROBLEM



The inverse problem.

$$d_{ij} = \sqrt{(y_j - y_i)^2 + (x_j - x_i)^2}$$

$$\alpha_{ij} = \tan^{-1} \frac{(y_j - y_i)}{(x_j - x_i)} + C$$

C = 0° in the 1st quadrant (Δy +ve, Δx +ve)

C = 180 ° in the 2nd quadrant (Δy +ve, Δx -ve)

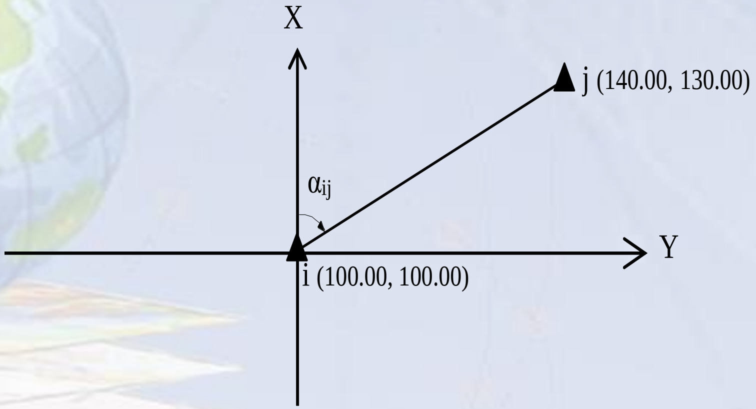
C = 180 ° in the 3rd quadrant (Δy -ve, Δx -ve)

C = 360 ° in the 4th quadrant (Δy -ve, Δx +ve)

EXAMPLES ON THE FOUR CASES:

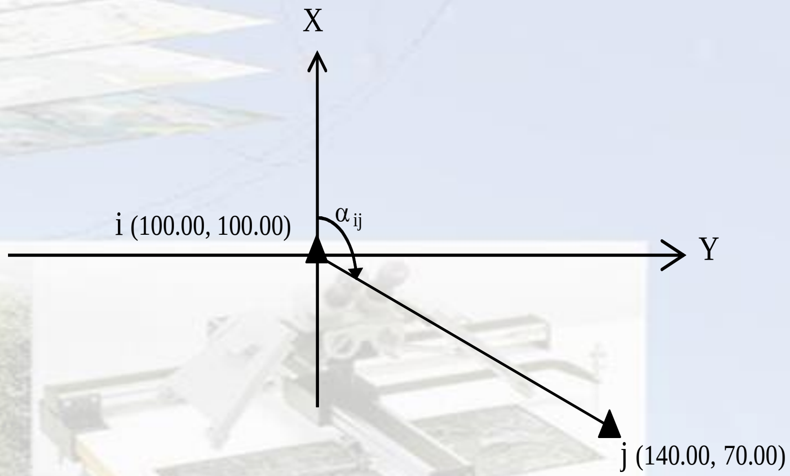
1) $\Delta Y = 140.00 - 100.00 = +40.00$
$\Delta X = 130.00 - 100.00 = +30.00$
ΔY +ve, ΔX +ve $\Rightarrow C = 0°$

$$\alpha_{ij} = \tan^{-1}\frac{\Delta Y}{\Delta X} = \tan^{-1}\frac{+40}{+30} + 0$$
$$= 53° \, 07' \, 48''$$



2) $\Delta Y = 140.00 - 100.00 = +40.00$
$\Delta X = 70.00 - 100.00 = -30.00$
ΔY +ve, ΔX -ve $\Rightarrow C = 180°$

$$\alpha_{ij} = \tan^{-1}\frac{\Delta Y}{\Delta X} = \tan^{-1}\frac{+40}{-30} + 180$$
$$= 126° \, 52' \, 12''$$



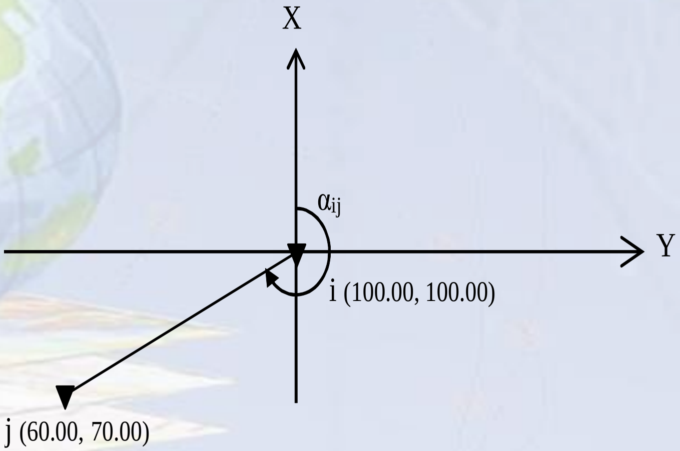
3) ΔY = 60.00 – 100.00 = -40.00

ΔX = 70.00 – 100.00 = -30.00

ΔY -ve, ΔX -ve $\Rightarrow$ C = 180°

$$\alpha_{ij} = \tan^{-1}\frac{\Delta Y}{\Delta X} = \tan^{-1}\frac{-40}{-30} + 180$$

$$= 233°\ 07'\ 48''$$



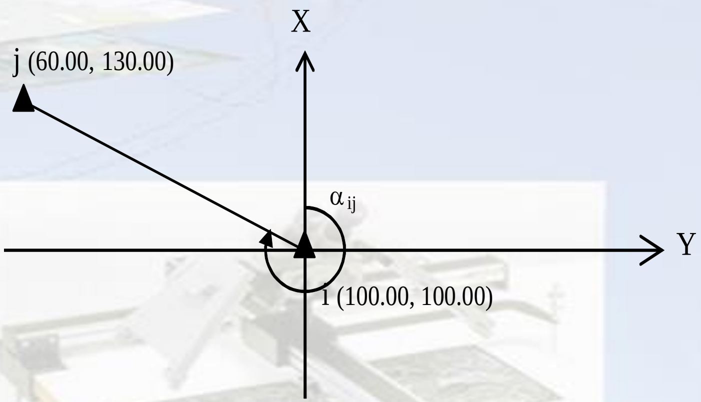
4) ΔY = 60.00 – 100.00 = -40.00

ΔX = 130.00 – 100.00 = +30.00

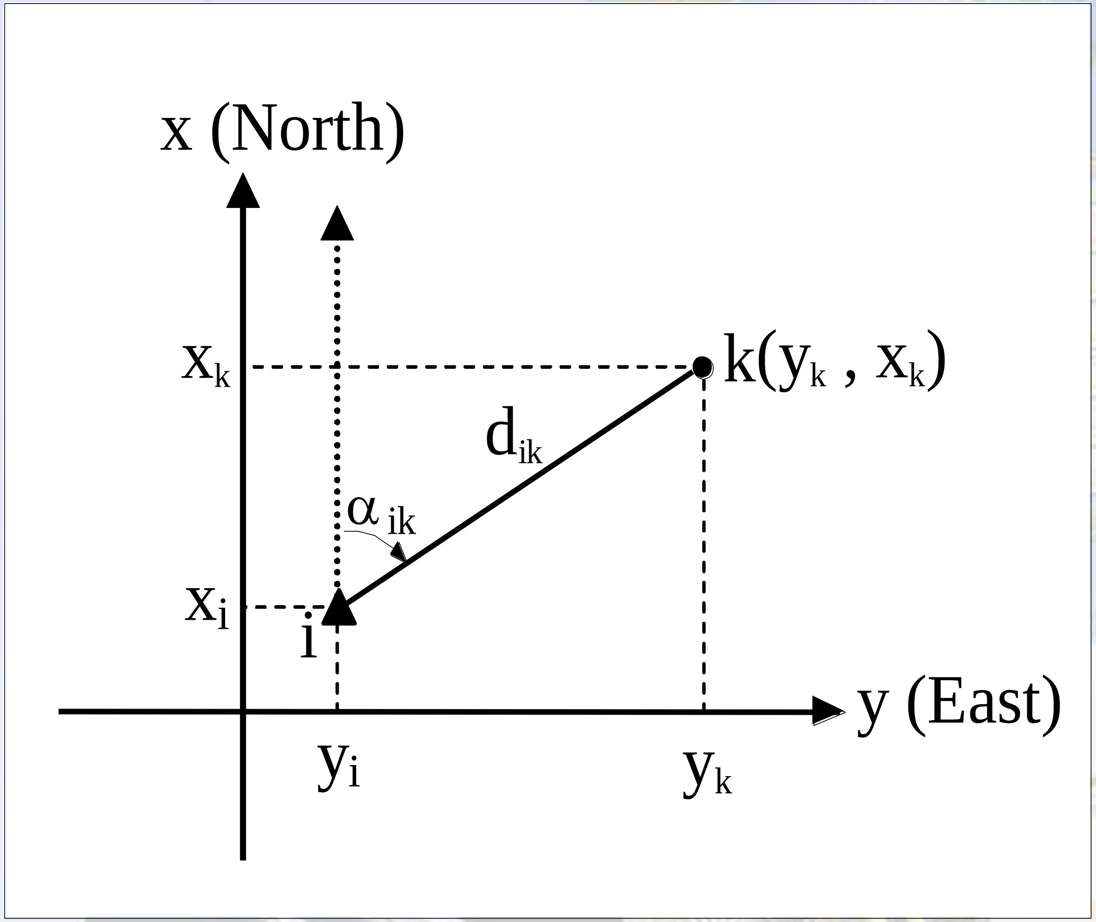
ΔY -ve, ΔX +ve $\Rightarrow$ C = 360°

$$\alpha_{ij} = \tan^{-1}\frac{\Delta Y}{\Delta X} = \tan^{-1}\frac{-40}{+30} + 360$$

$$= 306°\ 52'\ 12''$$



LOCATION BY ANGLE AND DISTANCE

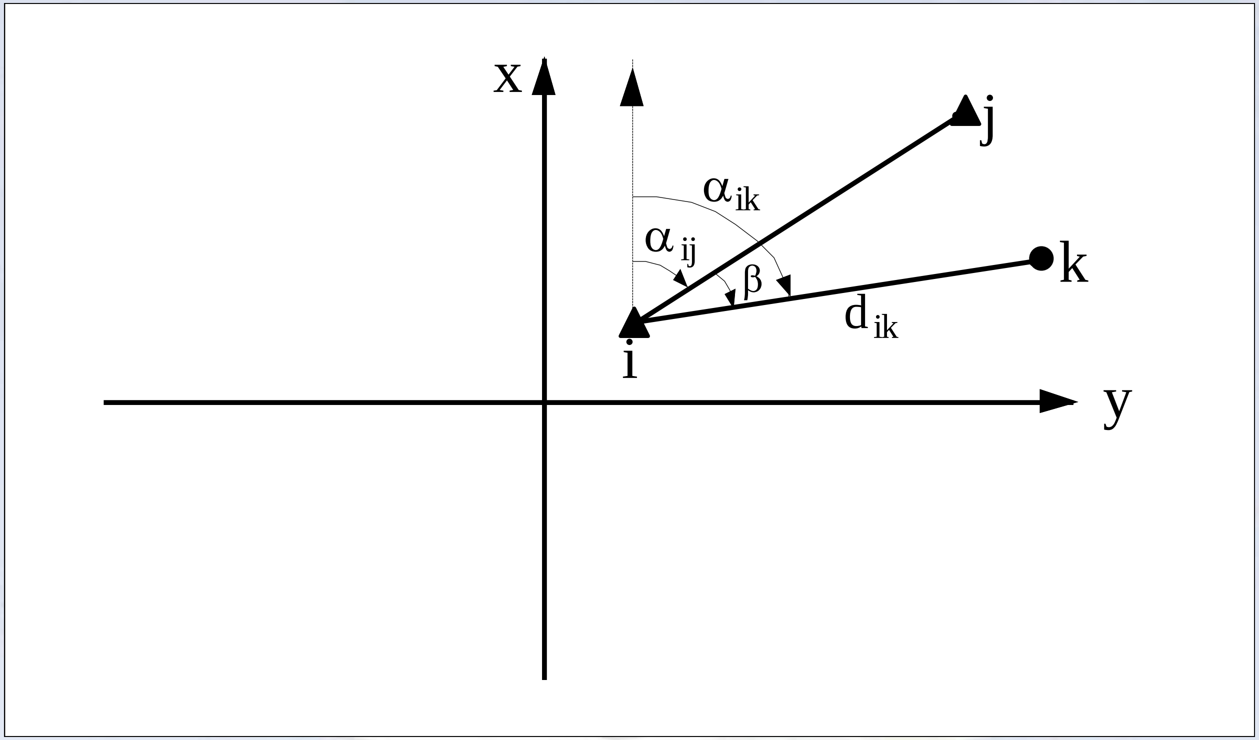


$$y_k = y_i + d_{ik} \sin \alpha_{ik}$$

$$x_k = x_i + d_{ik} \cos \alpha_{ik}$$

To compute the coordinates of point k, three things need to be known:

1) Coordinates of point i,
2) Distance (d_{ik}), and
3) Azimuth α_{ik}

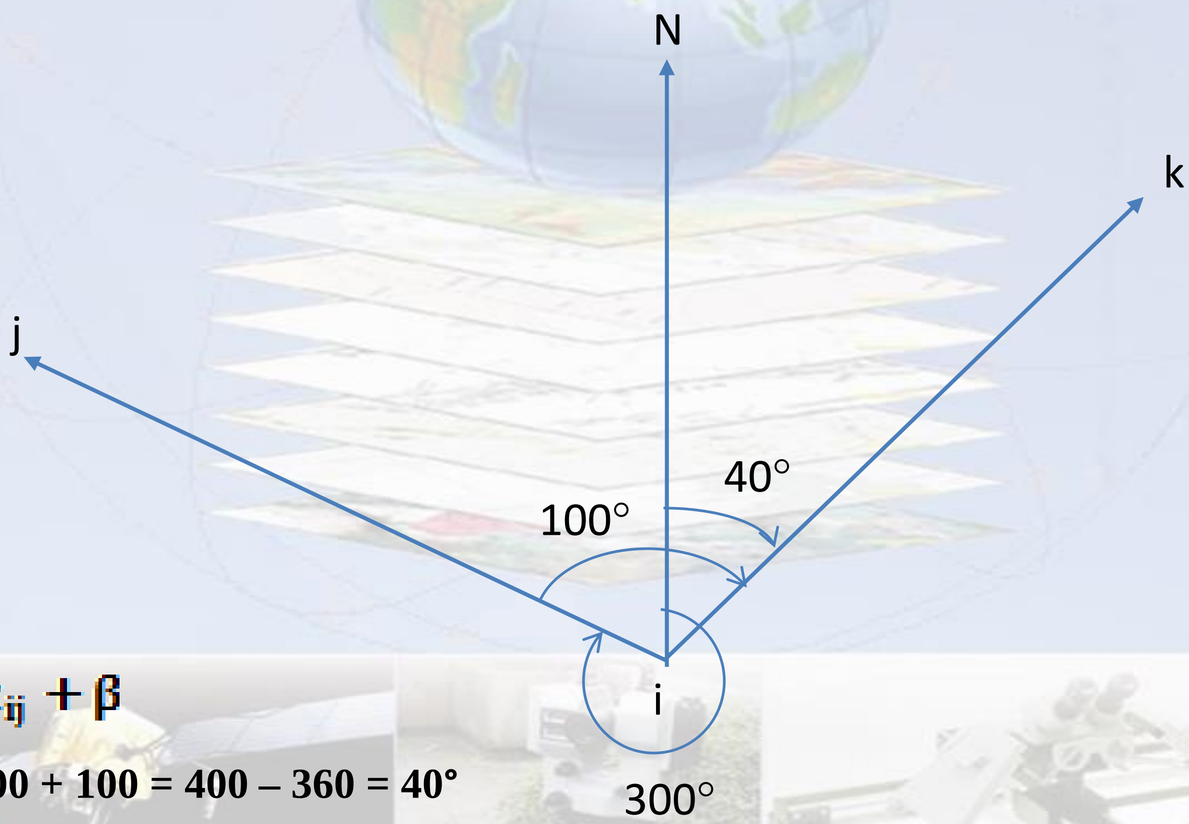


Set up the total station at station i and make the horizontal circle to read zero while sighting the known point j. Then measure the horizontal angle β and the horizontal distance d_{ik}.

$$\alpha_{ik} = \alpha_{ij} + \beta$$

If α_{ik} > 360°, then subtract 360° from it.

The coordinates of point k are calculated according to the equations in the previous slide.



$$\alpha_{ik} = \alpha_{ij} + \beta$$

= 300 + 100 = 400 – 360 = 40°

EXAMPLE:

A total station was set up at point *i* and directed towards point *j*. The following horizontal angle and distance were then measured to point *k*:

$\beta = 111° 27' 45''$, $d_{ik} = 318.10$ m, Compute the horizontal coordinates of point *k*.

SOLUTION:

$$\Delta y = y_j - y_i = 174205.31 - 174410.56 = -205.25 \text{ m}$$

$$\Delta x = x_j - x_i = 181810.22 - 181680.76 = 129.46 \text{ m}$$

Since Δy is negative and Δx is positive $\Rightarrow$ the line is in the 4th quadrant (c=360°),

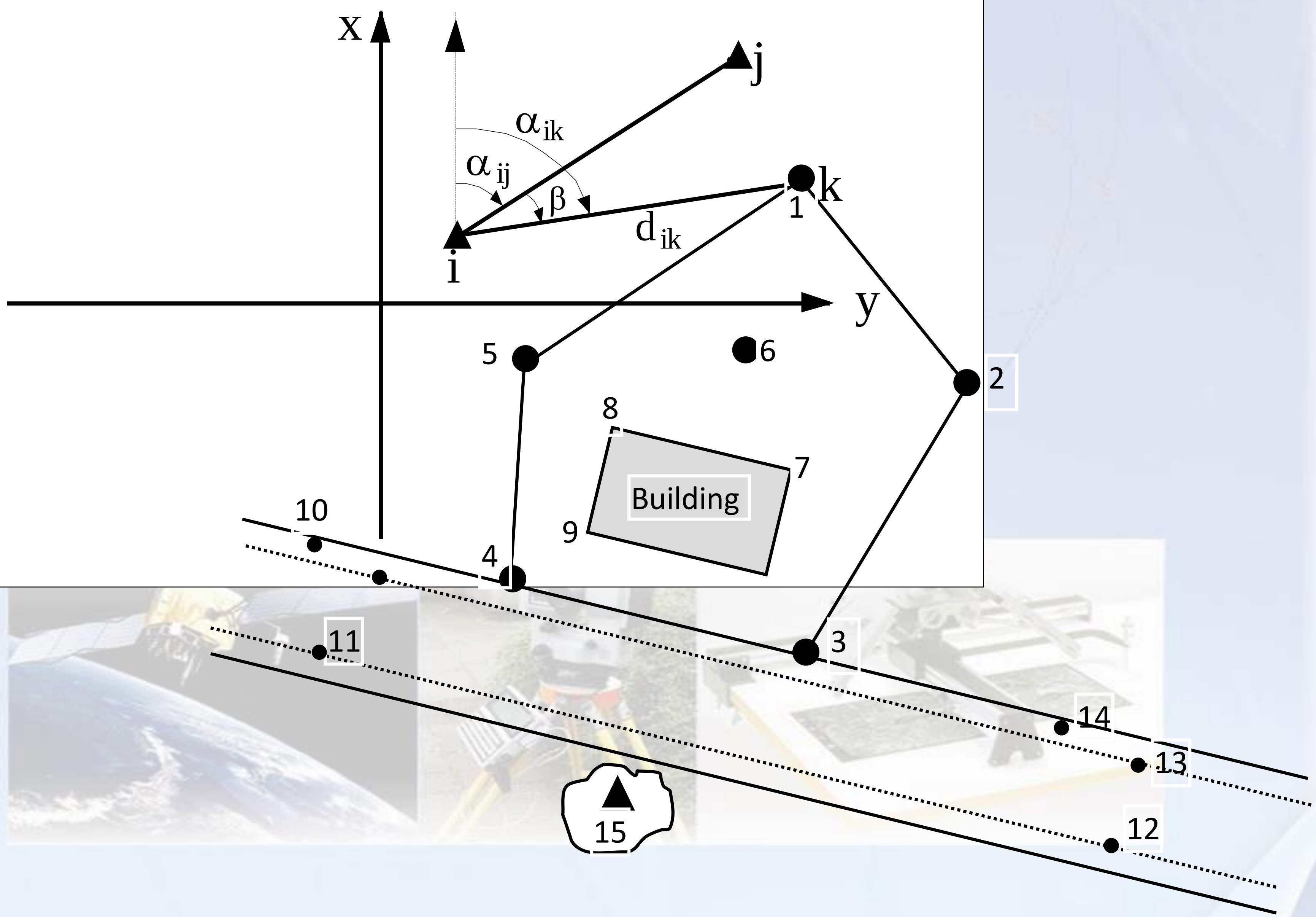
$$\Rightarrow \alpha_{ij} = \tan^{-1}\frac{\Delta y}{\Delta x} + c = \tan^{-1}\frac{-205.25}{129.46} + 360 = 302° 14' 29''$$

$$\alpha_{ik} = \alpha_{ij} + \beta = 302° 14' 29'' + 111° 27' 45'' = 413° 42' 14'' - 360°$$

$$= 53° 42' 14''$$

$$y_k = 174410.56 + 318.10 \sin(53° 42' 14'') = 174666.94 \text{ m}$$

$$x_k = 181680.76 + 318.10 \cos(53° 42' 14'') = 181869.06 \text{ m}$$

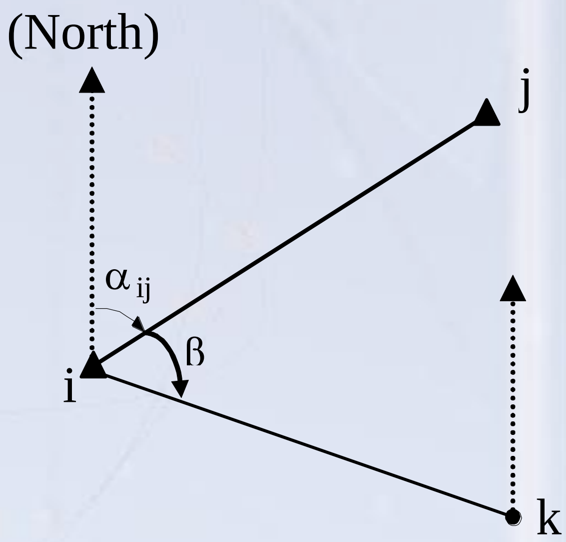

x
y
j
k
i
α_{ik}
α_{ij}
β
d_{ik}
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Building

LOCATING THE NORTH DIRECTION AT A POINT

1) To locate the north direction at the known point i:

   a) Set up the theodolite or total station at i and make the horizontal circle to read an angle equal to α_{ij} while sighting point j (use the Hold button for this purpose).

   b) Rotate the telescope counterclockwise until you read zero on the horizontal circle. The telescope will then be directed towards the north direction.



2) To locate the north direction at the unknown point k:

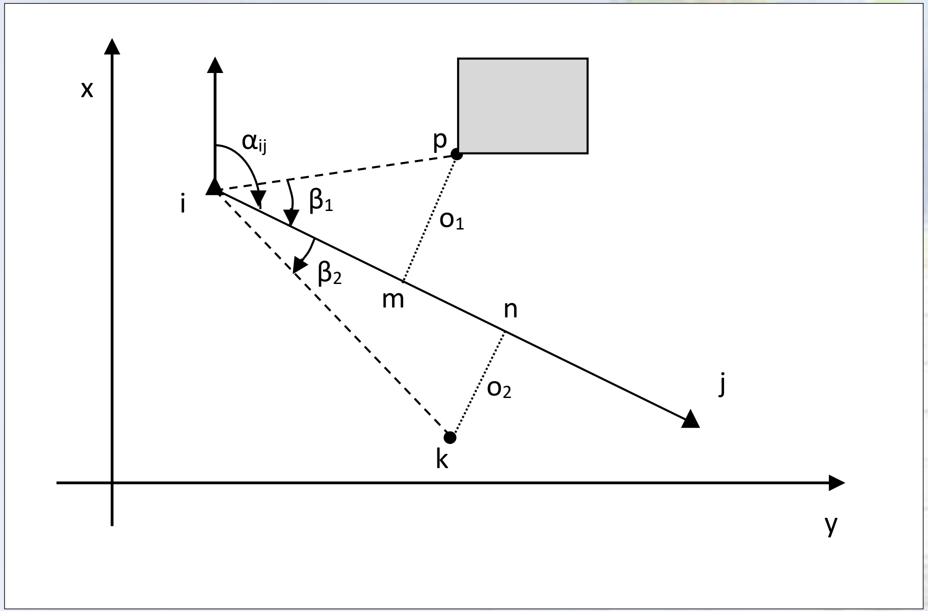
   a) Set up the theodolite or total station at i and make the horizontal circle to read zero while sighting point j, and then measure angle β. Calculate $\alpha_{ik} = \alpha_{ij} + \beta$.

   b) Calculate the back azimuth of line ik ($\alpha_{ki} = \alpha_{ik} \pm 180°$).

   c) Set up the theodolite or total station at k and make the horizontal circle to read an angle equal to α_{ki} while sighting point i (use the Hold button for this purpose).

   d) Rotate the telescope in a clockwise direction until you read zero on the horizontal circle (or 360°). The telescope will then be directed towards the north direction

LOCATION BY DISTANCE AND OFFSET



1) The point is to the left of line ij (point p):

$$d_{ip} = \sqrt{d_{im}^2 + o_1^2} \;,\quad \beta_1 = \tan^{-1}\frac{o_1}{d_{im}}$$

$$\alpha_{ip} = \alpha_{ij} - \beta_1$$

$$y_p = y_i + d_{ip}.\sin\alpha_{ip}$$

$$x_p = x_i + d_{ip}.\cos\alpha_{ip}$$

2) The point is to the right of line ij (point k):

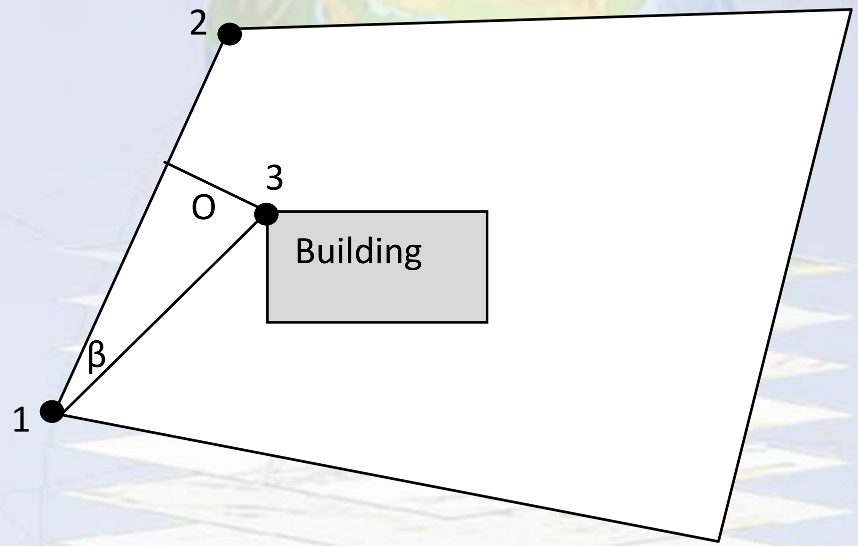
$$d_{ik} = \sqrt{d_{in}^2 + o_2^2} \;,\quad \beta_2 = \tan^{-1}\frac{o_2}{d_{in}}$$

$$\alpha_{ik} = \alpha_{ij} + \beta_2$$

$$y_k = y_i + d_{ik}.\sin\alpha_{ik}$$

$$x_k = x_i + d_{ik}.\cos\alpha_{ik}$$

SETBACK OF A BUILDING CORNER



1) Set up the total station at a convenient location and make angle and distance measurements to compute the coordinates of points 1, 2 and 3.

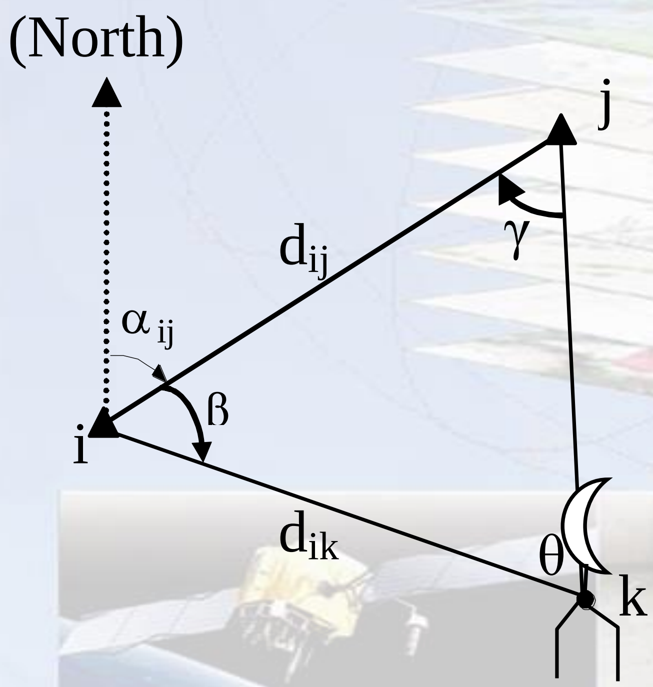
2) Compute the azimuths of lines 1-2 & 1-3 from the coordinates.

3) Compute distance 1-3 from the coordinates.

4) Angle β = Azimuth of line 1-3 - Azimuth of line 1-2.

5) The setback (O) = $d_{1-3} . \sin \beta$

INTERSECTION BY ANGLES



Known: Coordinates of i & j

Measured: angles β and γ

Procedure to compute y_k and x_k :

1) Compute α_{ij} and d_{ij}
2) Compute $\theta = 180° - \beta - \gamma$
3) Compute d_{ik} from the sine law
4) Compute $\alpha_{ik} = \alpha_{ij} + \beta$
5) Compute the coordinates of point k:

$$y_k = y_i + d_{ik} \sin \alpha_{ik}$$

$$x_k = x_i + d_{ik} \cos \alpha_{ik}$$

EXAMPLE:

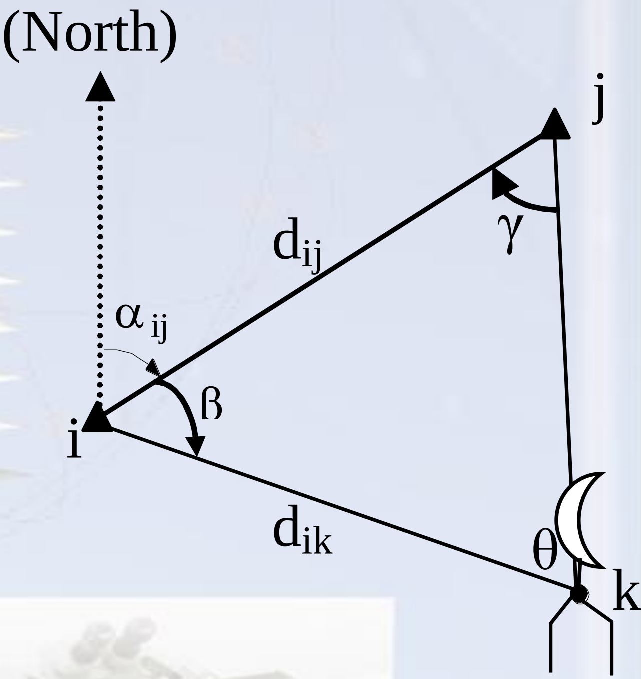
Referring to the next figure, let *i* & *j* be two points of known coordinates, and point *k* be the crescent of a minaret whose coordinates are to be calculated. Given:

y_i = 175329.41 m x_i = 184672.66 m

y_j = 176321.75 m x_j = 185188.24 m

β = 31° 26' 30" γ = 42° 33' 41"

Compute the horizontal coordinates y_k and x_k.



SOLUTION:

$y_j - y_i = 176321.75 - 175329.41 = 992.34$ m

$x_j - x_i = 185188.24 - 184672.66 = 515.58$ m

$$\Rightarrow d_{ij} = \sqrt{(992.34)^2 + (515.58)^2} = 1118.29 \text{ m}$$

$$\alpha_{ij} = \tan^{-1}\frac{992.34}{515.58} + 0 = 62° 32' 44" \text{ (1st quadrant)}$$

$\alpha_{ik} = \alpha_{ij} + \beta$ = 62° 32' 44" + 31° 26' 30" = 93° 59' 14"

$180° - \beta - \gamma$ = 180° - 31° 26' 30" - 42° 33' 41" = 105° 59' 49"

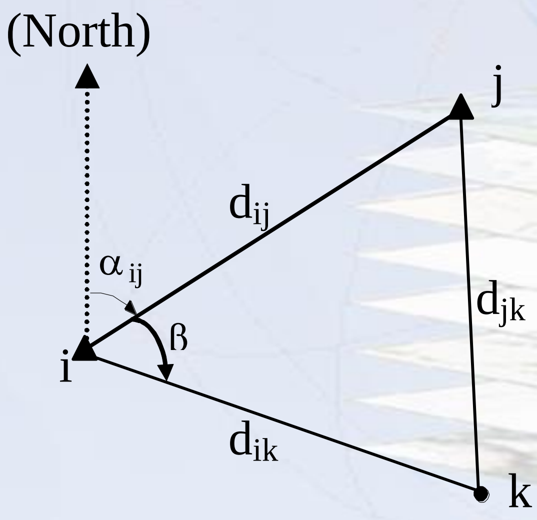
$$d_{ik} = \frac{1118.29 \sin(42° 33' 41")}{\sin(105° 59' 49")} = 786.86 \text{ m}$$

Then,

y_k = 175329.41 + 786.86 sin(93° 59' 14") = 176114.37 m

x_k = 184672.66 + 786.86 cos(93° 59' 14") = 184617.95 m

INTERSECTION BY DISTANCES



Known: Coordinates of i & j

Measured: distances d_{ik} and d_{jk}.

Procedure to compute y_k and x_k :

1) Compute α_{ij} and d_{ij}

2) Compute β from the cosine law:

$$\beta = \cos^{-1}\left[\frac{d_{ij}^2 + d_{ik}^2 - d_{jk}^2}{2d_{ij} \cdot d_{ik}}\right]$$

3) Compute $\alpha_{ik} = \alpha_{ij} + \beta$

4) Compute the coordinates of point k:

$$y_k = y_i + d_{ik} \sin \alpha_{ik}$$

$$x_k = x_i + d_{ik} \cos \alpha_{ik}$$

EXAMPLE:

Let *i* & *j* be two known points with the following coordinates:

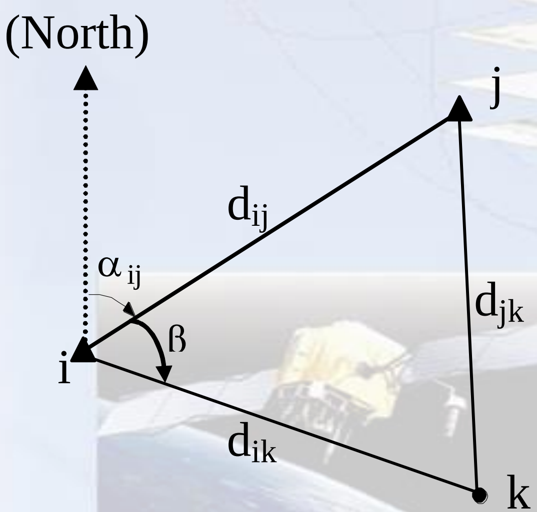
y_i = 175329.41 m x_i = 184672.66 m

y_j = 176321.75 m x_j = 185188.24 m

The following distance measurements were made from points *i & j* to point *k:*

$\mathbf{d_{ik}}$ = 888.86 m , $\mathbf{d_{jk}}$ = 950.55 m, compute the horizontal coordinates y_k and x_k .

SOLUTION:



$$\beta = \cos^{-1}\left[\frac{d_{ij}^{2} + d_{ik}^{2} - d_{jk}^{2}}{2d_{ij} \cdot d_{ik}}\right] = \cos^{-1}\left[\frac{1118.29^2 + 888.86^2 - 950.55^2}{2x1118.29x888.86}\right]$$

$$= 55^\circ\, 06'\, 42''$$

$$\alpha_{ij} = \tan^{-1}\frac{(176321.75 - 175329.41)}{(185188.24 - 184672.66)} + 0 = 62°\ 32'\ 44''\ \ (1^{st}\ \text{quadrant})$$

$$\alpha_{ik} = \alpha_{ij} + \beta = 62°\ 32'\ 44'' + 55°\ 06'\ 42'' = 117°\ 39'\ 26''$$

Then,

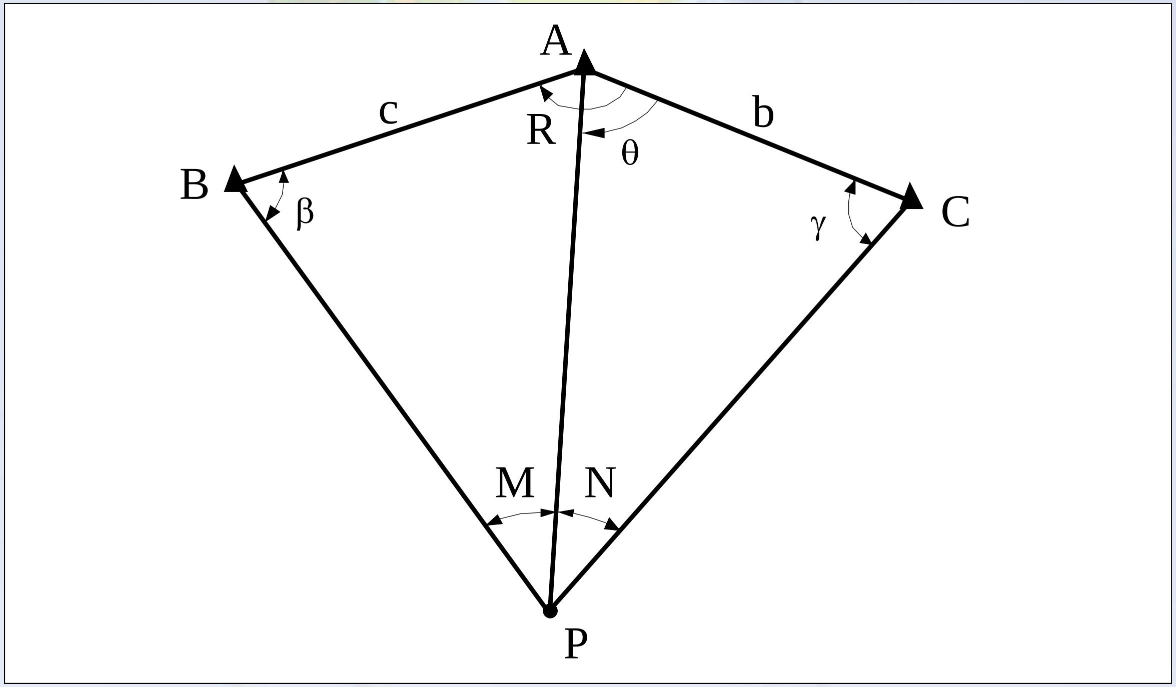
$$y_k = y_i + d_{ik} \sin \alpha_{ik} = 175329.41 + 888.86 \sin(117°\ 39'\ 26'')$$

$$= 176116.71 \text{ m}$$

$$x_k = x_i + d_{ik} \cos \alpha_{ik} = 184672.66 + 888.86 \cos(117°\ 39'\ 26'')$$

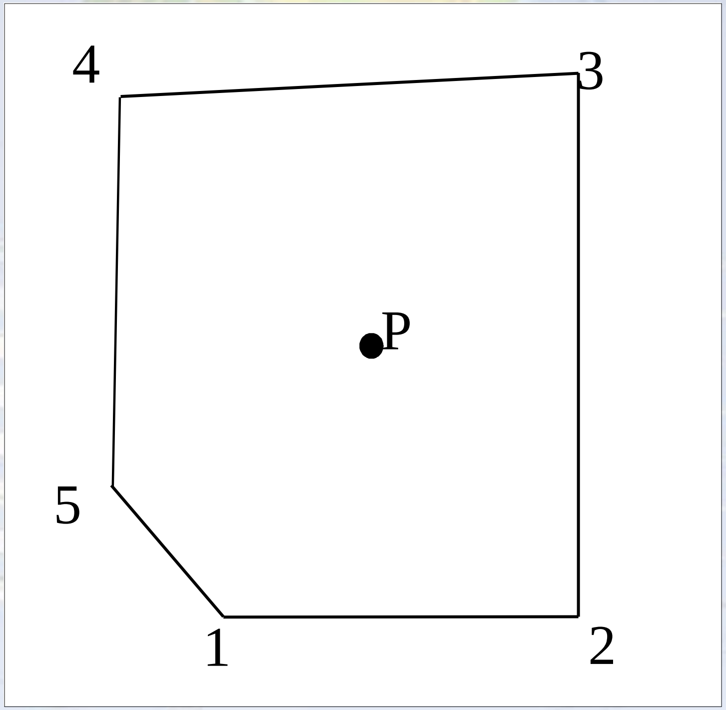
$$= 184260.07 \text{ m}$$

RESECTION



Here the theodolite or total station is set up at the unknown point P. Two angles M & N are measured to three seen control points that can be several kilometers away. For more about the mathematics, refer to the textbook page 200.

MAPPING DETAILS USING EDM



A sketch for a land parcel.

This is a repeated application of location by angle and distance to all the details' points. If the angle and distance to any point are measured, then the coordinates of this point can be computed as explained earlier. Knowing the coordinates of all the points, a map can be prepared either manually or using AutoCad.

A table for booking measurements done by the EDM

Station No. ____________________ H.I. (ارتفاع الجهاز) ____________ (m)

/ / : التاريخ ____ : اسم المساح

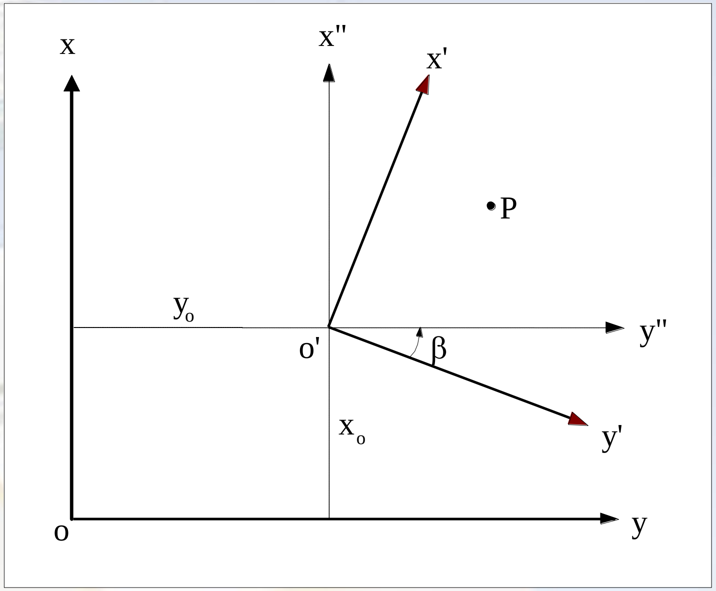
رقم النقطة Point #	المسافة المائلة S.D. (m)	المسافة الأفقية H.D. (m)	ΔH' (m)	(H.A.) الزاوية الأفقية			(Z.A.) الزاوية السمتية			ارتفاع العاكس HT (m)	ملاحظات Notes
				°	'	"	°	'	"		

TRANSFORMATION OF COORDINATES

- Used to transform coordinates from one coordinate system to another.
- Two points that are common to both coordinate systems are needed.

$$y_p = y_0 + s\left(y'_p \cdot \cos\beta + x'_p \cdot \sin\beta\right)$$

$$x_p = x_0 + s\left(-y'_p \cdot \sin\beta + x'_p \cdot \cos\beta\right)$$



TRAVERSE SURVEYING

- Traverse surveying is a measurement procedure used for determining the horizontal relative positions (y & x coordinates) of a number of survey points.
- It is a repeated application of location by angle and distance starting from a known point and direction.

PURPOSE OF THE TRAVERSE

The traverse serves several purposes among which are:

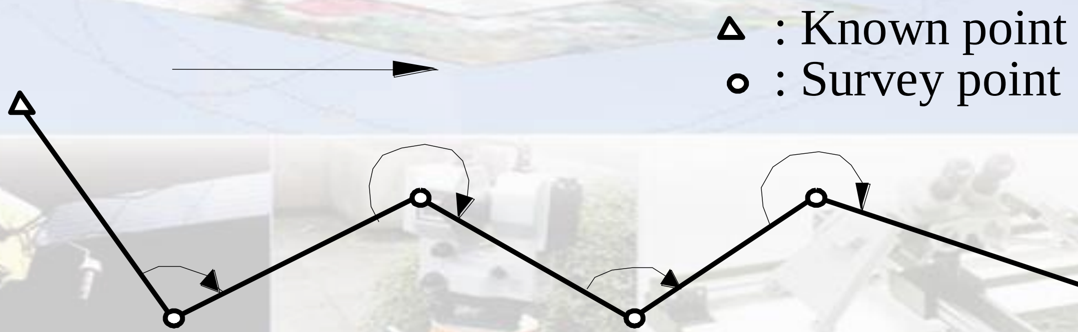
1. Property surveys to establish boundaries.

2. Location and construction layout surveys for highways, railways and other works.

3. Providing Ground control points for photogrammetric mapping.

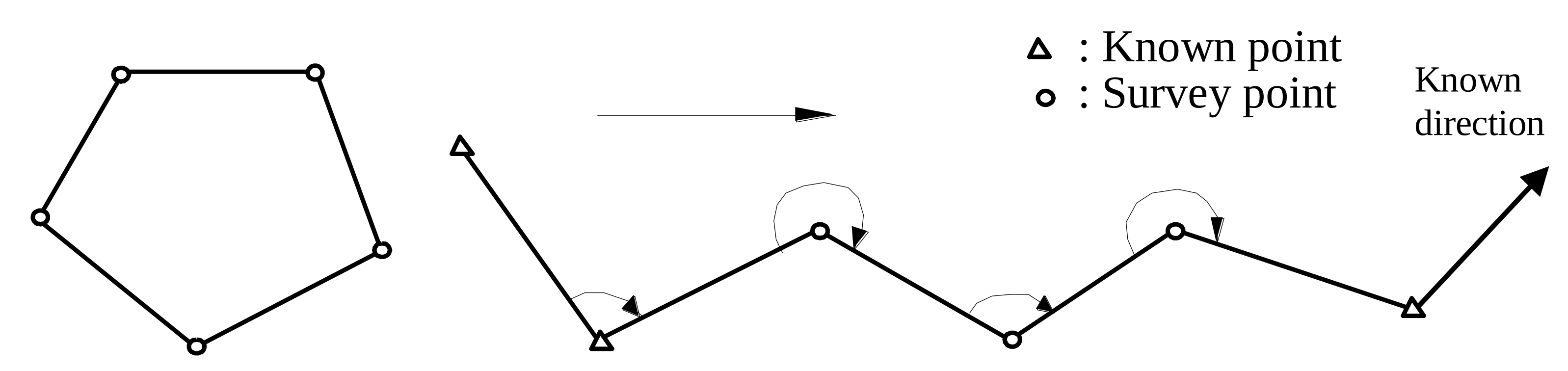
TYPES OF TRAVERSE

- ***Open traverse***: this originates at a point that could be of known or unknown position and ends at a different point of unknown position.



- ***Closed traverse***: this type originates at a point (of known or assumed position) and terminates at the same point yielding a closed loop traverse (Figure a),

  or originates at a line of known coordinates and ends at another line of known coordinates (or a coordinate and direction) yielding a closed connecting traverse (Figure b).



(a) Loop traverse

(b) Connecting traverse

CHOICE OF TRAVERSE STATIONS

- Traverse stations should be located so that:

1. Traverse lines should be as close as possible to the details to be surveyed.
2. Distances between traverse stations should be approximately equal and the shortest line should be greater than one third of the longest line.
3. Stations should be chosen on firm ground, or monumented in a way to make sure that they are not easily lost or damaged.
4. When standing on one station, it should be easy to see the backsight and foresight stations.

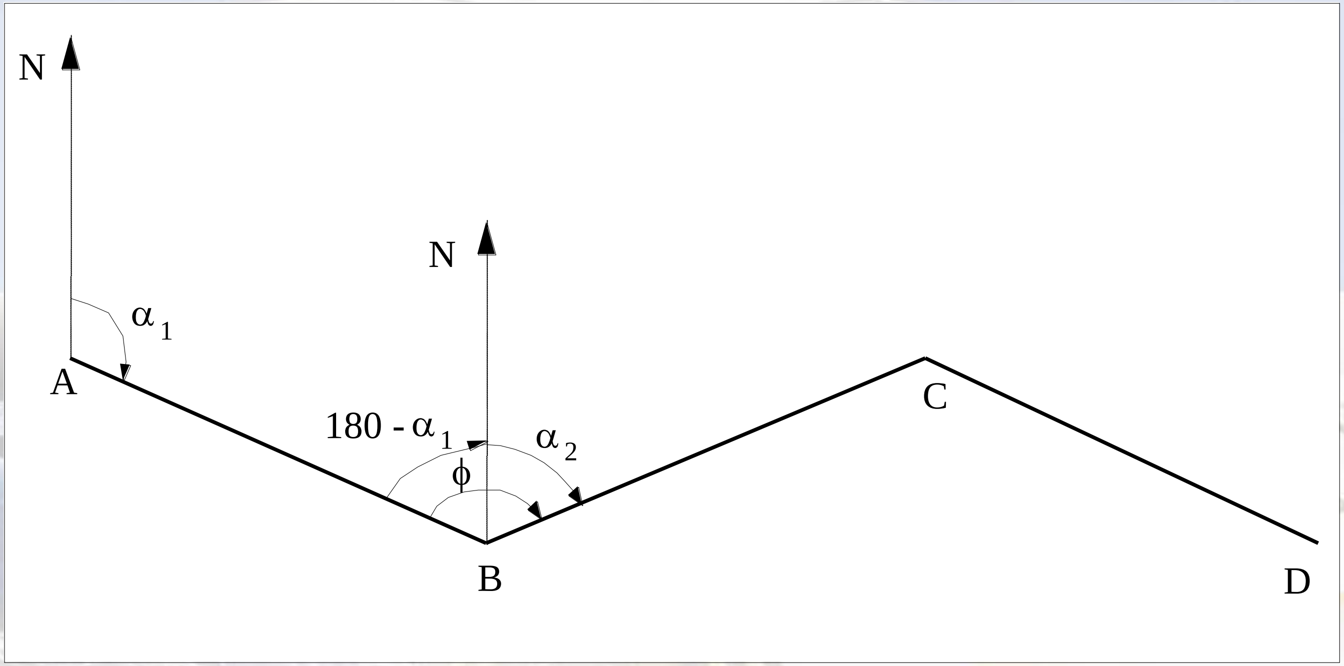
TRAVERSE COMPUTATIONS AND CORRECTION OF ERRORS

A) Azimuth of a line:

1. When $(\alpha_1 + \phi) > 180°$:

$$\alpha_2 = \phi - (180° - \alpha_1)$$

$$\Rightarrow \alpha_2 = \phi + \alpha_1 - 180°$$

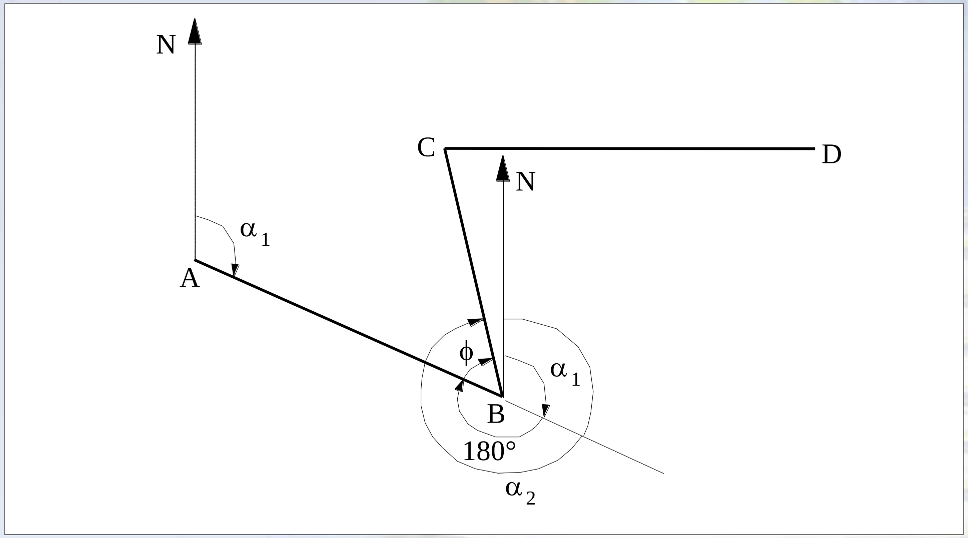


Case (1): $\alpha_1 + \phi > 180°$

2. When $(\alpha_1 + \phi) < 180°$:

$\alpha_2 = \phi + 180° + \alpha_1$

$\Rightarrow \alpha_2 = \phi + \alpha_1 + 180°$



Case (2): $\alpha_1 + \phi < 180°$

In general,

$$\alpha_2 = \alpha_1 + \phi \pm 180°$$

Where α_2 = the azimuth of the following line,

α_1 = the azimuth of the previous line, and

ϕ = the clockwise angle between the previous line and the following line.

If $\alpha_1 + \phi < 180°$ $\Rightarrow$ add 180° to get α_2

If $\alpha_1 + \phi > 180°$ $\Rightarrow$ subtract 180° to get α_2

B) Checks and Correction of Errors:

(1) ***Angle Correction***: This can be done in two ways:

a) Closed loop traverse**.** For a closed loop traverse of *n* sides:

Theoretical sum of internal angles = (n - 2) x 180

As an example: For a 5-sided closed loop traverse:

Theoretical sum = (5 - 2) x 180° = 540°

If the measured sum = 540° 00' 15“, Error = +15"

If this error is acceptable, then

No. of internal angles = 5

Correction for each angle = -15"/5 = -3"

Subtract 3" from each angle

b) ***For both loop and connecting closed traverses*:** If the azimuth of the last line in the traverse is known, this azimuth is compared with the calculated azimuth (as explained earlier), and the error is distributed between the angles.

Assume that the known azimuth of last line is α_n , calculated azimuth α_c , then:
Error (ε_α) = $\alpha_c - \alpha_n$
For n measured angles: Correction/angle = $-\varepsilon_\alpha/n$
This correction is added to each angle in the traverse and the azimuths of all lines are recalculated.

Alternatively and easier is to apply the correction directly to the computed azimuths:

Let α'_i be the initially computed azimuth of the *i-th* line in the traverse, then the corrected azimuth α_i of this line is:

$$\alpha_i = \alpha'_i - i \cdot \left(\frac{\varepsilon_\alpha}{n}\right)$$

(2) *Position Correction*:

Given that the length and azimuth of each line in the traverse are known, calculate the coordinates of all the traverse points including the last known point.

Assume that the calculated and known coordinates of the last point are (y_c, x_c) and (y_n, x_n) respectively. Then:

Closure error in the y-direction $(\varepsilon_y) = y_c - y_n$

Closure error in the x-direction $(\varepsilon_x) = x_c - x_n$

Closure error in the position of the last point: $\varepsilon = \sqrt{\varepsilon_y^2 + \varepsilon_x^2}$

Compass Rule (Bowditch Rule):

The preliminary computed coordinates of the traverse points can be corrected directly in the same manner like the azimuths. This is performed as follows:

$$y_{i_f} = y_{i_c} - \frac{L_i}{\sum d_j} \boldsymbol{\varepsilon_y}$$

$$x_{i_f} = x_{i_c} - \frac{L_i}{\sum d_j} \boldsymbol{\varepsilon_x}$$

Where y_{i_f} and x_{i_f} are the final corrected coordinates of point i,

y_{i_c} and x_{i_c} are the computed coordinates of point i,

$\boldsymbol{\varepsilon_y}$ and $\boldsymbol{\varepsilon_x}$ are the errors in the y and x errors,

L_i is the cumulative distance up to point i

$\sum d_j$ is the sum of measured distances in the traverse.

ALLOWABLE ERRORS IN TRAVERSE SURVEYING

- The Department of Surveying in the West Bank allows the following errors in traverse surveying:

	Allowable error	
	Important areas (example: urban areas)	Less important areas (example: rural areas)
Measured distances	$\Delta\ell = 0.0005\ell + 0.03$ m	$\Delta\ell = 0.0007\ell + 0.03$ m
Measured angles	$\Delta = 60'' \sqrt{n}$	$\Delta = 90'' \sqrt{n}$
Closure error	$\varepsilon = 0.0006\ \Sigma\ell + 0.20$ m	$\varepsilon = 0.0009\ \Sigma\ell + 0.20$ m

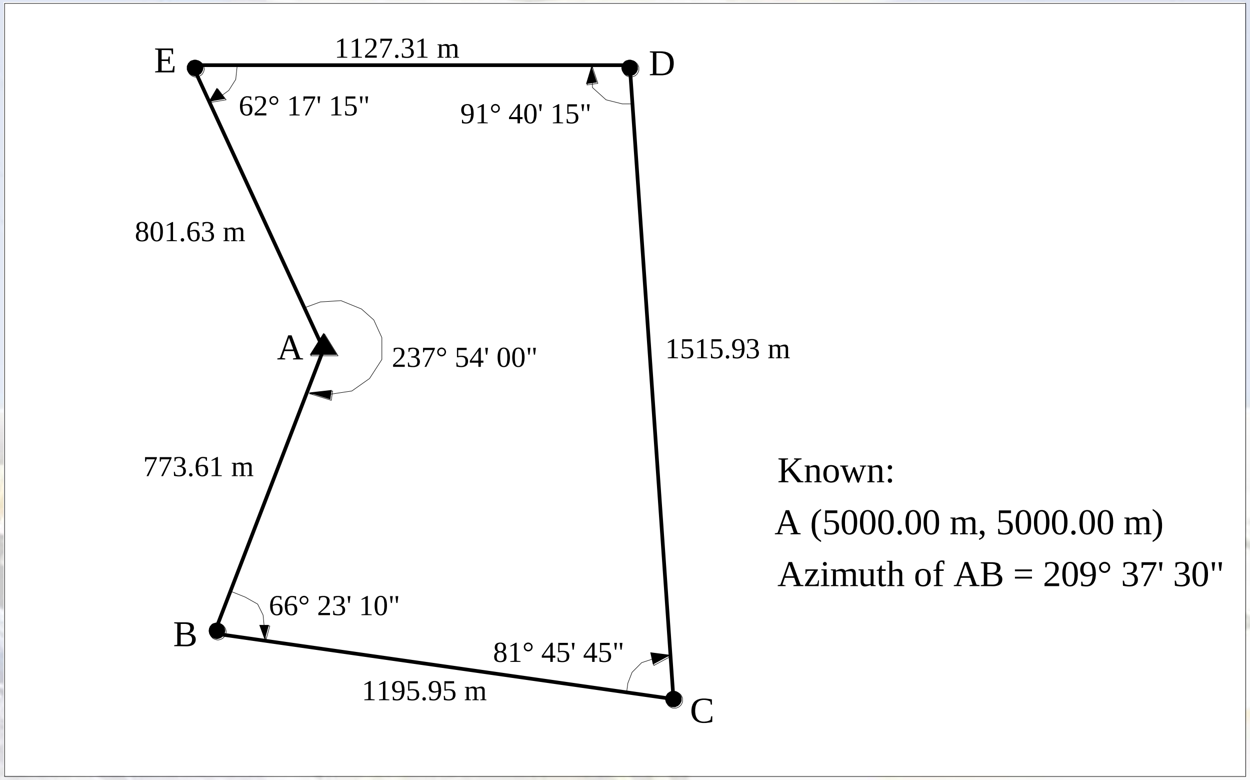
Where ℓ = measured length, Δ = angle closure error in seconds

n = number of measured angles, $\varepsilon = \sqrt{\varepsilon_y^2 + \varepsilon_x^2}$

$\Delta\ell$ = allowable error in the measured distance

EXAMPLE

- Balance the following closed loop traverse and calculate the final corrected coordinates, azimuths and distances.



SOLUTION:

Note: It is also possible to correct the angles by comparing their sum to (n-2).180 = 540°, correcting the angles individually, and then recalculating the azimuths again.

Line		Preliminary Azimuth	Correction	Corrected Azimuth
AB		209° 37' 30"		
	+ B̂	66 23 10		
		276 00 40		
		- 180		
BC		96 00 40	- 5"	96° 00' 35"
	+ Ĉ	81 45 45		
		177 46 25		
		+180		
CD		357 46 25	- 10"	357° 46' 15"
	+ D̂	91 40 15		
		449 26 40		
		-180		
DE		269 26 40	- 15"	269° 26' 25"
	+ Ê	62 17 15		
		331 43 55		
		-180		
EA		151 43 55	- 20"	151° 43' 35"
	+ Â	237 54 00		
		389 37 55		
		-180		
AB		209 37 55	- 25"	209° 37' 30"
		209 37 30		

Closure error = + 25"

Correction per angle = -25"/5 = -5"

Preliminary coordinates:

Station	Corrected Azimuth (α_{ij})	Distance d_{ij} (m)	Departure $\Delta y_{ij} = d_{ij} \sin\alpha_{ij}$	Latitude $\Delta x_{ij} = d_{ij} \cos\alpha_{ij}$	Preliminary coordinates y	x
A					5000.00	5000.00
	209° 37′ 30″	773.61	-382.41	-672.48		
B					4617.59	4327.52
	96° 00′ 35″	1195.95	1189.38	-125.21		
C					5806.97	4202.31
	357° 46′ 15″	1515.93	-58.96	1514.78		
D					5748.01	5717.09
	269° 26′ 25″	1127.31	-1127.26	-11.01		
E					4620.75	5706.08
	151° 43′ 35″	801.63	379.72	-705.99		
A					5000.47	5000.09

$$\sum d_{ij} = 5414.43 \text{ m}$$

Closure error: $\varepsilon_y = +0.47$ m

$\varepsilon_x = +0.09$ m

$$\text{Linear error of closure} = \sqrt{(0.47)^2 + (0.09)^2} = 0.48 \text{ m}$$

$$\text{Relative error of closure} = \frac{1}{5414.43/0.48} \cong \frac{1}{11{,}000}$$

Corrected coordinates:

Station	Cumulative Distance	y-coordinate			x-coordinate		
		Preliminary	Correction	Final	Preliminary	Correction	Final
A	0	5000.00	0	5000.00	5000.00	0	5000.00
B	773.61	4617.59	-0.07	4617.52	4327.52	-0.01	4327.51
C	1969.56	5806.97	-0.17	5806.80	4202.31	-0.03	4202.28
D	3485.49	5748.01	-0.30	5747.71	5717.09	-0.06	5717.03
E	4612.80	4620.75	-0.40	4620.35	5706.08	-0.08	5706.00
A	5414.43	5000.47	-0.47	5000.00	5000.09	-0.09	5000.00

Final Results:

Line	Final Distance	Final Azimuth	Final Reduced Bearing
A – B	773.65	209° 37′ 45″	S 29° 37′ 45″ W
B – C	1195.86	96° 00′ 39″	S 83° 59′ 21″ E
C – D	1515.90	357° 45′ 57″	N 2° 14′ 03″ W
D – E	1127.41	269° 26′ 22″	S 89° 26′ 22″ W
E – A	801.60	151° 43′ 52″	S 28° 16′ 08″ E